Finite Automata Theory and Formal Languages

(Week 1, Lecture 2)

Descriptive Definition

- The language is defined, describing the conditions imposed on its words.

Examples:

- The language L of strings of odd length, defined over $\Sigma=\{a\}$, can be written as $L=\{a, aaa, aaaaa, \ldots\}$.

Descriptive Definition

- The language L of strings that does not start with a, defined over Σ ={a,b,c}, can be written as L = {Λ, b, c, ba, bb, bc, ca, cb, cc, …}.
- The language L of strings of length 2, defined over Σ ={0,1,2}, can be written as L= {00, 01, 02,10, 11,12,20,21,22}.
- The language L of strings ending in 0, defined over Σ ={0,1}, can be written as L= {0,00,10,000,010,100,110,…}.

Descriptive Definition

- The language EQUAL, of strings with number of a’s equal to number of b’s, defined over Σ={a,b}, can be written as {Λ ,ab,aabb,abab,baba,abba,…}.
- The language EVEN-EVEN, of strings with even number of a’s and even number of b’s, defined over Σ={a,b}, can be written as {Λ, aa, bb, aaaa,aabb,abab, abba, baab, baba, bbaa, bbbb,…}.

Descriptive Definition

- The language INTEGER, of strings defined over $\Sigma=\{-,0,1,2,3,4,5,6,7,8,9\}$, can be written as INTEGER = $\{\ldots,-2,-1,0,1,2,\ldots\}$.
- The language EVEN, of stings defined over $\Sigma=\{-,0,1,2,3,4,5,6,7,8,9\}$, can be written as EVEN = $\{\ \ldots,-4,-2,0,2,4,\ldots\}$.

Descriptive Definition

- The language EVEN-EVEN, of strings with even number of a's and even number of b's, defined over $\Sigma=\{a,b\}$, can be written as {Λ, aa, bb, aaaa,aabb,abab, abba, baab, baba, bbaa, bbbb,…}.
- The language INTEGER, of strings defined over $\Sigma=\{-,0,1,2,3,4,5,6,7,8,9\}$, can be written as INTEGER = {…,-2,-1,0,1,2,…}.

Descriptive Definition

- The language EVEN, of stings defined over $\Sigma=\{-,0,1,2,3,4,5,6,7,8,9\}$, can be written as EVEN = { …,-4,-2,0,2,4,…}.
- The language $\{a^n b^n\}$, of strings defined over $\Sigma=\{a,b\}$, as $\{a^n b^n : n=1,2,3,\ldots\}$, can be written as {ab, aabb, aaabbb, aaaabbbb,…}.
- The language $\{a^n b^n a^n\}$, of strings defined over $\Sigma=\{a,b\}$, as $\{a^n b^n a^n: n=1,2,3,\ldots\}$, can be written as {aba, aabbaa, aaabbbaaa,aaaabbbbaaaa,…}.

Descriptive Definition

- The language factorial, of strings defined over $\Sigma=\{0,1,2,3,4,5,6,7,8,9\}$ i.e. $\{1,2,6,24,120,\ldots\}$.
- The language PRIME, of strings defined over $\Sigma=\{a\}$, as $\{a^p : p \text{ is prime}\}$, can be written as {aa, aaa, aaaaa, aaaaaaa, aaaaaaaaaaa…}.
- For $\Sigma=\{a,b\}$, PALINDROME={Λ , a, b, aa, bb, aaa, aba, bab, bbb, ...}.

Recursive Definition

- The three steps used:
  - Some basic words are specified in the language.
  - Rules for constructing more words are defined in the language.
  - No strings except those constructed in above, are allowed to be in the language.

Recursive Definition

Examples:

- Defining language of INTEGER

  Step 1: 1 is in INTEGER.

  Step 2: If x is in INTEGER then x+1 and x−1 are also in INTEGER.

  Step 3: No strings except those constructed in above, are allowed to be in INTEGER.

Recursive Definition

- Defining language of EVEN

  Step 1: 2 is in EVEN.

  Step 2: If x is in EVEN then x+2 and x−2 are also in EVEN.

