Q1. Suppose an implementation of insertion sort and merge sort on the same machine requires $8n^2$ steps and $64n\lg n$ steps respectively, where n is input size. Find the values of n, where insertion sort beats merge sort.

Q2. Two algorithms, A and B have running time $100n^2$ and $2n$ respectively, on the same machine. Find the smallest value of n for which A beats B.

Q3. For the following functions of n:

$F1(n)= n^2$

$F2(n)= n^2 + 100n$

$F3(n)=$ n if n is odd

$\quad\;\; =$ n^3 if n is even

$F4(n)=$ n if n <= 100

$\quad\;\; =$ n^3 if n >100

For each distinct pair j and k, indicate whether $f_j(n)$ is $O(f_k(n))$ and whether $f_j(n)$ is $(f_k(n))$

Q4. Order the following functions by growth rate:

$$n,\ \sqrt{n},\ \log n,\ \log\log n,\ \log^2 n,\ n\log n,\ \sqrt{n}\log^2 n,\ (1/3)^n,\ (3/2)^n,\ 17$$

Q5. Give the time complexity of the following procedures as a function of n.

```
Procedure matmpy ( n : integer)
    var
        i, j, k : integer
    begin
        for i= 1 to n do
            for j = 1 to n do begin
                C[i, j] = 0;
                for k = 1 to n do
                    C[i, j] = C [i, j] + A [i, k] * B[k, j]
            end
    end
```

```
Procedure mystery ( n : integer)
    var
        i, j, k : integer
    begin
        for i = 1 to n-1 do
        for j = i+1 to n do begin
            for k = 1 to j do
                {some O(1) statement }
    end
```

```
procedure veryodd ( n : integer)
        var
                i, j, x, y : integer
        begin
                for i = 1 to n-1 do
                        if odd(i) then begin
                                for j = i to n do
                                        x = x + 1;
                                for j = 1 to i do
                                        y = y + 1;
                        end
        end
```

Q6. Assume the parameter n in the procedure below is a positive power of 2, i.e., n = 2, 4, 8, 16, Give the formula that expresses the value of the variable count in terms of the value of n when the procedure terminates.

```
procedure mystery ( n : integer)
        var
                x, count : integer
        begin
                count = 0;
                x = 2;
                while x < n do begin
                        x = 2 * x;
                        count = count +1
                end;
                cout<<count;
        end
```

Q7. For each function f(n) and time t in the following table, determine the largest size n of the problem that can be solved in time t, assuming that the algorithm to solve the problem takes f(n) milliseconds.

	1 second	1 minute	1 day	1 month
lg n				
√n				
N				
N lg n				
N^2				
N^3				
2^n				
N!				

Q8. Consider sorting n numbers stored in an array A by first finding the smallest element of A and exchanging it with the element in A[1]. Then find the second smallest element of A, and exchange it with A[2]. Continue in this manner for the first n-1 elements of A. Write pseudo code for this algorithm, which is known as selection sort. Give the best case and the worst case running times of selection sort in theta notation.

Q9. Write the pseudo code for iterative binary search and argue that its worst case running time is O(lgn).

Q10. Come up with a O (n lg n) algorithm that, given a set S of n integers and another integer determines whether there exist two integers in S whose sum is exactly x.

Q11. Does the statement the running time of algorithm A is at least $O(n^2)$ make any sense. If not, why
not?

Q12. Indicate, for each pair of expressions (A, B) in the table below, whether A is O, Ω or Θ of B. Assume k >= 1 and c > 1 are constants. Answer in the form of "yes" or "no".

A	B	O	Ω	Θ
n^k	c^n			
2^n	$2^{n/2}$			
$\lg(n!)$	$\lg(n^n)$			