Question 1

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Design a lowpass filter using the window design technique, where $G_p = 2, \delta = 0.01, \omega_p = \pi/8$ and $\omega_s = \pi/4$.

Define intial values

```
delta_spec=.01;
Gp=2;
ws=pi/4;
wp=pi/8;
delta = delta_spec/Gp;
```

Select window

```
A=-20*log10(delta)
% A=46.02, select Hamming
M=6.27*pi/(ws-wp)
% M=50.16, use 50
M=round(M);


A =

  46.020599913279625


M =

  50.159999999999989
```

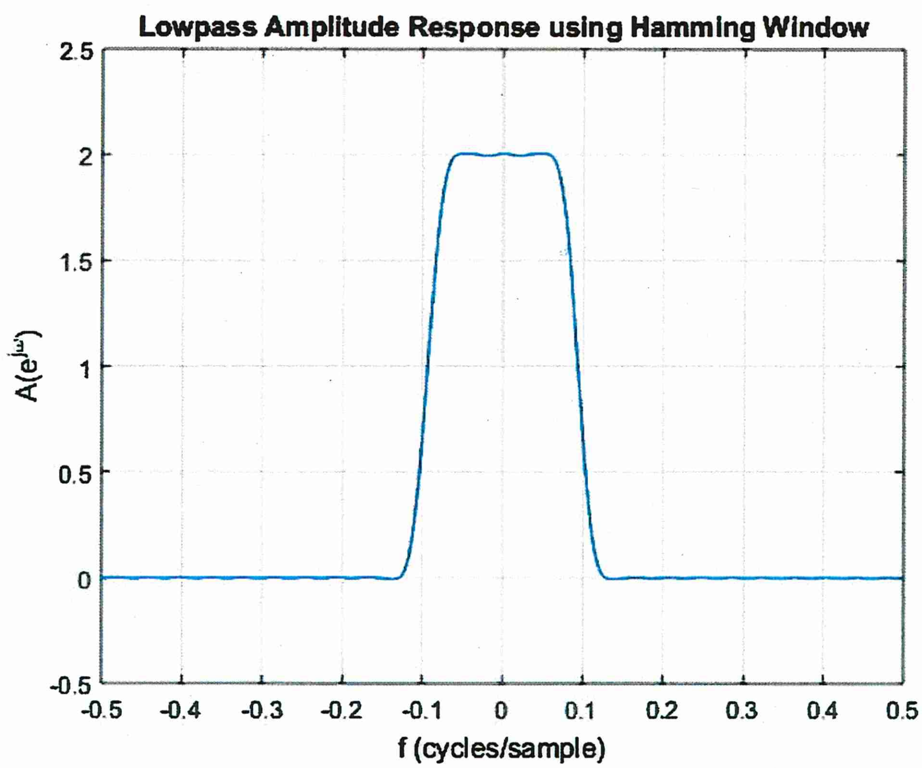
Generate the impulse response

```
n=0:M;
f2=((wp+ws)/2)/(2*pi);
f1=0;
hd=2*f2*sinc(2*f2*(n-M/2))-2*f1*sinc(2*f1*(n-M/2));
hd=Gp*hd;
win=hamming(M+1);
```

```
h=hd.*win.';
```

Plot the amplitude response

```
L=1024;
hz=[h,zeros(1,L-length(h))];
H=fft(hz);
k=0:L-1;
% M is even - thus this a type 1 filter with a linear
% phase response of -(M/2)*2*pi*f, where f=k/L
W=exp(-j*(-(M/2)*2*pi)*k/L);
A=H.*W;
A=real(A);
figure(1)
clf
plot(k/L-0.5,fftshift(A))
ylabel('A(e^{j\omega})')
title('Lowpass Amplitude Response using Hamming Window')
xlabel('f (cycles/sample)')
grid
```



Questions Answered

The peak ripple measured on the plot is approximately 2.0048. This peak ripple is less than .01, because a Hamming window is used which results in a 53 dB stopband attenuation. Though the nominal passband gain is 2, which will decrease the stopband attenuation by 6 dB. Thus, since, $A - 6 = 20log(\delta)$, the expected $\delta = 10^{-47/20} = 0.0045$. This is close to the measured value.

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3a) This a bandpass filter, the two stop bands have ripple of 0.005 and 0.001. The passband ripple specification is 0.1. Taking the minimum

$$\delta = .001$$

$$A = -20\log(\delta) = -20\log(.001) = 60\,dB$$

There are two transition bands one of width $0.4\pi - 0.25\pi = 0.15\pi$ and the other $0.75\pi - 0.65\pi = 0.1\pi$. Using the smallest transition band

$$M = \frac{A-8}{2.285(\omega_s - \omega_p)} = \frac{60-8}{2.285(0.1\pi)}$$

$$= 72.44$$

Use $M = 73$ (a type 2 filter)

$A > 50$, thus

$$\beta = 0.1102(A - 8.7)$$

$$= 0.1102(60 - 8.7)$$

$$= 5.65$$

3 b) The phase of a type 2 filter is

$$\Theta(\omega) = -\frac{M}{2}\omega$$

The input signal is delayed by

$$\frac{M}{2} = \frac{73}{2} = 36.5 \text{ samples.}$$

As an example, assume the input is $x[n] = \cos\omega_1 n$.

