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Linear Algebra
Assignment # 3

Ex 4.6

Q5 $x_1 - 3x_2 + x_3 = 0$
$2x_1 - 6x_2 + 2x_3 = 0$
$3x_1 - 9x_2 + 3x_3 = 0$

$$\begin{bmatrix} 1 & -3 & 1 \\ 2 & -6 & 2 \\ 3 & -9 & 3 \end{bmatrix}$$

$R_2 - 2R_1$

$$\begin{bmatrix} 1 & -3 & 1 \\ 0 & 0 & 0 \\ 3 & -9 & 3 \end{bmatrix}$$

$R_2 \Leftrightarrow R_3$

$$\begin{bmatrix} 1 & -3 & 1 \\ 3 & -9 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_2 - 3R_1$

$$\begin{bmatrix} 1 & -3 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_3 = t$
$x_2 = s$
$x_1 - 3s + t = 0$

$x_1 = 3s - t$

$(x_1, x_2, x_3) = 3s - t, s, t \Rightarrow$

$$s\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} \quad t\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Dimensions = 2

Ex 4.5

Q4 Show that the following polynomials form a basis for P_3.

$1+x$, $1-x$, $1-x^2$, $1-x^3$

First thing to verify is linear independency.

$c_1(1+x)+c_2(1-x)+c_3(1-x^2)+c_4(1-x^3)=0$

$(c_1+c_2+c_3+c_4)+(c_1-c_2)x-c_3x^2-c_4x^3=0$

$c_1+c_2+c_3+c_4=0$, $c_1-c_2=0$, $-c_3=0$, $-c_4=0$

From these equations

1 $c_3=0$

$c_4=0$

$c_1-c_2=0 \Rightarrow c_1=c_2$

$c_1+c_2+c_3+c_4=0 \Rightarrow c_1+c_2=0$, $2c_1=0 \Rightarrow c_1=c_2=0$

As $c_1=c_2=c_3=c_4=0$ so the polynomials are independent

* The polynomials $1+x, 1-x, 1-x^2, 1-x^3$ span P_3 are linearly independent. Therefore they form a basis for P_3.

Ex 5.1

Q9 Find characteristic eqn, eigenvalues and basis for the eigen-spaces of the matrix.

$$\begin{bmatrix} 6 & 3 & -8 \\ 0 & -2 & 0 \\ 1 & 0 & -3 \end{bmatrix}$$

$$A.\lambda I = \begin{bmatrix} 6-\lambda & 3 & -8 \\ 0 & -2-\lambda & 0 \\ 1 & 0 & -3-\lambda \end{bmatrix}$$

$$= -2-\lambda \begin{bmatrix} 6-\lambda & -8 \\ 1 & -3-\lambda \end{bmatrix}$$

$$= -2-\lambda\left[(6-\lambda)(-3-\lambda)-(-8\times 1)\right]$$

$$-2-\lambda\left[(6-\lambda)(-3-\lambda)+8\right]$$

$$-2-\lambda\left[-18\lambda-6\lambda+3\lambda+\lambda^2+8\right]$$

$$-2\lambda^2+6\lambda+20-\lambda^3+3\lambda^2+10\lambda$$

$$-\lambda^3-\lambda^2+16\lambda+20$$

$$(\lambda+2)^2(\lambda-5)=0 \quad \text{Characteristic eqn}$$

Eigenvalues -2 and 5

$$\begin{bmatrix} 8 & 3 & -8 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \div 8$

$$\begin{bmatrix} 1 & 3/8 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

$R_2 \Leftrightarrow R_3$

$$\begin{bmatrix} 1 & 3/8 & -1 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Subtract R_1 from R_2

$$\begin{bmatrix} 1 & \frac{3}{8} & -1 \\ 0 & -\frac{3}{8} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_2 \times -\frac{8}{3}$

$$\begin{bmatrix} 1 & \frac{3}{8} & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_1 - \frac{3}{8}R_2$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{aligned} x_3 &= t \\ x_2 &= 0 \\ x_1 &= t \end{aligned} \quad \Rightarrow \quad t(1,0,1)$$

Basis for -2

Basis for S

$$\begin{bmatrix} 1 & 3 & -8 \\ 0 & -7 & 0 \\ 1 & 0 & -8 \end{bmatrix}$$

$R_3 - R_1$

$$\begin{bmatrix} 1 & 3 & -8 \\ 0 & -7 & 0 \\ 0 & -3 & 0 \end{bmatrix}$$

$R_2 \div -7$

$$\begin{bmatrix} 1 & 3 & -8 \\ 0 & 1 & 0 \\ 0 & -3 & 0 \end{bmatrix}$$

$R_1 - 3R_2$

$$\begin{bmatrix} 1 & 0 & -8 \\ 0 & 1 & 0 \\ 0 & -3 & 0 \end{bmatrix}$$

$R_3 + 3R_2$

$$\begin{bmatrix} 1 & 0 & -8 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_3 = t$

$x_2 = 0$

$x_1 - 8t = 0$

$x_1 = 8t$

$(x_1, x_2, x_3) = (8t, 0, t)$

$\Rightarrow t(8, 0, 1)$

$\boxed{\text{Basis}}$