Identification of critical transition to extreme events using visibility graph theory

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(Dated: today)

I. INTRODUCTION

Problem statement: Which indicator can be used to identify the critical transitions from periodic or chaotic behavior to sudden large-scale expansions?
Solution: In this study, we employed the visibility graph theory to detect transitions based on network characteristics such as average degree $\langle k \rangle$, average path length $\langle l \rangle$, clustering coefficient (C) and maximum degree ($d_{\max}$).

Depending on the driving parameter, dynamical systems can exhibit a variety of complex behaviors, including periodic, quasi-periodic, chaotic, and hyperchaotic dynamics. By changing the parameter, systems can evolve from order states to disorder states. Each of these dynamics plays a crucial role in mimicking complex phenomena observed in nature, ranging from ecological systems [1] to climate science [2, 3]. In natural systems, one particularly intriguing area of research focuses on the transition from periodic, quasi-periodic, or chaotic states to extreme events via various routes. Numerous studies have documented these critical transitions to these nonrecurrent, large-amplitude events due to their significant implications in natural and physical systems. Identification of extreme events has been studied in various systems, including neuronal systems [4, 5], hydrometrological systems [6], power grids [7], random walks [8–10], and complex networks [11–17]. Since predicting such extreme events is crucial to preventing potentially catastrophic outcomes, researchers have employed a wide range of methods and tools to recognize these transitions. Although, due to the intrinsic nature of nonrecurrent and large magnitude from the natural state, it is very difficult to predict these events using normal statistical approaches. Early warning indicators have been used to identify different transitions, yet not many studies have been used to detect these large-amplitude transitions. Recent research indicates that combining two types of entropy—permutation entropy and distance entropy—into a Shannon entropy framework can effectively identify transitions from chaotic behavior to large expansions. In this article, we also aim at these critical transitions and propose an innovative and efficient approach to detect them using graph visibility theory.

A recently developed novel approach called graph visibility theory is now used by many researchers to understand the behavior of time series.

II. METHOD

Graph visibility theory maps a time series into a graph by defining visibility criteria between time points. Given a time series $\{(t_i, y_i)\}_{i=1}^{n}$, each data point becomes a node. Two nodes (t_i, y_i) and (t_j, y_j) have an edge if for all t_k such that $t_i < t_k < t_j$, the following condition holds:

$$y_k < y_i + \frac{y_j - y_i}{t_j - t_i}(t_k - t_i)$$

This ensures that the straight line connecting (t_i, y_i) and (t_j, y_j) is above all intermediate points (t_k, y_k).

III. FHN MODEL

$$\begin{aligned}
\frac{du}{dt} &= u - \gamma \sinh(\rho u) - v + A\sin(2\pi\omega t), \\
\frac{dv}{dt} &= \delta(u + a - bv),
\end{aligned}$$

$(u_0, v_0) = (-0.2, -0.002)$, $a = 0.7$, $b = 0.8$, $\delta = 0.1$, $\gamma = 2.682 \times 10^{-4}$, $\omega = 0.17$

IV. LINARD NETWORK MODEL

$$\begin{aligned}
\frac{dx}{dt} &= y, \\
\frac{dy}{dt} &= -\alpha xy - \beta x^3 - \gamma x + f\sin(\Omega t),
\end{aligned}$$

where α, β, γ, f= 0.45, 0.5, -0.5, 0.2. $\Omega = 0.7314$ and 0.6423.

