Homework2

Discrete Math

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13-10-2020

1 Task 1

a)
Translate the negation of the following formula into CNF:

$$(p \to (q \to r)) \to ((p \to \neg r) \to (p \to \neg q)) =$$
$$= (\neg p \vee \neg q \vee r) \to ((\neg p \vee \neg r) \to (\neg p \vee \neg q)) =$$
$$= (p \wedge q \wedge \neg r) \vee (p \wedge r) \vee (\neg p \vee \neg q) =$$

Now, let's take the negation of the formulae above:

$$(\neg p \vee \neg q \vee r) \wedge (\neg p \vee \neg r) \wedge p \wedge q$$

b) Using resolution find whether it is satisfiable or not:

$$(\neg p \vee \neg q \vee r) \wedge (\neg p \vee \neg r) \wedge p \wedge q$$

resolution law:

$$\frac{A \vee \neg p \quad\quad p \vee B}{A \vee B}$$

In our case, we have $(\neg p \vee \neg r)$ and p, which gives $\neg r$. Then we have $(\neg p \vee \neg q \vee r)$ and q which gives us $\neg p \vee r$. As a result, we can get r from $\neg p \vee r$ and p

$$(\neg p \vee \neg q \vee r) \wedge (\neg p \vee \neg r) \wedge p \wedge q \wedge \neg r \wedge (\neg p \vee r) \wedge r$$

There are r and not r. Hence, it is not satisfiable.

2 Task 2

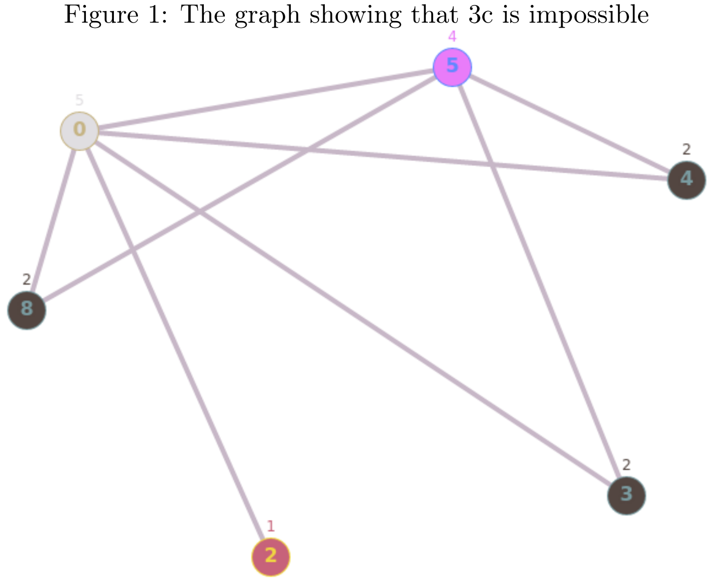
No, it is not possible. We know how to construct CNF from the truth table, and having $\neg p \vee \neg q \vee \neg r$ in CNF means that we somehow got 0 (False) when p, q, r = 1(True). The question becomes can we get 0(False) with p,q,r = 1 and using only $\vee, \to, \wedge$. The answer is No.Because, conjunction and disjunction of variables with values 1(True) always give you 1(True). Meanwhile, implication gives you false, when antecedent is 1(True) and consequent 0(False). However, since we can't get False value, our implication also always give us 1(True). Thus, we can't get 0 with p,q,r all being True and using only $\vee, \to, \wedge$, so CNF for A can't have $\neg p \vee \neg q \vee \neg r$

3 Task 3

Could there exist a graph with the following degrees of vertices:
Note that: The sum of all the degrees is equal to twice the number of edges.

a)4 3 3 1 - No, impossible, since the sum of degrees has to be even.
b) 3, 3, 2, 2 - possible, see figure 2(page 3)
c)5, 4, 4, 2, 2, 1 - impossible. We have 6 vertices and 9 edges. Since one of the vertices has degree of 5, all other vertices have to be connected to one vertex. So, now we already have 5 edges in our graph(9-5=4 edges left). Let's continue and try to get the vertex with degree of 4, now, we have to connect to it 3 vertices(not 4, because it is already connected to vertex with the degree of 5). Hence, now we have 5 + 3 = 8 edges, it means we have only 1 edge to use. For now the degree of vertices are following: 5,4,2,2,2,1. It is impossible to get 5,4,4,2,2,1 with 1 edge(it is impossible at all).

