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Lecturer, 03009602412 / 220532
Govt. A. M. College
SARGODHA

CH#10

Higher Order Linear Differential Eqs

Diff eqs of Ist Degree, but not of Ist order. OR

Linear Diff eqs with constant coefficients.

$$a_0\frac{d^ny}{dx^n}+a_1\frac{d^{n-1}y}{dx^{n-1}}+a_2\frac{d^{n-2}y}{dx^{n-2}}+\cdots+a_{n-1}\frac{dy}{dx}+a_ny=F(x)$$

If $F(x)=0$, then Homogeneous Linear Diff. Eq.

If $F(x)\neq 0$, then NonHomogeneous Linear Diff Eq.

1) **Real and Distinct Roots** If $m_1, m_2, \ldots m_n$ are distinct real roots then General Sol is $y=c_1e^{m_1x}+c_2e^{m_2x}+c_3e^{m_3x}+\cdots+c_ne^{m_nx}$.

2) **Real and Equal Roots** If $m_1=m_2$ are roots then G. Sol is $y=(c_1+c_2x)e^{m_1x}$

If $m_1=m_2=m_3$ are real roots then G. Sol is $y=(c_1+c_2x+c_3x^2)e^{m_1x}$

3) **Imaginary & Distinct Roots** If Roots are $\alpha\pm i\beta$ then G. Sol is $y=e^{\alpha x}(c_1\cos\beta x+c_2\sin\beta x)$

4) **Imaginary & Repeated Roots** If Roots are $\alpha\pm i\beta$, & $\alpha\pm i\beta$ the G. Sol $y=e^{\alpha x}\{(c_1+c_2x)\cos\beta x+(c_3+c_4x)\sin\beta x\}$

$\boxed{\text{Ex 10.1}}$

Solve

① $(9D^2-12D+4)y=0$

$9D^2-12D+4=0$ Characteristic Eq or Auxiliary Eq.

$D=\frac{12\pm\sqrt{144-4.9.4}}{18}=\frac{12\pm\sqrt{144-144}}{18}$

$D=\frac{12}{18}+0,\ \frac{12}{18}-0=\boxed{\frac{2}{3},\frac{2}{3}}$ (case II)

$\therefore$ G. Sol is $y=(c_1+c_2x)e^{\frac{2}{3}x}$

② $(75D^2+50D+12)y=0$

$75D^2+50D+12=0$ Characteristic Eq

$D=\frac{-50\pm\sqrt{2500-}}{2(75)}=\frac{-50\pm\sqrt{2500-3600}}{150}$

$=\frac{-50\pm\sqrt{-1100}}{150}=\frac{-50\pm\sqrt{-1(100)}}{150}=\frac{-50\pm10\sqrt{11}}{150}$

$=-10\left(\frac{-5\pm\sqrt{-11}}{150}\right)=\frac{-5\pm i\sqrt{11}}{15}=\boxed{-\frac{1}{3}\pm\frac{i\sqrt{11}}{15}}$

G. Sol $y=e^{-\frac{1}{3}x}\left(c_1\cos\frac{\sqrt{11}}{15}x+c_2\sin\frac{\sqrt{11}}{15}x\right)$

③ $(D^3-4D^2+D+6)y=0$

$D^3-4D^2+D+6=0$ Characteristic Eq.

-1	1	-4	1	6
	↓	-1	5	-6
	1	-5	6	$\lfloor 0$

$D^2-5D+6=0$ Depressed Eq

$(D-2)(D-3)=0$

$\boxed{D=2,3,-1}$

G. Sol is $y=c_1e^{-1x}+c_2e^{2x}+c_3e^{3x}$

④ $(D^3+D^2+D+1)y=0$

$D^3+D^2+D+1=0$ Characteristic Eq

-1	1	1	1	1
	↓	-1	0	-1
	1	0	1	$\lfloor 0$

$D^2+0D+1=0$

$D^2=-1$

$D=\pm i$

G. Sol $y=$ $\therefore D=-1,\pm i$

G. Sol is $y=c_1e^{-1x}+e^{0x}(c_2\cos x+c_3\sin x)$

$y=c_1e^{-x}+c_2\cos x+c_3\sin x$

Solution of Non Homogeneous Linear Diff Eq of order 'n'.

$$(a_0D^n + a_1D^{n-1} + a_2D^{n-2} + \cdots + a_{n-1}D + a_n)Y = F(x)$$

The solution consist of two parts

(i) Complementary Function (C.F) :- It is sol of Homogeneous L. Diff Eq i.e $(a_0D^n + a_1D^{n-1} + \cdots + a_n)Y = 0$. It is denoted by Y_c

(ii) Particular Integral (P.I) :- It is sol of $\frac{1}{a_0D^n + a_1D^{n-1} + \cdots a_n}F(x)$. It is denoted by Y_p

$\therefore$ General Sol $Y = Y_c + Y_p$

Properties of Differential Operator 'D' $= \frac{d}{dx}$

i) $D^n(ae^{bx}) = a(b)^n e^{bx}$ 'D' is replaced by 'b'

ii) $F(D)(ae^{bx}) = aF(b)e^{bx}$ 'D' is replaced by 'b'

iii) $D^n(e^{bx})U = e^{bx}(D+b)^n U$ 'b' is added in D in D

iv) $F(D)e^{bx}U = e^{bx}F(D+b)U$ b is added in D

v) $\frac{1}{F(D)}ae^{bx} = \frac{ae^{bx}}{F(b)}$ 'D' is replaced by 'b'

vi) $\frac{1}{F(D)}e^{bx}.U = e^{bx}\frac{1}{F(D+b)}.U$, b is added in D

vii) $\frac{1}{F(D^2)}\sin(ax) = \frac{\sin ax}{F(-a^2)}$

viii) $\frac{1}{F(D^2)}\cos(ax) = \frac{\cos ax}{F(-a^2)}$

D^2 is replaced by $(-a^2)$ only for D^2

ix) $\frac{1}{F(D)}\sin bx = \text{Im}\frac{1}{F(D)}e^{ibx} = \text{Im}\frac{e^{ibx}}{F(ib)}$

x) $\frac{1}{F(D)}\cos bx = \text{Re}\frac{1}{F(D)}e^{ibx} = \text{Re}\frac{e^{ibx}}{F(ib)}$

xi) $D^2\cos bx = (-b^2)\cos bx$

xii) $D^2\sin bx = (-b^2)\sin bx$

only for D^2

xiii) $\frac{1}{D^2}\cos bx = \frac{1}{-b^2}\cos bx$

xiv) $\frac{1}{D^2}\sin bx = \frac{1}{-b^2}\sin bx$

only for D^2

Imp Note

1) when, $\frac{1}{F(D)}(ae^{bx}) = \frac{ae^{bx}}{F(b)}$ if $F(b)=0$

then $\frac{1}{F(D)}ae^{bx} = \frac{x}{F'(D)}(ae^{bx})$

$= \frac{x}{F'(b)}ae^{bx}$ if $F'(b)=0$

then $\frac{x}{F'(D)}ae^{bx} = \frac{x^2}{F''(D)}ae^{bx}$

$= \frac{x^2}{F''(b)}ae^{bx}$ & so on. if $F''(b) \neq 0$

$$D(\sin x) = \frac{d}{dx}(\sin x) = \cos x$$

$$\frac{1}{D}(\sin x) = \int \sin x\,dx = -\cos x$$

B. Series. If n is -ive or fraction.

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{\lfloor 2}x^2 + \cdots$$

We apply B. Series when $F(x)$ is other than $\sin, \cos$ or e^{bx} see Q4, 8, 9,