

# POWER SYSTEM ENGINEERING

**Second Edition**



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I J NAGRATH**





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# Preface to the Second Edition

The excellent response to the first edition of the book by students and faculty of Indian and foreign universities and the practicing engineers has motivated the authors to venture for the second edition. The main aim was to include the latest developments in the field of power system engineering.

A large number of teaching appendices of the first edition has now been rewritten as full-fledged chapters, since all these topics continue to be important and are part of the curriculum. We hope students and teachers will welcome this.

This edition covers a wide variety of topics in power system engineering which are normally not found in a single volume. Since the appearance of the first edition in 1994, the overall energy situation has changed considerably and this has generated great interest in nonconventional and renewable energy sources, energy conservation, energy management, power reforms and restructuring and distributed/dispersed generation. Chapter 1 has been therefore, enlarged and completely re-written. In addition, the influences of environmental constraints are also discussed.

In Chapter 6, load flow under power electronic control, that is, AC-DC-LF has been added. In Chapter 7, maintenance scheduling, power system reliability, have been included. For the first time, unit commitment has been further elaborated as an appendix to Chapter 7. In Chapter 8, AGC of restructured power system is added, keeping in line with the latest changes in the power sector.

In Chapter 4, a few more sections/topics such as power transformer have been added. In chapters 2 and 3, magnetic field induction and electrostatic induction have been added, respectively. In Chapter 5, voltage control topic has been boosted by including control by midline boosters. Two appendices, K and I of the first edition on lightning phenomenon and neutral grounding, have now been brought in the main chapters as per the wishes of readers for completeness and clarity. In Chapter 14, new topics such as isolators, fuses and contractors, kilometric faults have now been included as per the review reports. In Chapter 15, numerical (digital) relay has now been introduced along with new trends.

The present edition, like the earlier one, is designed for a two-semester course at the undergraduate level or for first-semester postgraduate study.

With all these features, this is an indispensable text for electrical engineering students. AMIE, GATE, and UPSC Engineering services and IAS candidates along with practicing engineers would also find this book extremely valuable as a text/reference book.

A first-level PG course may be taught from sections (1.17, 1.18) chapters 6, 7, 8, sections 9.6, 9.7, 11.7, chapters 12, 13, 15, 20, 22. For UG courses a combination of chapters may be chosen depending on the syllabus of a university and type of the course.

## Salient Features

- Recent developments in various power system topics included
- Computational algorithm for various system studies presented
- Large number of solved examples and unsolved problems with answers presented at the end of each chapter for practice and self-evaluation
- New chapter added on voltage stability
- Old appendices of the first edition have been enlarged into full-fledged chapters 16–21 such as HVDC and Distribution Systems
- New appendices on:
  - MATLAB and SIMULINK demonstrating their use in problem solving
  - Real time computer control of power systems
  - Power Quality

MATLAB and SIMULINK ideal programs for power system analysis are included in this book as an appendix along with 18 solved examples illustrating their use in solving representative power system problems.

A new chapter on voltage stability has been added. A new appendix on real time computer control of power systems has also been added to more students aware of latest methods of power system control and monitoring in load dispatch centers. A new appendix on power quality has also been included.

Tata McGraw-Hill and the authors would like to thank the following reviewers of this edition: Prof. S. Dasgupta of S.I.T. Siliguri, Prof. P. R. Bijwe of I.I.T. Delhi, Prof. S. Roy of B.I.T.S. Pilani, PG and doctoral students of the first author, Mr. Sunil Bhat of VNIT Nagpur, Mr. Jayaprakash of GCEK Kerala, Mr. Praveen Verma, Mr. Abhishek Rathore, Mr. Jitender Malik, Mr. Vijay Pratap, Mr. Rajeev Ranjan Kumar, Mr. Sivananda, Dr. Subir Sen of Power Grid, Dr. Shekhar of Alstrom, Mr. K.P. Singh of NPTI Guwahati for their help in preparing the manuscript.

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While revising the text, we have been greatly encouraged by many colleagues, students and practicing engineers, reviewers who used the earlier edition of this book. All these individuals have influenced this edition. We express our thanks and appreciation to them. We hope this support/response would continue in the future also.

We also thank TMH personnel and our families who supported us during this period and given all possible help so that this book can see the light of the day.

New Delhi

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**D.P. Kothari  
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# Preface to the First Edition

Mathematical modelling and solutions on digital computers constitute an extremely viable approach to system analysis and planning studies for a modern-day power system with its large size and complex and integrated nature. A stage has, therefore, been reached where an undergraduate must be trained in the latest techniques of analysis of large-scale power systems. A similar need also exists in the industry where a practicing power system engineer is constantly faced with the challenge of the rapid advances in the field. This book has been designed to fulfil this need by integrating the basic principles of power system analysis illustrated through the simplest system structure with analysis techniques for practical size systems. In this book large-scale system analysis follows as a natural extension of the basic principles. The form and level of some of the well-known techniques are presented in such a manner that undergraduates can easily grasp and appreciate them.

The book covers a wide variety of topics in power system engineering which are normally not found in a single volume. The book is written in such a comprehensive manner that at least three courses on power systems can be designed—one at the postgraduate level and a two-semester sequence at the undergraduate level.

The reader is expected to have a prior grounding in circuit theory and electrical machines. He should also have been exposed to Laplace transform, linear differential equations, elementary optimization techniques and a first course in control theory. Matrix analysis is applied throughout the book. However, a knowledge of simple matrix operations would suffice and these are summarized in an appendix for quick reference.

The digital computer is an indispensable tool for power system analysis, and therefore, computational algorithms for various system studies such as load flow, fault level analysis, stability, etc. have been included at appropriate places in the book. The students should be encouraged to design computer programs for these studies using the algorithms provided. Further, the students can be asked to pool the various programs for more advanced and sophisticated studies such as optimal scheduling. A novel feature of the book is the inclusion of current trends that are practically useful such as unit commitment, generation reliability, optimal thermal scheduling, optimal hydrothermal scheduling and decoupled load flow.

The introductory chapter presents a discussion of various methods of electrical energy generation including renewable energy sources and their techno-economic comparison. The reader is also exposed to the Indian power scenario.

Chapters 2 and 3 provide the transmission line parameters and these are included for the sake of completeness of the text. Chapter 4 on the representation of power system components highlights the steady state model of the synchronous machine and the circuit models of composite power systems along with the per unit method.

Chapter 5 deals with the performance of transmission lines. The load flow problem is introduced at this stage through the simple two-bus system and basic concepts of watt and var control are illustrated. A brief treatment of circle diagrams is included as this forms an excellent teaching aid for putting across the concept of load flow and line compensation.

Chapter 6 elaborates on power network modelling and important techniques of load flow analysis like Gauss–Siedel, Newton–Raphson and decoupled load flow. Chapter 7 deals with optimal system operation for both thermal and hydrothermal systems. A rigorous treatment for thermal system is also presented.

Chapter 8 deals with load frequency control wherein both conventional and modern control approaches have been adopted for analysis and design. The chapter also covers the treatment of generation rate constraint. Voltage control is also discussed briefly.

Chapters 9–11 discuss fault studies (abnormal system operation). The synchronous machine model for transient studies is heuristically introduced to the reader.  $Z_{BUS}$  algorithm is presented and its use illustrated for both symmetrical and unsymmetrical faults.

Chapter 12 elaborates upon the concepts of various types of stability in power system. In particular, the concept of transient stability is well illustrated through the equal area criterion. The classical numerical solution technique of the swing equation as well as the algorithm for large system stability are also dealt with. A step-by-step solution of a 3-machine stability problem is presented.

Chapter 13 deals with power system transients. Topics such as traveling waves or propagation of surges, generation of over-voltages on lines, insulation coordination are discussed at length.

Chapter 14 presents a detailed account of various types of circuit breakers including HVDC breakers. Methods of testing circuit breakers are also explained.

Chapter 15 covers the important topic of power system protection. It includes an exhaustive survey of relaying schemes and also deals with the different types of protective relays used for the protection of various parts of power systems along with their theory, threshold characteristics as well as their merits and demerits. Microprocessor based relaying is also briefly explained.

A large number of appendices have been provided to deal with topics such as Cables, Insulators, Sag and Tension, Neutral Grounding, Corona, Lightning Phenomena, HVDC and Distribution Systems. These topics are still being taught and continue to be very important.

Every concept and technique presented is supported through examples employing a two-bus structure while at times, three- and four-bus illustrations have also been used. A large number of unsolved problems with their answers are included at the end of each chapter. These have been so selected that apart from providing a drill they help the reader to develop a deeper insight and illustrate some points beyond what is directly covered by the text.

The organization of various chapters is flexible and permits the teacher to mould them to the particular needs of the class and curriculum. If desired, some of the advanced level topics could be bypassed without loss of continuity. The style of writing is amenable to self-study.

