Discussed so far:
- Descriptive Definitions
- Recursive Definitions
- FA: DFA, NFA
- RE, TG, GTG
- Kleene Theorem
- FA with Output
- Decidability

Context Free Grammar (CFG)

Terminologies:
- Terminals (T): Symbols that cannot be replaced by anything
- Non-Terminals (NT): Symbols that are replaced by NT or T or combination of Ts and/or NTs
- Productions: Grammatical Rules

- Terminals are usually represented by lowercase letters
- Non-Terminals are usually represented by uppercase letters

CFG is a collection of the following:
1. A set of input letters Σ called terminals from which the input string is formed.
2. A set of symbols called non-terminals, including S (Means START).
3. A finite set of productions of the form
   NT → Finite string of Ts and/or NTs

Note: The language generated by CFG (Context Free Grammar) is called CFL (Context Free Language).

λ Null/Empty String Ɛ Null/Empty String aλ = a λa = a aλb = ab

Σ = {a,b}
Productions:
1. S → aS
2. S → λ

S – P2 – λ
S – P1 – aS – P2 – aλ – a
S – P1 – aS – P1 – aaS – P2 – aaλ – aa
S – P1 – aS – P1 – aaS – P1 – aaaS – P2 – aaaλ – aaa
L = {λ,a,aa,aaa,aaaa,aaaaa, …}, RE: a^*

Σ = {a}
Productions:
1. S → SS
2. S → a
3. S → λ

S – P3 – λ
S – P2 – a
S – P1 – SS – P3 – λS – S – P3 – λ
S – P1 – SS – P3 – Sλ – S – P3 – λ

S – P1 – SS – P3 – λS – S – P2 – a
S – P1 – SS – P3 – Sλ – S – P2 – a
S – P1 – SS – P2 – aS – P2 – aa
S – P1 – SS – P2 – Sa – P2 – aa
S – P1 – SS – P1 – SSS – P2 – aSS – P2 – aaS – P2 – aaa
L = {λ,a,aa,aaa,aaaa,aaaaa, …}, RE: a^*

Note:
- A string can be generated through multiple paths
- Multiple CFGs may generate the same language

Σ = {a}
Productions:
1. S → SS
2. S → a

S – P2 – a
S – P1 – SS – P2 – aS – P2 – aa
S – P1 – SS – P1 – SSS – P2 – aSS – P2 – aaS – P2 – aaa
L = {a,aa,aaa,aaaa,aaaaa, …}, RE: a^+

Σ – {a,b}
Productions:
1. S → X
2. S → Y
3. X → λ
4. Y → aY
5. Y → bY
6. Y → a
7. Y → b

S – P1 – X – P3 – λ
S – P2 – Y – P6 – a
S – P2 – Y – P7 – b
S – P2 – Y – P4 – aY – P6 – aa
S – P2 – Y – P4 – aY – P7 – ab
S – P2 – Y – P5 – bY – P6 – ba
S – P2 – Y – P5 – bY – P7 – bb
S – P2 – Y – P4 – aY – P4 – aaY – P6 – aaa
L = {λ,a,b,aa,ab,ba,bb,aaa,aab,aba,abb,baa,bab,bba,bbb, …}, RE: $(a+b)^*$

Σ = {a,b}
Productions:
1. S → aS
2. S → bS
3. S → a
4. S → b
5. S → λ

L = {λ,a,b,aa,ab,ba,bb,aaa,aab,aba,abb,baa,bab,bba,bbb, …}, RE: (a+b)*
How? Derive at least first nine words from the set L.

$\Sigma = \{a,b\}$
Productions:
1. $S \rightarrow XaaX$
2. $X \rightarrow aX$
3. $X \rightarrow bX$
4. $X \rightarrow \lambda$

S – P1 – XaaX – P4 – λaaX – aaX – P4 – aaλ – aa
S – P1 – XaaX – P2 – aXaaX – P4 – aλaaX – aaaX – P4 – aaaλ – aaa
S – P1 – XaaX – P3 – bXaaX – P4 – bλaaX – baaX – P4 – baaλ – baa
L = {aa,aaa,baa,aab, …}, RE: $(a+b)^{*}aa(a+b)^{*}$ -- Containing aa

$\Sigma = \{a,b\}$
Productions:
1. $S \rightarrow SS$
2. $S \rightarrow XS$
3. $S \rightarrow \lambda$
4. $S \rightarrow YSY$
5. $X \rightarrow aa$
6. $X \rightarrow bb$
7. $Y \rightarrow ab$
8. $Y \rightarrow ba$

L = {λ,aa,bb,aaaa,abba,baab,aabb,bbaa,bbbb, …}, RE: $(aa+bb+(ab+ba)(aa+bb)^{*}(ab+ba))^{*}$
-- EVEN-EVEN
How? Derive at least first nine words from the set L.