Neutron Star Physics and the QCD Phase Structure

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Introduction to Particle Physics

What is Particle Physics?

Particle physics explores the fundamental constituents of matter and the interactions that govern them. It underpins our understanding of the universe at the smallest scales.

Fundamental Particles

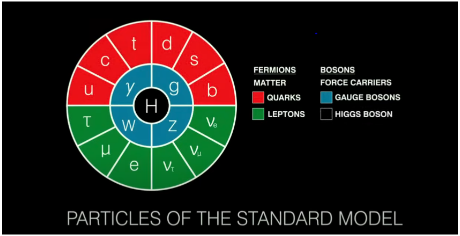
- **Leptons**: e, μ, τ and their neutrinos
- **Quarks**: u, d, s, c, b, t
- **Gauge Bosons**: $\gamma, W^{\pm}, Z^0, g$
- **Higgs Boson**: Gives mass via the Higgs field

Fundamental Forces

- **Strong**: Holds quarks together in nucleons
- **Electromagnetic**: Acts on charged particles
- **Weak**: Causes beta decay and neutrino interactions
- **Gravitational**: Weakest, acts on mass

The Standard Model

- Explains EM, Weak, and Strong interactions.
- Excludes gravity.
- Gauge symmetry: $SU(3)_C \times SU(2)_L \times U(1)_Y$

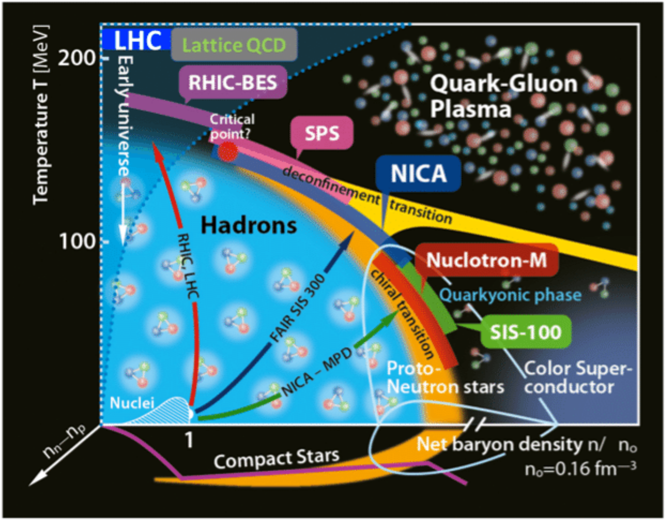


Big Bang Theory and Early Universe Timeline

Timeline from the Big Bang to Nuclei Formation

- **Time = 0:** The Big Bang — beginning of space, time, and energy.
- **10^{-43} s (Planck Time):** Quantum gravity dominates.
- **10^{-36} s:** Cosmic inflation begins — rapid exponential expansion.
- **10^{-32} s:** End of inflation; universe filled with quark-gluon plasma.
- **10^{-6} s:** Quarks combine to form hadrons (protons and neutrons).
- **3 min:** Nucleosynthesis begins — formation of light nuclei (H, He, Li).
- **380,000 years:** Recombination era — atoms form; CMB released.

QCD Phase Diagram

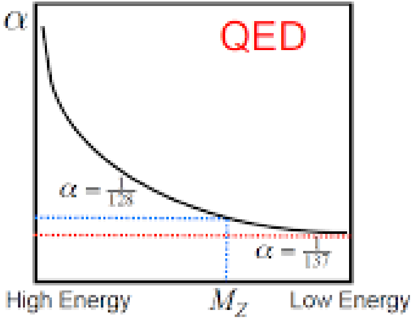


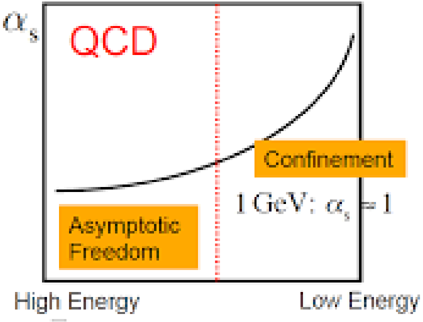
- T (Temperature) vs. μ_B (Baryon Chemical Potential)
- Phases: Hadronic, Quark-gluon plasma, Color superconducting

QCD Concept: Confinement

Definition

Quarks cannot be isolated due to strong interaction. Color-neutral hadrons form.