If the frequency response of the filter is $H(e^{j\omega}) = A(e^{j\omega})\, e^{j\Theta(\omega)}$, then if the input is put in the form $\cos\omega_1 n = \frac{e^{j\omega_1 n} + e^{-j\omega_1 n}}{2}$, the output is

$$y[n] = \tfrac{1}{2} e^{j\omega_1 n} H(e^{j\omega_1}) + \tfrac{1}{2} e^{-j\omega_1 n} H(e^{-j\omega_1})$$

$$= \frac{A(e^{j\omega_1})}{2} e^{j\omega_1 n} e^{-j\frac{M}{2}\omega_1} + \frac{A(e^{-j\omega_1})}{2} e^{-j\omega_1 n} e^{-j\frac{M}{2}(-\omega_1)}$$

$$= \frac{A(e^{j\omega_1})}{2} e^{j(\omega_1 n - \frac{M}{2}\omega_1)} + \frac{A(e^{-j\omega_1})}{2} e^{-j(\omega_1 n - \frac{M}{2}\omega_1)}$$

since $A(e^{j\omega_1}) = A(e^{-j\omega_1}) \longrightarrow$

$$= A(e^{j\omega_1}) \cos(\omega_1 n - \tfrac{M}{2}\omega_1) = A(e^{j\omega_1}) \cos(\omega_1 n + \Theta(\omega_1))$$

$$= A(e^{j\omega_1}) \cos(\omega_1(n - \tfrac{M}{2})) = A(e^{j\omega_1}) \cos(\omega_1(n + \tfrac{\Theta(\omega_1)}{\omega_1})).$$

$\tau_p = -\frac{\Theta(\omega_1)}{\omega} = \frac{M}{2}$ is called the phase delay with units samples.

5c) $$h_d[n] = 2f_2 \operatorname{sinc}(2f_2(n - \tfrac{M}{2})) - 2f_1 \operatorname{sinc}(2f_1(n - \tfrac{M}{2}))$$

where $f_1 = \frac{0.25\pi + 0.4\pi}{2(2\pi)} = \frac{0.65\pi}{4\pi} = 0.1625$

$f_2 = \frac{(0.65\pi + 0.75\pi)}{2(2\pi)} = \frac{1.4\pi}{4\pi} = 0.35$

Question 5

Contents

Define initial values

```
wp=0.4*pi;
ws=0.5*pi;
delta=0.001;
A=-20*log10(delta)
```

```
A =

    60
```

Kaiser window design

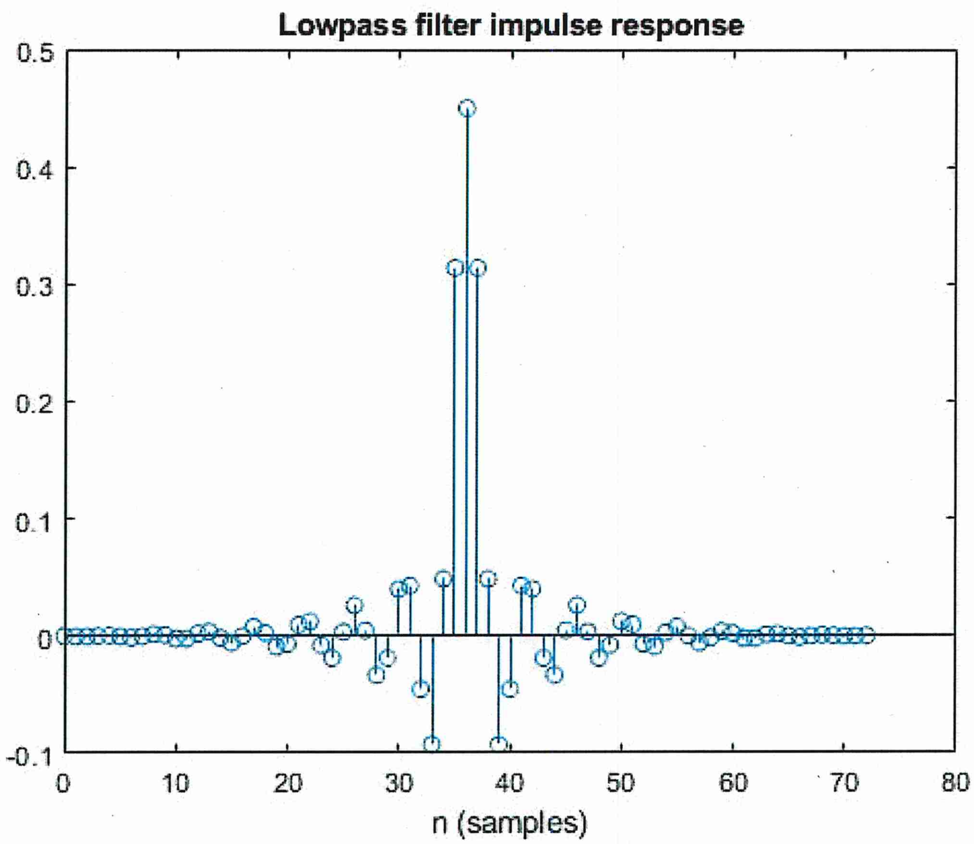
```
M=(A-8)/(2.285*(ws-wp))
M=round(M);
beta = .1102*(A-8.7);
if A<=50
display(['A=',num2str(A),', an incorrect expression was used for beta'])
end
wc=((wp+ws)/2);
h=fir1(M,wc/pi,'low',kaiser(M+1,beta));
```

