AP# 4- Dinâmica de uma partícula. Trabalho e Energia

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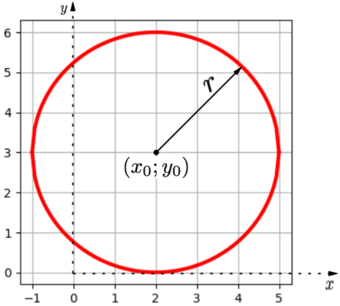


Eq. da circunferência:

$$\boxed{\frac{(x-x_0)^2}{r^2}+\frac{(y-y_0)^2}{r^2}=1}$$

$$\boxed{\frac{(x-2)^2}{3^2}+\frac{(y-3)^2}{3^2}=1}$$

$$x(t)=2+3cost \quad \wedge \quad y(t)=3+3sint$$

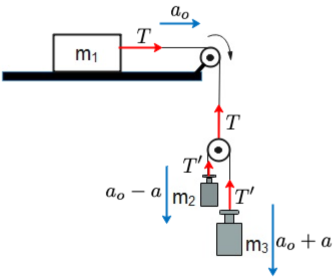
$$\vec{r}=(2+3cost)\vec{i}+(3+3sint)\vec{j}$$

$$\vec{v}=\frac{d\vec{r}}{dt} \rightsquigarrow \vec{v}=-3sint\vec{i}+3cost\vec{j}$$

$$\vec{a}=\frac{d\vec{v}}{dt} \rightsquigarrow -3cost\vec{i}-3sint\vec{j}$$

$\vec{F}=m\vec{a}$ Lei f. da dinâmica

Prob.7



$$2T' = T \tag{1a}$$

$$\begin{aligned} T &= m_1 a_o \\ -T' + m_1 g &= m_2(a_o - a) \\ -T' + m_3 g &= m_3(a_o + a) \end{aligned} \tag{1b}$$

Usando Eq.1a e Eqs.1b temos:

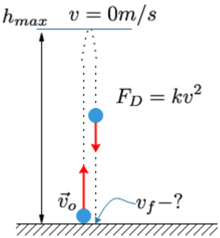
$$\begin{aligned} a_o - a &= g - \frac{m_1 a_o}{2m_2} \\ a_o + a &= g - \frac{m_1 a_o}{2m_3} \end{aligned} \tag{1c}$$

Adicionando-se as duas Eqs.1c obtém-se a_o que é aceleração da massa 1:

$$\boxed{a_o = \left(\frac{4m_2m_3}{4m_2m_3 + m_1m_3 + m_1m_2} \right) g}$$

Prob.10

Na subida:



$$
\begin{aligned}
-Fg - F_D &= ma \\
-mg\left(1 + \frac{k}{mg}v^2\right) &= m\frac{dv}{dt} \\
-g\left(1 + \frac{v^2}{v_t^2}\right) &= \frac{dv}{dy}\frac{dy}{dt} \\
-g\left(1 + \frac{v^2}{v_t^2}\right) &= \frac{vdv}{dy}
\end{aligned}
\tag{3a}
$$

velocidade terminal:

$$
\begin{aligned}
F_D &= Fg \\
\Rightarrow v_t &= \sqrt{\frac{mg}{k}}
\end{aligned}
\tag{2}
$$

Separar variáveis e integrar fica:

$$\int_{v_0}^{0} \frac{vdv}{v_t^2 + v^2} = -\frac{g}{v_t^2}\int_{0}^{h_{max}} dy$$

Prob.10 - Cont.

$$\frac{1}{2}\int_{v_0}^{0}\frac{d(v_t^2+v^2)}{v_t^2+v^2} = -\frac{g}{v_t^2}\int_{0}^{h_{max}} dy \Rightarrow h_{max} = -\frac{v_t^2}{2g} ln\left(\frac{v_t^2}{v_t^2+v_0^2}\right) \qquad (4)$$

Na descida: ______________________________

$$Fg - F_D = ma$$
$$mg\left(1-\frac{k}{mg}v^2\right) = m\frac{dv}{dt}$$
$$g\left(1-\frac{v^2}{v_t^2}\right) = \frac{dv}{dy}\frac{dy}{dt}$$
$$g\left(1-\frac{v^2}{v_t^2}\right) = \frac{vdv}{dy}$$

Separar variáveis e integrar fica:

$$-\frac{1}{2}\int_{0}^{v_f}\frac{d(v_t^2-v^2)}{v_t^2-v^2} = \frac{g}{v_t^2}\int_{0}^{h_{max}} dy$$
$$\rightsquigarrow h_{max} = -\frac{v_t^2}{2g} ln\left(\frac{v_t^2-v_f^2}{v_t^2}\right) \qquad (6)$$

Prob.10 - Cont.

