1(a) This is a type 1 filter so

$$\theta(\omega) = -\alpha\omega$$

where $\alpha = \frac{M}{2}$

$$M = N-1 = 5-1 = 4$$

$$\theta(\omega) = -2\alpha$$

1(b) $H(e^{j\omega}) = h_0 + h_1 e^{-j\omega} + h_2 e^{-j2\omega} + h_3 e^{-j3\omega} + h_4 e^{-j4\omega}$

$$= e^{-j2\omega}\left(h_0 e^{j2\omega} + h_1 e^{j\omega} + h_2 + h_3 e^{-j\omega} + h_4 e^{-j2\omega}\right)$$

This is a type I filter thus $h[n] = h[M-n]$

$$= e^{-j2\omega}\left(h_0 e^{j2\omega} + h_1 e^{j\omega} + h_2 + h_1 e^{-j\omega} + h_0 e^{-j2\omega}\right)$$

$$= e^{-j2\omega}\left(h_0\left(e^{j2\omega} + e^{-j2\omega}\right) + h_1\left(e^{j\omega} + e^{-j\omega}\right) + h_2\right)$$

$$= e^{-j2\omega}\left(\left(h_0\, 2\cos 2\omega\right) + h_1\, 2\cos\omega + h_2\right)$$

$$= \left(2h_0 \cos 2\omega + 2h_1 \cos\omega + h_2\right) e^{-j2\omega}$$

Thus

$$A(e^{j\omega}) = 2h_0 \cos 2\omega + 2h_1 \cos\omega + h_2.$$

EE461 Assignment Question 3

This m-file script generates the length 31 impulse response using impulse response truncation to approximate an ideal LPF filter with a cutoff frequency of f=1/8 cycles/sample.