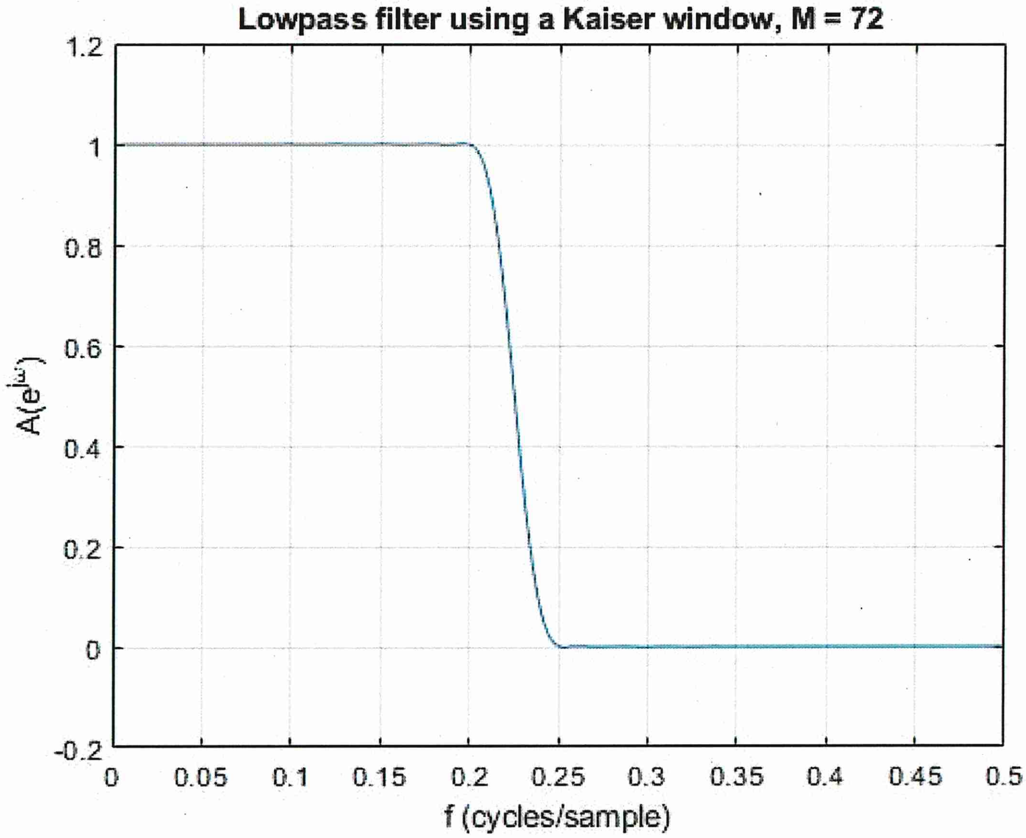
```
M =

  72.438136024319974
```

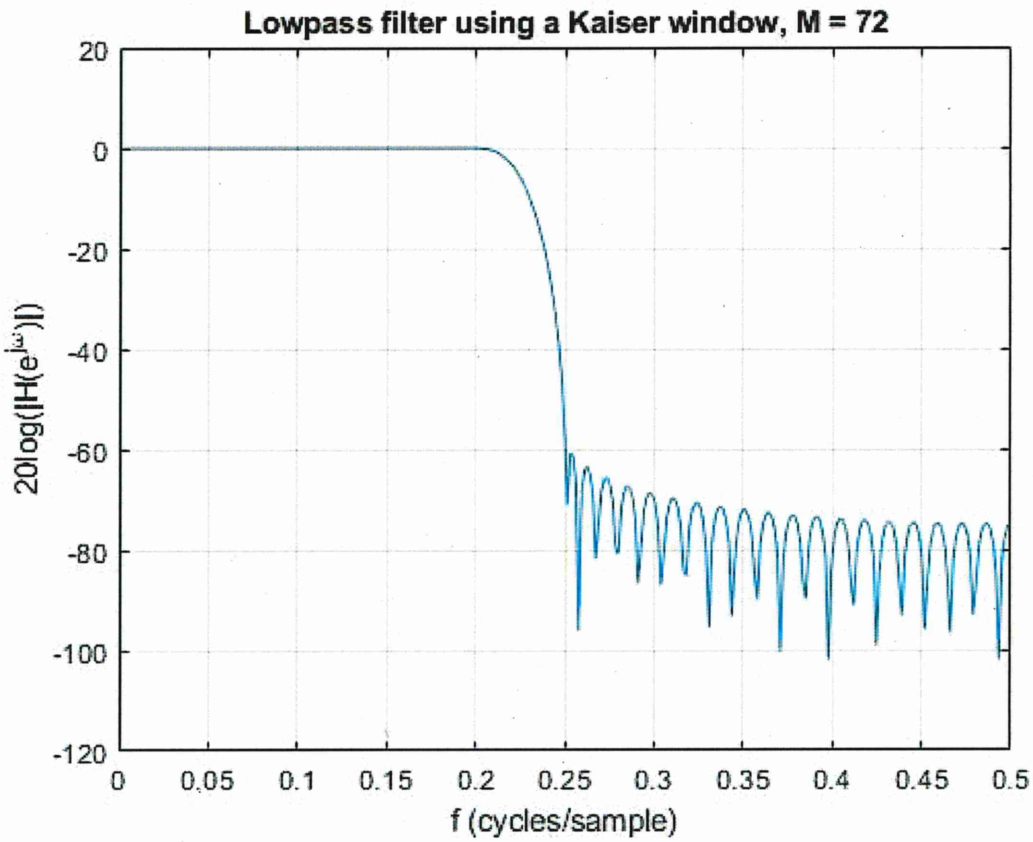
Plot Kaiser filter results

```
[H,w]=freqz(h,1,[0:.01:pi]);
W=exp(-j*(-M/2)*w);
A=H.*W;
A=real(A);
figure(1)
stem([0:M],h);
title('Lowpass filter impulse response')
xlabel('n (samples)')
```

```
figure(2)
clf
plot(w/(2*pi),A)
title(['Lowpass filter using a Kaiser window, M = ',num2str(M)])
xlabel('f (cycles/sample)')
ylabel('A(e^{j\omega})')
grid
figure(3)
clf
plot(w/(2*pi),20*log10(abs(H)))
title(['Lowpass filter using a Kaiser window, M = ',num2str(M)])
xlabel('f (cycles/sample)')
ylabel('20log(|H(e^{j\omega})|)')
grid
```