Figure 1: The graph showing that 3c is impossible



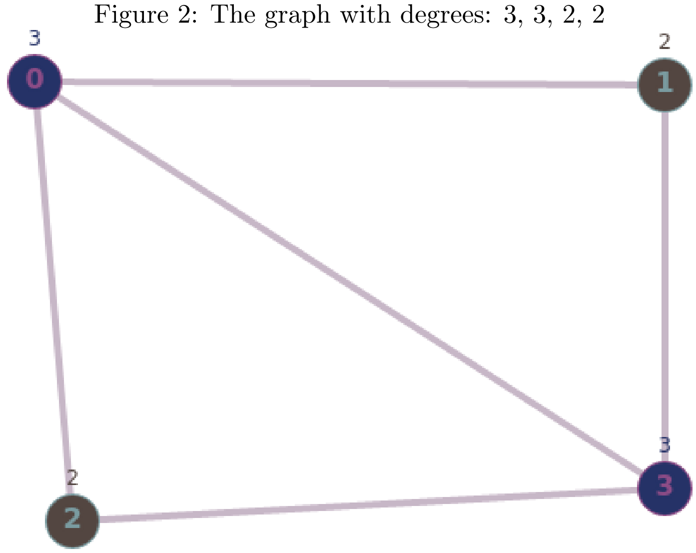
4 task 4

Construct a graph with 10 vertices such that every vertex has degree 3 and any two vertices are connected by a path of not more than 2 edges.
So, we have 15 edges and 10 vertices. see figure 3(pre-last page - page 4)

5 task 5

given:
A graph has two vertices of degree 1 and several vertices of degree 10. Prove that the vertices of degree1 are connected by a path in this graph. (Hint: suppose the contrary.)

Figure 2: The graph with degrees: 3, 3, 2, 2



Solution: Let's assume that we have overall n vertices. Let's calculate overall sum of degrees:

$$sumofdegree = 1 * 1 + 1 * 1 + 10 * (n - 2) = 10n - 18 = 2 * (5n - 9)$$

Now, let's assume that the vertices with degree 1 doesn't have path to each other. It means that we have 2 connected components in the graph and let's name them CC1 and CC2. Let's assume that there are m vertices in CC1, so n - m vertices in CC2. Now, we can calculate the sum of degrees in CC1 and in CC2:

$$CC1 : degree = 1 * 1 + (m - 1) * 10 = 10m - 9$$

$$CC2 : degree = 1 * 1 + (n - m - 1) * 10 = 10n - 10m - 9 = 10(n - m) - 9$$

Both of which $10m - 9$ and $10(n - m) - 9$ are always odd, hence such graph can't exist. Contradiction.Thus, there is a path between 2 vertices of degree 1.

6 Task 6

Brief Idea: if you see at ith position 1, replace it with x and check ith position from the back and if it is 1, also replace it with x. The same goes to 0 element, but replace 0 with y. See figure 4(last page - page 5) for more detailed answer. q0 is initial position. Note that in figure 4: $1 \to x, R$ means if you see 1, replace(write) x and move right

Figure 3: task 4

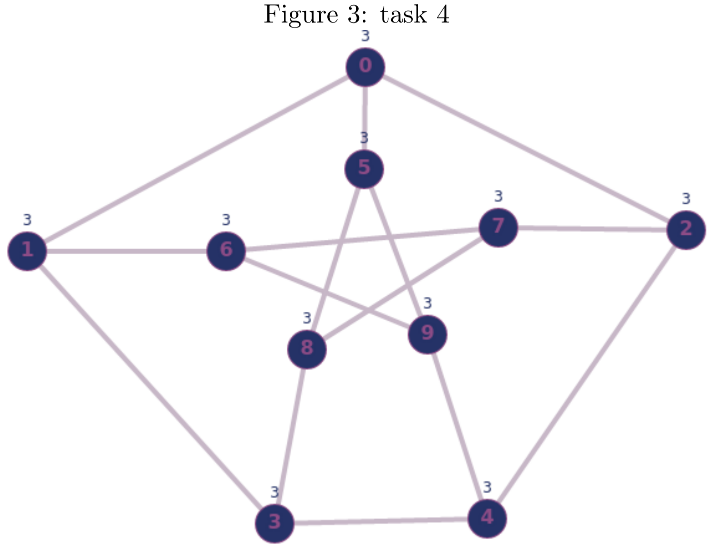


Figure 4: Turing machine