We are indebted to our colleagues at the Birla Institute of Technology and Science, Pilani, and the Indian Institute of Technology, Delhi, for their encouragement and various useful suggestions. We are also grateful to the authorities at BITS, Pilani, and IIT, Delhi, for providing the facilities necessary for writing this book. Further, we would like to thank the National Book Trust for subsidizing the production of the book and thereby facilitating its availability at an affordable price. We welcome any constructive criticism and will be grateful for any appraisal by the readers.

**I.J. Nagrath  
D.P. Kothari**

# **Chapter 1**

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## **Introduction**

### **1.1 ELECTRIC POWER SYSTEM**

We are in need of energy for our industrial, commercial and day-to-day activities, and we use energy in different forms. Out of all the forms of energy, electric energy is the most important one as it can be generated (actually converted from other forms of energy) efficiently, transmitted easily and utilized ultimately at a very reasonable cost. The ease of transmission of electric energy gives rise to a possibility of generating (converting) electric energy in bulk at a centralized place and transmit it over a long distance to be used ultimately by a large number of users. If we generate in small scale, say for example, just to light a house, we can perhaps intuitively make the connections needed for a reasonably reliable and efficient operation. But when we have generation in bulk, transmission over a long distance and utilization by a number of distributed users; we cannot do by intuition. We need to follow systematic methodology to have reliable, efficient, economic and safe use of electric energy. The components needed for generation, transmission and large-scale distribution of electric energy form a huge complex system termed as **Electric Power System**. Power system is the branch of Electrical Engineering where we study in depth for its design, operation, maintenance and analysis.

Electric power systems are a technical wonder and as per one opinion [27], electricity and its accessibility are the greatest engineering achievements of the 20th century, ahead of computers and airplanes. A modern society cannot exist without electricity. As will be explained in Secs 1.17 and 1.18, todays centralized (regulated) utilities will be distributed (deregulated) when tomorrow utilities (SEBs) have been forced to breakup in separate generation, and T/D companies. There is DG (distributed generation) by IPP (independent power producers), who can generate electric power by whatever means and must be allowed access (open access) to the power grid to sell power to consumers. The breakup has been encouraged by tremendous benefits of deregulation in communication and airline industries resulting in fierce competition leading to economy and better consumer service. In India, some

states are pursuing this deregulation and unbundling aggressively and some more cautiously. The aim is that the independent Transmission System Operators (TSO) wheel power for a charge from anywhere and anyone to the customer site.

Reliable Operation is ensured by the TSOs and the financial transactions are governed by real time bidding to buy and sell power to earn profit in the spot market. (Buying at lower prices and selling at higher prices).

## 1.2 INDIAN POWER SECTOR

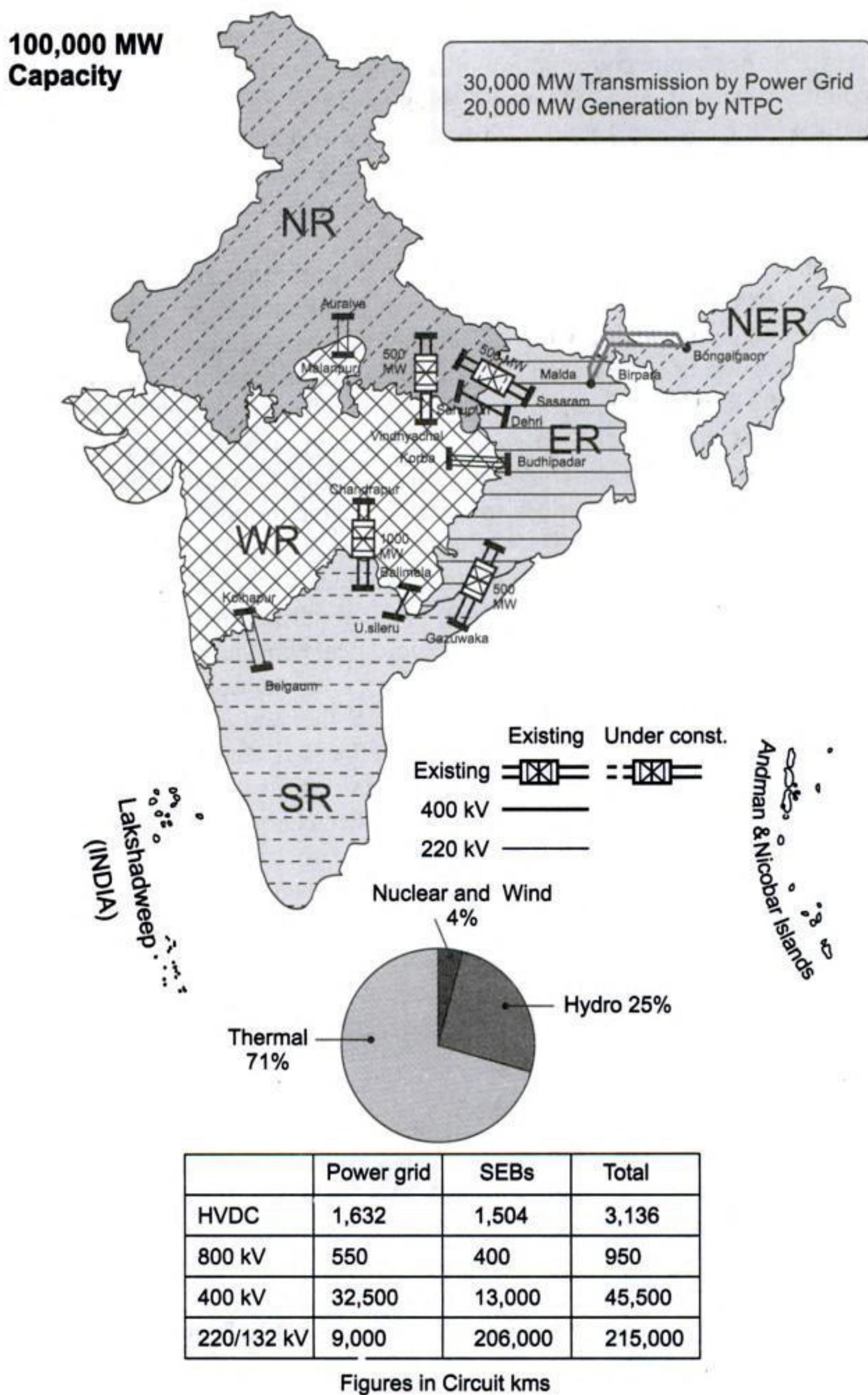
### Historical Background

More than 100 years back in 1890s first hydro power plant was commissioned in Darjeeling. At the time of independence, total installed capacity was 1360 MW (mostly owned by private companies in cities). After the enactment of Electricity (Supply) Act in 1948, barring few licenses, the entire Power Sector was owned by State Governments and largely managed by vertically integrated State Electricity Boards (SEBs). In 1975, Central Government, through Central Public Sector Undertakings, (NTPC etc.) also entered in the field of Generation and Transmission to supplement the efforts of cash starved State Electricity Boards. In 1989 Power Grid Corporation of India was formed to develop the transmission network and grid. In 1990 first HVDC bi-pole line was made operative. In 1990, power generation was opened to private sector. In 1998 Electricity Regulatory Commission Act was enacted for establishing Regulatory Commissions. Tariff is now obtained through competitive bidding to be adopted by the regulator. Congestion is managed by e-bidding.

In 1998 first 765 kV Transmission line was erected which was initially charged at 400 kV. In 2003 Electricity Act 2003 was enacted to have open access in transmission. In 2005–06 National Electricity Plan was finalized. In 2006 a big step was taken for formulation of national grid [Fig. 1.1(a)] by way of synchronization of NR with ER-NER-WR. In 2007, 765 kV transmission will be a reality. In 2010–11, 800 kV HVDC bi-pole line will start operating. Challenge is evacuation of power along with generation addition and also from surplus to deficit area. As of today (2007) India has an energy shortage of 7.8% and peak shortage of 11.23%.

## 1.3 A CONTEMPORARY PERSPECTIVE

Electric energy is an essential ingredient for the industrial and all-round development of any country. It is a coveted form of energy, because it can be generated centrally in bulk and transmitted economically over long distances. Further, it can be adapted easily and efficiently to domestic and industrial



**Fig. 1.1(a)** Development of Indian Grid (Courtesy ALSTOM)

applications, particularly for lighting purposes and mechanical work\*, e.g. drives. The per capita consumption of electrical energy is a reliable indicator of a country's state of development—figures for 2006 are 650 kWh for India and 5600 kWh for UK and 15000 kWh for USA, world average is 3000 kWh.