Question Answer

No, the filter does not meet spec, as the calculation below shows, the pass band ripple is too high. The filter order must be increased.

```
delta_estimate = max(A) - 1
```

```
delta_estimate =

   0.001145965563221
```

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Question 7

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Define initial values

```
wp=0.7*pi;
ws=0.8*pi;
deltas=0.0004;
deltap=0.001;
A=-20*log10(min(deltas,deltap))
% Choose Blackman


A =

  67.958800173440750
```

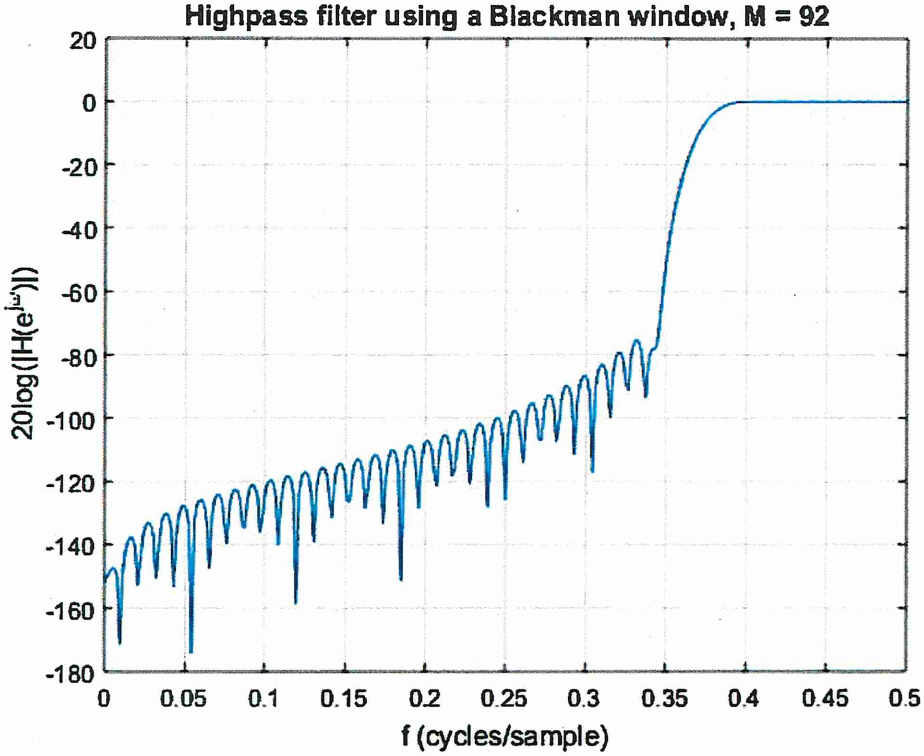
Blackman window design

```
M=9.19*pi/(ws-wp)
Mb=round(M);
wc=((wp+ws)/2);
hb=fir1(Mb,wc/pi,'high',blackman(Mb+1));


M =

  91.899999999999991
```

Plot Blackman filter magnitude response

```
[H,w]=freqz(hb,1,[0:.01:pi]);
figure(1)
clf
plot(w/(2*pi),20*log10(abs(H)))
title(['Highpass filter using a Blackman window, M = ',num2str(Mb)])
xlabel('f (cycles/sample)')
ylabel('20log(|H(e^{j\omega})|)')
```

```
grid
```



