

Homework 1

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Exercise 2

1, 2, 1.5, 1.6666667, 1.6, 1.625, 1.6153846, 1.6190476, 1.6176471, 1.6181818, 1.6179775, 1.6180556,
1.6180258, 1.6180371, 1.6180328, 1.6180344, 1.6180338, 1.6180341, 1.618034, 1.618034, 1.618034, 1.618034,
1.618034, 1.618034, 1.618034, 1.618034, 1.618034, 1.618034, 1.618034

(b) The golden ratio is $\frac{1+\sqrt{5}}{2} \approx 1.618034$. This does seem to be the value that the sequence in part (a) is converging to. We will prove that this is the case.

$$\frac{f_n}{f_{n-1}} = \frac{f_{n-1} + f_{n-2}}{f_{n-1}} = 1 + \frac{f_{n-2}}{f_{n-2} + f_{n-3}} = 1 + \frac{1}{\frac{f_{n-2} + f_{n-3}}{f_{n-2}}}.$$
$$\frac{f_n}{f_{n-1}} = \frac{1}{1 + \frac{1}{1 + \frac{1}{\ddots}}}$$
$$1 + \frac{1}{x} = x \implies x^2 - x - 1 = 0 \implies x = \frac{1 \pm \sqrt{1 - 4 * 1 * -1}}{2 * 1} = \frac{1 \pm \sqrt{5}}{2}.$$

Exercise 3

Checking in R gives us that answer = (3, 21, 23, 6, 11, 15, 12, 22, 30, 24, 13, 29, 17, 26, 27, 3). Note here that the last element of the vector is the same as the first. By just inspecting the sequence of numbers,

the pattern is not easy to see, however if we were to know (or guess) that the next element in the sequence is only dependent on the element preceding it, then we can predict that the successive element of the sequence is 21, as this is also the value of the second element.