Conventionally, electric energy is obtained by conversion from fossil fuels (coal, oil, natural gas), nuclear and hydro sources. Heat energy released by burning fossil fuels or by fission of nuclear material is converted to electricity by first converting heat energy to the mechanical form through a thermocycle and then converting mechanical energy through generators to the electrical form. Thermocycle is basically a low efficiency process—highest efficiencies for modern large size plants range up to 40%, while smaller plants may have considerably lower efficiencies. The earth has fixed non-replenishable resources of fossil fuels and nuclear materials, with certain countries overendowed by nature and others deficient. Hydro energy, though replenishable, is also limited in terms of power. The world's increasing power requirements can only be partially met by hydro sources. Furthermore, ecological and biological factors place a stringent limit on the use of hydro sources for power production. (The USA has already developed around 50% of its hydro potential and hardly any further expansion is planned because of ecological considerations.)

With the ever increasing per capita energy consumption and exponentially rising population, technologists already see the end of the earth's non-replenishable fuel resources.† The oil crisis of the 1970s has dramatically drawn attention to this fact. In fact, we can no longer afford to use oil as a fuel for generation of electricity. In terms of bulk electric energy generation, a distinct shift is taking place across the world in favour of coal and in particular nuclear sources for generation of electricity. Also, the problems of air and thermal pollution caused by power generation have to be efficiently tackled to avoid ecological disasters. A coordinated worldwide action plan is, therefore, necessary to ensure that energy supply to humanity at large is assured for a long time and at low economic cost. Some of the factors to be considered and actions to be taken are:

### ***Curtailment of Energy Consumption***

The energy consumption of most developed countries has already reached a level, which this planet cannot afford. There is, in fact, a need to find ways and means of reducing this level. The developing countries, on the other hand, have

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\* Electricity is a very inefficient agent for heating purposes, because it is generated by the low efficiency thermocycle from heat energy. Electricity is used for heating purposes for only very special applications, say an electric furnace.

† Varying estimates have been put forth for reserves of oil, gas and coal and fissionable materials. At the projected consumption rates, oil and gases are not expected to last much beyond 50 years; several countries will face serious shortages of coal after 2200 A.D. while fissionable materials may carry us well beyond the middle of the next century. These estimates, however, cannot be regarded as highly dependable.

to intensify their efforts to raise their level of energy production to provide basic amenities to their teeming millions. Of course, in doing so they need to constantly draw upon the experiences of the developed countries and guard against obsolete technology.

### ***Intensification of Efforts to Develop Alternative Sources of Energy Including Unconventional Sources like Solar, Tidal Energy, etc.***

Distant hopes are pitched on fusion energy but the scientific and technological advances have a long way to go in this regard. Fusion when harnessed could provide an inexhaustible source of energy. A break-through in the conversion from solar to electric energy could provide another answer to the world's steeply rising energy needs.

### ***Recycling of Nuclear Wastes***

Fast breeder reactor technology is expected to provide the answer for extending nuclear energy resources to last much longer.

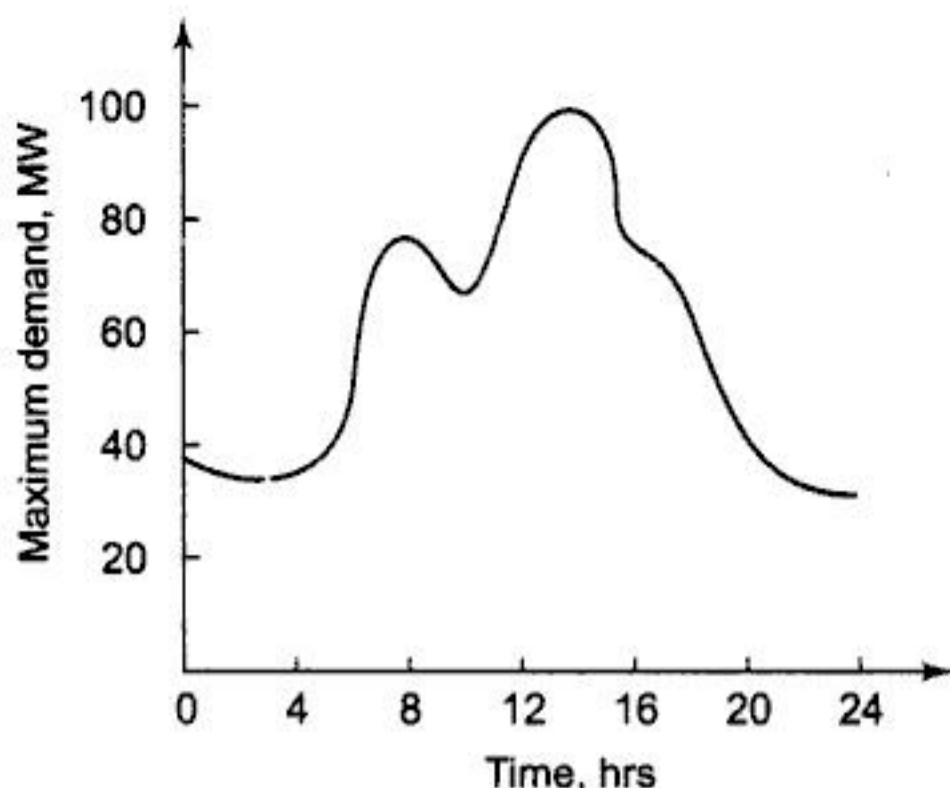
### ***Development and Application of Antipollution Technologies***

In this regard, the developing countries already have the example of the developed countries whereby they can avoid going through the phases of intense pollution in their programmes of energy development. Bulk power generating stations are more easily amenable to control of pollution since centralized one-point measures can be adopted.

Electric energy today constitutes about 30% of the total annual energy consumption on a worldwide basis. This figure is expected to rise as oil supply for industrial uses becomes more stringent. Transportation can be expected to go electric in a big way in the long run, when non-conventional energy resources are well developed or a breakthrough in fusion is achieved.

To understand some of the problems that the power industry faces let us briefly review some of the characteristic features of generation and transmission. Electricity, unlike water and gas, cannot be stored economically (except in very small quantities—in batteries), and the electric utility can exercise little control over the load (power demand) at any time. The power system must, therefore, be capable of matching the output from generators to the demand at any time at a specified voltage and frequency. The difficulty encountered in this task can be imagined from the fact that load variations over a day comprises three components—a steady component known as **base load**; a varying component whose daily pattern depends upon the time of day; weather, season, a popular festival, etc.; and a purely randomly varying component of relatively small amplitude. Figure 1.1(b) shows a typical daily load curve. The characteristics of a daily load curve on a gross basis are indicated by **peak load**, and the time of its occurrence and **load factor** defined as

$$\frac{\text{average load}}{\text{maximum (peak) load}} = \text{less than unity}$$



**Fig. 1.1(b)** Typical daily load curve

The average load determines the energy consumption over the day, while the peak load along with considerations of standby capacity determines plant capacity for meeting the load.

A high load factor helps in drawing more energy from a given installation. As individual load centres have their own characteristics, their peaks in general have a time diversity, which when utilized through transmission interconnection, greatly aids in jacking up load factors at an individual plant—excess power of a plant during light load periods is evacuated through long distance high voltage transmission lines, while a heavily loaded plant receives power.

## Diversity Factor

This is defined as the sum of individual maximum demands on the consumers, divided by the maximum load on the system. This factor gives the time diversification of the load and is used to decide the installation of sufficient generating and transmission plant. If all the demands came at the same time, i.e. unity diversity factor, the total installed capacity required would be much more. Luckily, the factor is much higher than unity, especially for domestic loads.

A high diversity factor could be obtained by:

1. Giving incentives to farmers and/or some industries to use electricity in the night or light load periods.
2. Using day-light saving as in many other countries.
3. Staggering the office timings.
4. Having different time zones in the country like USA, Australia, etc.
5. Having two-part tariff in which consumer has to pay an amount dependent on the maximum demand he makes, plus a charge for each unit of energy consumed. Sometimes consumer is charged on the basis of kVA demand instead of kW to penalize loads of low power factor.

Two other factors used frequently are:

### *Plant capacity factor*

$$= \frac{\text{Actual energy produced}}{\text{maximum possible energy that could have been produced} \\ (\text{based on installed plant capacity})}$$

$$= \frac{\text{Average demand}}{\text{Installed capacity}}$$

### *Plant use factor*

$$= \frac{\text{Actual energy produced (kWh)}}{\text{plant capacity (kW)} \times \text{Time (in hours) the plant has been in operation}}$$

### **Tariffs**

The cost of electric power is normally given by the expression  $(a + b \times \text{kW} + c \times \text{kWh})$  per annum, where  $a$  is a fixed charge for the utility, independent of the power output;  $b$  depends on the maximum demand on the system and hence on the interest and depreciation on the installed power station; and  $c$  depends on the units produced and therefore on the fuel charges and the wages of the station staff.

Tariff structures may be such as to influence the load curve and to improve the load factor.

