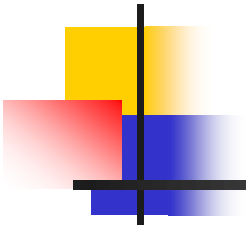
CSC 2204 Finite Automata Theory and Formal Languages

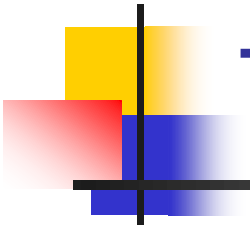
Department of Computer Science
SZABIST (Islamabad Campus)

Week 7 (Lecture 2)



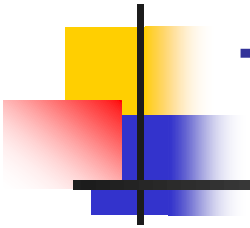
Equivalent Machines

- Two machines are said to be equivalent if they print the same output string when the same input string is run on them.
- Note:
  - Two Moore machines may be equivalent. Similarly two Mealy machines may also be equivalent, but a Moore machine can’t be equivalent to any Mealy machine.
  - Ignoring the extra character printed by the Moore machine, there exists a Mealy machine which is equivalent to the Moore machine.



Theorem

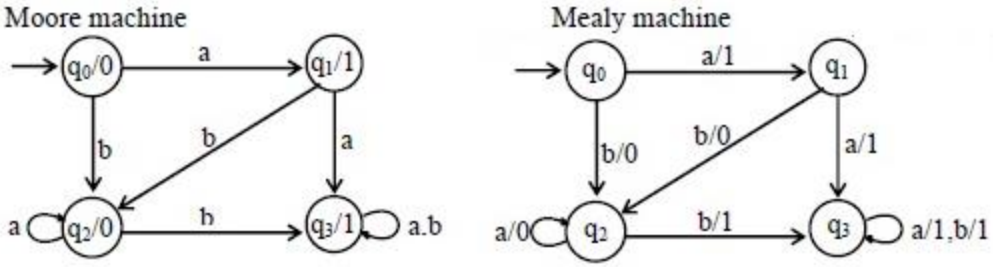
- Statement: For every Moore machine there is a Mealy machine that is equivalent to it (ignoring the extra character printed by the Moore machine).
- Proof:
  - Let M be a Moore machine, then shifting the output characters corresponding to each state to the labels of corresponding in coming transitions, machine thus obtained will be a Mealy machine equivalent to M.



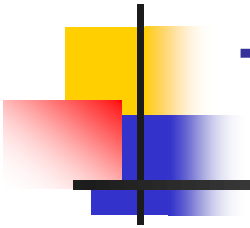
Theorem

- Note:
  - While converting a Moore machine in to an equivalent Mealy machine, the output character of a state will be ignored if there is no incoming transition at that state.
  - A loop at a state is also supposed to be an incoming transition.

Theorem (Example)

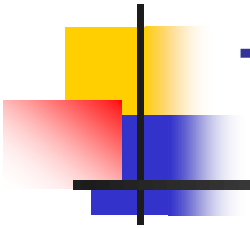


Input		a	b	b	a	b	b	b	a
States	q_0	q_1	q_2	q_3	q_3	q_3	q_3	q_3	q_3
Moore	0	1	0	1	1	1	1	1	1
Mealy		1	0	1	1	1	1	1	1



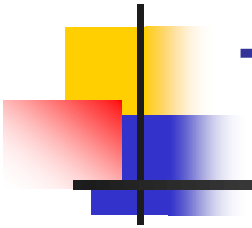
Theorem

- Statement: For every Mealy machine there is a Moore machine that is equivalent to it (ignoring the extra character printed the Moore machine).
- Proof:
  - Let M be a Mealy machine. At each state there are two possibilities for incoming transitions.
    - The incoming transitions have the same output character.
    - The incoming transitions have different output characters.