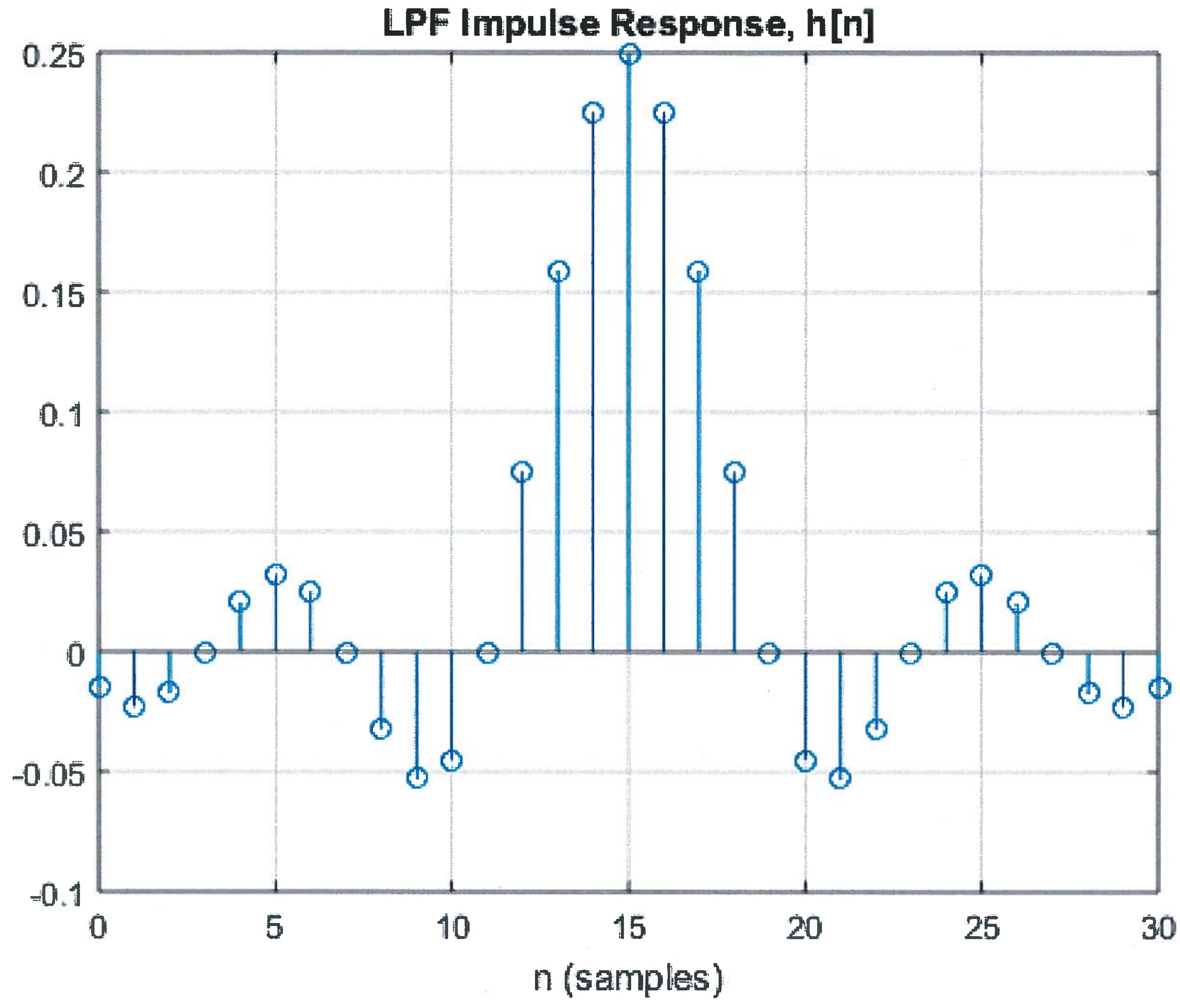
Contents

Define intial values

```
fc=1/8; % cutoff freq (cycles/sample)
M=30; % order of filter
n=0:M; % sample indices
```

Generate and plot the impulse response, h

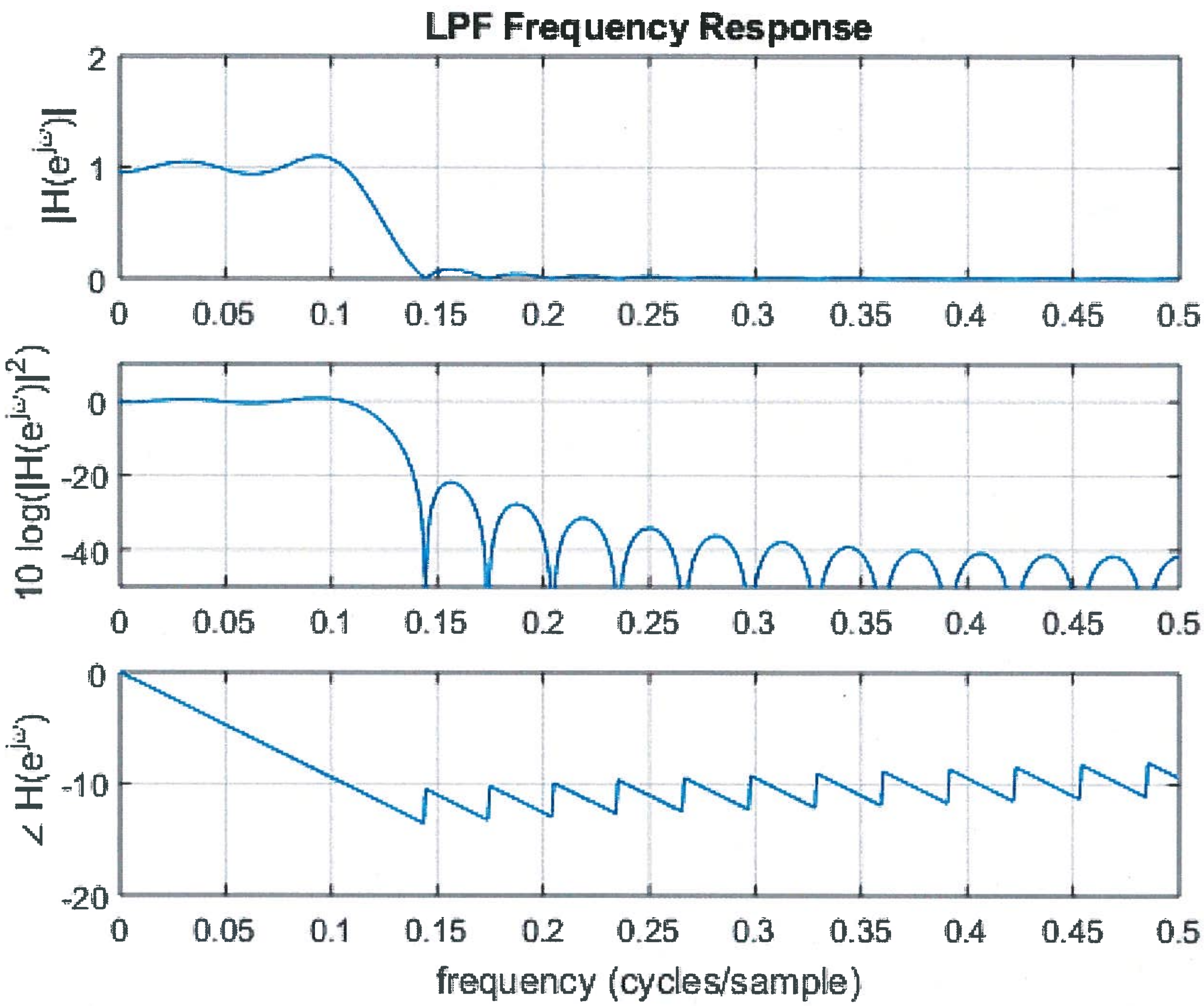
```
h=2*fc*sinc(2*fc*(n-M/2));
figure(1)
clf
stem(n,h)
title('LPF Impulse Response, h[n]')
xlabel('n (samples)')
grid
```