Confinement and Asymptotic Freedom

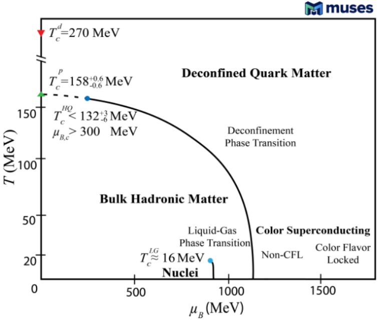
Confinement

Quarks are never found in isolation; they are always confined within hadrons (like protons and neutrons). This is because the strong force, mediated by gluons, becomes stronger as quarks move farther apart.

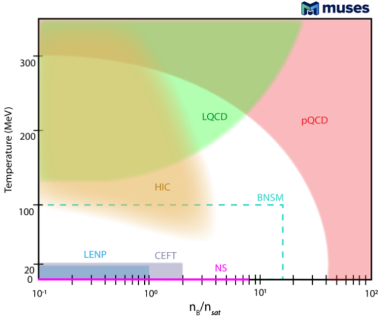
Asymptotic Freedom

At very short distances (or high energies), quarks interact weakly and behave almost like free particles. This property, predicted by quantum chromodynamics (QCD), explains why quarks inside hadrons appear nearly free in high-energy collisions.

QCD Phase Diagram Models- MUSES



R.K., et al., Phys. Rev. D 109, 074008 (2024)



R.K., et al., Living Rev. Rel. 27, 3 (2024)

- MUSES: modeling nuclear EOS
- Locate critical endpoint and crossover

Introduction to the Walecka Model

What is the Walecka Model?

The Walecka Model is a relativistic quantum field theory used to describe the properties of nuclear matter. It models nucleons (protons and neutrons) interacting via the exchange of mesons.

Key Features

- Based on **mean-field approximation**.
- Uses scalar (σ) and vector (ω) mesons to model attraction and repulsion.
- Captures saturation properties of nuclear matter.
- Forms the basis for studying dense matter, such as in neutron stars.

Saturation Properties of Nuclear Matter

What is Saturation?

Nuclear matter exhibits a saturation point where the binding energy per nucleon is minimized and remains roughly constant for large nuclei.

Key Properties

- **Saturation Density** (ρ_0): $\approx 0.16\,\mathrm{fm}^{-3}$; typical density of nuclear matter in heavy nuclei.
- **Binding Energy per Nucleon**: $\approx -16\,\mathrm{MeV}$ at ρ_0; energy required to remove a nucleon.
- **Compressibility (K)**: Measures the stiffness of nuclear matter; typical value $K \approx 200 - 300\,\mathrm{MeV}$.

Lagrangian of the Walecka Model

Lagrangian Density

$$\mathcal{L} = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - m_N + g_{\sigma}\sigma - g_{\omega}\gamma^{\mu}\omega_{\mu})\psi + \frac{1}{2}(\partial^{\mu}\sigma\partial_{\mu}\sigma - m_{\sigma}^2\sigma^2) - \frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} + \frac{1}{2}m_{\omega}^2\omega$$

Explanation of Terms

- ψ: Dirac spinor for nucleons (protons neutrons)
- m_N: Mass of the nucleon
- σ: Scalar meson mediating attraction (g_{σ}: coupling constant)
- ω_{μ}: Vector meson mediating repulsion (g_{ω}: coupling constant)
- $\omega_{\mu\nu} = \partial_{\mu}\omega_{\nu} - \partial_{\nu}\omega_{\mu}$: Field tensor
- m_{σ}, m_{ω}: Meson masses

Mean-field Approximation

What is Mean-field Approximation?

In dense nuclear matter, meson fields are replaced by their classical average values. Quantum fluctuations are neglected, simplifying the field equations.

