Elimination of unit productions from the grammar:

- Unit Productions are of the form Non-Terminal → Non-Terminal

EXAMPLE 1:

1. S → A | bb
2. A → B | b
3. B → S | a

Unit Productions:	Non-Unit Productions:
S → A	S → bb
A → B	A → b
B → S	B → a

S → A → b   means S → b
S → A → B → a   means S → a

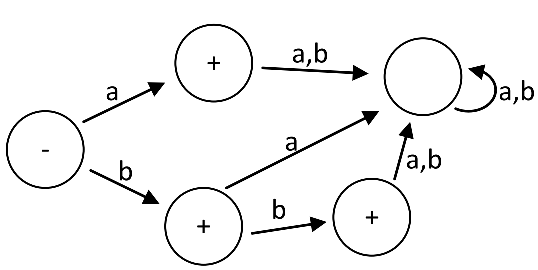
A → B → a   means A → a
A → B → S → bb   means A → bb

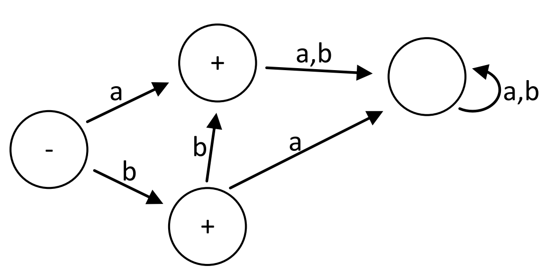
B → S → bb   means B → bb
B → S → A → b   means B → b

Final Grammar:
S → a | b | bb
A → a | b | bb   **Useless**
B → a | b | bb   **Useless**

Final: S → a | b | bb
Language: {a,b,bb}

DFA: There MUST be EXACTLY one outgoing transition of EACH input letter/alphabet from EACH state.





EXAMPLE 2:

```
I → a | b | Ia | Ib | I0 | I1
F → I | (E)
T → F | T*F
E → T | E+T
```

Rearranging Grammar:

```
E → T | E+T
T → F | T*F
F → I | (E)
I → a | b | Ia | Ib | I0 | I1
```

Unit Production:	Non-Unit Productions:
E → T	E → E+T
T → F	T → T*F
F → I	F → (E)
	I → a | b | Ia | Ib | I0 | I1

```
E → T → T*F                              means E → T*F
E → T → F → (E)                          means E → (E)
E → T → F → I → a | b | Ia | Ib | I0 | I1 means E → a | b | Ia | Ib | I0 | I1

T → F → (E)                              means T → (E)
T → F → I → a | b | Ia | Ib | I0 | I1    means T → a | b | Ia | Ib | I0 | I1

F → I → a | b | Ia | Ib | I0 | I1        means F → a | b | Ia | Ib | I0 | I1
```

Final Grammar:

```
E → E+T | T*F | (E) | a | b | Ia | Ib | I0 | I1
T → T*F | (E) | a | b | Ia | Ib | I0 | I1
F → (E) | a | b | Ia | Ib | I0 | I1
I → a | b | Ia | Ib | I0 | I1
```

Simplification:

- Eliminate useless symbols and productions.
- Eliminate λ-productions.
- Eliminate unit-productions.
- Eliminate useless symbols and productions.

Task: Complete the simplification of grammars discussed in the previous lecture.

Properties of CFL:

CFG: Context Free Grammar
CFL: Context Free Language (Generated by the CFG) – **Constructing PDA is possible**.

1. CFL is closed under Union: Union of two CFLs is also CFL.
2. CFL is closed under Concatenation: Concatenation of two CFLs is also CFL.
3. CFL is closed under Star-Closure: Star-Closure of a CFLs is also CFL.
4. CFL is NOT closed under Intersection: Intersection of two CFLs is NOT CFL.
5. CFL is NOT closed under Complementation: Complement of a CFLs is NOT CFL.

CFL is closed under Union (Order does not matter in the Newly introduced production):

Assume CFG for CFL_1:	Assume CFG for CFL_2:	Union of CFL_1 and CFL_2:	Because we have a CFG after union, it means that we have a CFL too.
$S_1 \rightarrow \ldots$ …	$S_2 \rightarrow \ldots$ …	$S \rightarrow S_1 \mid S_2$ $S_1 \rightarrow \ldots$ … $S_2 \rightarrow \ldots$ …	

Example:

GFG_1:	GFG_2:	Union:	
$X \rightarrow AB$ $A \rightarrow a \mid b$ $B \rightarrow c \mid d$	$Y \rightarrow EF$ $E \rightarrow e \mid f$ $F \rightarrow g \mid h$	$S \rightarrow X \mid Y$ $X \rightarrow AB$ $A \rightarrow a \mid b$ $B \rightarrow c \mid d$ $Y \rightarrow EF$ $E \rightarrow e \mid f$ $F \rightarrow g \mid h$	Note: Because the order does not matter, $X \mid Y$ and $Y \mid X$ generate the same language.

CFL is closed under Concatenation (Order Matters in the Newly introduced production):

Assume CFG for CFL_1:	Assume CFG for CFL_2:	Concatenation of CFL_1 and CFL_2:	Because we have a CFG after concatenation, it means that we have a CFL too.
$S_1 \rightarrow \ldots$ …	$S_2 \rightarrow \ldots$ …	$S \rightarrow S_1S_2$ $S_1 \rightarrow \ldots$ … $S_2 \rightarrow \ldots$ …	

Example:

GFG_1:	GFG_2:	Concatenation:	
$X \rightarrow AB$ $A \rightarrow a \mid b$ $B \rightarrow c \mid d$	$Y \rightarrow EF$ $E \rightarrow e \mid f$ $F \rightarrow g \mid h$	$S \rightarrow XY$ $X \rightarrow AB$ $A \rightarrow a \mid b$ $B \rightarrow c \mid d$ $Y \rightarrow EF$ $E \rightarrow e \mid f$ $F \rightarrow g \mid h$	Note: Because the order matters, XY and YX generate different languages.

CFL is closed under Star-Closure:

Assume CFG for CFL:	Star-Closure of CFL:	Because we have a CFG after Star-Closure, it means that we have a CFL too.
$S_1 \rightarrow \ldots$	$S \rightarrow S_1S \mid \lambda$	
…	$S_1 \rightarrow \ldots$	
	…	

Example:

GFG:	Closure CFG:
$X \rightarrow A$	$S \rightarrow SX \mid \lambda$
$A \rightarrow a \mid b$	$X \rightarrow A$
	$A \rightarrow a \mid b$

Tasks:

- Derive ALL words from the original grammar.
- Derive ALL words of length 3 or below from the "Closure CFG"