Generate and plot the frequency response

```
figure(2)
clf
[H,w]=freqz(h,1);
subplot(3,1,1)
plot(w/(2*pi),abs(H))
ylabel('|H(e^{j\omega})|')
title('LPF Frequency Response')
grid
subplot(3,1,2)
plot(w/(2*pi),10*log10(abs(H).^2))
grid
axis([0 .5 -50 10])
ylabel('10 log{(|H(e^{j\omega})|^2)}')
subplot(3,1,3)
plot(w/(2*pi),unwrap(angle(H)))
ylabel('\angle H(e^{j\omega})')
```

```
xlabel('frequency (cycles/sample)')
grid
```



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EE461 Assignment Question 5

This m-file script generates the length 31 impulse response using impulse response truncation to approximate an ideal LPF filter with a cutoff frequency of f=1/8 cycles/sample.

Contents

Define intial values

```
fc=1/8; % cutoff freq (cycles/sample)
M=30; % order of filter
n=0:M; % sample indices
```

Generate and plot the impulse response, h

```
h=2*fc*sinc(2*fc*(n-M/2));

% Convert the LPR to a HPF using technique 1
h=-h;
h(M/2+1)=1+h(M/2+1);

figure(1)
clf
stem(n,h)
title('HPF Impulse Response, h[n]')
xlabel('n (samples)')
grid
```