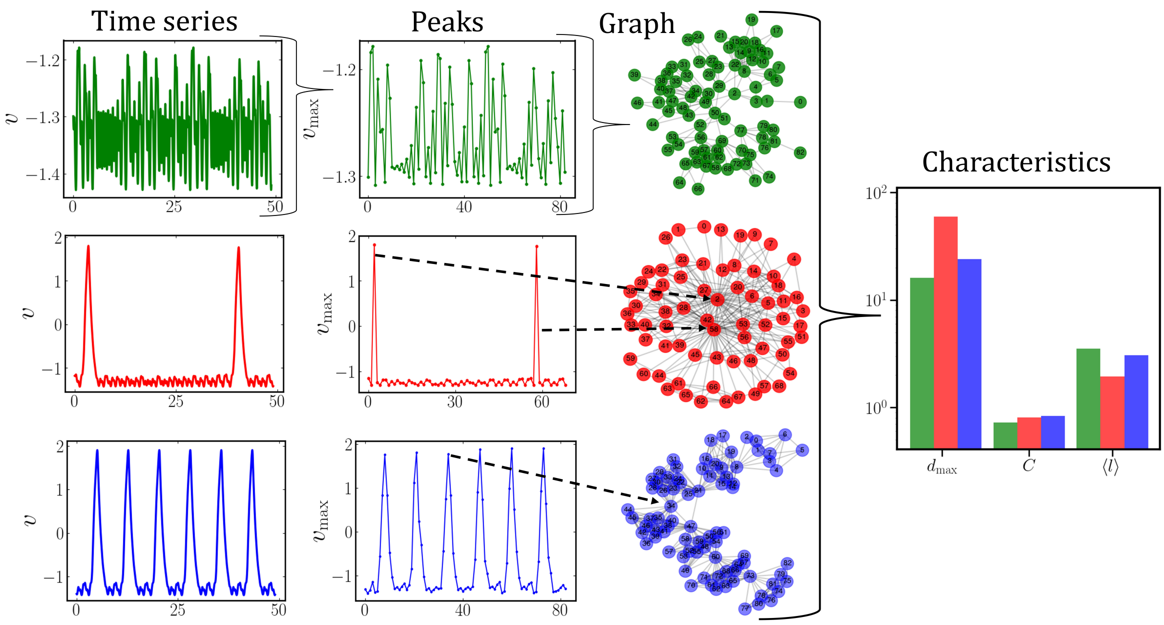

FIG. 1: FHN model.

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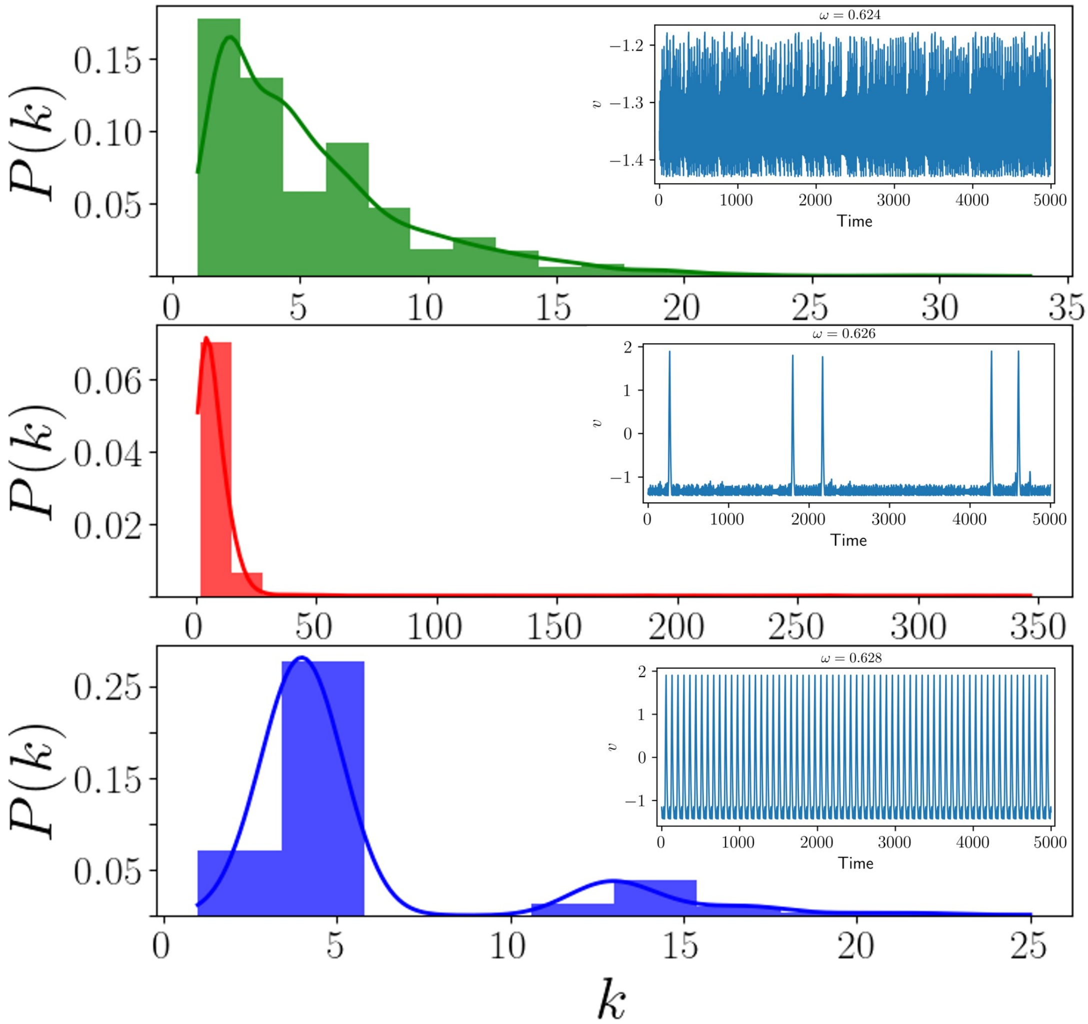