Mean-field Substitutions

- $\sigma(x) \to \langle \sigma \rangle = \bar{\sigma}$
- $\omega^\mu(x) \to \langle \omega^\mu \rangle = \delta^{\mu 0} \bar{\omega}$

Modified Dirac Equation

$$[i\gamma^\mu \partial_\mu - (m_N - g_\sigma \bar{\sigma}) - g_\omega \gamma^0 \bar{\omega}]\psi = 0$$

- $m^* = m_N - g_\sigma \bar{\sigma}$: Effective nucleon mass

Equations in Walecka Model

Mean-Field Equations of Motion

$$\frac{dU}{d\sigma} = g_\sigma \rho_s \quad \text{(Scalar Field Equation)}$$

$$\omega_0 = \frac{g_\omega}{m_\omega^2}\rho_B \quad \text{(Vector Field Solution)}$$

Densities

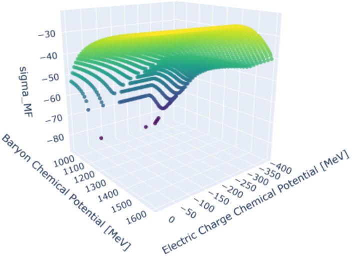
$$\rho_s = \langle \bar{\psi}\psi \rangle \quad \text{(Scalar Density)}, \quad \rho_B = \langle \psi^\dagger \psi \rangle \quad \text{(Baryon Density)}$$

Effective Mass and Chemical Potential

$$m^* = m - g_\sigma \sigma \quad \text{(Effective Mass)}$$

$$\mu^* = \mu - g_\omega \omega_0 \quad \text{(Effective Chemical Potential)}$$

Mean-Fields vs Chemical Potential



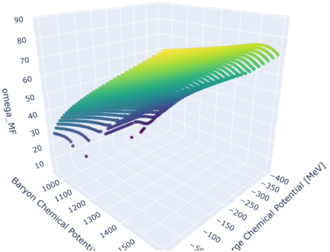
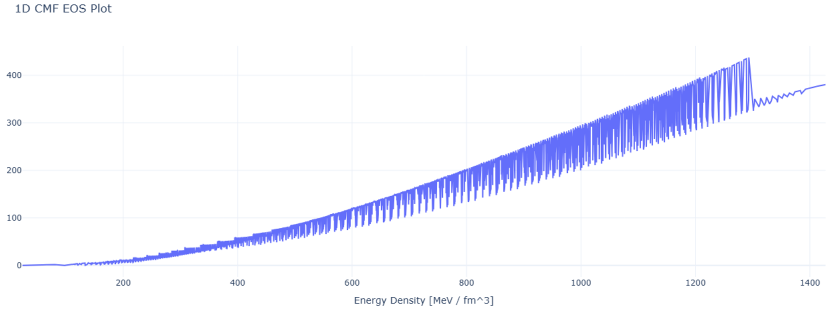


Figure 1 (left) and Figure 2 (right): Mean-field evolution with chemical potential.

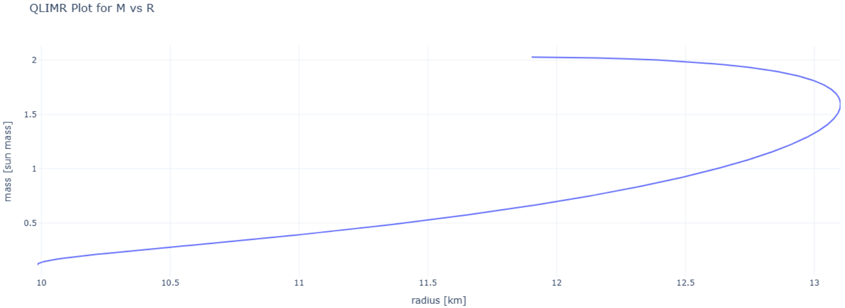
- Scalar field σ and vector field ω_0 reflect chiral symmetry restoration.

Equation of State: $P(\varepsilon)$



- Key input for solving TOV equations.

Mass-Radius Relation of Neutron Stars



- Solving TOV gives max mass and radius.
- EOS stiffness affects maximum mass.

Conclusion

- The structure of nuclear matter was explored using the QCD phase diagram and the Walecka model.
- The mean-field approximation simplified the modeling of interactions in dense nuclear systems.
- The equation of state was computed using the CMF model and applied to neutron star matter.
- Resulting mass-radius relationships align with observational constraints.
- The study highlights the importance of combining field-theoretical models with astrophysical data in high-density physics.