Kaiser window design

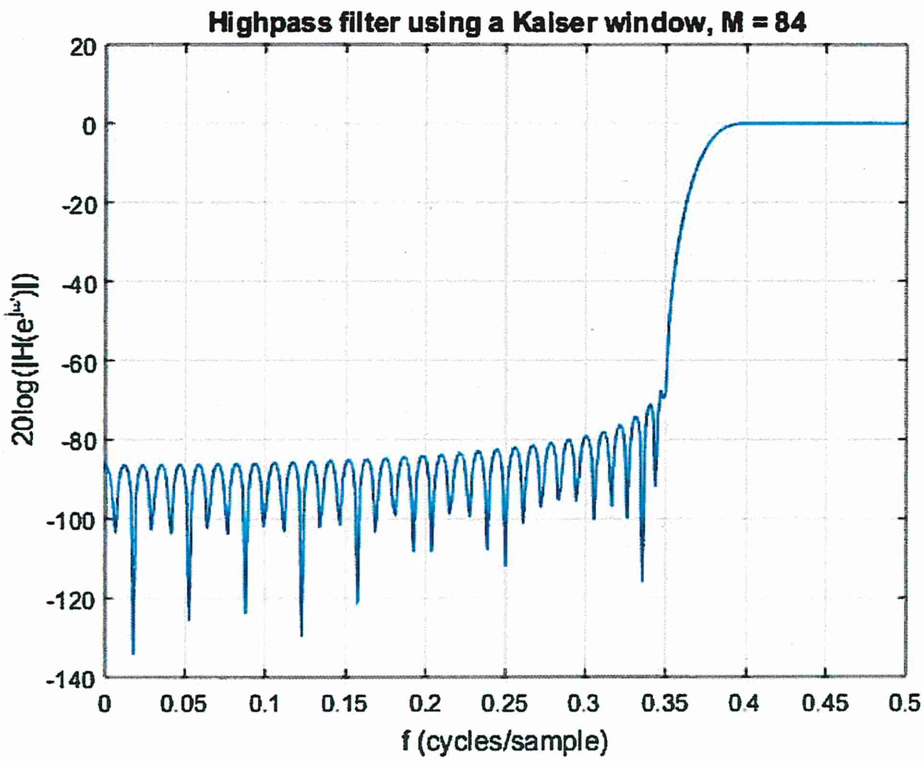
```
M=(A-8)/(2.285*(ws-wp))
Mk=round(M);
beta = .1102*(A-8.7);
if A<=50
display(['A=',num2str(A),', an incorrect expression was used for
 beta'])
end
hk=fir1(Mk,wc/pi,'high',kaiser(Mk+1,beta));
```

```
M =

  83.525071592667715
```

Plot Kaiser filter magnitude response

```
[H,w]=freqz(hk,1,[0:.01:pi]);
figure(2)
clf
plot(w/(2*pi),20*log10(abs(H)))
```

```
title(['Highpass filter using a Kaiser window, M = ',num2str(Mk)])
xlabel('f (cycles/sample)')
ylabel('20log(|H(e^{j\omega})|)')
grid
```



Question Answer

The Kaiser window filter is the better choice, since it has an order of 84 versus the Blackman window filter with an order of 92.

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Question 9 : Lowpass Filter Design Using Equiripple (Parks-McClellan) Method

Contents

Define intial values

```
wp = pi/4;
ws = 3*pi/8;
deltap = 0.02;
deltas = 0.01;
Gp = 1;
Gs = 0;
```

Generate filter order using 4 techniques

```
M_Bellanger = (2/3)*log10(1/(10*deltap*deltas))*(2*pi/abs(ws-wp))
M_Kaiser=(-20*log10(sqrt(deltap*deltas))-13)/(2.32*abs(wp-ws))
M_Harris=(-20*log10(deltas))/(22*abs(wp-ws)/(2*pi))
M_firpmord=firpmord([wp/pi,ws/pi],[1 0],[deltap,deltas])

M=round(M_Harris); % Use the Harris order
```

```
M_Bellanger =

  28.789013379584198


M_Kaiser =

  26.331581690983654


M_Harris =

  29.090909090909097


M_firpmord =
```

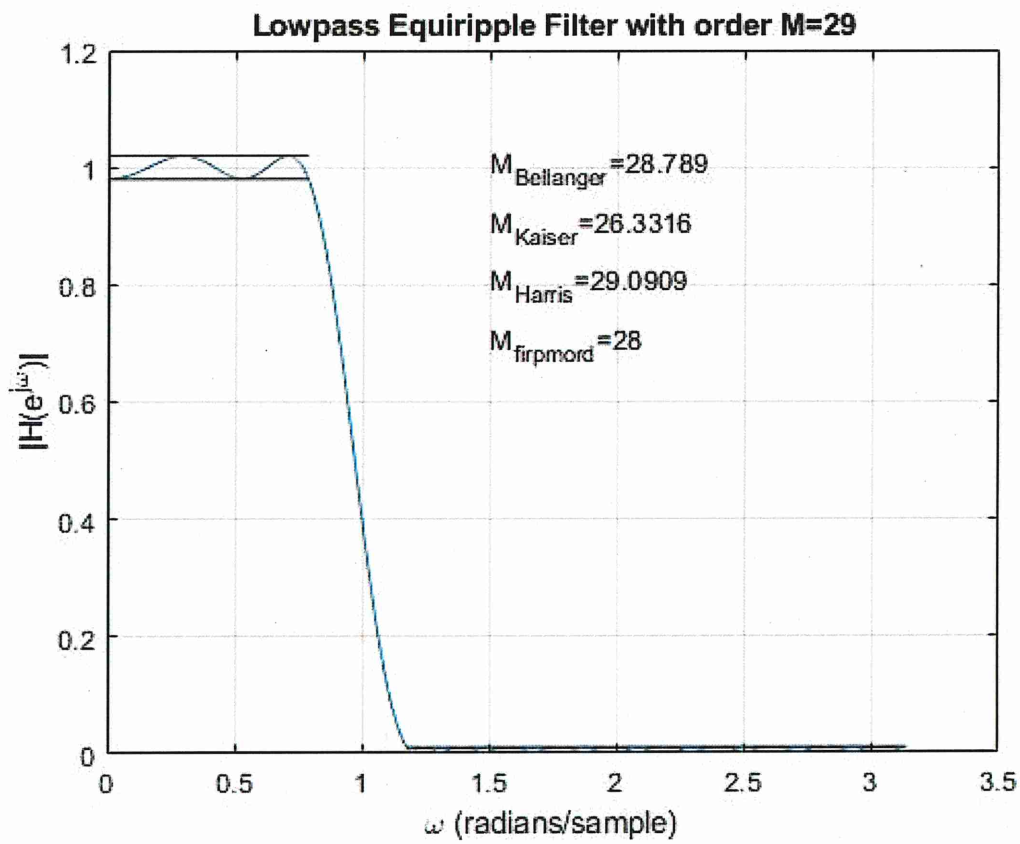
Design the equiripple filter

```
fb=[0,wp/pi,ws/pi,1];
a=[Gp,Gp,Gs,Gs];
wght=[1,deltap/deltas];
% or could use wght=[1/deltap,1/deltas];
b=firpm(M,fb,a,wght);
```

