

Parse Trees II

$S \rightarrow (S+S) \mid (S*S) \mid \text{number}$

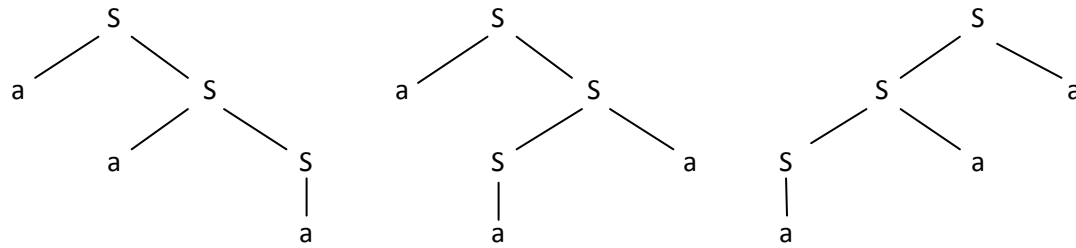
$a+b*c$ – Invalid expression

$((a+b)*c)$ and $(a+(b*c))$ – Valid expressions

Ambiguous Grammar: There exists more than one paths to derive a string/expression.

$S \rightarrow aS \mid Sa \mid a$

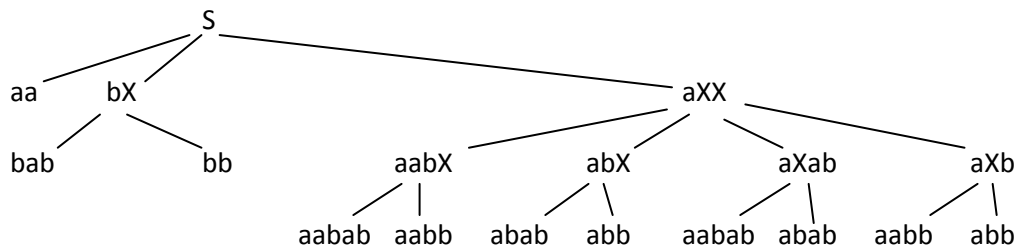
String: aaa



Total Language Tree

$S \rightarrow aa \mid bX \mid aXX$

$X \rightarrow ab \mid b$

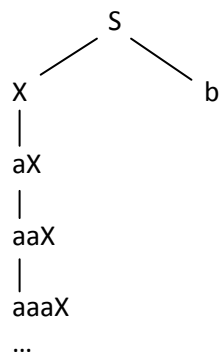


Language = {aa, bab, bb, aabab, aabb, abab, abb}

$S \rightarrow X \mid b$

$X \rightarrow aX$

Language = {b}



Leftmost Derivation: The derivation of a word, generated by the CFG, such that at each step, a production is applied to the leftmost non-terminal in the working string.

1. $S \rightarrow XY$
2. $X \rightarrow XX$
3. $X \rightarrow a$
4. $Y \rightarrow YY$
5. $Y \rightarrow b$

String: aaabb

$S \Rightarrow XY \Rightarrow XXY \Rightarrow aXY \Rightarrow aXXY \Rightarrow aaXY \Rightarrow aaaY \Rightarrow aaaYY \Rightarrow aaabY \Rightarrow aaabb$
 $S \Rightarrow XY \Rightarrow XXY \Rightarrow XXXY \Rightarrow aXXY \Rightarrow aaXY \Rightarrow aaaY \Rightarrow aaaYY \Rightarrow aaabY \Rightarrow aaabb$

1. $S \rightarrow YX$
2. $X \rightarrow XX$
3. $X \rightarrow b$
4. $Y \rightarrow YY$
5. $Y \rightarrow a$

String: abbbb

Leftmost Derivation:

$S \Rightarrow 1 \Rightarrow YX$
 $\Rightarrow 5 \Rightarrow aX$
 $\Rightarrow 2 \Rightarrow aXX$
 $\Rightarrow 3 \Rightarrow abX$
 $\Rightarrow 2 \Rightarrow abXX$
 $\Rightarrow 3 \Rightarrow abbX$
 $\Rightarrow 2 \Rightarrow abbXX$
 $\Rightarrow 3 \Rightarrow abbbX$
 $\Rightarrow 3 \Rightarrow abbbb$

