Finite Automata Theory and Formal Languages

(Week 3, Lecture 1)

Transition Graph (TG)

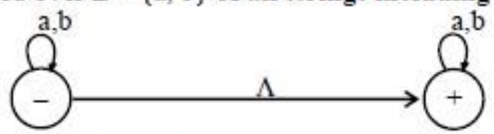
- A collection of the followings
  - Finite number of states, at least one of which is start state and some (maybe none) final states.
  - Finite set of input letters (Σ) from which input strings are formed.
  - Finite set of transitions that show how to go from one state to another based on reading specified substrings of input letters, possibly even the null string (Λ).

Transition Graph (TG)

- NOTE:
  - There may exist more than one paths for certain string, while there may not exist any path for certain string.
  - If there exists at least one path for a certain string, starting from initial state and ending in a final state, the string is supposed to be accepted by the TG.
  - The collection of accepted strings is the language accepted by the TG.

Transition Graph (TG)

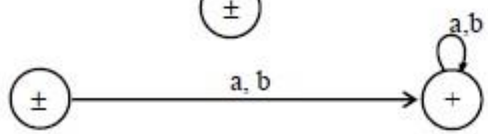
Consider the Language L , defined over $\Sigma = \{a, b\}$ of **all strings including Λ.** The language L may be accepted by the following TG



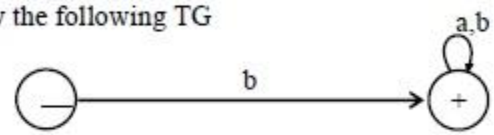
The language L may also be accepted by the following TG

TG_1

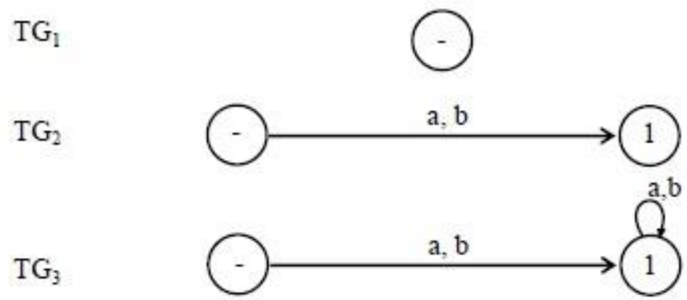
TG_2



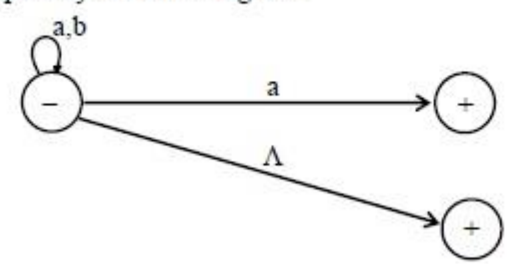
Consider the language L of strings, defined over $\Sigma=\{a, b\}$, **starting with b.** The language L may be expressed by RE $b(a + b)^*$, may be accepted by the following TG



Transition Graph (TG)

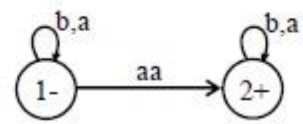


Consider the language L of strings, defined over $\Sigma=\{a, b\}$, **not ending in b.** The language L may be expressed by RE $\Lambda + (a + b)^* a$, may be accepted by the following TG

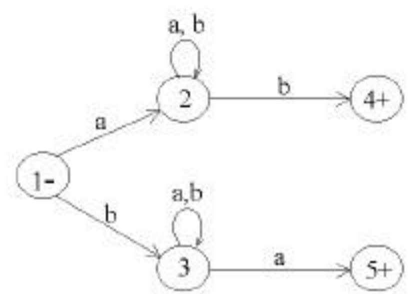


Transition Graph (TG)

Consider the Language L of strings, defined over $\Sigma = \{a, b\}$, **containing double a.**
The language L may be expressed by the following regular expression $(a+b)^* (aa) (a+b)^*$. This language may be accepted by the following TG

