

Real Analysis(H2) (MA4.101a)
IIIT-H, Semester Monsoon 22, Assignment 1

Submission deadline: 6th January 2023

1. The cross product of three vectors $\vec{X}_1, \vec{X}_2, \vec{X}_3$ is in general not associative, *i.e.*,

$$\vec{X}_1 \times (\vec{X}_2 \times \vec{X}_3) \neq (\vec{X}_1 \times \vec{X}_2) \times \vec{X}_3. \quad (1)$$

Prove this with an example.

2. Find the equation of a plane defined three points on a Cartesian coordinate, $(1, 2, 1), (-1, -1, -1), (1, -1, 0)$.
Find the equation of the line that is formed at the intersection of this plane and the plane defined by $z = 0$.
3. The components of a vector $\vec{X} = \hat{i}x_1 + \hat{j}x_2 + \hat{k}x_3$, transform under the rotation of the coordinate system as

$$\begin{pmatrix} x'_1 \\ x'_2 \\ x'_3 \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}. \quad (2)$$

What conditions must the “direction cosines” R_{ij} , where $i, j = 1, 2, 3$ satisfy ?

4. Find the work done by a force field $\vec{F} = \hat{i}x \sin y + \hat{j}y$ for moving an object along the curve $\vec{r} = (1+t)\hat{i} + t^3\hat{j}$ for $t = 1$ to $t = 2$.
5. Compute the integration

$$\int xy^2zds \quad (3)$$

- (a) over a line between two points $(0, 2, 1)$ and $(3, 1, 5)$.
- (b) over a line given by $\vec{r} = \hat{i}u + \hat{j}u^2 + \hat{k}u$, for $u = 1$ to $u = 2$.