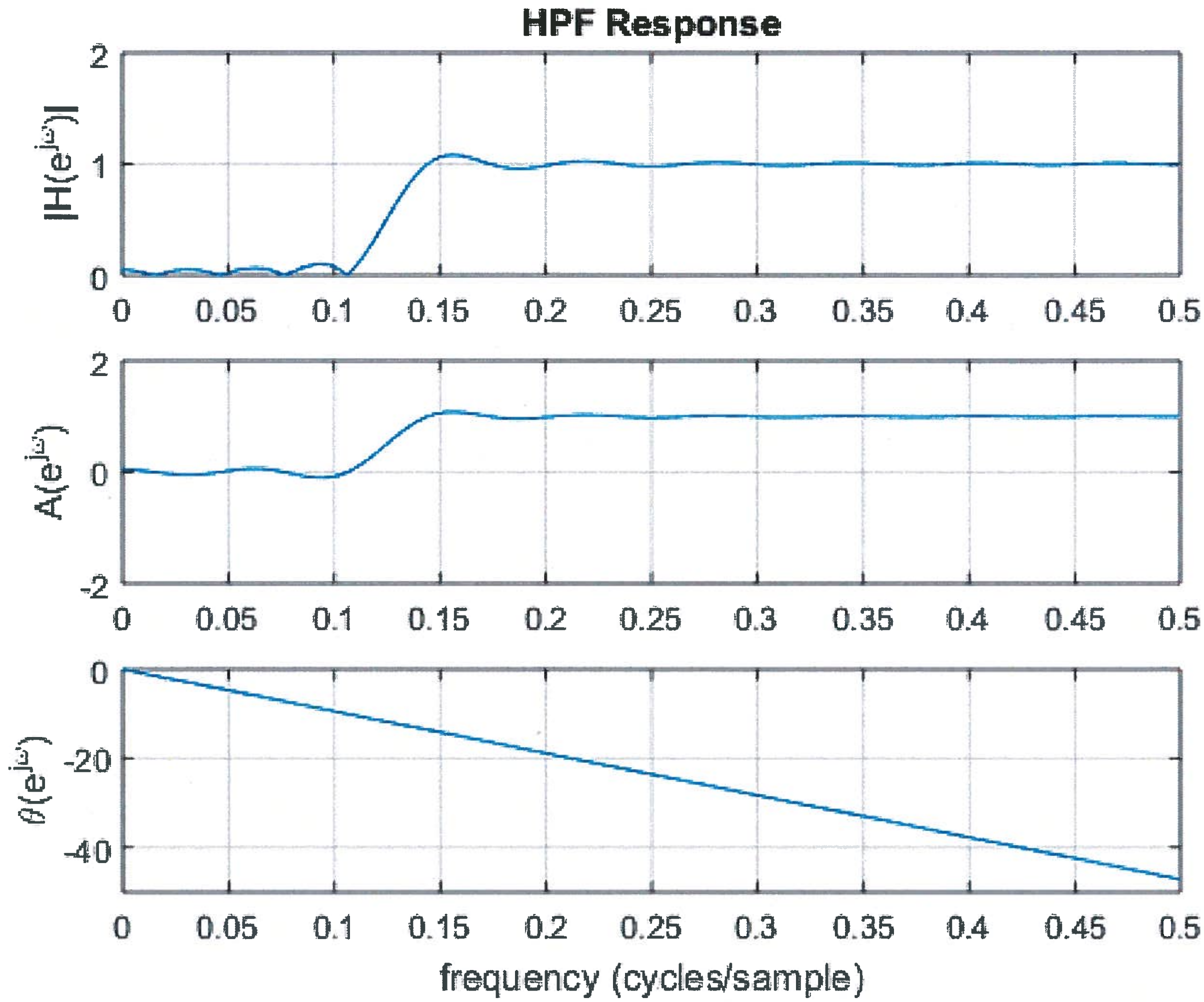
Generate and plot $10\log|H(e^{j\omega})|^2$ in dB

```
figure(2)
clf
[H,w]=freqz(h,1);
plot(w/(2*pi),10*log10(H.*conj(H)))
```

```
ylabel('10 log{(|H(e^{j\omega})|^2)}')
title('HPF Response')
grid
```

Generate complete response

```
A=H.*exp(j*w*M/2);
figure(3)
subplot(3,1,1)
plot(w/(2*pi),abs(H))
ylabel('|H(e^{j\omega})|')
title('HPF Response')
grid
subplot(3,1,2)
plot(w/(2*pi),real(A))
grid
ylabel('A(e^{j\omega})')
subplot(3,1,3)
plot(w/(2*pi),-(M/2)*w)
ylabel('\theta(e^{j\omega})')
xlabel('frequency (cycles/sample)')
grid
```



What is the cutoff frequency?

The cutoff frequency is the same as the LPF, fc = 1/8.