  Step 3: No strings except those constructed in above, are allowed to be in EVEN.

Recursive Definition

- Defining the language factorial

  Step 1: As 0!=1, so 1 is in factorial.

  Step 2: n!=n*(n-1)! is in factorial.

  Step 3: No strings except those constructed in above, are allowed to be in factorial.

Recursive Definition

- Defining the language $\{a^n b^n\}$, $n=1,2,3,\ldots$, of strings defined over $\Sigma=\{a,b\}$

  Step 1: ab is in $\{a^n b^n\}$

  Step 2: if x is in $\{a^n b^n\}$, then axb is in $\{a^n b^n\}$

  Step 3: No strings except those constructed in above, are allowed to be in $\{a^n b^n\}$

Recursive Definition

- Defining the language L, of strings beginning and ending in same letters , defined over Σ={a, b}

  Step 1: a and b are in L

  Step 2: (a)s(a) and (b)s(b) are also in L, where s belongs to Σ*

  Step 3: No strings except those constructed in above, are allowed to be in L

Recursive Definition

- Defining the language L, of strings containing aa or bb , defined over $\Sigma=\{a, b\}$

  Step 1: aa and bb are in L

  Step 2: s(aa)s and s(bb)s are also in L, where s belongs to Σ^*

  Step 3: No strings except those constructed in above, are allowed to be in L

Recursive Definition

- Defining the language L, of strings containing exactly aa, defined over $\Sigma=\{a, b\}$

  Step 1: aa is in L

  Step 2: s(aa)s is also in L, where s belongs to b*

  Step 3: No strings except those constructed in above, are allowed to be in L

Recursive Definition

- Defining the language L, of POLYNOMIAL

  Rule 1: Any number is in POLYNOMIAL.

  Rule 2: The variable x is in POLYNOMIAL.

  Rule 3: If p and q are in POLYNOMIAL, then so are p + q, p – q, (p), and pq.

  Q: Show that $3x^2 + 7x - 9$ is in POLYNOMIAL

Recursive Definition

By Rule I: 3 is in POLYNOMIAL.

By Rule 2: x is in POLYNOMIAL.

By Rule 3: 3x is in POLYNOMIAL.

By Rule 3: 3xx is in POLYNOMIAL; call it $3x^2$.

By Rule I: 7 is in POLYNOMIAL.

By Rule 3: 7x is in POLYNOMIAL.

By Rule 3: $3x^2+7x$ is in POLYNOMIAL.

By Rule 1: −9 is in POLYNOMIAL.

By Rule 3: $3x^2+7x+(-9) = 3x^2+7x-9$ is in POLYNOMIAL.

Recursive Definition

- Defining the language EVEN

  Rule I: 2 is in EVEN.

  Rule 2: If x is in EVEN , then so are x+2 and x−2.

  Rule 3: The only elements in the set EVEN are those that can be produced from the two rules above.

  Q: Show that 14 is in EVEN

Recursive Definition

By Rule I: 2 is in EVEN .

By Rule 2: 2 + 2 = 4 is in EVEN .

By Rule 2: 4 + 2 = 6 is in EVEN .

By Rule 2: 6 + 2 = 8 is in EVEN .

By Rule 2: 8 + 2 = 10 is in EVEN .

By Rule 2: 10 + 2 = 12 is in EVEN .

By Rule 2: 12 + 2 = 14 is in EVEN .

Recursive Definition

- Defining the language EVEN

  Rule I: 2 i s in EVEN .

  Rule 2: If x and y are both in EVEN, then so are x+y and x−y.

  Q: Show that 14 is in EVEN

Recursive Definition

By Rule I: 2 is in EVEN .

By Rule 2: $x=2$, $y=2$, $x+y=2+2=4$ is in EVEN .

By Rule 2: $x=2$. $y=4$, $x+y=2+4=6$ is in EVEN.

By Rule 2: $x=4$, $y=4$, $x+y=4+4=8$ is in EVEN .

By Rule 2: $x=6$, $y=8$, $x+y=6+8=14$ is in EVEN.