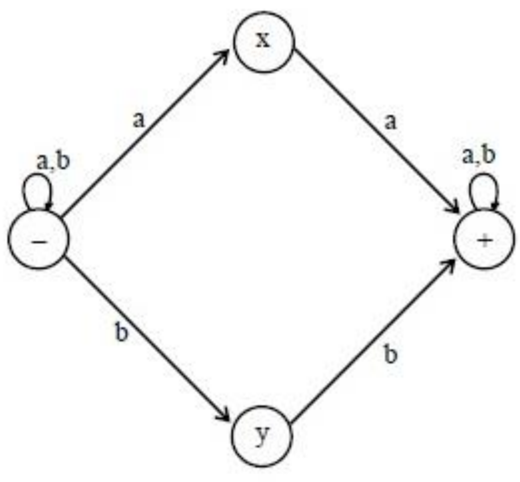


Consider the language L of strings, defined over $\Sigma = \{a, b\}$, **beginning and ending in different letters.**
The language L may be expressed by RE $a(a + b)^*b + b(a + b)^*a$
The language L may be accepted by the following TG

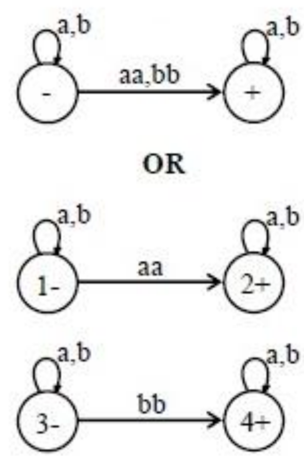


Transition Graph (TG)

Consider the language L of strings, defined over $\Sigma=\{a, b\}$, **having double a or double b**.
The language L can be expressed by RE $(a+b)^* (aa + bb) (a+b)^*$.
The above language may also be expressed by the following TGs.

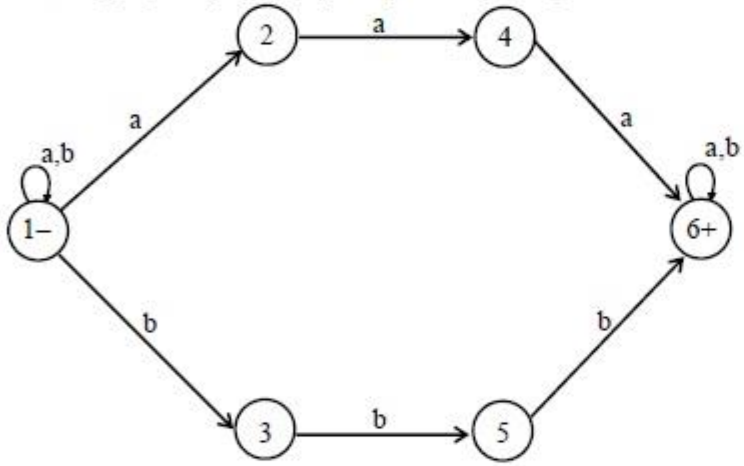


OR



Transition Graph (TG)

Consider the language L of strings, defined over $\Sigma = \{a, b\}$, **having triple a or triple b.** The language L may be expressed by RE $(a+b)^* (aaa + bbb) (a+b)^*$
This language may be accepted by the following TG



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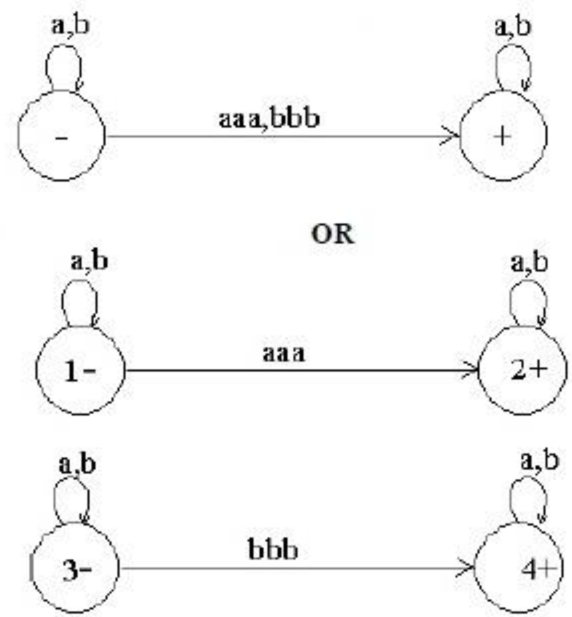
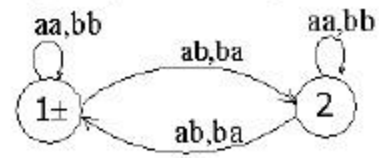
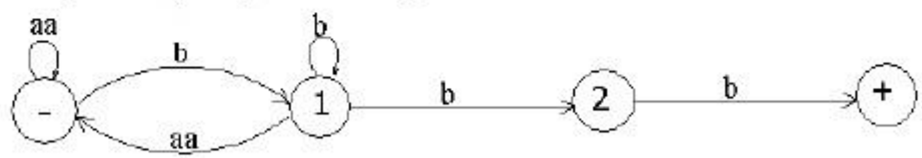