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7a) $A_d(e^{jw}) = \cos(\frac{w}{2})(1+\cos w)$

Using Eulers

$$= \left(\frac{e^{j\frac{w}{2}}+e^{-j\frac{w}{2}}}{2}\right)\left(1+\frac{e^{jw}+e^{-jw}}{2}\right)$$

$$= \frac{1}{2}e^{jw/2} + \frac{1}{2}e^{-jw/2} + \frac{1}{4}e^{j3w/2} + \frac{1}{4}e^{jw/2} + \frac{1}{4}e^{-j\frac{w}{2}} + \frac{1}{4}e^{-j3\frac{w}{2}}$$

$$= \left(\frac{1}{2}e^{-jw} + \frac{1}{2}e^{-j2w} + \frac{1}{4} + \frac{1}{4}e^{-jw} + \frac{1}{4}e^{-j2w} + \frac{1}{4}e^{-j3w}\right)e^{j3\frac{w}{2}}$$

$$A_d(e^{jw}) = \left(\frac{1}{4} + \frac{3}{4}e^{-jw} + \frac{3}{4}e^{-j2w} + \frac{1}{4}e^{-j3w}\right)e^{j3\frac{w}{2}}$$

Given $H_d(e^{jw}) = A_d(e^{jw})e^{-jw\frac{M}{2}}$

then $A_d(e^{jw}) = H_d(e^{jw})\,e^{jw\frac{M}{2}}$ where $M=3$.

The integral square error is

$$\epsilon = \frac{1}{2}\int_{-\pi}^{\pi}\{H_d(e^{jw}) - H(e^{jw})\}^2\,dw.$$

Thus ϵ will be 0 if $H(e^{jw}) = H_d(e^{jw})$,

let

$$H(e^{jw}) = H_d(e^{jw}) = \frac{1}{4} + \frac{3}{4}e^{-jw} + \frac{3}{4}e^{-j2w} + \frac{1}{4}e^{-j3w}$$

which gives

$$h(n) = \left[\frac{1}{4}, \frac{3}{4}, \frac{3}{4}, \frac{1}{4}\right]$$

b) $M = N-1 = 4-1 = 3$

c) $h[n] = h[3-n]$ and M is odd $\rightarrow$ Type 2

d) $\theta(w) = -\alpha w = -\frac{M}{2}w = -\frac{3}{2}w$

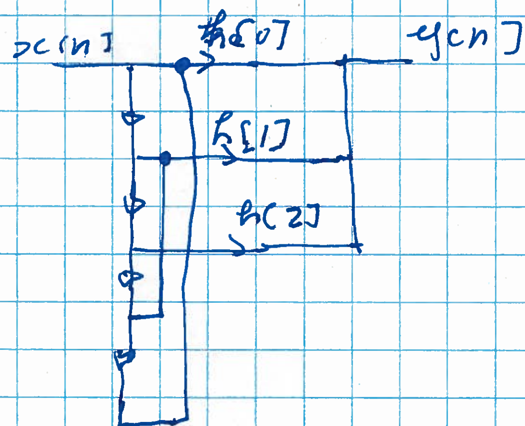
9) $h[n] = h[4-n] \quad 0 \le n \le 4$

$$y[n] = \sum_{k=0}^{4} h[k]\, x[n-k]$$

$$= h[0]x[n] + h[1]x[n-1] + h[2]x[n-2] + h[3]x[n-3] + h[4]x[n-4]$$

Using $h[0] = h[4]$, $h[1] = h[3]$ (from $h[n] = h[4-n]$)

$$y[n] = h[0]\left(x[n] + x[n-4]\right) + h[1]\left(x[n-1] + x[n-3]\right) + h[2]\,x[n-2]$$



a) Multipliers
Direct form - 5
Reduced Direct form - 3

b) Adders
Direct Form - 4
Reduced Direct Form - 4

11) a) Given $H(z) = a + bz^{-1} + cz^{-2}$

then $h[n] = [a, b, c]$

This is a type 3 filter, since

$$H(e^{j\pi}) = H(e^{j0}) = 0$$

A type 3 filter has an antisymmetric

$h[n]$, which force b to be 0.