Tariff should consider the pf (power factor) of the load of the consumer. If it is low, it takes more current for the same kWs and hence  $T$  and  $D$  (transmission and distribution) losses are correspondingly increased. The power station has to install either pf correcting (improvement) devices such as synchronous capacitors, SVC (Static Var Compensator) or voltage regulating equipment to maintain the voltages within allowed limits and thus total cost increases. One of the following alternatives may be used to avoid low pf:

- (i) to charge the consumers based on kVA rather than kW.
- (ii) a pf penalty clause may be imposed on the consumer.
- (iii) the consumer may be asked to use shunt capacitors for improving the power factor of his installations.

### **Availability Based Tariffs (ABT)\***

ABT comprises three main components viz. capacity charge, energy charge and charges for deviation from schedule.

1. Capacity charge, towards reimbursement of the fixed cost of the plant, linked to the plant's capacity to supply MWs.
2. Energy charge, to reimburse the fuel cost for scheduled generation and

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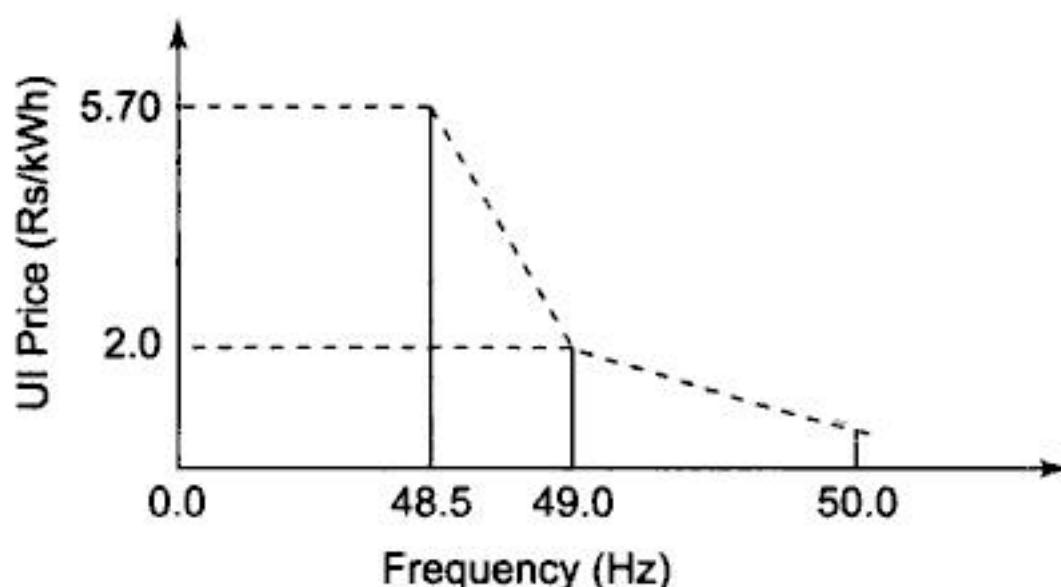
\* The term Availability Tariff, particularly in the Indian context, stands for a rational tariff structure for power supply from generating stations, on a contracted basis. In the availability tariff mechanism, the fixed and variable cost components are treated separately. The payment of fixed cost to the generation company is linked to availability of the plant, i.e. its capability to deliver MWs on a day-by-day basis.

The reader may refer to 'ABC of ABT by Mr. Bhanu Bhushan of Central Electricity Regulatory Commission (27.6.2005) for further details.

3. Payment for deviations from schedule at a rate dependent on system conditions. The last component would be negative in case the power plant is delivering less power than scheduled. For example: If a power plant delivers 600 MW while it was scheduled to supply only 500 MW, the energy charge payment would be for the scheduled generation (500 MW) only, and the excess generation (100 MW) would be paid for at a certain rate.

If the grid has surplus power at that time and frequency is above 50 Hz. The rate would be small. If the excess generation is at the time of generation deficit in the system (frequency below 50.0 Hz) the payment for extra generation would be at higher rate.

If frequency ( $F$ ) is 49 Hz or below, UI (unscheduled interchange) prices is maximum (570 paise per unit), and the price is minimum (zero paisa), when frequency is 50.5 Hz or above. [see Fig. 1.2].



**Fig. 1.2**

- If frequency is in between 49.0 Hz and 50 Hz, the UI prices varies linearly as, UI rate =  $187 - 3.7f$ .
- If frequency is in between 50 Hz and 50.5 Hz, the UI price is given by UI rate =  $202 - 4.0 \times f$ .
- Maximum value of ABT is fixed, according to the cost of generation of the costliest generating unit (diesel generating plants).

ABT has been successfully adopted for maintaining the grid discipline and already been implemented in Western Region w.e.f. 1.7.2002, in Northern Region w.e.f. 1.12.2002 and in Southern Region w.e.f. 1.1.2003.

**Example 1.1** A factory to be set up is to have a fixed load of 760 kW at 0.8 pf. The electricity board offers to supply energy at the following alternate rates:

- (a) LV supply at Rs 32/kVA max demand/annum + 10 paise/kWh
- (b) HV supply at Rs 30/kVA max demand/annum + 10 paise/kWh

The HV switchgear costs Rs 60/kVA and switchgear losses at full load amount to 5%. Interest, depreciation charges for the switchgear are 12% of the capital cost. If the factory is to work for 48 hours/week, determine the more economical tariff.

**Solution**

$$\text{Maximum demand} = \frac{760}{0.8} = 950 \text{ kVA}$$

Loss in switchgear = 5%

$$\therefore \text{Input demand} = \frac{950}{0.95} = 1000 \text{ kVA}$$

$$\text{Cost of switchgear} = 60 \times 1000 = \text{Rs } 60,000$$

$$\text{Annual charges on depreciation} = 0.12 \times 60,000 = \text{Rs } 7,200$$

$$\begin{aligned}\text{Annual fixed charges due to maximum demand corresponding to tariff (b)} \\ &= 30 \times 1,000 = \text{Rs } 30,000\end{aligned}$$

$$\begin{aligned}\text{Annual running charges due to kWh consumed} \\ &= 1000 \times 0.8 \times 48 \times 52 \times 0.10 \\ &= \text{Rs } 1,99,680\end{aligned}$$

$$\text{Total charges/annum} = \text{Rs } 2,36,880$$

$$\text{Max. demand corresponding to tariff (a)} = 950 \text{ kVA}$$

$$\text{Annual fixed charges} = 32 \times 950 = \text{Rs } 30,400$$

$$\begin{aligned}\text{Annual running charges for kWh consumed} \\ &= 950 \times 0.8 \times 48 \times 52 \times 0.10 \\ &= \text{Rs } 1,89,696 \\ \text{Total} &= \text{Rs } 2,20,096\end{aligned}$$

Therefore, tariff (a) is economical.

**Example 1.2** A region has a maximum demand of 500 MW at a load factor of 50%. The load duration curve can be assumed to be a triangle. The utility has to meet this load by setting up a generating system, which is partly hydro and partly thermal. The costs are as under:

Hydro plant: Rs 600 per kW per annum and operating expenses at 3p per kWh.

Thermal plant: Rs 300 per kW per annum and operating expenses at 13p per kWh.

Determine the capacity of hydro plant, the energy generated annually by each, and overall generation cost per kWh.

**Solution**

$$\begin{aligned}\text{Total energy generated per year} &= 500 \times 1000 \times 0.5 \times 8760 \\ &= 219 \times 10^7 \text{ kWh}\end{aligned}$$

Figure 1.3 shows the load duration curve. Since operating cost of hydro plant is low, the base load would be supplied from the hydro plant and peak load from the thermal plant.

Let the hydro capacity be  $P$  kW and the energy generated by hydro plant  $E$  kWh/year.

$$\text{Thermal capacity} = (5,00,000 - P) \text{ kW}$$

$$\text{Thermal energy} = (219 \times 10^7 - E) \text{ kWh}$$

Annual cost of hydro plant

$$= 600P + 0.03E$$

Annual cost of thermal plant

$$= 300(5,00,000 - P) + 0.13$$

$$(219 \times 10^7 - E)$$

$$\text{Total cost } C = 600P + 0.03E$$

$$+ 300(5,00,000 - P) + 0.13$$

$$(219 \times 10^7 - E)$$

$$\text{For minimum cost, } \frac{dC}{dP} = 0$$

$$\therefore 600 + 0.03 \frac{dE}{dP} - 300 - 0.13$$

$$\frac{dE}{dP} = 0$$

or

$$dE = 3000dP$$

But

$$dE = dP \times t$$

$\therefore$

$$t = 3000 \text{ hours}$$

From  $\Delta ADF$  and  $\Delta ABC$ ,

$$\frac{5,00,000 - P}{5,00,000} = \frac{3000}{8760}$$

$\therefore$

$$P = 328, \text{ say } 330 \text{ MW}$$

Capacity of thermal plant = 170 MW

$$\text{Energy generated by thermal plant} = \frac{170 \times 3000 \times 1000}{2}$$

$$= 255 \times 10^6 \text{ kWh}$$

Energy generated by hydro plant =  $1935 \times 10^6$  kWh

Total annual cost = Rs  $340.20 \times 10^6$ /year

$$\text{Overall generation cost} = \frac{340.20 \times 10^6}{219 \times 10^7} \times 100$$

$$= 15.53 \text{ paise/kWh}$$

**Example 1.3** A generating station has a maximum demand of 25 MW, a load factor of 60%, a plant capacity factor of 50%, and a plant use factor of 72%. Find (a) the daily energy produced, (b) the reserve capacity of the plant, and (c) the maximum energy that could be produced daily if the plant, while running as per schedule, were fully loaded.