Plot the magnitude response

```
figure(1)
clf
[H,w]=freqz(b,1,[0:.01:pi]);
plot(w,abs(H))
grid
hold
plot([0,wp],[Gp+deltap,Gp+deltap],'-k')
plot([0,wp],[Gp-deltap,Gp-deltap],'-k')
plot([ws,pi],[Gs+deltas,Gs+deltas],'-k')
title(['Lowpass Equiripple Filter with order M=',num2str(M)])
ylabel('|H(e^{j\omega})|')
xlabel('\omega (radians/sample)')
text(1.5,1,['M_{Bellanger}=',num2str(M_Bellanger)]);
text(1.5,0.9,['M_{Kaiser}=',num2str(M_Kaiser)]);
text(1.5,0.8,['M_{Harris}=',num2str(M_Harris)]);
text(1.5,0.7,['M_{firpmord}=',num2str(M_firpmord)]);
```

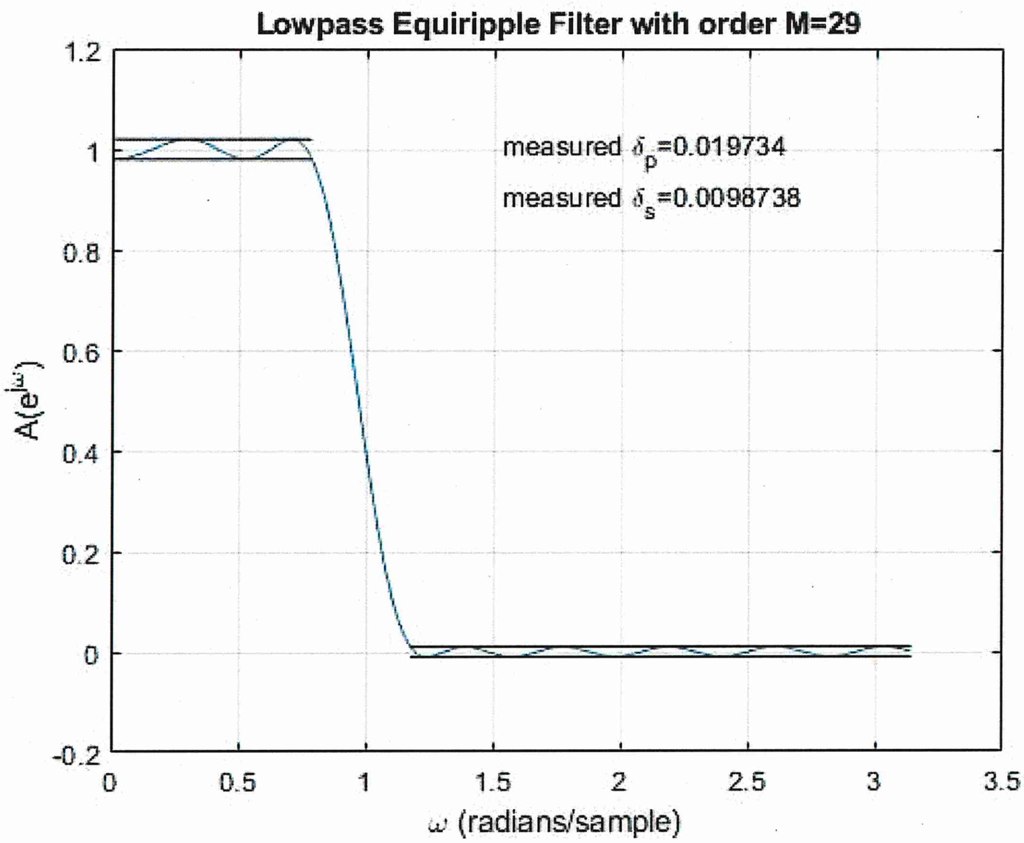
```
Current plot held
```



Plot the amplitude response

```
figure(2)
clf
W=exp(-j*((-M/2)*w));
A=H.*W;
A=real(A);
plot(w,A)
grid
hold
plot([0,wp],[Gp+deltap,Gp+deltap],'-k')
plot([0,wp],[Gp-deltap,Gp-deltap],'-k')
plot([ws,pi],[Gs+deltas,Gs+deltas],'-k')
plot([ws,pi],[Gs-deltas,Gs-deltas],'-k')
title(['Lowpass Equiripple Filter with order M=',num2str(M)])
ylabel('A(e^{j\omega})')
xlabel('\omega (radians/sample)')
text(1.5,1,['{\rm measured }{\delta_p}=',num2str(max(A)-1)]);
text(1.5,0.9,['{\rm measured }{\delta_s}=',num2str(-min(A))]);
```