Using $H(e^{j0}) = a + be^{-j0} + ce^{-j0} = 0$

$$= a + b + c = 0$$

with $b = 0$

$$a + c = 0 \qquad (1)$$

Using the impulse response has unit energy

$$a^2 + b^2 + c^2 = 1$$

with $b = 0$, $\quad a^2 + c^2 = 1 \qquad (2)$

Substituting (1) into (2)

$$a^2 + ((-a)^2) = \cancel{0}$$

$$a^2 = \frac{1}{2}$$

$$a = \frac{1}{\sqrt{2}}$$ ($a > 0$ as required

and $c = -\frac{1}{\sqrt{2}}$

$h[n] = [\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}]$

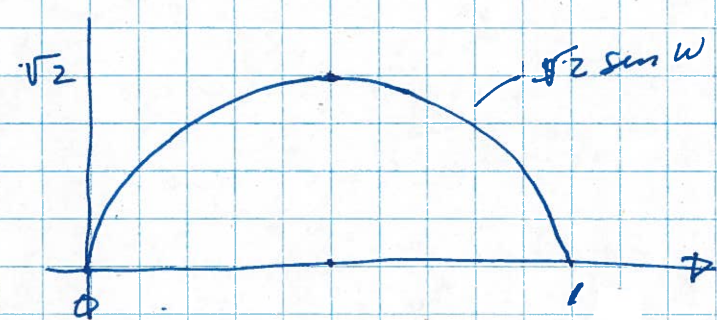
b) $H(e^{jw}) = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} e^{-j2w}$

$$= \left(\frac{1}{\sqrt{2}} e^{jw} - \frac{1}{\sqrt{2}} e^{-jw}\right) e^{-jw}$$

$$= \frac{2j}{\sqrt{2}} \sin(w)\, e^{-jw}$$

$$= \sqrt{2} \sin w \; e^{j\left(\frac{\pi}{2} - w\right)}$$

$A(e^{jw}) = \sqrt{2} \sin w$



c) $\theta(w) = \frac{\pi}{2} - w$

Solution to Question 13

Table of Contents

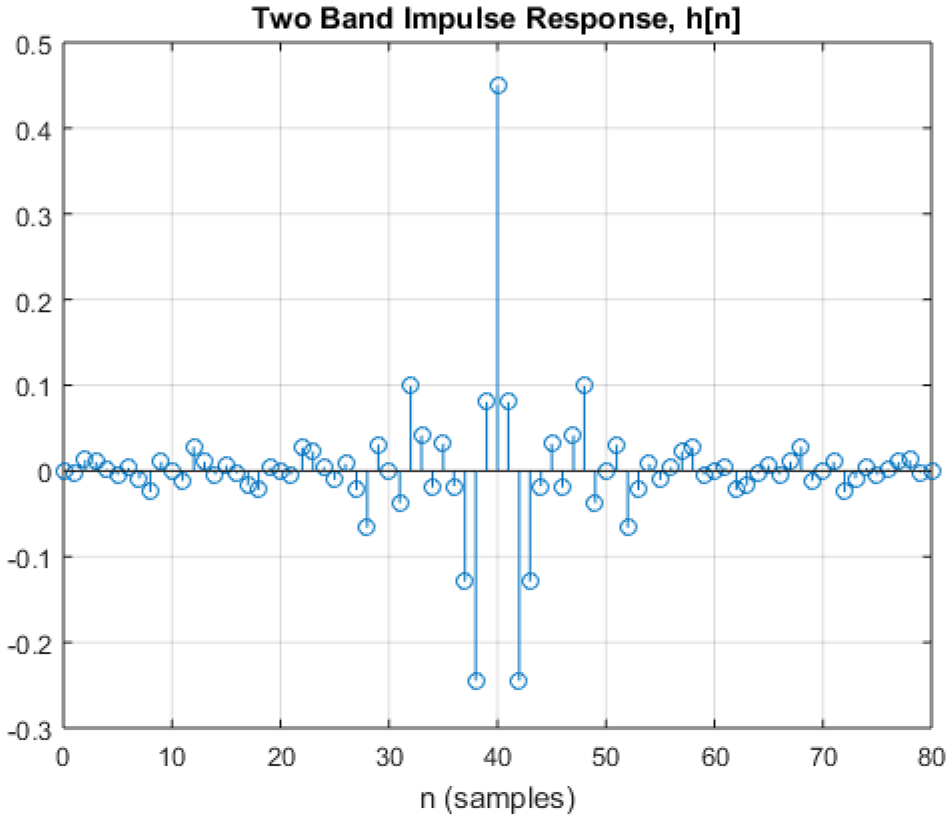
Design a two band filter, using the impulse response truncation techinue, with M=80, f11=0.1, f21=0.3, f12=0.35, f22=0.4, C1=1, C2=0.5. Plot the impulse response on one figure and the amplitude response on a second figure for f from -0.5 to 0.5 cycles/sample. Use the MATLAB function fft to generate the amplitude response.