FIG. 2: FHN model.

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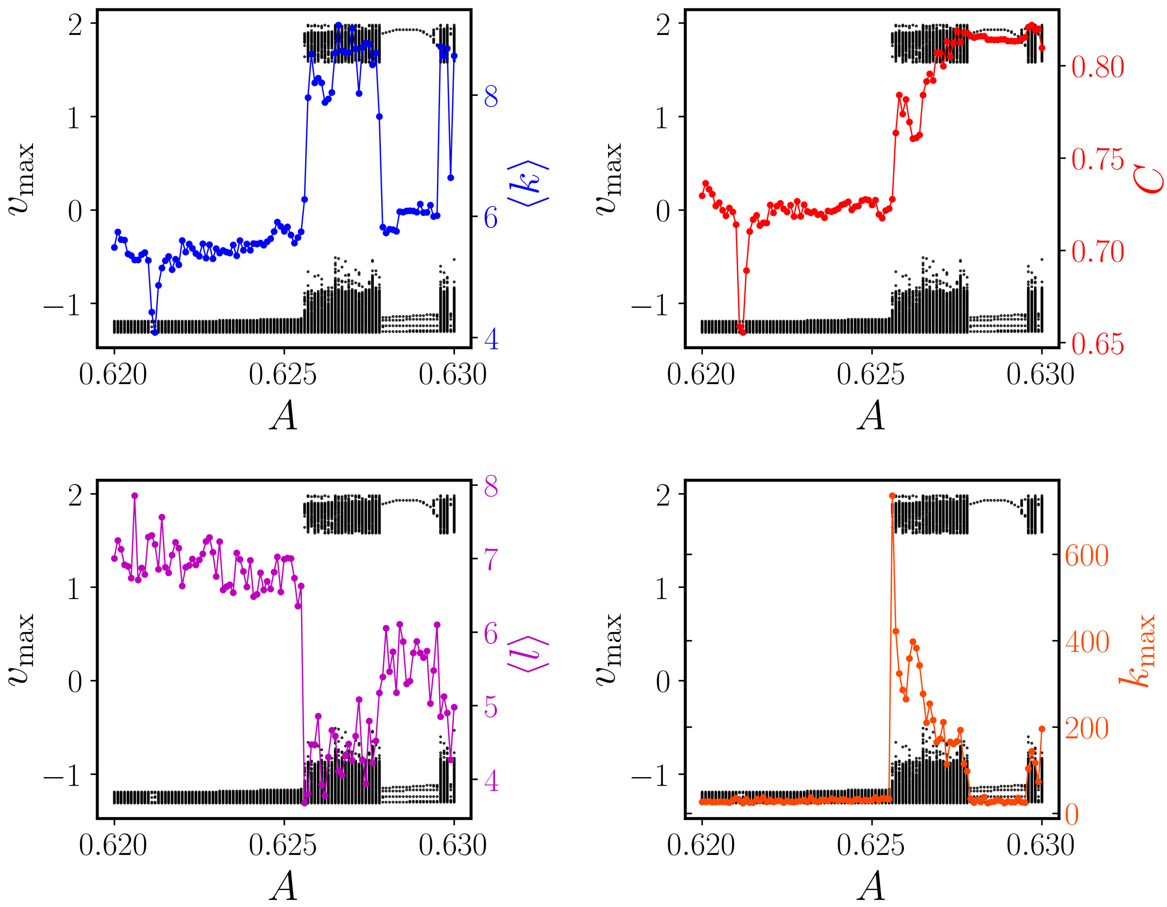