Igualando os resultados Eq.4 e Eq.6, temos:

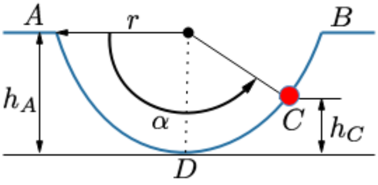
$$\frac{v_t^2 - v_f^2}{v_t^2} = \frac{v_t^2}{v_t^2 + v_0^2} \Rightarrow v_f^2 = v_t^2 - \frac{v_t^4}{v_t^2 + v_0^2}$$

$$\mathsf{v}_f = v_t\sqrt{1 - \frac{v_t^2}{v_t^2 + v_0^2}} \Rightarrow \boxed{v_f = \frac{v_0 v_t}{\sqrt{v_0^2 + v_t^2}}}$$

Prob.11

Para resolver este exercício poderá usar a lei de conservação de energia

$$\boxed{EM_A = EM_C}$$



a)

$$mgh_A = mgh_C + \frac{1}{2}mv_C^2$$

$$h_C = h_A - rsin\alpha$$

$$v_c = \sqrt{2rgsin\alpha}$$

b)

$$v_C = rw$$

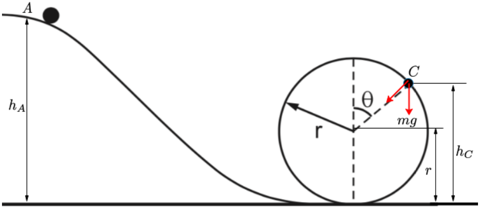
$$w = \sqrt{\frac{2gsin\alpha}{r}}$$

c)

$$EM_C = mgh_A$$

Prob.12

No ponto em que a massa perde contacto com a superfície, a reacção normal ao apoio se anula e a componente normal da força exercida pela massa é uma resultante centrípeta



$$mgcos\theta = m\frac{v^2}{r}$$

$$v^2 = grcos\theta$$

$$mgh_A = mgr(1 + cos\theta) + \frac{1}{2}mv^2$$

$$cos\theta = \left(\frac{2h_A}{r} - 1\right)\frac{1}{3}$$

$$\theta = arcos\left[\left(\frac{2h_A}{r} - 1\right)\frac{1}{3}\right]$$

Se $h_A > \frac{5r}{2}$ a massa não perderá contacto com a superficie !

Prob.13

$$m = 2kg$$
$$\vec{r} = 5t\vec{i} + \frac{10}{3}t^3\vec{j}$$
$$\vec{v} = 5\vec{i} + 10t^2\vec{j}$$
$$\tau\big|_{t=1.0s} - ?$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$
$$\vec{F} = m\frac{d\vec{v}}{dt}$$
$$\vec{F} = 40t\vec{j}$$
$$\vec{\tau} = 200t^2\vec{k}$$
$$\tau\big|_{t=1.0s} = 200mN$$

Prob.14

É análogo ao Prob.13. É aplicação das fórmulas.

Parte - II: Trabalho e Energia

4

$$dW = \vec{F}\, d\vec{r} \Rightarrow W = \int_A^B \vec{F}\, d\vec{r} = \vec{F} \cdot \vec{r}\Big|_A^B$$

$$W = \vec{F} \cdot \vec{r} = F \cdot r \cdot \cos\theta$$

$$= F_x r_x + F_y r_y + F_z r_z$$

$$= -100 - 105$$

$$= -205\,J$$

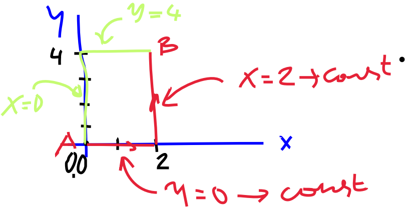
$$W = \Delta E_c \Rightarrow \Delta E_c = -205\,J$$

$$\Delta E_c = \frac{1}{2} m \left(v_B^2 - v_A^2\right)$$

$\vec{F_1}\,\vec{r}$ — ? $\hookrightarrow$ $\alpha = \arccos\left(\frac{F_{1x} r_x + F_{1y} r_y + F_{1z} r_z}{F_1 \cdot r}\right) \simeq 122^\circ$

