



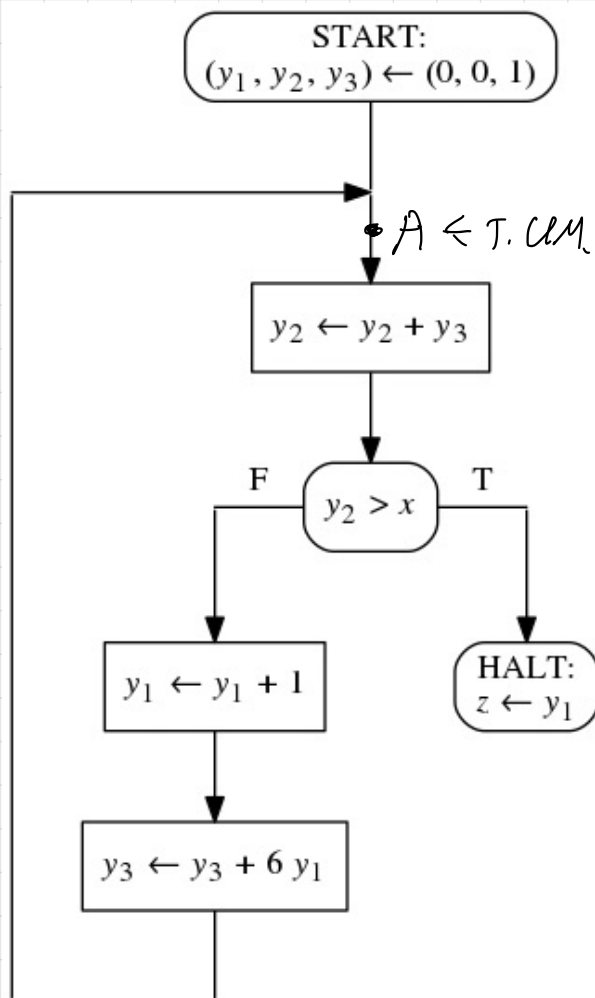
Задача 1.2.

$$V = \{ x, y_1, y_2, y_3, z \}$$

$$\phi(x) \equiv x \geq 0$$

$$\psi(x, z) \equiv \underline{z^3 \leq x < (z+1)^3}$$

$$\mathcal{D}_x = \mathcal{D}_{y_1} = \mathcal{D}_{y_2} = \mathcal{D}_{y_3} = \mathcal{D}_z = \mathbb{Z}$$



$$\underline{S-A} : x \geq 0 \rightarrow P_A[x, y_1, y_2, y_3]$$

$$\underline{A-A} : x \geq 0 \wedge P_A[x, y_1, y_2, y_3]$$

$$\wedge y_2 + y_3 \leq x \rightarrow P_A[x, y_1 + 1, y_2 + y_3, y_3 + 6y_1 + 6]$$

$$\underline{A-H} : x \geq 0 \wedge P_A[x, y_1, y_2, y_3]$$

$$\wedge (y_2 + y_3 > x) \rightarrow y_1^3 \leq x < (y_1 + 1)^3$$

$$\underline{\exists P_A = (x \geq y_1^3) \wedge \dots}$$

$$y_1^3 \leq x < (y_1 + 1)^3 \Leftrightarrow y_1^3 \leq x < y_1^3 + 3 \cdot y_1 + 3y_1^2 + 1$$

$$\Leftrightarrow y_1^3 \leq x < \underline{y_1^3 + 3y_1 + 3y_1^2 + 1}$$

$$P_A = (x \geq y_1^3) \wedge (y_2 = (y_1 + 1)^3 - y_3) \wedge$$

$$\wedge (y_3 = (y_1 + 1)^3 - y_1^3) \leftarrow$$

$$P_A = (x \geq y_1^3) \wedge (y_2 = (y_1+1)^3 - (y_1+1)^3 + y_1^3) \wedge$$

$$x = x \quad (x \geq y_1^3) \wedge (y_2 = y_1^3) \wedge (y_3 = (y_1+1)^3 - y_1^3)$$

S-A — irrelevant ✓

$$\underline{A-H} = (x \geq 0) \wedge (x \geq y_1^3) \wedge (y_2 = y_1^3) \wedge$$

$$\wedge (y_2 + y_3 > x) \rightarrow y_1^3 \leq x < (y_1+1)^3 \Leftarrow \vee$$

$$(x \geq 0) \wedge (x \geq y_1^3) \wedge (y_1^3 = y_2) \wedge (y_1^3 + y_3 > x)$$

$$\rightarrow y_1^3 \leq x < (y_1+1)^3 = y_2 + 3y_1^2 + 3y_1 + 1 \quad \checkmark$$

$$\underline{A-A} = (x \geq 0) \wedge (y_2 + y_3 \leq x) \wedge (x \geq y_1^3) \wedge$$

$$\wedge (y_2 = y_1^3) \wedge (y_2 = y_1^3) \wedge (y_3 = (y_1+1)^3 - y_1^3)$$

$$\rightarrow P_A(x, y_1+1, y_2+y_3, y_3+6y_1+6)$$

\Downarrow

$$(x \geq (y_1+1)^3) \wedge (y_2 + y_3 = (y_1+1)^3) \wedge$$

$$y_3 + 6y_1 + 6 = (y_1+2)^3 - (y_1+1)^3)$$

\Downarrow

$$(x \geq (y_1+1)^3) \wedge (y_2 + y_3 \leq (y_1+1)^3) \wedge (y_3 + 6y_1 + 6 =$$

$$= \cancel{y_1^3} + 6y_1^2 + 12y_1 + 8 - \cancel{y_1^3} - 3y_1^2 - 3y_1 - 1)$$

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$$(x \geq (y_1 + 1)^3) \wedge (y_2 + y_3 \leq (y_1 + 1)^3) \wedge$$

$$\wedge (y_3 + 6y_1 + 6 = 3y_1^2 + 9y_1 + 7)$$

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$$y_3 = 3y_1^2 + 3y_1 + 1$$

$$(y_1 + 1)^3 - y_1^3 = 3y_1^2 + 3y_1 + 1$$



$$(\cancel{y_1^3} + 3y_1^2 + 9y_1 + 1 - \cancel{y_1^3} = 3y_1^2 + 3y_1 + 1)$$