FIG. 3: FHN model.

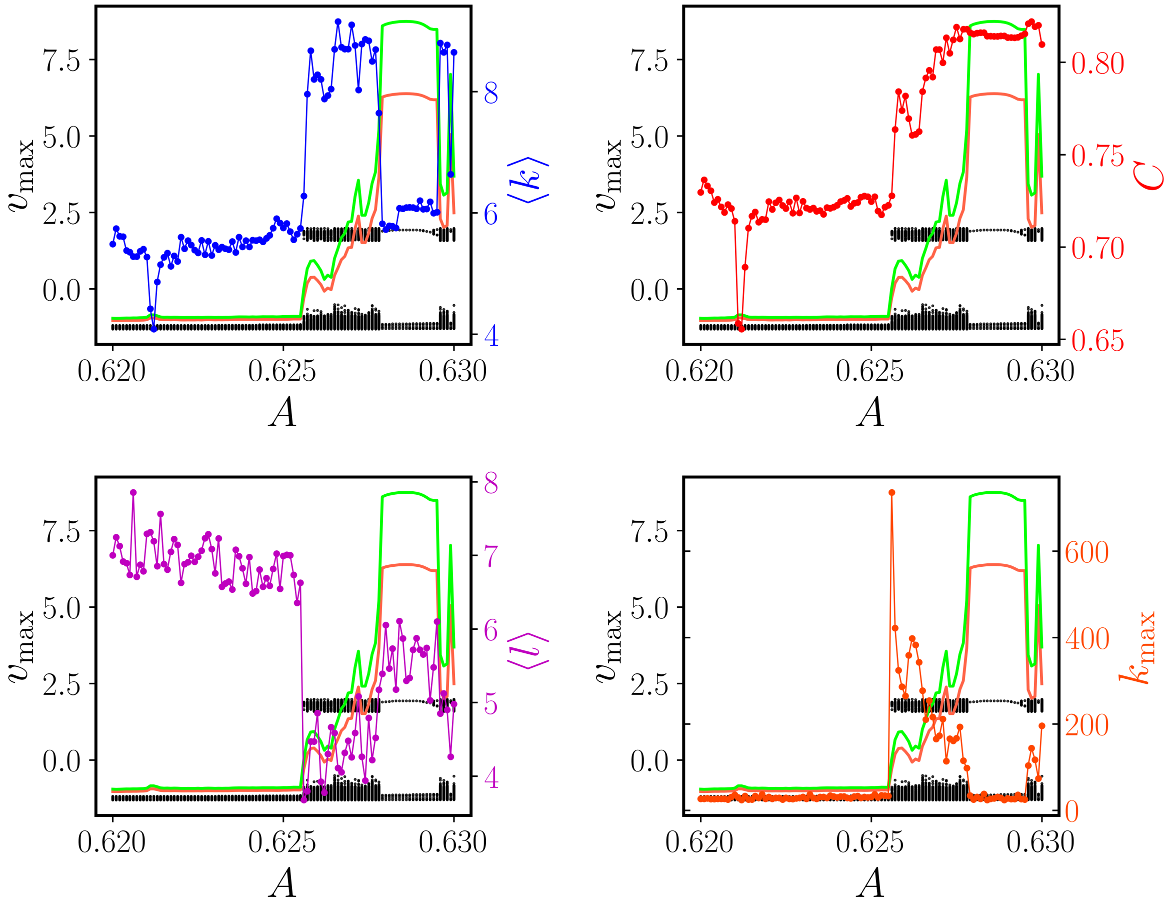

FIG. 4: FHN model.