$A \longrightarrow B$

(0;0) (2.4)



a) $W = \int_A^B \vec{F}\, d\vec{r}$

$$W = \int_{x_i;\, y=const}^{x_f} F_x\, dx + \int_{y_i,\, x=const}^{y_f} F_y\, dy$$

$$W = \int_{0,\, y=0}^{2} (y^2 - x^2)\, dx + \int_{0,\, x=2}^{4} 3xy\, dy$$

$$W = \int_0^2 -x^2\, dx + \int_0^4 6y\, dy$$

$$-\frac{8}{3} + 48 = 45.3\,J$$

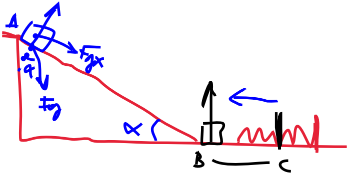
b) $W = \int_{0,\, x=0}^{4} 3xy\, dy + \int_{0;\, y=4}^{2} (y^2 - x^2)\, dx$

$$W = \int_0^2 (16 - x^2)\, dx = \left(16x - \frac{1}{3}x^3\right)\Big|_0^2$$

$$W = 32 - \frac{8}{3} = 29.3\,J$$

$\vec{F}$ — ? $\begin{cases} \text{cons.} \\ \bar{\text{n}}\ \text{cons.} \end{cases}$ ✓

$E_{pe} = \frac{1}{2}k(\Delta x)^2$



$W_{F_f} = -F_a\overline{BC} = -\mu mg\overline{BC} = -0.25 \times 3 \times 10 \times 4 = -30\,J$

$E_{pe} = E_{cB} - |W_{F_a}| \Rightarrow E_{pe} = mgh - 30 = 30\,J$

$\Delta x = \sqrt{\frac{2E_{pe}}{k}} \Rightarrow \Delta x = 6.32 \times 10^{-5}\,m$

e) $E_{pe} = |W_{F_a}| \Rightarrow E_{pe} = F_a \gamma \Rightarrow E_{pe} = \mu mg\gamma$

$\gamma = \frac{30}{0{,}25 \times 3 \times 10} = 4$

nabla

$$\nabla \equiv \underbrace{\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}}\quad \rightarrow \text{rect}$$

$$\nabla \equiv \frac{\partial}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial}{\partial \varphi}\vec{e}_\varphi + \frac{\partial}{\partial z}\vec{k} \quad \rightarrow \text{Cilin}$$

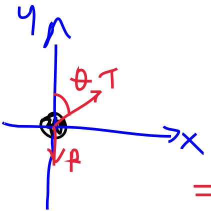
$$\nabla \equiv \frac{\partial}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial}{\partial \theta}\vec{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}\vec{e}_\varphi \rightarrow \text{Esf.}$$

$$\vec{F} = -\underbrace{\nabla U}_{\text{grad}} \quad \rightarrow \quad \underline{\text{rot}\,\vec{F} = 0} \quad \rightarrow \quad \underline{\nabla \times \vec{F} = 0}$$

$$\vec{F} = -\text{grad}\,U$$

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} = 0 \rightarrow \int_u^0 dU = -\int_r^{\infty} \vec{F}\cdot d\vec{r}$$

a)



$T\cos\theta = mg \rightarrow T = \dfrac{mg}{\cos\theta}$ (1)

$T\sin\theta = m\dfrac{v^2}{r}$ (2)

$\hookrightarrow mg\,tg\theta = m\dfrac{v^2}{r}$ (3)

$\Rightarrow g\,tg\theta = \dfrac{v^2}{\ell\sin\theta} \Rightarrow v = \sqrt{\ell g \sin\theta\, tg\theta}$

$\sin\theta = \dfrac{r}{\ell}$

$v = r\omega$

$v = \ell\omega\sin\theta$

$\omega = \dfrac{v}{\ell\sin\theta}$ [rad/s]

$\omega = \dfrac{30}{\pi}\,\dfrac{1}{\ell\sin\theta}\sqrt{\ell g\sin\theta\, tg\theta}$

$\boxed{\omega = 1.1 \times 10^2\ \text{rpm}}$ cqd

rpm

$1r = 2\pi\,\text{rad}$

$1\,\text{rad} = \dfrac{1}{2\pi}r$

$1\text{min} = 60s$

$1s = \dfrac{1}{60}\text{min}$

$\dfrac{\text{rad}}{s} = \dfrac{1}{2\pi}60 = \dfrac{30}{\pi}\text{rpm}$