CSC 2204 Finite Automata Theory and Formal Languages

Week 1 (Lecture 1)



Automaton

- Something that works automatically.
- Plural: Automata.

Alphabets

- Finite non-empty set of symbols.
- Symbol: Σ
- Examples:
 - Binary alphabets: Σ = {0,1}
 - Lower-case letters: $\Sigma = \{a,b,c, ...,z\}$
 - $\Sigma = \{a,b\}$
 - $\Sigma = \{a,bc\}$
 - $\Sigma = \{a, bcd, 00, 1\}$

Strings

- String: Concatenation of finite alphabets set of sequence of symbols chosen from the given set of alphabets Σ.
 - $\Sigma = \{0,1\}$, String s=0100
 - $\Sigma = \{a,bc\}$, String s=abc
 - $\Sigma = \{a,bcd,00,1\}$. String s=abcd00
- Empty/Null String
 - String with no symbols
 - Denoted by λ , Λ or \in

String Operations

Length

- $\Sigma = \{0,1\}, s=0100, |s| = 4$
- $\Sigma = \{a,bc\}, s=abc, |s| = ?$
- $\Sigma = \{a,bcd,00,1\}$. s=abcd00, |s| = ?
- $|\lambda| = 0$

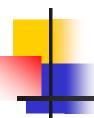
Reverse

- $\Sigma = \{0,1\}, s=0100, Rev(s) or s^r = 0010$
- $\Sigma = \{a,bc\}, s=abc, s^r = ?$
- $\Sigma = \{a,bcd,00,1\}$. s=abcd00, s^r = ?



Concatenation

- w = abc, v = cde, u = a
- uv = acde
- vw = cdeabc
- $\lambda w = w\lambda = w$
- |w| = 3, |v| = 3, |u| = 1
- |uv| = |u| + |v| = 1 + 3 = 4



- Substrings
 - String: abbababa
 - Substrings:
 - bba
 - ab
 - aba
 - bab
 - abab



- Prefix and Suffix
 - String: abbab

Prefix	Suffix
λ	abbab
а	bbab
ab	bab
abb	ab
abba	b
abbab	λ

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String Operations

Power

- $W^0 = \lambda$, $W^1 = W$, $W^2 = WW$, $W^3 = WWW$
- s = ab
 - $s^0 = \lambda$
 - s²=abab
 - s³=ababab

- * Operation
 - Σ^* = Set of all possible strings from Σ

 - Example: Σ = {a,b}
 - $\Sigma^0 = \lambda$
 - $\Sigma^1 = \{a, b\}$
 - $\Sigma^2 = \{aa, ab, ba, bb\}$
 - Σ^3 = {aaa, aab, aba, abb, baa, bab, bba, bbb}
 - Σ* = {λ, a, b, aa, ab, ba, bb, aaa, aab, aba, abb, baa, bab, bba, bbb, ...}

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- + Operation
 - Σ^+ = Set of all possible strings from Σ except λ

 - Example: Σ = {a,b}
 - $\Sigma^1 = \{a, b\}$
 - $\Sigma^2 = \{aa, ab, ba, bb\}$
 - Σ^3 = {aaa, aab, aba, abb, baa, bab, bba, bbb}
 - Σ⁺ = {a, b, aa, ab, ba, bb, aaa, aab, aba, abb, baa, bab, bba, bbb, ...}



Languages

- A subset of Σ*
- Example:
 - $\Sigma = \{a, b\}$
 - $\Sigma^* = \{\lambda, a, b, aa, ab, ba, bb, aaa, aab, ...\}$
 - Languages:
 - {λ}
 - {λ, a, b, aa, ab}
 - {λ, a, b, aa, ab, ...}

Finite Language

Finite Language

Infinite Language

Operations on Languages

Usual set operations

- {aa,ab} ∪ {a,aab} = {aa,ab,a,aab}
- {aa,ab} ∩ {aa,abb,aab} = {aa}
- {aa,ab,aaaa} {aa,abb,aab} = {ab,aaaa}
- \bot ' = Σ * \bot

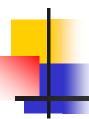
Concatenation

{a,ab,ba}{b,aa} = {ab,aaa,abb,abaa,bab,baaa}

Operations on Languages

Power

- $\{a,b\}^0 = \lambda$
- $\{a,b\}^1 = \{a,b\}$
- $\{a,b\}^2 = \{a,b\}\{a,b\} = \{aa,ab,ba,bb\}$
- $\{a,b\}^3 = \{a,b\}\{a,b\}\{a,b\}$
- $\{aa,ab\}^1 = \{aa,ab\}$
- {aa,ab}² = {aa,ab}{aa,ab} = {aaaa,aaab,abaa,abab}



Operations on Languages

- Star-Closure (Kleene *)
 - $L^* = L^0 \cup L^1 \cup L^2 \cup L^3 \cup \dots$
- Plus-Closure
 - $L^+ = L^1 \cup L^2 \cup L^3 \cup ...$