Theorem

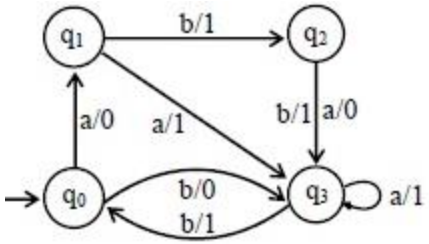
- Proof:
  - If all the transitions have same output characters, then shift that character to the corresponding state.
  - If all the transitions have different output characters, then the state will be converted to as many states as the number of different output characters for these transitions, which shows that if this happens at state q_i then q_i will be converted to q_i^1 for one character and q_i^2 for other character.



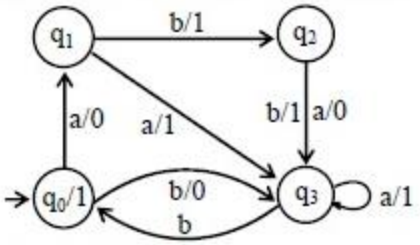
Theorem

- Proof:
  - Shift the output characters of the transitions to the corresponding new states q_i^1 and q_i^2. Moreover, these new states q_i^1 and q_i^2 should be have like qi as well.
  - Continuing the process, the machine thus obtained, will be a Moore machine equivalent to Mealy machine.

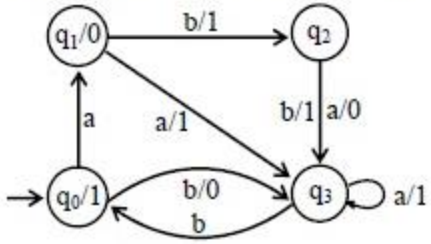
Theorem (Example)



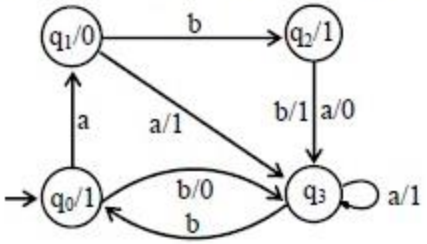
Shifting the output character 1 of transition b to q_0



Shifting the output character 0 of transition a to q_1

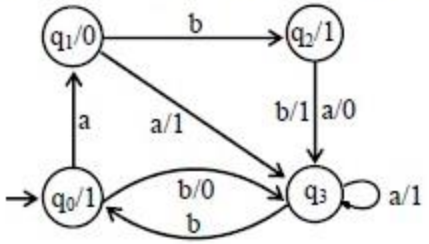


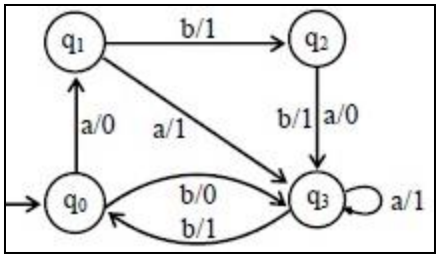
Shifting the output character 1 of transition b to q_2



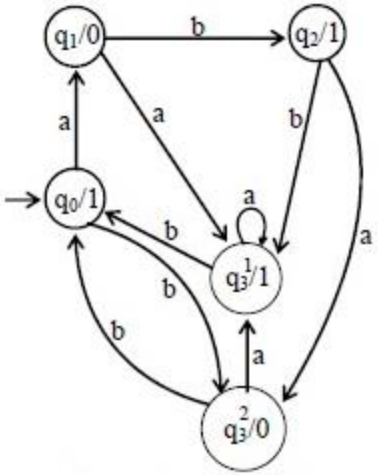
Theorem (Example)

Shifting the output character 1 of transition b to q_2





Splitting q_3 into q^1_3 and q^2_3



Input		a	b	b	a	b	b	b	a
States	q_0	q_1	q_2	q_3	q_3	q_0	q_3	q_0	q_1
Mealy		0	1	1	1	1	0	1	0
Moore	1	0	1	1	1	1	0	1	0