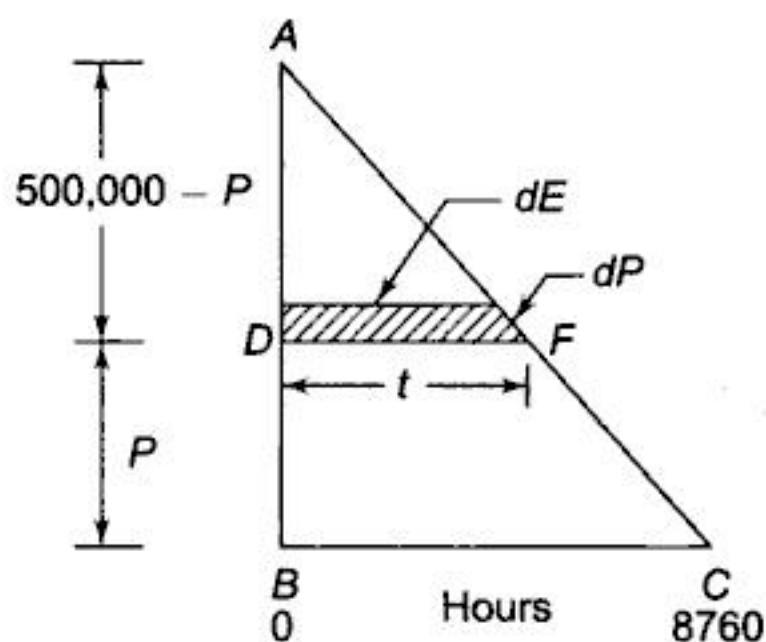


Fig. 1.3 Load duration curve

**Solution**

$$\text{Load factor} = \frac{\text{average demand}}{\text{maximum demand}}$$

$$0.60 = \frac{\text{average demand}}{25}$$

$$\therefore \text{Average demand} = 15 \text{ MW}$$

$$\text{Plant capacity factor} = \frac{\text{average demand}}{\text{installed capacity}}$$

$$0.50 = \frac{15}{\text{installed capacity}}$$

$$\therefore \text{Installed capacity} = \frac{15}{0.5} = 30 \text{ MW}$$

$$\begin{aligned}\therefore \text{Reserve capacity of the plant} &= \text{installed capacity} - \text{maximum demand} \\ &= 30 - 25 = 5 \text{ MW}\end{aligned}$$

$$\begin{aligned}\text{Daily energy produced} &= \text{average demand} \times 24 = 15 \times 24 \\ &= 360 \text{ MWh}\end{aligned}$$

$$\begin{aligned}\text{Energy corresponding to installed capacity per day} \\ &= 24 \times 30 = 720 \text{ MWh}\end{aligned}$$

Maximum energy that could be produced

$$= \frac{\text{actual energy produced in a day}}{\text{plant use factor}}$$

$$= \frac{360}{0.72} = 500 \text{ MWh/day}$$

**Example 1.4** From a load duration curve, the following data are obtained:

Maximum demand on the system is 20 MW. The load supplied by the two units is 14 MW and 10 MW. Unit No. 1 (base unit) works for 100% of the time, and Unit No. 2 (peak load unit) only for 45% of the time. The energy generated by Unit 1 is  $1 \times 10^8$  units, and that by Unit 2 is  $7.5 \times 10^6$  units. Find the load factor, plant capacity factor and plant use factor of each unit, and the load factor of the total plant.

**Solution**

$$\text{Annual load factor for Unit 1} = \frac{1 \times 10^8 \times 100}{14,000 \times 8760} = 81.54\%$$

The maximum demand on Unit 2 is 6 MW.

$$\text{Annual load factor for Unit 2} = \frac{7.5 \times 10^6 \times 100}{6000 \times 8760} = 14.27\%$$

Load factor of Unit 2 for the time it takes the load

$$= \frac{7.5 \times 10^6 \times 100}{6000 \times 0.45 \times 8760}$$

$$= 31.71\%$$

Since no reserve is available at Unit No. 1, its capacity factor is the same as the load factor, i.e. 81.54%. Also since Unit 1 has been running throughout the year, the plant use factor equals the plant capacity factor, i.e. 81.54%.

$$\text{Annual plant capacity factor of Unit 2} = \frac{7.5 \times 10^6 \times 100}{10 \times 8760 \times 1000} = 8.56\%$$

$$\text{Plant use factor of Unit 2} = \frac{7.5 \times 10^6 \times 100}{10 \times 0.45 \times 8760 \times 1000} = 19.02\%$$

$$\text{The annual load factor of the total plant} = \frac{1.075 \times 10^8 \times 100}{20,000 \times 8760} = 61.35\%$$

*Comments:* The various plant factors, the capacity of base and peak load units can thus be found out from the load duration curve. The load factor of the peak load unit is much less than that of the base load unit, and thus the cost of power generation from the peak load unit is much higher than that from the base load unit.

**Example 1.5** There are three consumers of electricity having different load requirements at different times. Consumer 1 has a maximum demand of 5 kW at 6 p.m. and a demand of 3 kW at 7 p.m. and a daily load factor of 20%. Consumer 2 has a maximum demand of 5 kW at 11 a.m., a load of 2 kW at 7 p.m. and an average load of 1200 W. Consumer 3 has an average load of 1 kW and his maximum demand is 3 kW at 7 p.m. Determine: (a) the diversity factor, (b) the load factor and average load of each consumer, and (c) the average load and load factor of the combined load.

**Solution**

(a) Consumer 1	MD 5 kW at 6 p.m.	3 kW at 7 p.m.	LF 20%
Consumer 2	MD 5 kW at 11 a.m.	2 kW at 7 p.m.	Average load 1.2 kW
Consumer 3	MD 3 kW at 7 p.m.		Average load 1 kW

Maximum demand of the system is 8 kW at 7 p.m.

Sum of the individual maximum demands =  $5 + 5 + 3 = 13 \text{ kW}$

$$\therefore \text{Diversity factor} = 13/8 = 1.625$$

(b) Consumer 1, Average load  $0.2 \times 5 = 1 \text{ kW}$ , LF = 20%

$$\text{Consumer 2, Average load } 1.2 \text{ kW, LF} = \frac{1.2}{5} \times 1000 = 24\%$$

Consumer 3, Average load 1 kW,  $LF = \frac{1}{3} \times 100 = 33.3\%$

(c) Combined average load  $= 1 + 1.2 + 1 = 3.2$  kW

$\therefore$  Combined load factor  $= \frac{3.2}{8} \times 100 = 40\%$

## Load Forecasting

As power plant planning and construction require a gestation period of four to eight years or even longer for the present day super power stations, energy and load demand forecasting plays a crucial role in power system studies.

This necessitates long range forecasting. While sophisticated probabilistic methods exist in literature [5, 16, 28], the simple extrapolation technique is quite adequate for long range forecasting. Since weather has a much more influence on residential than the industrial component, it may be better to prepare forecast in constituent parts to obtain total. Both power and energy forecasts are made. Multi factors involved render forecasting an involved process requiring experience and high analytical ability.

Yearly forecasts are based on previous year's loading for the period under consideration updated by factors such as general load increases, major loads and weather trends.

In short-term load forecasting, hour-by-hour predictions are made for the particular day under consideration. A minor forecast error on low side might necessitate the use of inefficient, oil-fired turbine generators or "peaking units" which are quite costly. On the other hand, a high side forecast error would keep excessive generation in hot reserve. Accuracy of the order of 1% is desirable. A temperature difference of  $2^{\circ}\text{C}$  can vary the total load by 1%. This indicates the importance of reliable weather forecast to a good load forecast. The short term forecast problem is not a simple one as often random factors such as unexpected storms, strikes, the sudden telecast of a good TV programme can upset the predictions. Regression analysis is often used for obtaining a short term load forecast which is very important and is required before solving unit commitment and economic load despatch problems discussed in Ch. 7. Owing to the great importance of load forecasting (an important input-before solving almost all power system problems), a full chapter is added in this book describing various methods of load forecasting (Ch. 16).

In India, energy demand and installed generating capacity are both increasing exponentially (so is population growth—a truly formidable combination). Power demand\* has been roughly doubling every ten years as in many other countries.

