UNIVERSITY OF SASKATCHEWAN
ELECTRICAL ENGINEERING 461.3

Questions

1. Given the impulse response of a Type 1 linear phase FIR filter,

$$H(e^{j\omega}) = h_0 + h_1 e^{-j\omega} + h_2 e^{-j2\omega} + h_3 e^{-j3\omega} + h_4 e^{-j4\omega}.$$

   (a) What is an expression for the phase response $\theta(\omega)$?

   (b) What is an expression for the amplitude response $A(e^{j\omega})$ (the expression should be a sum of three real terms).

2. Given the impulse response of a Type 4 linear phase FIR filter,

$$H(e^{j\omega}) = h_0 + h_1 e^{-j\omega} + h_2 e^{-j2\omega} + h_3 e^{-j3\omega}.$$

   (a) What is an expression for the phase response $\theta(\omega)$?

   (b) What is an expression for the amplitude response $A(e^{j\omega})$ (the expression should be a sum of two real terms).

3. Design a linear phase and causal FIR filter of length 31 using impulse response truncation to approximate an ideal LPF frequency response given by

$$H_d(e^{j\omega}) = \begin{cases} 1 & \text{if } 0 \leq |f| \leq 1/8, \\ 0 & \text{if } 1/8 < |f| < 0.5. \end{cases}$$

   Write a MATLAB program that plots the impulse response on one figure window and on another figure window $|H(e^{j\omega})|$, $|H(e^{j\omega})|^2$ in dB and the phase response (as the angle of $H(e^{j\omega})$) of this filter as subplots. Appropriately label the plots. Also use the unwrap command for the angle to better view the phase response. Publish your m-file as a pdf document with suitable section headings.

4. Design a linear phase and causal FIR filter of length 31 using impulse response truncation to approximate an ideal HPF frequency response given by

$$H_d(e^{j\omega}) = \begin{cases} 0 & \text{if } 0 \leq |f| < 1/8, \\ 1 & \text{if } 1/8 \leq |f| \leq 0.5. \end{cases}$$

   Write a MATLAB program that plots the impulse response on one figure window, $|H(e^{j\omega})|^2$ in dB on a second figure window and on a third figure window $|H(e^{j\omega})|$, the amplitude response (use the fft in the process of calculating the amplitude response) and phase response (as a straight line) of this filter as subplots. Appropriately label the plots. Publish your m-file as a pdf document with suitable section headings.

5. Convert the LPF in question 3 to a HPF. Use technique 1 (involving an all pass filter).

   (a) What is the cutoff frequency of the HPF? Write a MATLAB program that plots the impulse response on one figure window, $|H(e^{j\omega})|^2$ in dB on a second figure window and on a third figure window $|H(e^{j\omega})|$, the amplitude response (use freqz in the process of calculating the amplitude response) and phase response (as a straight line) of this filter as subplots. Appropriately label the plots. Publish your m-file as a pdf document with suitable section headings.

6. Convert the LPF in question 3 to a HPF. Use technique 2 (involving modulation).
   (a) What is the cutoff frequency of the HPF?
   (b) Write a MATLAB program that plots the impulse response on one figure window, $|H(e^{j\omega})|^2$ in dB on a second figure window and on a third figure window $|H(e^{j\omega})|$, the amplitude response (use freqz in the process of calculating the amplitude response) and phase response (as a straight line) of this filter as subplots. Appropriately label the plots. Publish your m-file as a pdf document with suitable section headings.

7. Design a linear-phase FIR filter with the desired amplitude response

$$A_d(e^{j\omega}) = cos(\omega/2)(1 + cos(\omega))$$

   The order of the filter, M, is required to be no larger than 8. Subject to this requirement, the filter must have the smallest possible integral of square error ϵ.
   (a) Find the filter's coefficients $h[n]$ and the associated value of ϵ. (Hint: You should solve this without using any integrals, instead use Euler's formula.)
   (b) What is the order of the filter?
   (c) What type (1,2,3 or 4) of linear phase filter is $h[n]$.
   (d) What is the phase response $\theta(\omega)$ of the filter?

8. Design a linear-phase FIR filter with the desired amplitude response

$$A_d(e^{j\omega}) = sin(\omega/2)(2 + cos(\omega))$$

   The order of the filter, M, is required to be no larger than 8. Subject to this requirement, the filter must have the smallest possible integral of square error ϵ.
   (a) Find the filter's coefficients $h[n]$ and the associated value of ϵ. (Hint: You should solve this without using any integrals, instead use Euler's formula.)
   (b) What is the order of the filter?
   (c) What type (1,2,3 or 4) of linear phase filter is $h[n]$.
   (d) What is the phase response $\theta(\omega)$ of the filter?

9. An FIR filter can be implemented using the direct form structure. Using the direct form block diagram as a starting point, reduce the number of multiplications, such that the number is a minimum for the FIR filter whose impulse response is

$$h[n] = h[4 - n]; \quad 0 \leq n \leq 4.$$

   (a) Compare the number of multipliers in the direct form and the reduced multiplier direct form.
   (b) Compare the number of adders in the direct form and the reduced multiplier direct form.

10. An FIR filter can be implemented using the direct form structure. Using the direct form block diagram as a starting point, reduce the number of multiplications, such that the number is a minimum for the FIR filter whose impulse response is

$$h[n] = h[5 - n]; \quad 0 \leq n \leq 5.$$

(a) Compare the number of multipliers in the direct form and the reduced multiplier direct form.

(b) Compare the number of adders in the direct form and the reduced multiplier direct form.

11. An LTI system has generalized linear phase and system function $H(z) = a + bz^{-1} + cz^{-2}$. The impulse response has unit energy, $a \geq 0$, and $H(e^{j\pi}) = H(e^{j0}) = 0$.

    (a) Determine the impulse response $h[n]$.

    (b) Plot by hand the amplitude response.

    (c) What is the phase response?

12. An LTI system has generalized linear phase and system function $H(z) = a + bz^{-1}$. The impulse response has unit energy, $a \geq 0$, and $H(e^{j0}) = 0$.

    (a) Determine the impulse response $h[n]$.

    (b) Plot by hand the amplitude response.

    (c) What is the phase response?

13. Design a symmetric linear phase FIR filter with order 80 using impulse response truncation and ideal magnitude response

$$H_d(e^{j\omega}) = \begin{cases} 1 & \text{if } 0.2\pi \leq |\omega| \leq 0.6\pi, \\ 0.5 & \text{if } 0.7\pi < |\omega| \leq 0.8\pi, \\ 0 & \text{elsewhere.} \end{cases}$$

    Write a MATLAB program that plots the impulse response on one figure window, the magnitude response in dB on a second figure window and on a third figure window the magnitude response, amplitude response and phase response of this filter as subplots. Appropriately label the plots. Use an appropriate range for the figure y-axis values. Publish your m-file as a pdf document with suitable section headings.

14. Design a symmetric linear phase FIR filter using impulse response truncation with phase delay $M/2 = 40$ and ideal magnitude response

$$H_d(e^{j\omega}) = \begin{cases} 1 & \text{if } 0 \leq |\omega| \leq \pi/3, \\ 0 & \text{if } \pi/3 < |\omega| < 2\pi/3, \\ 0.5 & \text{if } 2\pi/3 \leq |\omega| \leq \pi. \end{cases}$$

    Write a MATLAB program that plots the impulse response on one figure window, the magnitude response in dB on a second figure window and on a third figure window the magnitude response, amplitude response and phase response of this filter as subplots. Appropriately label the plots. Use an appropriate range for the figure y-axis values. Publish your m-file as a pdf document with suitable section headings.