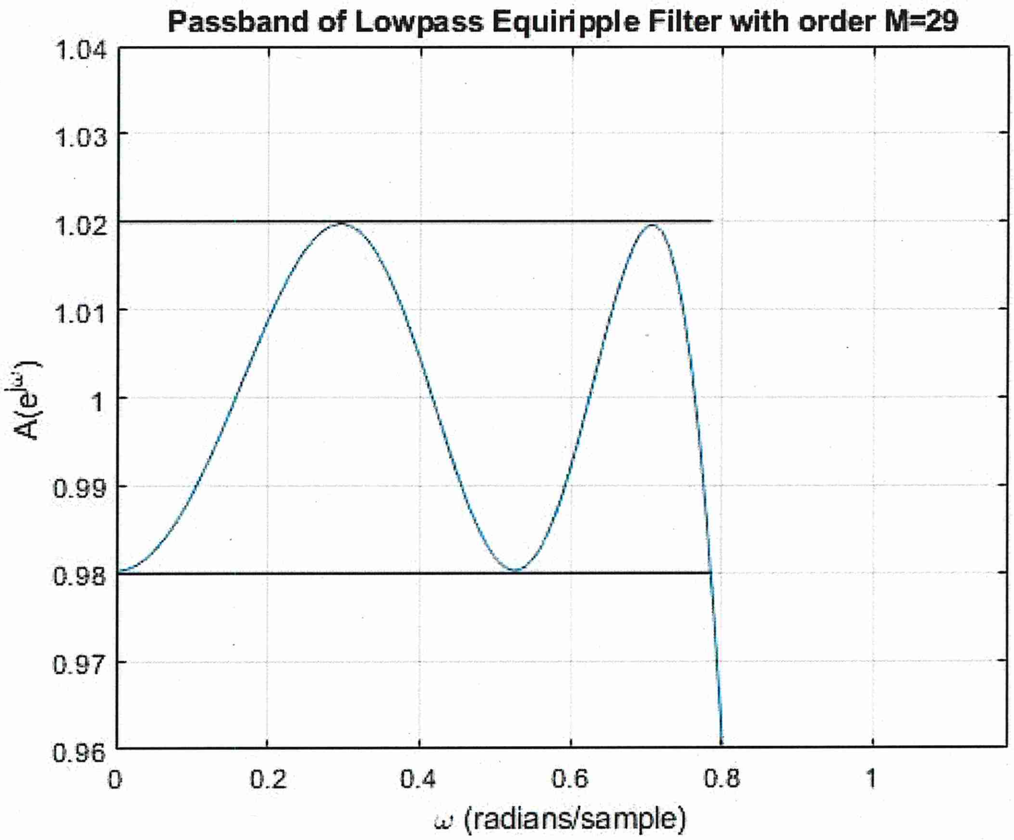
```
Current plot held
```



Plot the pass band of the amplitude response

```
figure(3)
clf
plot(w,A)
grid
hold
plot([0,wp],[Gp+deltap,Gp+deltap],'-k')
plot([0,wp],[Gp-deltap,Gp-deltap],'-k')
axis([0,ws,Gp-2*deltap,Gp+2*deltap])
title(['Passband of Lowpass Equiripple Filter with order M=',num2str(M)])
ylabel('A(e^{j\omega})')
xlabel('\omega (radians/sample)')
```

