

Simplification of CFG:

- Elimination of useless symbols from the grammar. These are the symbols that do NOT play any role in any derivation of any string/expression.
- Eliminating λ -productions. These are of the form $A \rightarrow \lambda$.
- Eliminating unit-productions. These are of the form $A \rightarrow B$ (Both are Non-Terminals).

Elimination of useless symbols from the grammar:

EXAMPLE 1: $S \rightarrow aX \mid b$

b

$S \rightarrow aX \mid aaX \mid aaaX \dots$ Never generates a string – Useless

Eliminating the useless symbol/production: $S \rightarrow b$

EXAMPLE 2:

1. $S \rightarrow AB \mid a$
2. $A \rightarrow b$

$S \rightarrow a$ S generates a

$A \rightarrow b$ A generates b

Useless Symbol: B

$S \rightarrow AB$ Useless because it does not play any role in the generation of any string – Because of B

Eliminating the useless Symbols/Productions: $S \rightarrow a$

Eliminating λ -productions:

1. $S \rightarrow aA \mid bB \mid \lambda$
2. $A \rightarrow aa \mid \lambda$
3. $B \rightarrow aS$

λ -productions:

$S \rightarrow \lambda$

$A \rightarrow \lambda$

EXAMPLE 2:

1. $S \rightarrow XaY \mid YY \mid aX \mid ZYX$
2. $X \rightarrow Za \mid bZ \mid ZZ \mid Yb$
3. $Y \rightarrow Ya \mid XX \mid \lambda$
4. $Z \rightarrow aX \mid YYY$

Null-Production(s): $Y \rightarrow \lambda$

Nullable-Production(s): $S \rightarrow YY, Z \rightarrow YYY, X \rightarrow ZZ$

1. $S \rightarrow XaY \mid YY \mid aX \mid ZYX$

1a: XaY Length(RHS) = 3, We consider No λ , One λ and Two λ s (Length-1)

- XaY No λ
- $\lambda aY - aY$ One λ
- $Xa\lambda - Xa$ One λ
- $\lambda a\lambda - a$ Two λ s

So, $S \rightarrow XaY$ becomes $S \rightarrow XaY \mid aY \mid Xa \mid a$

1b: YY Length(RHS) = 2, We consider No λ and One λ (Length-1)

- YY No λ
- $\lambda Y - Y$ One λ
- $Y\lambda - Y$ One λ

So, $S \rightarrow YY$ becomes $S \rightarrow YY \mid Y$

1c: aX Length(RHS) = 2, We consider No λ and One λ (Length-1)

- aX No λ
- $a\lambda - a$ One λ

So, $S \rightarrow aX$ becomes $S \rightarrow aX \mid a$

1d: ZYX Length(RHS) = 3, We consider No λ , One λ and Two λ s (Length-1)

- ZYX No λ
- $\lambda YX - YX$ One λ
- $Z\lambda X - ZX$ One λ
- $ZY\lambda - ZY$ One λ
- $\lambda\lambda X - X$ Two λ s
- $\lambda Y\lambda - Y$ Two λ s
- $Z\lambda\lambda - Z$ Two λ s

So, $S \rightarrow ZYX$ becomes $S \rightarrow ZYX \mid YX \mid ZX \mid ZY \mid X \mid Y \mid Z$

So, $S \rightarrow XaY \mid YY \mid aX \mid ZYX$ becomes

$S \rightarrow XaY \mid aY \mid Xa \mid a \mid YY \mid Y \mid aX \mid ZYX \mid YX \mid ZX \mid ZY \mid X \mid Z$

2. $X \rightarrow Za \mid bZ \mid ZZ \mid Yb$

2a: $X \rightarrow Za$ becomes $X \rightarrow Za \mid a$ (No λ and One λ)

2b: $X \rightarrow bZ$ becomes $X \rightarrow bZ \mid b$ (No λ and One λ)

2c: $X \rightarrow ZZ$ becomes $X \rightarrow ZZ \mid Z$ (No λ and One λ)

2d: $X \rightarrow Yb$ becomes $X \rightarrow Yb \mid b$ (No λ and One λ)

