Neutron Star Physics and the QCD Phase Structure

Honour Uche
University of Lagos
Summer Internship in Computational Nuclear Astrophysics
M.R.P.D Government College Talwara, India
July 2025

1. Introduction to Particle Physics

Particle physics investigates the fundamental constituents of matter and the forces that govern their interactions. It introduces a variety of elementary particles, including leptons, quarks, gauge bosons, and the Higgs boson. These particles interact via four fundamental forces: the strong, electromagnetic, weak, and gravitational forces. Understanding these particles and forces provides a foundation for exploring more complex structures such as nuclear matter and neutron stars.

2. The Standard Model

The Standard Model is a successful theoretical framework that unifies the electromagnetic, weak, and strong interactions under a gauge symmetry described by the group structure $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. While it does not incorporate gravity, it remains one of the most experimentally validated theories in modern physics.

3. Big Bang Theory and Early Universe Timeline

The Big Bang theory outlines the evolution of the universe from an extremely hot and dense state. Key stages include Planck time, inflation, quark-gluon plasma formation, hadron formation, nucleosynthesis, and recombination. These early processes laid the groundwork for matter structures observed today.

4. The QCD Phase Diagram

Quantum Chromodynamics (QCD) describes the behavior of strongly interacting matter. The QCD phase diagram maps different phases of nuclear matter in terms of temperature T and baryon chemical potential μ_B. Understanding this diagram is crucial for modeling matter in extreme environments like neutron stars.

5. Confinement and Asymptotic Freedom

Confinement ensures that quarks cannot exist freely due to the strength of the strong force increasing with distance. Asymptotic freedom describes how quarks interact weakly at high energies or short distances. These properties are key features of QCD.

6. Modeling Nuclear Matter: The Walecka Model

The Walecka Model, or Quantum Hadrodynamics (QHD), models nucleons interacting through meson exchange. It uses a scalar meson (σ) for attraction and a vector meson (ω) for repulsion, successfully reproducing nuclear saturation properties and forming the basis for studying dense astrophysical matter.

7. Saturation Properties of Nuclear Matter

Saturation occurs when the binding energy per nucleon is minimized. Typical properties include saturation density $\rho_0 \approx 0.16\,\mathrm{fm}^{-3}$, binding energy per nucleon $\approx -16\,\mathrm{MeV}$, and compressibility $K \approx 200{-}300\,\mathrm{MeV}$. These are important for validating nuclear models.

8. Lagrangian of the Walecka Model

The model's Lagrangian density includes nucleon and meson fields:

$$\begin{aligned}\mathcal{L} &= \bar{\psi}(i\gamma^\mu\partial_\mu - m_N + g_\sigma\sigma - g_\omega\gamma^\mu\omega_\mu)\psi \\ &\quad + \frac{1}{2}(\partial^\mu\sigma\partial_\mu\sigma - m_\sigma^2\sigma^2) - \frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu\end{aligned}$$

Each term corresponds to kinetic, mass, and interaction contributions for nucleons and mesons.

9. Mean-field Approximation

To simplify the model, the mean-field approximation replaces meson fields with their average values. This leads to a modified Dirac equation with an effective nucleon mass $m^* = m_N - g_\sigma\bar{\sigma}$ and reduces the problem to solvable equations.

10. Equations in the Walecka Model

The key equations under mean-field approximation are:

$$\begin{aligned}\frac{dU}{d\sigma} &= g_\sigma\rho_s, & \omega_0 &= \frac{g_\omega}{m_\omega^2}\rho_B \\ \rho_s &= \langle\bar{\psi}\psi\rangle, & \rho_B &= \langle\psi^\dagger\psi\rangle \\ m^* &= m - g_\sigma\sigma, & \mu^* &= \mu - g_\omega\omega_0\end{aligned}$$

These describe scalar and vector field dynamics, effective mass, and chemical potential.

11. From Model to Astrophysics: CMF and EOS

The Chiral Mean Field (CMF) model builds on the Walecka framework to compute the equation of state (EoS), $P(\epsilon)$, which is essential for solving the Tolman-Oppenheimer-Volkoff (TOV) equations. These predict the structure of neutron stars under general relativity.

12. Neutron Star Predictions and Mass-Radius Relation

Solving the TOV equations with the derived EoS yields mass-radius relations. A stiffer EoS supports higher neutron star masses. Observations of stars above $2M_{\odot}$ place constraints on the EoS and validate the theoretical model.

13. Conclusion

This report explored nuclear matter through the QCD phase diagram and the Walecka model. The mean-field approximation provided solvable equations for dense matter. Using the CMF model, the EoS was applied to neutron stars, producing results consistent with observations. This underscores the importance of combining theory with astrophysical data.