Rightmost Derivation:

$S \Rightarrow 1 \Rightarrow YX$
 $\Rightarrow 2 \Rightarrow YXX$
 $\Rightarrow 3 \Rightarrow YXb$
 $\Rightarrow 2 \Rightarrow YXXb$
 $\Rightarrow 3 \Rightarrow YXbb$
 $\Rightarrow 2 \Rightarrow YXXbb$
 $\Rightarrow 3 \Rightarrow YXbbb$
 $\Rightarrow 3 \Rightarrow Ybbbb$
 $\Rightarrow 5 \Rightarrow abbbb$

Rightmost Derivation: The derivation of a word, generated by the CFG, such that at each step, a production is applied to the rightmost non-terminal in the working string.

Chomsky Normal Form (CNF):

CNF is a CFG that has productions of the following types ONLY

- $NT \rightarrow NT_1NT_2$ String of two Non-Terminals ONLY
- $NT \rightarrow T$ One Terminal ONLY

EXAMPLE 1:

$S \rightarrow aSa \mid bSb \mid a \mid b \mid aa \mid bb$ Not CNF

Correct:

$S \rightarrow a$

$S \rightarrow b$

Introduce new productions:

$A \rightarrow a$ Correct

$B \rightarrow b$ Correct

Result:

$S \rightarrow aa$ becomes $S \rightarrow AA$ Correct

$S \rightarrow bb$ becomes $S \rightarrow BB$ Correct

$S \rightarrow aSa$ becomes $S \rightarrow ASA$ Incorrect

$S \rightarrow bSb$ becomes $S \rightarrow BSB$ Incorrect

Introduce new productions:

$C \rightarrow AS$ Correct

$D \rightarrow BS$ Correct

Result:

$S \rightarrow ASA$ becomes $S \rightarrow CA$ Correct

$S \rightarrow BSB$ becomes $S \rightarrow DB$ Correct

Final Grammar (CNF):

1. $S \rightarrow a$
2. $S \rightarrow b$
3. $A \rightarrow a$
4. $B \rightarrow b$
5. $S \rightarrow AA$
6. $S \rightarrow BB$
7. $C \rightarrow AS$
8. $D \rightarrow BS$
9. $S \rightarrow CA$
10. $S \rightarrow DB$

EXAMPLE 2:

1. $S \rightarrow ABAB$ Not CNF
2. $A \rightarrow a \mid \lambda$
3. $B \rightarrow b \mid \lambda$

Nullable Production is $S \rightarrow ABAB$

$S \rightarrow BAB \mid AAB \mid ABB \mid ABA \mid AA \mid AB \mid BA \mid BB \mid A \mid B$

$S \rightarrow BAB \mid AAB \mid ABB \mid ABA \mid AA \mid AB \mid BA \mid BB \mid a \mid b$

$S \rightarrow AA$	Correct
$S \rightarrow AB$	Correct
$S \rightarrow BA$	Correct
$S \rightarrow BB$	Correct
$S \rightarrow a$	Correct
$S \rightarrow b$	Correct
$A \rightarrow a$	Correct
$B \rightarrow b$	Correct

$S \rightarrow BAB$	Incorrect
$S \rightarrow AAB$	Incorrect
$S \rightarrow ABB$	Incorrect
$S \rightarrow ABA$	Incorrect

Introduce new production:

$C \rightarrow AB$	Correct
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Result:

$S \rightarrow BAB$ becomes $S \rightarrow BC$	Correct
$S \rightarrow AAB$ becomes $S \rightarrow AC$	Correct
$S \rightarrow ABB$ becomes $S \rightarrow CB$	Correct
$S \rightarrow ABA$ becomes $S \rightarrow CA$	Correct

Final Grammar (CNF):

1. $S \rightarrow a$
2. $S \rightarrow b$
3. $A \rightarrow a$
4. $B \rightarrow b$
5. $S \rightarrow AA$
6. $S \rightarrow AB$
7. $S \rightarrow BA$
8. $S \rightarrow BB$
9. $C \rightarrow AB$
10. $S \rightarrow BC$
11. $S \rightarrow AC$
12. $S \rightarrow CB$
13. $S \rightarrow CA$