$X \rightarrow Za \mid bZ \mid ZZ \mid Yb$ becomes

$X \rightarrow Za \mid a \mid bZ \mid b \mid ZZ \mid Z \mid Yb$

$$3. Y \rightarrow Ya \mid XX \mid \lambda$$

$$3a: Y \rightarrow Ya \quad \text{becomes } Y \rightarrow Ya \mid a \quad (\text{No } \lambda \text{ and One } \lambda)$$

$$3b: Y \rightarrow XX \quad \text{becomes } Y \rightarrow XX \mid X \quad (\text{No } \lambda \text{ and One } \lambda)$$

$$Y \rightarrow Ya \mid XX \mid \lambda \text{ becomes}$$

$$Y \rightarrow Ya \mid a \mid XX \mid X$$

$$4. Z \rightarrow aX \mid YYY$$

$$4a: Z \rightarrow aX \quad \text{becomes } Z \rightarrow aX \mid a \quad (\text{No } \lambda \text{ and One } \lambda)$$

$$4b: Z \rightarrow YYY \quad \text{becomes } Z \rightarrow YYY \mid YY \mid Y \quad (\text{No } \lambda, \text{ One } \lambda \text{ and Two } \lambda\text{s})$$

$$Z \rightarrow aX \mid YYY \text{ becomes}$$

$$Z \rightarrow aX \mid a \mid YYY \mid YY \mid Y$$

Final Grammar:

$$1. S \rightarrow XaY \mid aY \mid Xa \mid a \mid YY \mid Y \mid aX \mid ZYX \mid YX \mid ZX \mid ZY \mid X \mid Z$$

$$2. X \rightarrow Za \mid a \mid bZ \mid b \mid ZZ \mid Z \mid Yb$$

$$3. Y \rightarrow Ya \mid a \mid XX \mid X$$

$$4. Z \rightarrow aX \mid a \mid YYY \mid YY \mid Y$$

Contains Unit-Production(s)

Contains Unit-Production(s)

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EXAMPLE 3:

$$1. S \rightarrow XY$$

$$2. X \rightarrow Zb$$

$$3. Y \rightarrow bW$$

$$4. Z \rightarrow AB$$

$$5. W \rightarrow Z$$

$$6. A \rightarrow aA \mid bA \mid \lambda$$

$$7. B \rightarrow Ba \mid Bb \mid \lambda$$

Null-Production(s): $A \rightarrow \lambda, B \rightarrow \lambda$

Nullable Productions: $Z \rightarrow AB, W \rightarrow Z$

$$1. S \rightarrow XY$$

$$2. X \rightarrow Zb$$

$$\text{becomes } X \rightarrow Zb \mid b$$

$$3. Y \rightarrow bW$$

$$\text{becomes } Y \rightarrow bW \mid b$$

$$4. Z \rightarrow AB$$

$$\text{becomes } Z \rightarrow AB \mid A \mid B$$

$$5. W \rightarrow Z$$

Contains Unit-Production(s)

Contains Unit-Production(s)

$$6. A \rightarrow aA \mid bA \mid \lambda$$

$$\text{becomes } A \rightarrow aA \mid a \mid bA \mid b$$

$$7. B \rightarrow Ba \mid Bb \mid \lambda$$

$$\text{becomes } B \rightarrow Ba \mid a \mid Bb \mid b$$

Final Grammar:

1. $S \rightarrow XY$
2. $X \rightarrow Zb \mid b$
3. $Y \rightarrow bW \mid b$
4. $Z \rightarrow AB \mid A \mid B$
5. $W \rightarrow Z$
6. $A \rightarrow aA \mid a \mid bA \mid b$
7. $B \rightarrow Ba \mid a \mid Bb \mid b$

Contains Unit-Production(s)

Contains Unit-Production(s)

Task:

1. $S \rightarrow AB$
2. $A \rightarrow aAA \mid \lambda$
3. $B \rightarrow bBB \mid \lambda$

Null-Production(s): $A \rightarrow \lambda, B \rightarrow \lambda$

Nullable-Production(s): $S \rightarrow AB$

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|-------------------------------------|--|
| 1. $S \rightarrow AB$ | becomes $S \rightarrow AB \mid A \mid B$ |
| 2. $A \rightarrow aAA \mid \lambda$ | becomes $A \rightarrow aAA \mid aA \mid a$ |
| 3. $B \rightarrow bBB \mid \lambda$ | becomes $B \rightarrow bBB \mid bB \mid b$ |

Final Grammar:

1. $S \rightarrow AB \mid A \mid B$
2. $A \rightarrow aAA \mid aA \mid a$
3. $B \rightarrow bBB \mid bB \mid b$

EXAMPLE1 was

1. $S \rightarrow aA \mid bB \mid \lambda$
2. $A \rightarrow aa \mid \lambda$
3. $B \rightarrow aS$

Null-Production(s): $S \rightarrow \lambda, A \rightarrow \lambda$

Nullable-Production(s): None

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|--|---|
| 1. $S \rightarrow aA \mid bB \mid \lambda$ | becomes $S \rightarrow aA \mid a \mid bB$ |
| 2. $A \rightarrow aa \mid \lambda$ | becomes $A \rightarrow aa$ |
| 3. $B \rightarrow aS$ | becomes $B \rightarrow aS \mid a$ |

Final Grammar:

1. $S \rightarrow aA \mid a \mid bB$
2. $A \rightarrow aa$
3. $B \rightarrow aS \mid a$