(CS1.301) Advanced Algorithm Design (Monsoon 2023)

Practice Problem Set 4

Instructions:

- Discussions amongst the students are not discouraged.
- Referring sources other than the lecture notes and textbooks is discouraged as the corresponding solutions available on the internet need not be accurate.
- Please attend tutorials to ensure that you understand the problem correctly.

Question 1 Edit distance between two strings is defined as the minimum number of letter insertions, substitutions and deletions to convert one word to another. For example, the edit distance between the words GOOD and FORTH is 4. Construct an algorithm to compute the edit distance between two given strings and give its time complexity.

Question 2 Suppose you're working with a company that specializes in image processing, and they've tasked you with implementing a seam carving algorithm. Seam carving is a technique used for resizing images while preserving important content. The energy of a pixel in the image is defined as the sum of the absolute differences in color intensity between that pixel and its adjacent pixels.

Given an image (energy values) represented as m x n grid of pixels, write an algorithm to find the seam curve with the minimum energy. The seam curve is defined as a connected path of pixels from the leftmost column to the rightmost column, such that the sum of energies is minimized. Note that a valid path is considered only for movement from (x,y) to (x+1,y-1), (x+1,y), (x+1, y+1).

Consider an example of an image grid of 3x3 pixels with energy values:

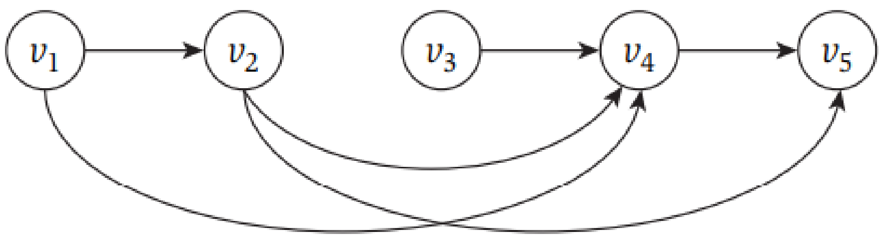
2	8	4
5	1	3
9	3	2

Provide an algorithm to solve the same and give its time complexity.

Question 3 Let G = (V, E) be a directed graph with nodes $v1, ..., vn$. We say that G is an ordered graph if it has the following properties.

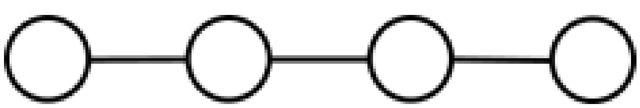
(i) Each edge goes from a node with a lower index to a node with a higher index. That is, every directed edge has the form (v_i, v_j) with $i < j$.

(ii) Each node except v_n has at least one edge leaving it. That is, for every node $V_i (i = 1, 2, ..., n - 1)$, there is at least one edge of the form (v_i, v_j). The length of a path is the number of edges in it. The goal in this question is to solve the following problem (see figure for an example).



Give an efficient algorithm that takes an ordered graph G and returns the length of the longest path that begins at v_1 and ends at v_n. (Again, the length of a path is the number of edges in the path.)

Question 4 Let G = (V, E) be an undirected path graph with n nodes. A graph is a path if the graph is connected, and the nodes can be put in a row so that all edges are between adjacent nodes. For example, here's a path graph on 4 nodes:



A subset of nodes is called an independent set if no two nodes in the subset are connected by an edge. For example, in the path graph above the first and third nodes form an independent set, but the first and second nodes do not.

We will also associate with each node a positive integer weight (note that these weights are attached to nodes, not to edges). For example, here's a path graph with node weights:

In the path weighted independent set problem, we have:
Input: an undirected, unweighted path graph G = (V, E) with |V | = n nodes, and integer weight $w_i > 0$ on each node i
Output: a subset of nodes S such that no two nodes in S have an edge between them, and the total weight of the nodes in S, Weight is maximized

Give an algorithm that takes an n-node path G with weights and returns an independent set of maximum total weight. The running time should be polynomial in n, independent of the values of the weights.

Question 5 Given two integer arrays array1 and array2, write the integers of array1 and array2 (in the order they are given) on two separate horizontal lines. A straight line connecting two numbers array1[i] and array2[j] is called a connecting line if:

A)array1[i] == array2[j], and B)the line drawn does not intersect any other connecting (non-horizontal) line.

Note that a connecting line cannot intersect even at the endpoints (i.e., each number can only belong to one connecting line).

Give an algorithm that takes two integer arrays, array1 and array2 and returns the maximum number of connecting lines that can be drawn between the two arrays.

Question 6 The greatest common divisor of two positive integer x and y, denoted gcd(x, y), is the largest integer d such that both x/d and y/d are integers. Euclid describes the following recursive approach.

(i) If x= y then return x.
(ii) Else if x > y, compute gcd($x - y$, y).
(iii) Else, compute gcd(x, $y - x$).

Prove that the above procedure correctly computes gcd(x, y). Prove that gcd(x, y) divides both x and y. Prove that every divisor of x and y is a divisor of gcd(x, y).

Question 7 A string w of parentheses (and) and brackets [and] is balanced if it satisfies one of the following conditions:
(i) w is the empty string. (ii) w = (x) for some balanced string x. (iii) w = [x] for some balanced string x (iv) w = xy for some balanced strings x and y.

Describe and analyze an algorithm to determine whether a given string of parentheses and brackets is balanced.

Question 8 There is an m x n grid. A robot is initially located at the top-left corner (i.e., grid[0][0]). The robot tries to move to the bottom-right corner (i.e., grid[m - 1][n - 1]). An obstacle in the path of the robot is marked by a '1', I.e, if grid[i][j] is 1, the robot cannot move through that space. The robot can move through grid cells marked with a 0. The robot can only move either down or right at any point in time.

Given two integers m and n, write an algorithm that returns the number of possible unique paths that the robot can take to reach the bottom-right corner.

Question 9 Given an array of non-negative integers representing values at each point, give an algorithm which determines the maximum sum that can be obtained without choosing consecutive elements.