Transition Graph (TG)

Consider the **EVEN-EVEN** language, defined over $\Sigma = \{a, b\}$. As discussed earlier that **EVEN-EVEN** language can be expressed by a regular expression $(aa+bb+(ab+ba)(aa+bb)^*(ab+ba))^*$
The language **EVEN-EVEN** may be accepted by the following TG

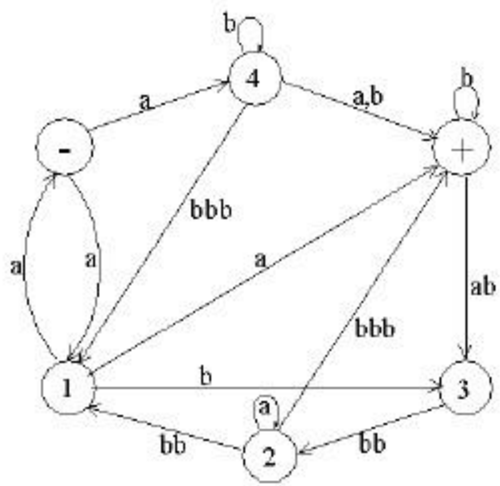


Consider the language L, defined over $\Sigma=\{a, b\}$, in which **a's occur only in even clumps and that ends in three or more b's**. The language L can be expressed by its regular expression $(aa)^*b(b^*+(aa(aa)^*b)^*)bb$ OR $(aa)^*b(b^*+((aa)^+b)^*)bb$.
The language L may be accepted by the following TG



Transition Graph (TG)

Consider the following TG



Consider the string abbbabbbabba. It may be observed that the above string traces the following three paths, (using the states)

1) (a)(b) (b) (b) (ab) (bb) (a) (bb) (a)
   (-)(4)(4)(+)(+)(3)(2)(2)(1)(+)
2) (a)(b) ((b)(b)) (ab) (bb) (a) (bb) (a)
   (-)(4)(+)(+)(+)(3)(2)(2)(1)(+)
3) (a)((b) (b)) (b) (ab) (bb) (a) (bb) (a)
   (-) (4)(4)(4)(+) (3)(2)(2)(1)(+)

Which shows that all these paths are successful, (*i.e.* the path starting from an initial state and ending in a final state).

Generalized Transition Graph (GTG)

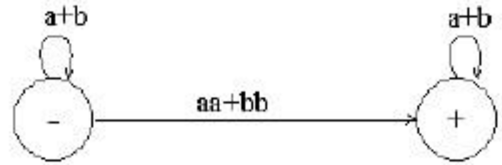
- A collection of the following
  - Finite number of states, at least one of which is start state and some (maybe none) final states.
  - Finite set of input letters (Σ) from which input strings are formed.
  - Directed edges connecting some pair of states labeled with regular expression.
- Note: The labels of transition edges are corresponding regular expressions.

Generalized Transition Graph (GTG)

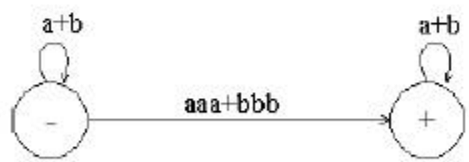
- Note:
  - The labels of transition edges are corresponding regular expressions.
  - TGs and GTGs provide certain relaxations, i.e. there may exist more than one path for a certain string or there may not be any path for a certain string, this property creates nondeterminism.

Generalized Transition Graph (GTG)

Consider the language L of strings, defined over $\Sigma = \{a,b\}$, containing **double a or double b**. The language L can be expressed by the following regular expression $(a+b)^* (aa + bb) (a+b)^*$
The language L may be accepted by the following GTG.



Consider the language L of strings, defined over $\Sigma = \{a, b\}$, **having triple a or triple b.** The language L may be expressed by RE $(a+b)^* (aaa + bbb) (a+b)^*$
This language may be accepted by the following GTG



OR