

Real Analysis(H2) (MA4.101a)
IIIT-H, Semester Monsoon 22, Assignment 2

Submission deadline: 21th January 2023

1. Show that $\vec{V} = (2xy + z^2)\hat{i} + (2yz + x^2)\hat{j} + (2xz + y^2)\hat{k}$ is a conservative vector field and calculate a scalar field ϕ such that $\vec{V} = \vec{\nabla}\phi$.
2. Prove the divergence theorem for a right circular cylinder. *Hint: Use cylindrical polar coordinate system. Make the axis of cylinder along the z axis.*
3. Prove the identity $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \vec{\nabla}^2 \vec{A}$.
4. Show that for a complex number z

$$\lim_{z \rightarrow \infty} \frac{2z^3 - 1}{z^2 + 1} = \infty \quad (1)$$

5. Show that for a function of complex variable $f(z) = \sqrt{r}e^{i\theta/2}$, the derivative is

$$f'(z) = \frac{1}{2f(z)}. \quad (2)$$

6. Show that for a function of complex variable $f(z) = e^{-\theta} \cos(\ln r) + ie^{-\theta} \sin(\ln r)$, where $r > 0, 0 \leq \theta \leq 2\pi$, the derivative is

$$f'(z) = i \frac{f(z)}{z}. \quad (3)$$