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\* 38% of the total power required in India is for industrial consumption. Generation of electricity in India was around 530 billion kWh in 2000–2001 A.D. compared to less than 200 billion kWh in 1986–87.

On 31.3.06 the total installed generation capacity in India is 1,24,287 MW \*\*. As per the present indications, by the time we enter the 2nd decade of the 21st century it would be nearing 2,00,000 MW—a stupendous task indeed. This, in turn, would require a corresponding development in coal resources. Development of a coalmine takes a little over four years.

#### 1.4 STRUCTURE OF POWER SYSTEMS

Generating stations, transmission lines and the distribution systems are the main components of an electric power system. Generating stations and a distribution system are connected through transmission lines, which also connect one power system (grid, area) to another. A distribution system connects all the loads in a particular area to the transmission lines.

For economical and technological reasons (which will be discussed in detail in later chapters), individual power systems are organized in the form of electrically connected areas or regional grids (also called power pools). Each area or regional grid operates technically and economically independently, but these are eventually interconnected\* to form a national grid (which may even form an international grid) so that each area is contractually tied to other areas in respect to certain generation and scheduling features. India is now heading for a national grid.

The siting of hydro stations is determined by the natural water power sources. The choice of site for coal fired thermal stations is more flexible. The following two alternatives are possible:

1. Power stations may be built close to coal mines (called pit head stations) and electric energy is evacuated over transmission lines to the load centres.
2. Power stations may be built close to the load centres and coal is transported to them from the mines by rail road.

\*\*Comprising 32,326 MW hydro, 82,411 MW thermal 33106 MW nuclear and 6,191 MW wind/RES.

\* Interconnection has the economic advantage of reducing the reserve generation capacity in each area. Under conditions of sudden increase in load or loss of generation in one area, it is immediately possible to borrow power from adjoining interconnected areas. Interconnection causes larger currents to flow on transmission lines under faulty condition with a consequent increase in capacity of circuit breakers. Also, the synchronous machines of all interconnected areas must operate stably and in a synchronized manner. The disturbance caused by a short circuit in one area must be rapidly disconnected by circuit breaker openings before it can seriously affect adjoining areas. It permits the construction of larger and more economical generating units and the transmission of large chunk of power from the generating plants to major load centres. It provides capacity savings by seasonal exchange of power between areas having opposing winter and summer requirements. It permits capacity savings from time zones and random diversity. It facilitates transmission of off-peak power. It also gives the flexibility to meet unexpected emergency loads.

In practice, however, power station siting will depend upon many factors—technical, economical and environmental. As it is considerably cheaper to transport bulk electric energy over extra high voltage (EHV) transmission lines than to transport equivalent quantities of coal over rail road, the recent trends in India (as well as abroad) is to build super (large) thermal power stations near coal mines. Bulk power can be transmitted to fairly long distances over transmission lines of 400/765 kV and above. However, the country's coal resources are located mainly in the eastern belt and some coal fired stations will continue to be sited in distant western and southern regions.

As nuclear stations are not constrained by the problems of fuel transport and air pollution, a greater flexibility exists in their siting, so that these stations are located close to load centres while avoiding high density pollution areas to reduce the risks, however remote, of radioactivity leakage.

In India, as of now, about 75% of electric power used is generated in thermal plants (including nuclear). 23% from mostly hydro stations and 2% come from renewables and others. Coal is the fuel for most of the steam plants, the rest depends upon oil/natural gas and nuclear fuels.

Electric power is generated at a voltage of 11 to 25 kV which then is stepped up to the transmission levels in the range of 66 to 765 kV (or higher). As the transmission capability of a line is proportional to the square of its voltage, research is continuously being carried out to raise transmission voltages. Some of the countries are already employing 765 kV. The voltages are expected to rise to 800 kV in the near future. In India, several 400 kV lines are already in operation. Several 765 kV lines have been built so far in India.

For very long distances (over 600 km), it is economical to transmit bulk power by DC transmission (see Ch. 20.) It also obviates some of the technical problems associated with very long distance AC transmission. The DC voltages used are 400 kV and above, and the line is connected to the AC systems at the two ends through a transformer and converting/inverting equipment (silicon controlled rectifiers are employed for this purpose). Several DC transmission lines have been constructed in Europe and the USA. In India several HVDC transmission line (bipolar) have already been commissioned and several others are being planned. Four back to back HVDC systems are in operation (for details, see Ch. 20).

The first stepdown of voltage from transmission level is at the bulk power substation, where the reduction is to a range of 33 to 132 kV, depending on the transmission line voltage. Some industries may require power at these voltage levels. This stepdown is from the transmission and grid level to subtransmission level.

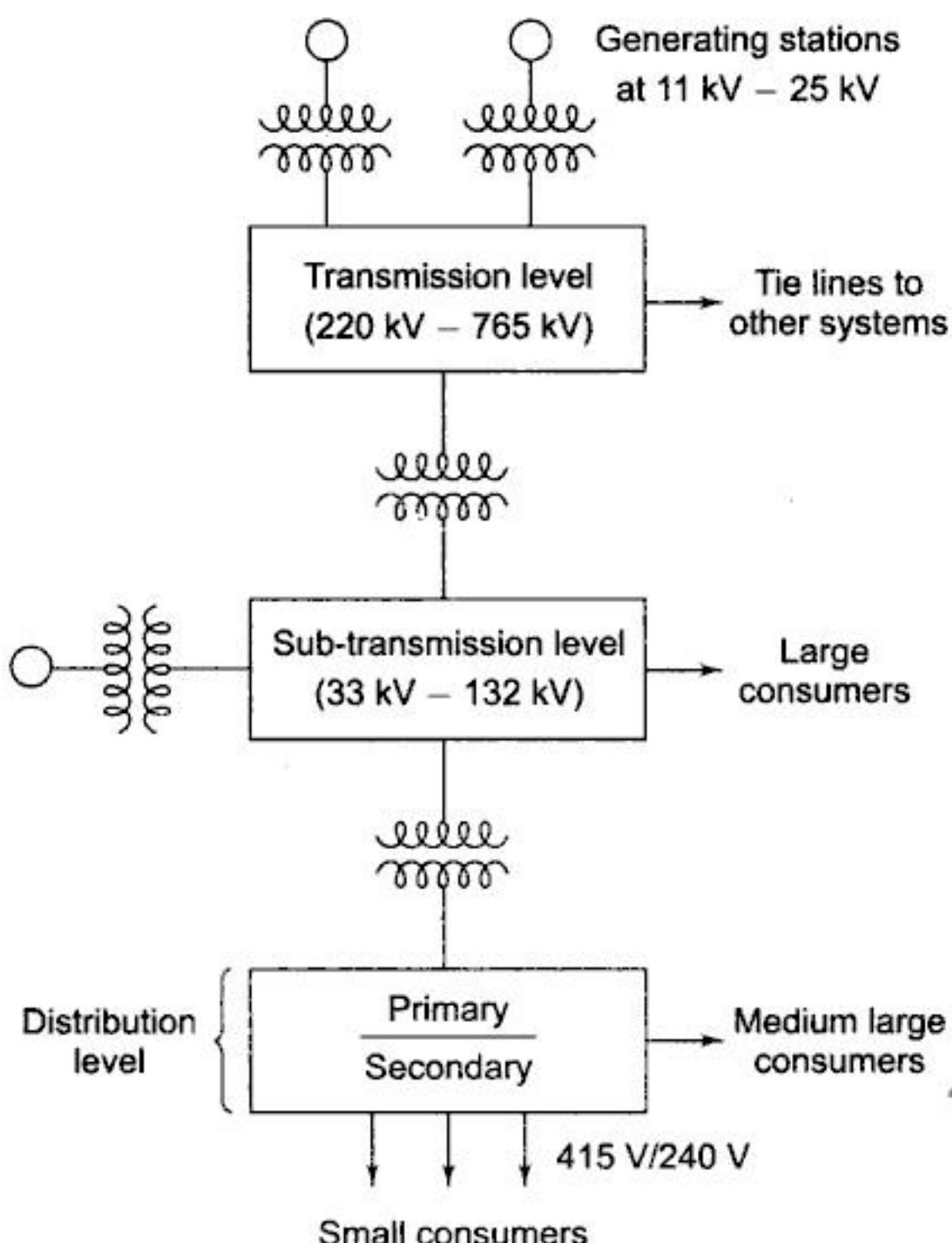
The next stepdown in voltage is at the distribution substation. Normally, two distribution voltage levels are employed: (see Ch. 21).

1. The primary or feeder voltage (11 kV).
2. The secondary or consumer voltage (415 V three phase/230 V single phase).

The distribution system, fed from the distribution transformer stations, supplies power to the domestic or industrial and commercial consumers.

