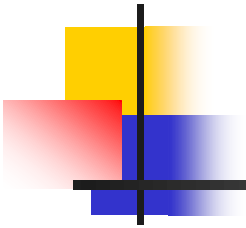
CSC 2204 Finite Automata Theory and Formal Languages

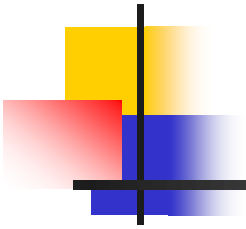
Department of Computer Science
SZABIST (Islamabad Campus)

Week 7 (Lecture 1)



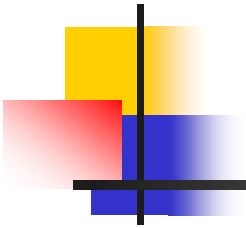
FA with Output

- A simple FA is just associated with the RE or the language.
- There exist an FA which generates an output string corresponding to each input string. Such machines are called machines with output.
- There are two types of machines with output.
  - Moore machine.
  - Mealy machine.



Moore Machine

- A Moore machine consists of the following
  - A finite set of states q0, q1, q2, … where q0 is the initial state.
  - An alphabet of letters Σ = {a,b,c,…} from which the input strings are formed.
  - An alphabet Γ={x,y,z,…} of output characters from which output strings are generated.
  - A transition table that shows for each state and each input letter what state is entered the next.
  - An output table that shows what character is printed by each state as it is entered.



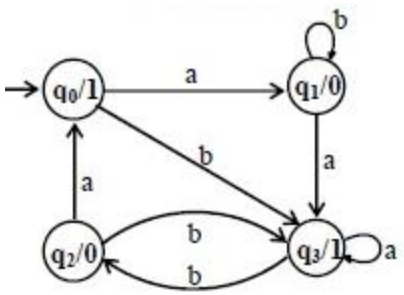
Moore Machine

- Note
  - Since in Moore machine no state is designated to be a final state, so there is no question of accepting any language by Moore machine.
  - In some cases the relation between an input string and the corresponding output string may be identified by the Moore machine.
  - The state to be initial is not important as if the machine is used several times and is restarted after sometime, the machine will be started from the state where it was left off.

Moore Machine (Example 1)

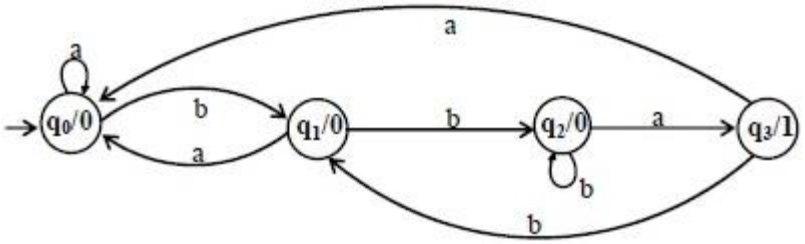
Consider the following Moore machine having the states q_0, q_1, q_2, q_3 where q_0 is the start state and
$\Sigma = \{a,b\}$,
$\Gamma=\{0,1\}$

Old States	New States after reading		Characters to be printed
	a	b	
q_0-	q_1	q_3	1
q_1	q_3	q_1	0
q_2	q_0	q_3	0
q_3	q_3	q_2	1



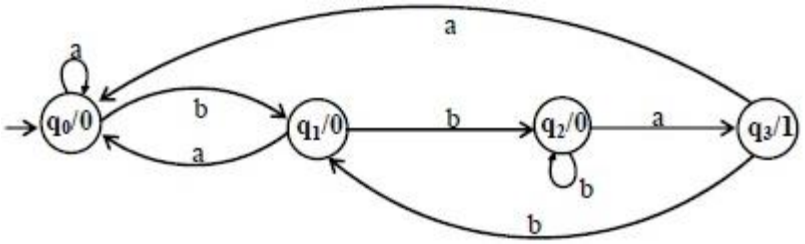
Input		a	b	b	a	b	b	b	a
State	q_0	q_1	q_1	q_1	q_3	q_2	q_3	q_2	q_0
output	1	0	0	0	1	0	1	0	1

Moore Machine (Example 2)

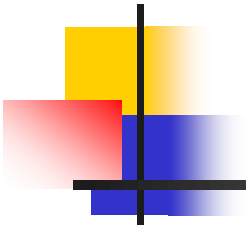


Run String bbbabaabbaa

Moore Machine (Example 2)

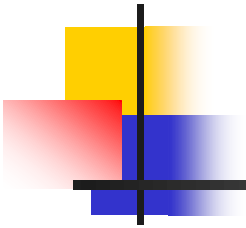


Input		b	b	b	a	b	a	a	b	b	a	a
State	q_0	q_1	q_2	q_2	q_3	q_1	q_0	q_0	q_1	q_2	q_3	q_0
output	0	0	0	0	1	0	0	0	0	0	1	0



Mealy Machine

- A Mealy machine consists of the following
  - A finite set of states q0,q1,q2,… where q0 is the initial state.
  - An alphabet of letters Σ={a,b,c,…} from which the input strings are formed.
  - An alphabet Γ={x,y,z,…} of output characters from which output strings are generated.
  - A pictorial representation with states and directed edges labeled by an input letter along with an output character. The directed edges also show how to go from one state to another corresponding to every possible input letter.

