**CFL is NOT closed under Intersection: Intersection of two CFLs is NOT CFL:**

$L_1 = \{a^m b^n c^n \mid m,n \geq 1\}$ ex: abc, abbcc, abbbccc, aabbbccc etc.

$L_2 = \{a^n b^n c^m \mid m,n \geq 1\}$ ex: abc, aabbc, aaabbbc, aaabbbcc etc.

PDA $L_1$: Push b, Pop when c and compare count – Only one stack is required

PDA $L_2$: Push a, Pop when b and compare count – Only one stack is required

$L_1 \cap L_2 = \{a^n b^n c^n \mid n \geq 1\}$

PDA is not possible for $L_1 \cap L_2$ because we cannot use one stack.

Because PDA of intersection is not possible, it is not a CFL.

**CFL is NOT closed under Complementation: Complement of a CFL is NOT CFL:**

Let us assume that complement of a CFL is also a CFL.

| | |
|---|---|
| If $L_1$ is CFL then $L_1'$ is CFL | **As per Assumption** |
| If $L_2$ is CFL then $L_2'$ is CFL | **As per Assumption** |
| $L_1' \cup L_2'$ is CFL | **Property of CFL** |
| $(L_1' \cup L_2')'$ is CFL | **As per Assumption** |

By DeMorgan Law: $(L_1' \cup L_2')' = L_1 \cap L_2$ -- So, I shows that $L_1 \cap L_2$ is also CFL.

BUT, CFL is NOT closed under Intersection. So, our assumption was wrong and complement of a CFL is not CFL. Hence, CFL is **NOT** closed under Complementation.