Thus, the power system operates at various voltage levels separated by transformer. Figure 1.4 depicts schematically the structure of a power system.

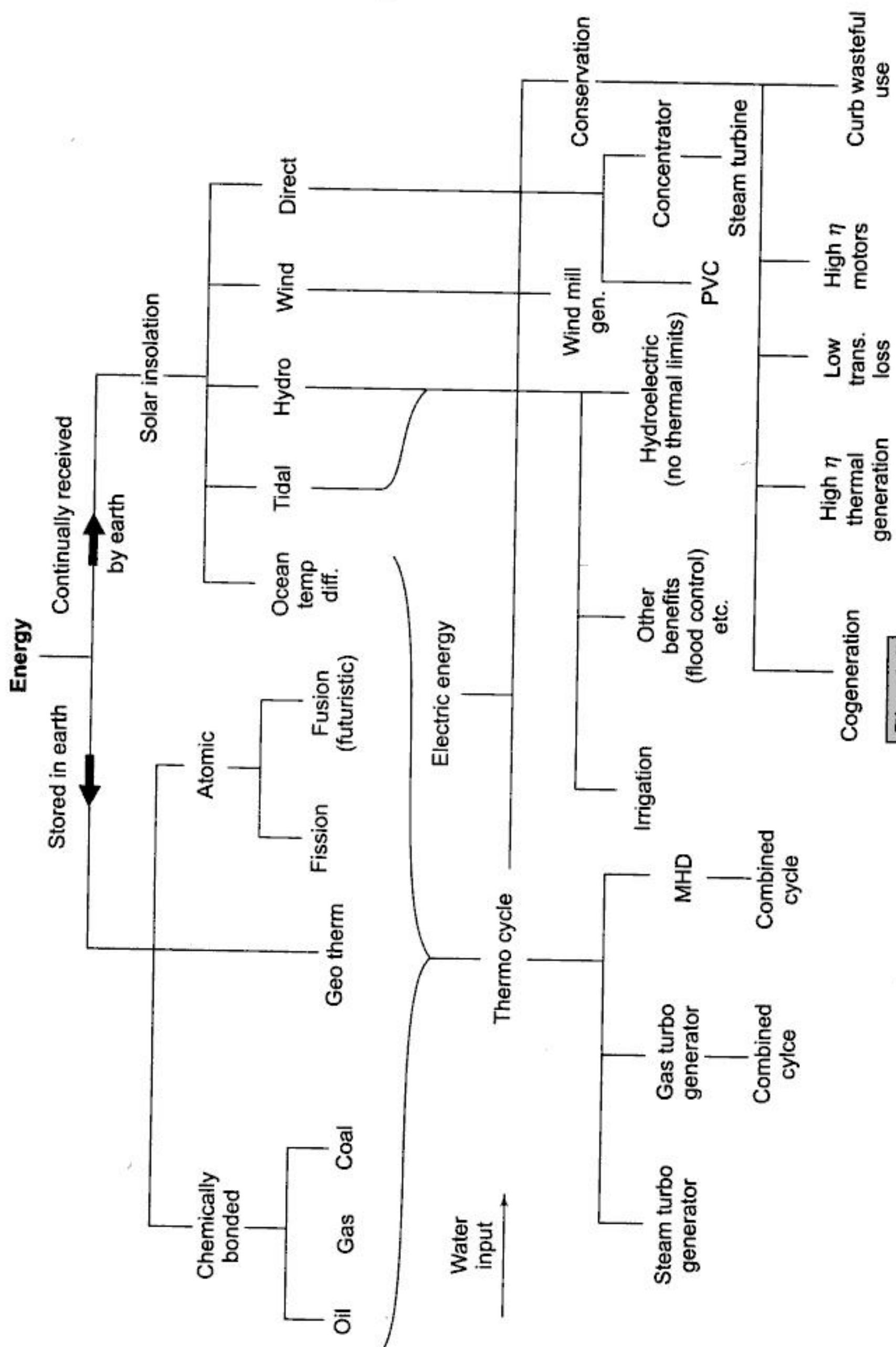
Though the distribution system design, planning and operation are subjects of great importance, we are compelled, for reasons of space, to exclude them from the scope of this book.



**Fig. 1.4** Schematic diagram depicting power system structure

## 1.5 CONVENTIONAL SOURCES OF ELECTRIC ENERGY

Thermal (coal, oil, nuclear) and hydro generations are the main conventional sources of electric energy. The necessity to conserve fossil fuels has forced scientists and technologists across the world to search for nonconventional sources of electric energy. Some of the sources being explored are solar, wind and tidal sources. The conventional and some of the nonconventional sources and techniques of energy generation are briefly surveyed here with a stress on future trends, particularly with reference to the Indian electric energy scenario. A panoramic view of energy conversion to electrical form is presented in Fig. 1.5.