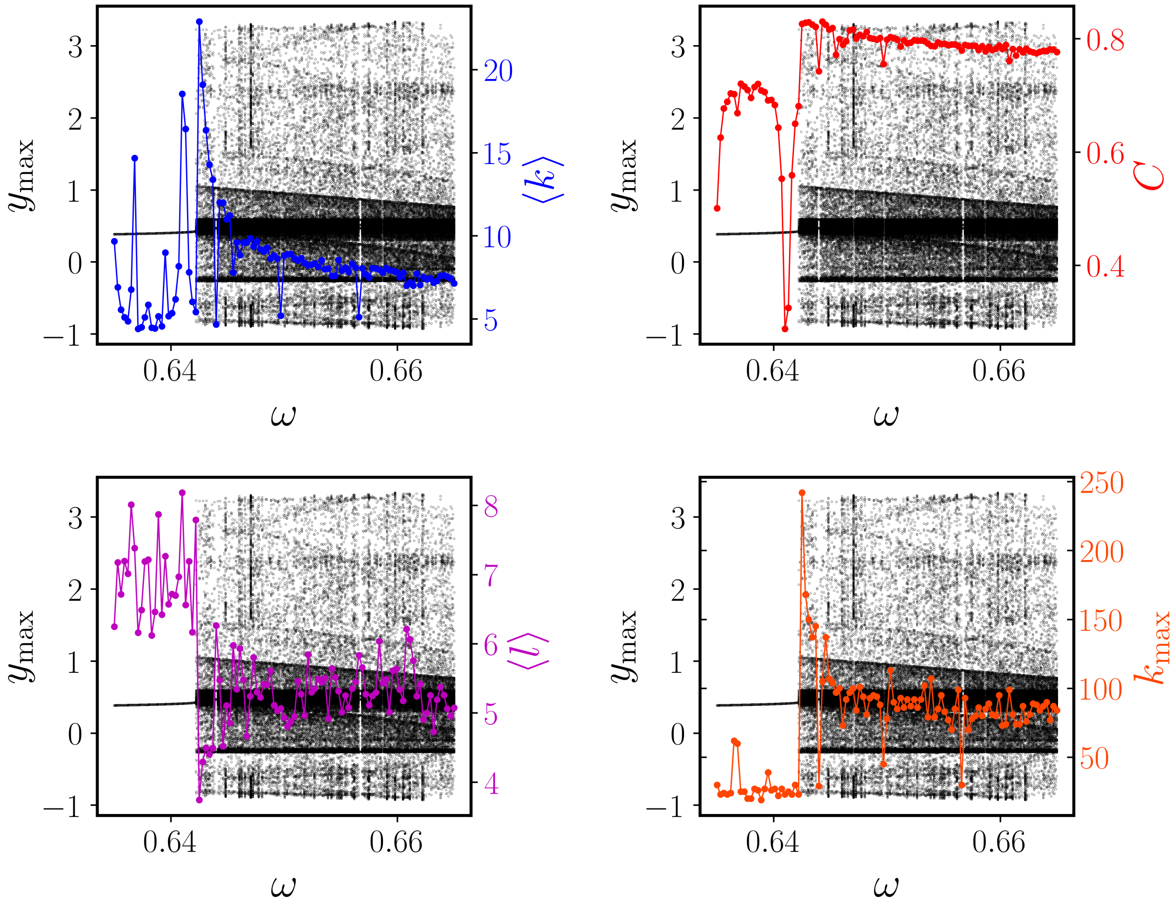

FIG. 5: Lineard model.