AP# 5- Dinâmica de sistema de partículas.

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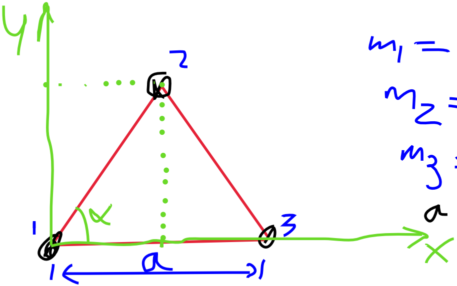

$m_1 = 1\,kg$

$m_2 = 2\,kg$

$m_3 = 3\,k$

$a = 1\,m$

$X_{cm} - ?$

Y_{CM}

$$X_{CM} = \frac{\sum m_i x_i}{\sum m_i} \Rightarrow X_{cm} = \frac{1 * 0 + 2 * \frac{1}{2}a + 3 * a}{1+2+3} = \frac{4}{6} = \frac{2}{3}\,m$$

$$Y_{CM} = \frac{\sum m_i y_i}{M} = \frac{\cancel{1 * 0} + \cancel{2} * a \sin 60^\circ + \cancel{3 * 0}}{\cancel{6}\,3} = \frac{\sqrt{3}}{6}\,m$$

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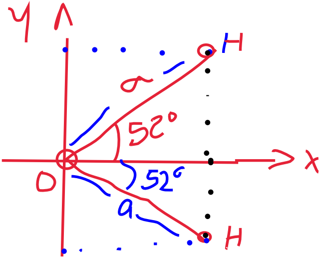


$m_O = 16 m_H$ | $\boxed{m = m_H}$

$$x_{cm} = \frac{16m * 0 + 2m * a\cos 52^\circ}{2m + 16m} = \frac{a}{9}\cos 52^\circ$$

$$y_{cm} = \frac{16m * 0 + m * a\sin 52^\circ - m * a\sin 52^\circ}{18m} = 0$$

(3)

$$v_{CM} = \frac{\sum m_i v_i}{M} \qquad \underline{v_i = v_i' + v_{CM}}$$

$$\hookrightarrow v_i' = v_i - v_{CM}$$

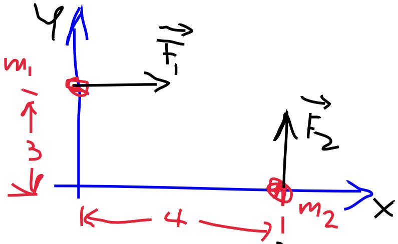
$$v_{CM} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2};$$

$$v_1' = v_1 - \frac{m_1 v_1 + m_2 v_2}{M} \Rightarrow v_1' = \frac{m_2 (v_1 - v_2)}{M}$$

$$v_1 - v_2 = v_{12} \quad \hookrightarrow v_1' = \frac{m_2 v_{12}}{M}$$

$$v_2' = -\frac{m_1 v_{12}}{M} \rightsquigarrow v_2 - v_1 = -v_{12}$$

(4)



$$\vec{a}_1 = \frac{\vec{F}_1}{m_1} \Rightarrow \vec{a}_1 = \frac{8}{10}\vec{i}$$

$$\boxed{\vec{a}_1 = \frac{4}{5}\vec{i}\ (m/s)}$$

$$\vec{a}_1 = \frac{d\vec{v}}{dt} \Rightarrow$$

$$d\vec{v}_1 = \vec{a}_1\, dt \Rightarrow \int_0^{\vec{v}_1} d\vec{v}_1 = \frac{4}{5}\int_0^t dt \Rightarrow \boxed{\vec{v}_1 = \frac{4}{5}t\,\vec{i}}$$

$$\vec{v}_1 = \frac{d\vec{r}_1}{dt} \Rightarrow d\vec{r}_1 = \vec{v}_1\, dt \Rightarrow \int_{3\vec{j}}^{\vec{v}_1} d\vec{r}_1 = \left(\frac{4}{5}\int_0^t t\, dt\right)\vec{i}$$

$$\vec{r}_1 - 3\vec{j} = \frac{2}{5}t^2\,\vec{i} \Rightarrow \boxed{\vec{r}_1 = \frac{2}{5}t^2\vec{i} + 3\vec{j}}\ *$$

$$4t^2\vec{i} + 30\vec{j}$$

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$4\vec{i}$$

④ $\vec{a}_2 = \frac{\vec{F}_2}{m_2} \Rightarrow \boxed{\vec{a}_2 = \frac{6}{6}\vec{j} = \vec{j}\ (m/s^2)}$

$$d\vec{v}_2 = \vec{a}_2\,dt \Rightarrow \int_0^{\vec{v}_2} d\vec{v}_2 = \vec{a}_2\int_0^t dt \Rightarrow \boxed{\vec{v}_2 = t\vec{j}}$$

$$d\vec{r}_2 = \vec{v}_2\,dt \Rightarrow \int_{4\vec{i}}^{\vec{r}_2} d\vec{r}_2 = \left(\int_0^t t\,dt\right)\vec{j}$$

$$\Rightarrow \vec{r}_2 - 4\vec{i} = \frac{1}{2}t^2\vec{j} \Longrightarrow \boxed{\vec{r}_2 = 4\vec{i} + \frac{1}{2}t^2\vec{j}}$$

$$\vec{r}_{cm}(t) = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

$$\vec{r}_{cm}(t) = \left(\frac{6+t^2}{4}\right)\vec{i} + \left(\frac{30+3t^2}{16}\right)\vec{j}$$

$$\vec{v}_{cm} = \frac{\sum m_i\vec{v}_i}{M}$$

$$\boxed{\vec{r}_{CM}(t) = \vec{r}_{CM}\Big|_{t=0} + \vec{v}_{CM}\Big|_{t=0} \cdot t + \frac{1}{2}\vec{a}_{CM} \cdot t^2}$$

$$\vec{r}_{CM} = \frac{3}{2}\vec{i} + \frac{15}{8}\vec{j} + \frac{1}{4}t^2\vec{i} + \frac{3}{16}t^2\vec{j}$$

$$= \left(\frac{3}{2} + \frac{1}{4}t^2\right)\vec{i} + \left(\frac{15}{8} + \frac{3}{16}t^2\right)\vec{j}$$

$$\vec{F}_R = 8\vec{i} + 6\vec{j} \quad \Big| \quad M = 16$$

$$\vec{a}_{CM} = \frac{1}{2}\vec{i} + \frac{3}{8}\vec{j}$$

$$\vec{F}_R = M\vec{a}_{CM}$$

$$\vec{v}_{CM_0} = \frac{m_1\vec{v}_{1_0} + m_2\vec{v}_{2_0}}{m_1+m_2}$$

$$\frac{30\vec{j} + 24\vec{i}}{16}$$

$$\vec{r}_{CM_0} = \frac{3}{2}\vec{i} + \frac{15}{8}\vec{j}$$

$$\vec{P}_t = M\vec{v}_{CM}$$