

Homework 1

Kenneth Ruiter

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Exercise 2

1, 2, 1.5, 1.6666667, 1.6, 1.625, 1.6153846, 1.6190476, 1.6176471, 1.6181818, 1.6179775, 1.6180556,
1.6180258, 1.6180371, 1.6180328, 1.6180344, 1.6180338, 1.6180341, 1.618034, 1.618034, 1.618034, 1.618034,
1.618034, 1.618034, 1.618034, 1.618034, 1.618034, 1.618034, 1.618034

(b) The golden ratio is $\frac{1+\sqrt{5}}{2} \approx 1.618034$. This does seem to be the value that the sequence in part (a) is converging to. We will prove that this is the case.

$$\frac{f_n}{f_{n-1}} = \frac{f_{n-1} + f_{n-2}}{f_{n-1}} = 1 + \frac{f_{n-2}}{f_{n-2} + f_{n-3}} = 1 + \frac{1}{\frac{f_{n-2} + f_{n-3}}{f_{n-2}}}.$$
$$\frac{f_n}{f_{n-1}} = \frac{1}{1 + \frac{1}{1 + \frac{1}{\ddots}}}$$
$$1 + \frac{1}{x} = x \implies x^2 - x - 1 = 0 \implies x = \frac{1 \pm \sqrt{1 - 4 * 1 * -1}}{2 * 1} = \frac{1 \pm \sqrt{5}}{2}.$$

Exercise 3

Checking in R gives us that answer = (3, 21, 23, 6, 11, 15, 12, 22, 30, 24, 13, 29, 17, 26, 27, 3). Note here that the last element of the vector is the same as the first. By just inspecting the sequence of numbers,

the pattern is not easy to see, however if we were to know (or guess) that the next element in the sequence is only dependent on the element preceding it, then we can predict that the successive element of the sequence is 21, as this is also the value of the second element.

If statements

Exercise 4

```
interest <- function(P, periods){  
  if(periods <= 3){  
    i <- 0.04 #Annual interest for a term 3 years or less  
  } else {  
    i <- 0.05 #Annual interest for a term more than 3 years  
  }  
  
  I <- P*((1+i)^periods - 1)  
  return(I)  
}
```

Exercise 5

```
mortgage <- function(n, P, open){  
  if(open){  
    i <- 0.005 #Interest rate when the mortgage term is open  
  } else {  
    i <- 0.004 #Interest rate when the mortgage term is closed  
  }  
  
  R <- P*i/(1-(1+i)^(-n))  
  return(R)  
}
```

While statements

Exercise 2

```
Fibonacci <- c(1,1)
while(Fibonacci[length(Fibonacci)] + Fibonacci[length(Fibonacci)-1] < 300){
  Fibonacci <- c(Fibonacci, Fibonacci[length(Fibonacci)] + Fibonacci[length(Fibonacci)-1])
}
```

Exercise 4

```
i <- 0.006
iOld <- -1
while(abs(i - iOld) > 0.000001){
  iOld <- i
  i <- (1-(1+i)^(-20))/19
}
```

Using a starting guess of $i = 0.006$ results in an iteratively calculated value of $i = 0.0049541$. The result will be about the same (with a maximal difference of 0.0000001), for any starting value that is not close to 0. This is the case as 0 is also a fixed point, however it is unstable, unlike the value we find for i . Because of our initial choice of $iOld = -1$, a starting value close to -1 would also not give a desired result, however a negative guess for i would not make sense for this example anyway.

Exercise 5

```
i <- 0.006
iOld <- -1
iterations <- 0
while(abs(i - iOld) > 0.000001){
  iterations <- iterations + 1
  iOld <- i
  i <- (1-(1+i)^(-20))/19
}
```

Again starting with $i = 0.006$, we find our desired estimate of i in 74 iterations.