Define intial values

```
f11=.1; % (cycles/sample)
f21=.3;
f12=.35;
f22=.4;
C1=1;
C2=.5;
M=80; % order of filter
n=0:M; % sample indices
```

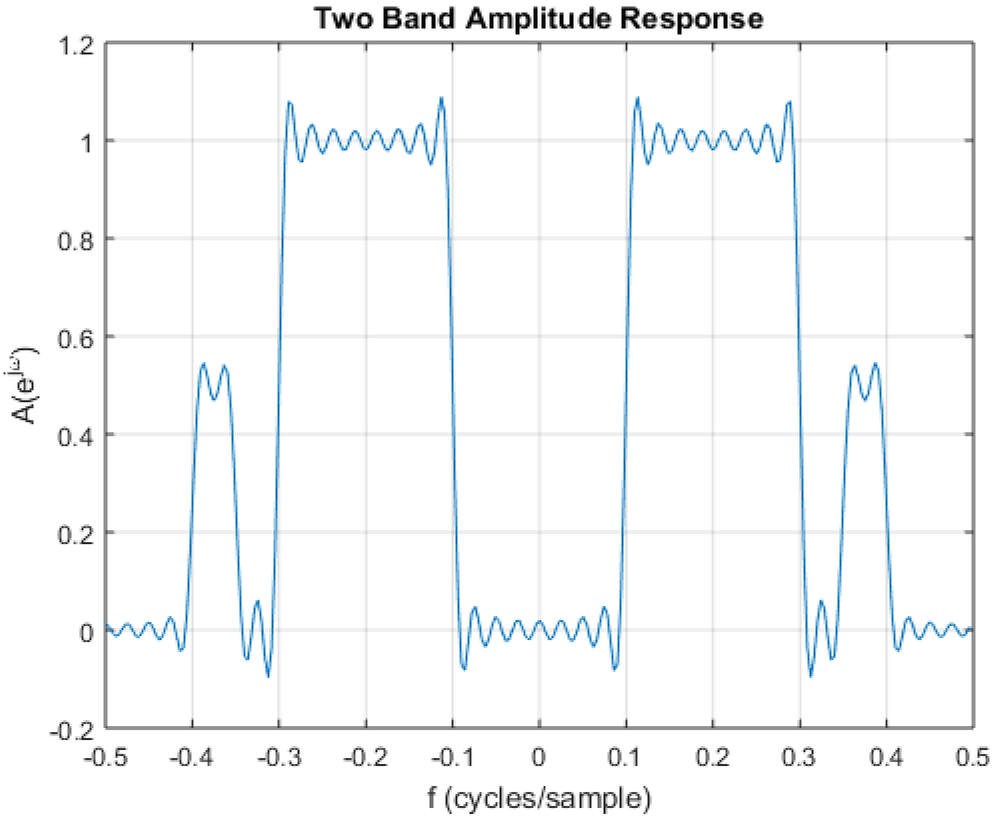
Generate and plot the impulse response, h

```
h1=2*f21*sinc(2*f21*(n-M/2))-2*f11*sinc(2*f11*(n-M/2));
h2=2*f22*sinc(2*f22*(n-M/2))-2*f12*sinc(2*f12*(n-M/2));
h=C1*h1+C2*h2;
figure(1)
clf
stem(n,h)
title('Two Band Impulse Response, h[n]')
xlabel('n (samples)')
grid
```



Generate and plot the amplitude response

```
figure(2)
clf
L=256;
hz=[h,zeros(1,L-length(h))];
H=fft(hz);
k=0:L-1;
% M is even - thus this a type 1 filter with a linear phase response
 of
% -(M/2)*2*pi*f, where f=k/L
W=exp(-j*(-(M/2)*2*pi)*k/L);
A=H.*W;
A=real(A);
plot(k/L-0.5,fftshift(A))
ylabel('A(e^{j\omega})')
title('Two Band Amplitude Response')
xlabel('f (cycles/sample)')
grid
```



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