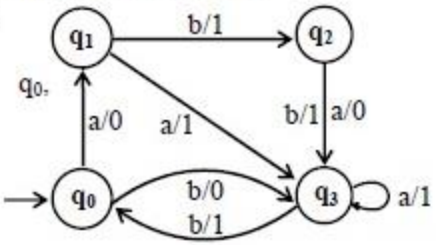


Mealy Machine

- Note:
  - It is not possible to give transition table in this case.
  - No state is designated to be a final state, so there is no question of accepting any language by Mealy machine.
  - In some cases the relation between an input string and the corresponding output string may be identified by the Mealy machine.
  - The state to be initial is not important as if the machine is used several times and is restarted after some time, the machine will be started from the state where it was left off.

Mealy Machine (Example 1)

Consider the Mealy machine shown aside, having the states q_0, q_1, q_2, q_3 , where q_0 is the start state and
$\Sigma = \{a,b\}$,
$\Gamma=\{0,1\}$

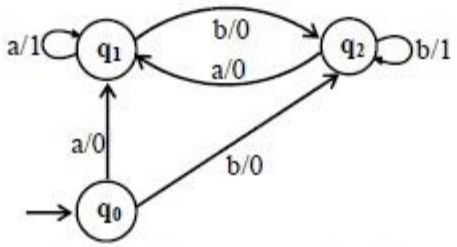


Old State	New State			
	Reading a		Reading b	
	Output	State	Output	State
Q0-	0	Q1	0	Q3
Q1	1	Q3	1	Q2
Q2	0	Q3	1	Q3
Q3	1	Q3	1	Q0

Input		a	b	b	a	b	b	b	a
States	q_0	q_1	q_2	q_3	q_3	q_0	q_3	q_0	q_1
output		0	1	1	1	1	0	1	0

Mealy Machine (Example 2)

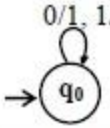
Consider the following Mealy machine having the states q_0, q_1, q_2, where q_0 is the start state and
$\Sigma = \{a,b\}$,
$\Gamma = \{0,1\}$



It is observed that in the above Mealy machine, if in the output string the nth character is 1, it shows that the nth letter in the input string is the second in the pair of double letter.
For babaababba as input string the machine will print 0000100010.

Mealy Machine (Example 3)

Consider the following Mealy machine having the only state q_0 as the start state and
$\Sigma = \{0,1\}$,
$\Gamma = \{0,1\}$



If 0011010 is run on this machine then the corresponding output string will be 1100101.
This machine is called **Complementing machine.**

Mealy Machine (Example 4)

Before the incrementing machine is constructed, consider how 1 is added to a binary number.
Since, if two numbers are added, the addition is performed from right to left, so while increasing the binary number by 1, the string (binary number) must be read by the corresponding Mealy machine from right to left, and hence the output string (binary number) will also be generated from right to left.

a)
$$\begin{array}{r} 100101110 \\ +\ 1 \\ \hline 100101111 \end{array}$$

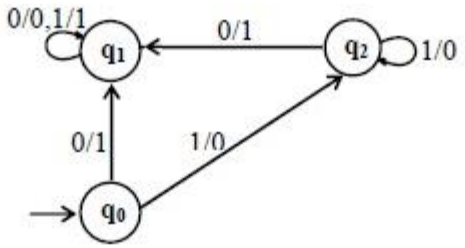
b)
$$\begin{array}{r} 1001100111 \\ +\ 1 \\ \hline 1001101000 \end{array}$$

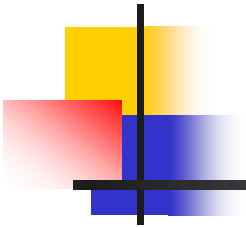
It may be observed from the above that

a) If the right most bit of binary number, to be incremented, is 0, the output binary number can be obtained by converting the right most bit to 1 and remaining bits unchanged.

b) If the right most bit of binary number is 1 then the output can be obtained, converting that 1 along with all its concatenated 1's to 0's, then converting the next 0 to 1 and remaining bits unchanged.

The observations (a) and (b) help to construct the following Incrementing (Mealy) machine.
The Mealy machine have the states q_0, q_1, q_2, where q_0 is the start state and $\Sigma = \{0,1\}$, $\Gamma = \{0,1\}$





Example

- Subtraction:
  - Complementing Machine,
  - Incrementing Machine,
  - Adding Machine.