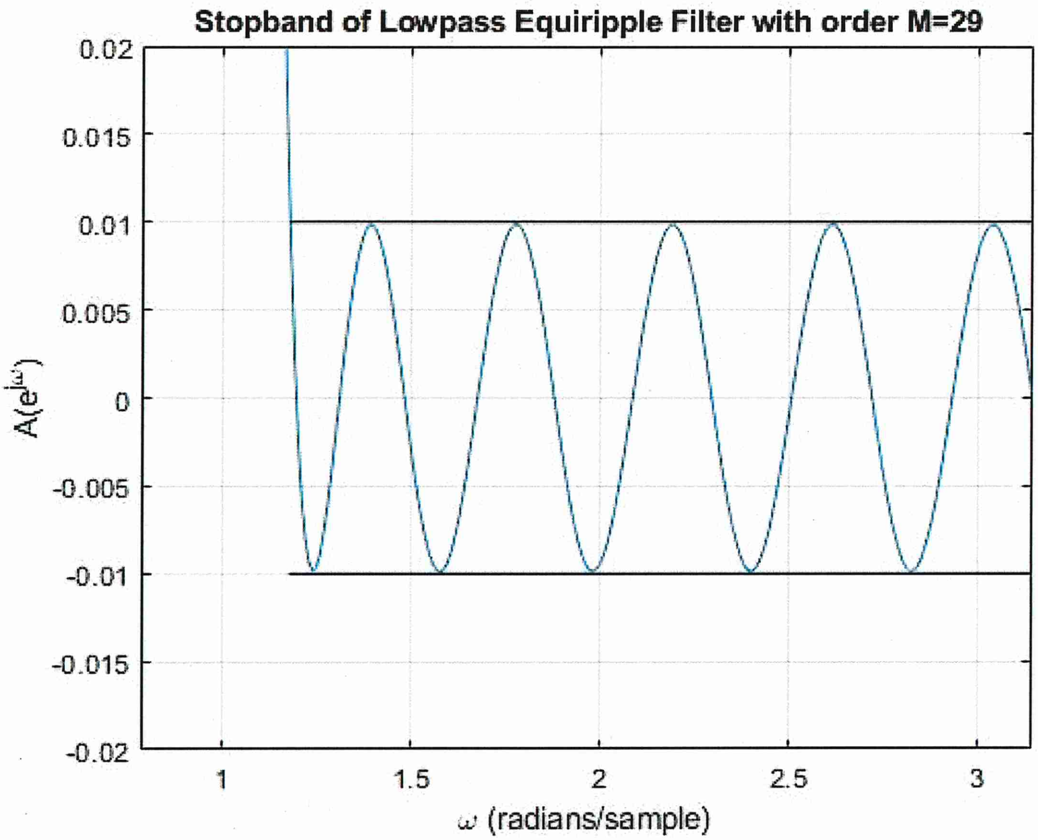
```
Current plot held
```



Plot the stop band of the amplitude response

```
figure(4)
clf
plot(w,A)
grid
hold
plot([ws,pi],[Gs+deltas,Gs+deltas],'-k')
plot([ws,pi],[Gs-deltas,Gs-deltas],'-k')
axis([wp,pi,Gs-2*deltas,Gs+2*deltas])
title(['Stopband of Lowpass Equiripple Filter with order M=',num2str(M)])
ylabel('A(e^{j\omega})')
xlabel('\omega (radians/sample)')
```

```
Current plot held
```

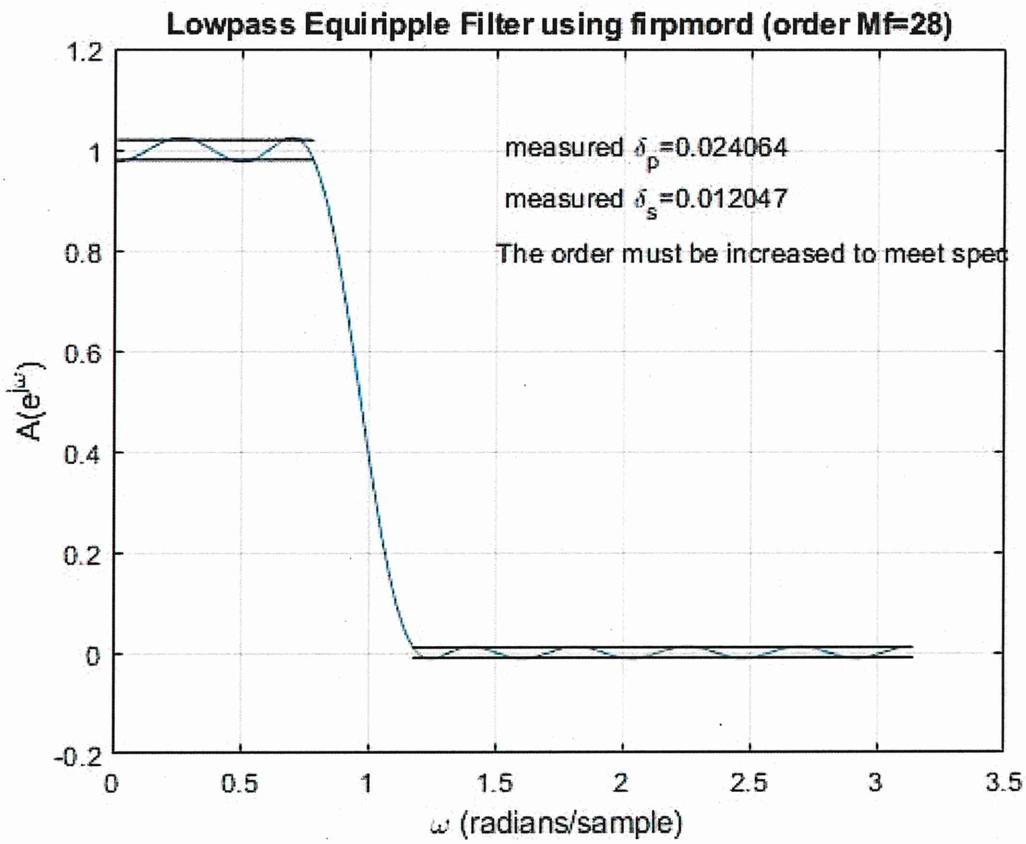


Using firpmord to generate all vectors for firpm

The frequency, amplitude and weighting vectors generated by firpmord are are the same as the ones generated by hand in the approach used above.

```
figure(5)
clf
[Mf,fo,ao,wo]=firpmord([wp/(2*pi),ws/(2*pi)],[Gp,Gs],[deltap,deltas],1);
%[Mf,fo,ao,wo]=firpmord([wp/(pi),ws/(pi)],[Gp,Gs],[deltap,deltas],2);
bf=firpm(Mf,fo,ao,wo);
[Hf,wf]=freqz(bf,1,[0:.01:pi]);
Wf=exp(-j*((-Mf/2)*wf));
Af=Hf.*Wf;
Af=real(Af);
plot(w,Af)
hold
plot([0,wp],[Gp+deltap,Gp+deltap],'-k')
plot([0,wp],[Gp-deltap,Gp-deltap],'-k')
plot([ws,pi],[Gs+deltas,Gs+deltas],'-k')
plot([ws,pi],[Gs-deltas,Gs-deltas],'-k')
title(['Lowpass Equiripple Filter using firpmord (order Mf=',num2str(Mf),')'])
ylabel('A(e^{j\omega})')
xlabel('\omega (radians/sample)')
grid
text(1.5,1,['{\rm measured }{\delta_p}=',num2str(max(Af)-1)]);
text(1.5,0.9,['{\rm measured }{\delta_s}=',num2str(-min(Af))]);
text(1.5,0.8,['The order must be increased to meet spec']);
```

```
Current plot held
```



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