CSC 2204 Finite Automata Theory and Formal Languages

Week 1 (Lecture 1)

Automaton

- Something that works automatically.
- Plural: Automata.

Alphabets

- Finite non-empty set of symbols.
- Symbol: Σ
- Examples:
  - Binary alphabets: $\Sigma = \{0,1\}$
  - Lower-case letters: $\Sigma = \{a,b,c, \dots ,z\}$
  - $\Sigma = \{a,b\}$
  - $\Sigma = \{a,bc\}$
  - $\Sigma = \{a,bcd,00,1\}$

Strings

- String: Concatenation of finite alphabets set of sequence of symbols chosen from the given set of alphabets Σ.
  - Σ = {0,1}, String s=0100
  - Σ = {a,bc}, String s=abc
  - Σ = {a,bcd,00,1}. String s=abcd00

- Empty/Null String
  - String with no symbols
  - Denoted by λ, Λ or ∈

String Operations

- Length
  - $\Sigma = \{0,1\}$, $s=0100$, $|s| = 4$
  - $\Sigma = \{a,bc\}$, $s=abc$, $|s| = ?$
  - $\Sigma = \{a,bcd,00,1\}$. $s=abcd00$, $|s| = ?$
  - $|\lambda| = 0$

- Reverse
  - $\Sigma = \{0,1\}$, $s=0100$, Rev(s) or $s^r = 0010$
  - $\Sigma = \{a,bc\}$, $s=abc$, $s^r = ?$
  - $\Sigma = \{a,bcd,00,1\}$. $s=abcd00$, $s^r = ?$

String Operations

- Concatenation
  - $w = abc$, $v = cde$, $u = a$
  - $uv = acde$
  - $vw = cdeabc$
  - $\lambda w = w\lambda = w$

  - $|w| = 3$, $|v| = 3$, $|u| = 1$
  - $|uv| = |u| + |v| = 1 + 3 = 4$

String Operations

- Substrings
  - String: abbababa
  - Substrings:
    - bba
    - ab
    - aba
    - bab
    - abab

String Operations

- Prefix and Suffix
  - String: abbab

Prefix	Suffix
λ	abbab
a	bbab
ab	bab
abb	ab
abba	b
abbab	λ

String Operations

- Power
  - $w^0 = \lambda$, $w^1 = w$, $w^2 = ww$, $w^3 = www$
  - $s = ab$
    - $s^0 = \lambda$
    - $s^2 = abab$
    - $s^3 = ababab$

String Operations

- $*$ Operation
  - Σ^* = Set of all possible strings from Σ
  - $\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \Sigma^4 \cup \ldots$

  - Example: $\Sigma = \{a,b\}$
    - $\Sigma^0 = \lambda$
    - $\Sigma^1 = \{a, b\}$
    - $\Sigma^2 = \{aa, ab, ba, bb\}$
    - $\Sigma^3 = \{aaa, aab, aba, abb, baa, bab, bba, bbb\}$
    - $\Sigma^* = \{\lambda, a, b, aa, ab, ba, bb, aaa, aab, aba, abb, baa, bab, bba, bbb, \ldots\}$

String Operations

- $^+$ Operation
  - Σ^+ = Set of all possible strings from Σ except λ
  - $\Sigma^+ = \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \Sigma^4 \cup \ldots$

  - Example: $\Sigma = \{a,b\}$
    - $\Sigma^1 = \{a, b\}$
    - $\Sigma^2 = \{aa, ab, ba, bb\}$
    - $\Sigma^3 = \{aaa, aab, aba, abb, baa, bab, bba, bbb\}$
    - $\Sigma^+ = \{a, b, aa, ab, ba, bb, aaa, aab, aba, abb, baa, bab, bba, bbb, \ldots\}$

Languages

- A subset of Σ^*
- Example:
  - $\Sigma = \{a, b\}$
  - $\Sigma^* = \{\lambda, a, b, aa, ab, ba, bb, aaa, aab, \ldots\}$
  - Languages:
    - $\{\lambda\}$ Finite Language
    - $\{\lambda, a, b, aa, ab\}$ Finite Language
    - $\{\lambda, a, b, aa, ab, \ldots\}$ Infinite Language

Operations on Languages

- Usual set operations
  - $\{aa,ab\} \cup \{a,aab\} = \{aa,ab,a,aab\}$
  - $\{aa,ab\} \cap \{aa,abb,aab\} = \{aa\}$
  - $\{aa,ab,aaaa\} - \{aa,abb,aab\} = \{ab,aaaa\}$
  - $L' = \Sigma^* - L$
- Concatenation
  - $\{a,ab,ba\}\{b,aa\} = \{ab,aaa,abb,abaa,bab,baaa\}$

Operations on Languages

- Power
  - $\{a,b\}^0 = \lambda$
  - $\{a,b\}^1 = \{a,b\}$
  - $\{a,b\}^2 = \{a,b\}\{a,b\} = \{aa,ab,ba,bb\}$
  - $\{a,b\}^3 = \{a,b\}\{a,b\}\{a,b\}$

  - $\{aa,ab\}^1 = \{aa,ab\}$
  - $\{aa,ab\}^2 = \{aa,ab\}\{aa,ab\} = \{aaaa,aaab,abaa,abab\}$

Operations on Languages

- Star-Closure (Kleene *)
  - $L^* = L^0 \cup L^1 \cup L^2 \cup L^3 \cup \ldots$
- Plus-Closure
  - $L^+ = L^1 \cup L^2 \cup L^3 \cup \ldots$