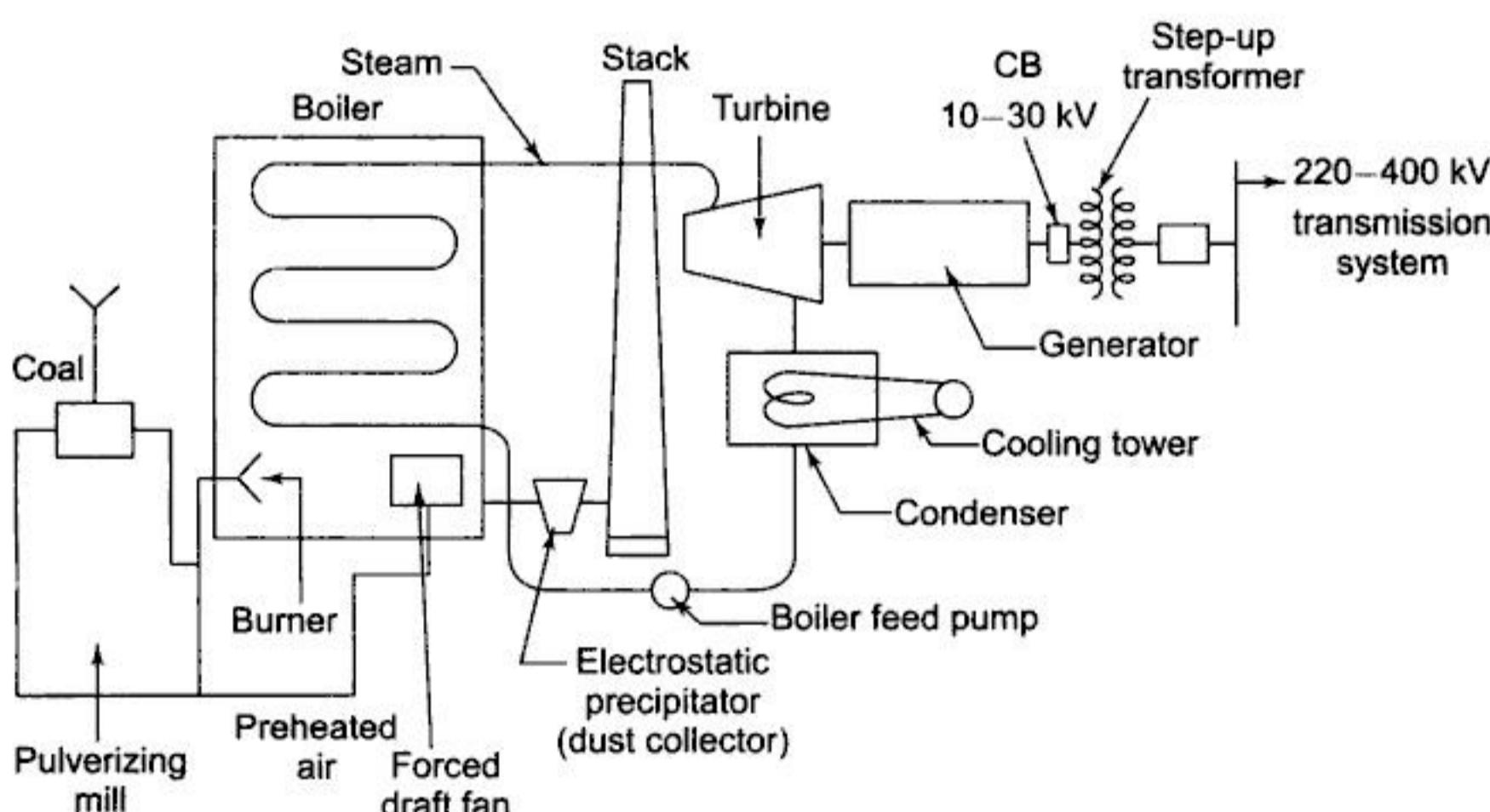


**Fig. 1.5**

## Thermal Power Stations—Steam/Gas-based

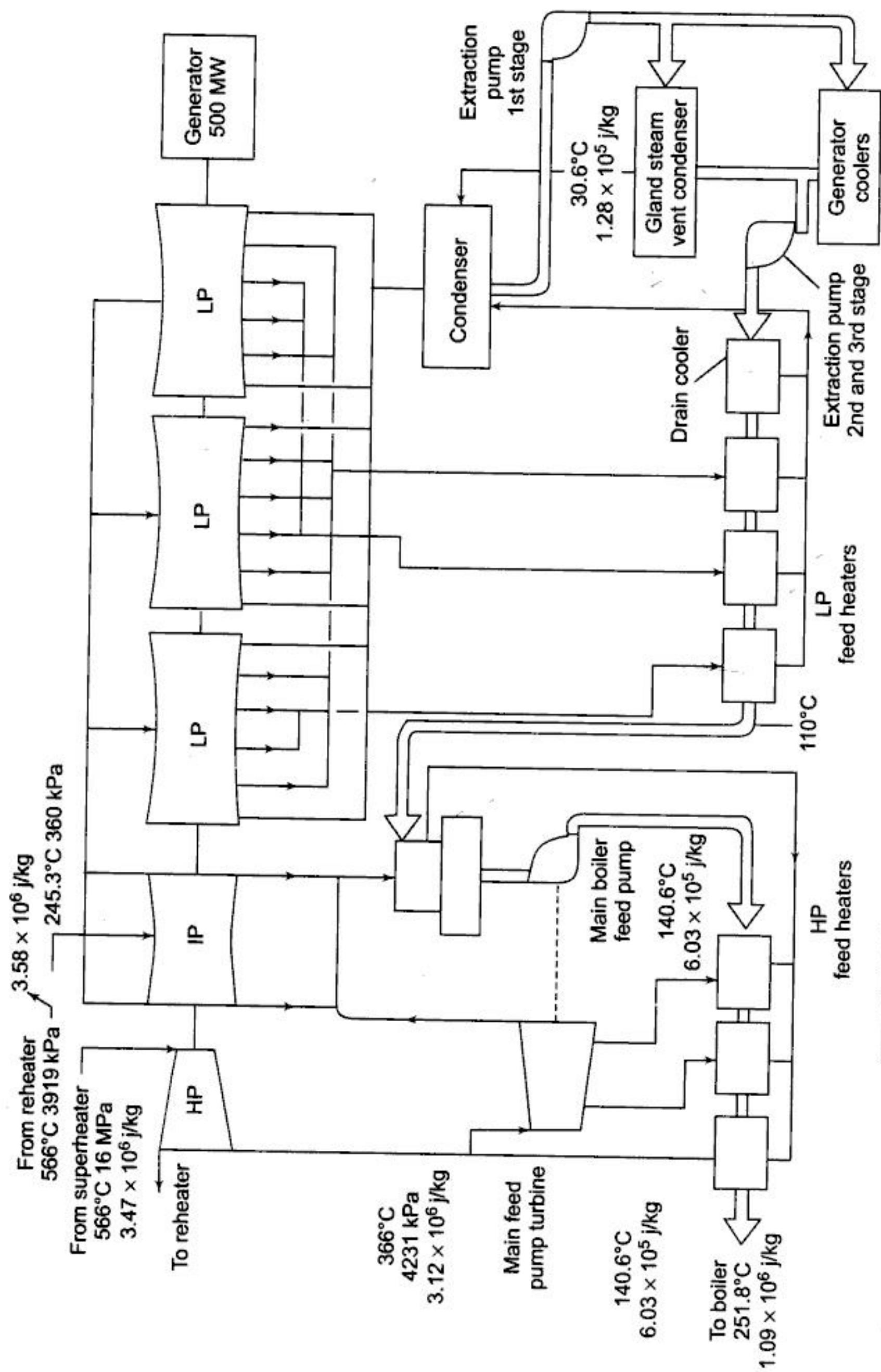
The heat released during the combustion of coal, oil or gas is used in a boiler to raise steam. In India heat generation is mostly coal based except in small sizes, because of limited indigenous production of oil. Therefore, we shall discuss only coal-fired boilers for raising steam to be used in a turbine for electric generation. Natural gas is India's most important potential alternative to coal. India is planning to use natural gas in power generation and in the industrial and residential sectors. Our heavy reliance on highly polluting coal makes development and installation of clean coal technology (cct) a high priority.

The chemical energy stored in coal is transformed into electric energy in thermal power plants. The heat released by the combustion of coal produces steam in a boiler at high pressure and temperature, which when passed through a steam turbine gives off some of its internal energy as mechanical energy. The axial-flow type of turbine is normally used with several cylinders on the same shaft. The steam turbine acts as a prime mover and drives the electric generator (alternator). A simple schematic diagram of a coal fired thermal plant is shown in Fig. 1.6(a).



**Fig. 1.6(a)** Schematic diagram of a coal fired steam plant

The efficiency of the overall conversion process is poor and its maximum value is about 40% because of the high heat losses in the combustion gases and the large quantity of heat rejected to the condenser which has to be given off in cooling towers or into a stream/lake in the case of direct condenser cooling. The steam power station operates on the Rankine cycle, modified to include superheating, feed-water heating, and steam reheating as shown in Fig. 1.6(b). The thermal efficiency (conversion of heat to mechanical energy) can be increased by using steam at the highest possible pressure and



**Fig. 1.6(b)** Energy flow diagram for a 500 MW turbogenerator

temperature. With steam turbines of this size, additional increase in efficiency is obtained by reheating the steam after it has been partially expanded by an external heater. The reheated steam is then returned to the turbine where it is expanded through the final stages of bleeding.

To take advantage of the principle of economy of scale (which applies to units of all sizes), the present trend is to go in for larger sizes of units. Larger units can be installed at much lower cost per kilowatt. They are also cheaper to operate because of higher efficiency. They require lower labour and maintenance expenditure. According to Kashkari [3] there may be a saving of as high as 15% in capital cost per kilowatt by going up from a 100 to 250 MW unit size and an additional saving in fuel cost of about 8% per kWh. Since larger units consume less fuel per kWh, they produce less air, thermal and waste pollution, and this is a significant advantage in our concern for environment. The only trouble in the case of a large unit is the tremendous shock to the system when outage of such a large capacity unit occurs. This shock can be tolerated so long as this unit size does not exceed 10% of the on-line capacity of a large grid.

In India, in 1970s the first 500 MW superthermal unit had been commissioned at Trombay. Bharat Heavy Electricals Limited (BHEL) has produced several turbogenerator sets of 500 MW capacity. Today's maximum generator unit size is (nearly 1200 MW) limited by the permissible current densities used in rotor and stator windings. Efforts are on to develop *super conducting* machines where the winding temperature will be nearing absolute zero. Extreme high current and flux densities obtained in such machines could perhaps increase unit sizes to several GWs which would result in better generating economy.

Air and thermal pollution is always present in a coal fired steam plant. The air polluting agents (consisting of particulates and gases such as NOX, CO, CO<sub>2</sub>, SOX, etc.) are emitted via the exhaust gases and thermal pollution is due to the rejected heat transferred from the condenser to cooling water. Cooling towers are used in situations where the stream/lake cannot withstand the thermal burden without excessive temperature rise. The problem of air pollution can be minimized through scrubbers and electrostatic precipitators and by resorting to minimum emission dispatch [32] and Clean Air Act has already been passed in Indian Parliament.

### ***Fluidized-bed Boiler***

The main problem with coal in India is its high ash content (up to 40% max). To solve this, *fluidized bed combustion technology* is being developed and perfected. The fluidized-bed boiler is undergoing extensive development and is being preferred due to its lower pollutant level and better efficiency. Direct ignition of pulverized coal is being introduced but initial oil firing support is needed.

## Cogeneration

Considering the tremendous amount of waste heat generated in thermal power generation, it is advisable to save fuel by the simultaneous generation of electricity and steam (or hot water) for industrial use or space heating. Now called cogeneration, such systems have long been common, here and abroad. Currently, there is renewed interest in these because of the overall increase in energy efficiencies which are claimed to be as high as 65%.

Cogeneration of steam and power is highly energy efficient and is particularly suitable for chemicals, paper, textiles, food, fertilizer and petroleum refining industries. Thus these industries can solve energy shortage problem in a big way. Further, they will not have to depend on the grid power which is not so reliable. Of course they can sell the extra power to the government for use in deficient areas. They may also sell power to the neighbouring industries, a concept called *wheeling power*.

As on 31.12.2000, total co-generation potential in India is 19,500 MW and actual achievement is 273 MW as per MNES (Ministry of Non-Conventional Energy Sources, Government of India) Annual Report 2000–01.

There are two possible ways of cogeneration of heat and electricity: (i) Topping cycle, (ii) Bottoming cycle. In the topping cycle, fuel is burnt to produce electrical or mechanical power and the waste heat from the power generation provides the process heat. In the bottoming cycle, fuel first produces process heat and the waste heat from the processes is then used to produce power.

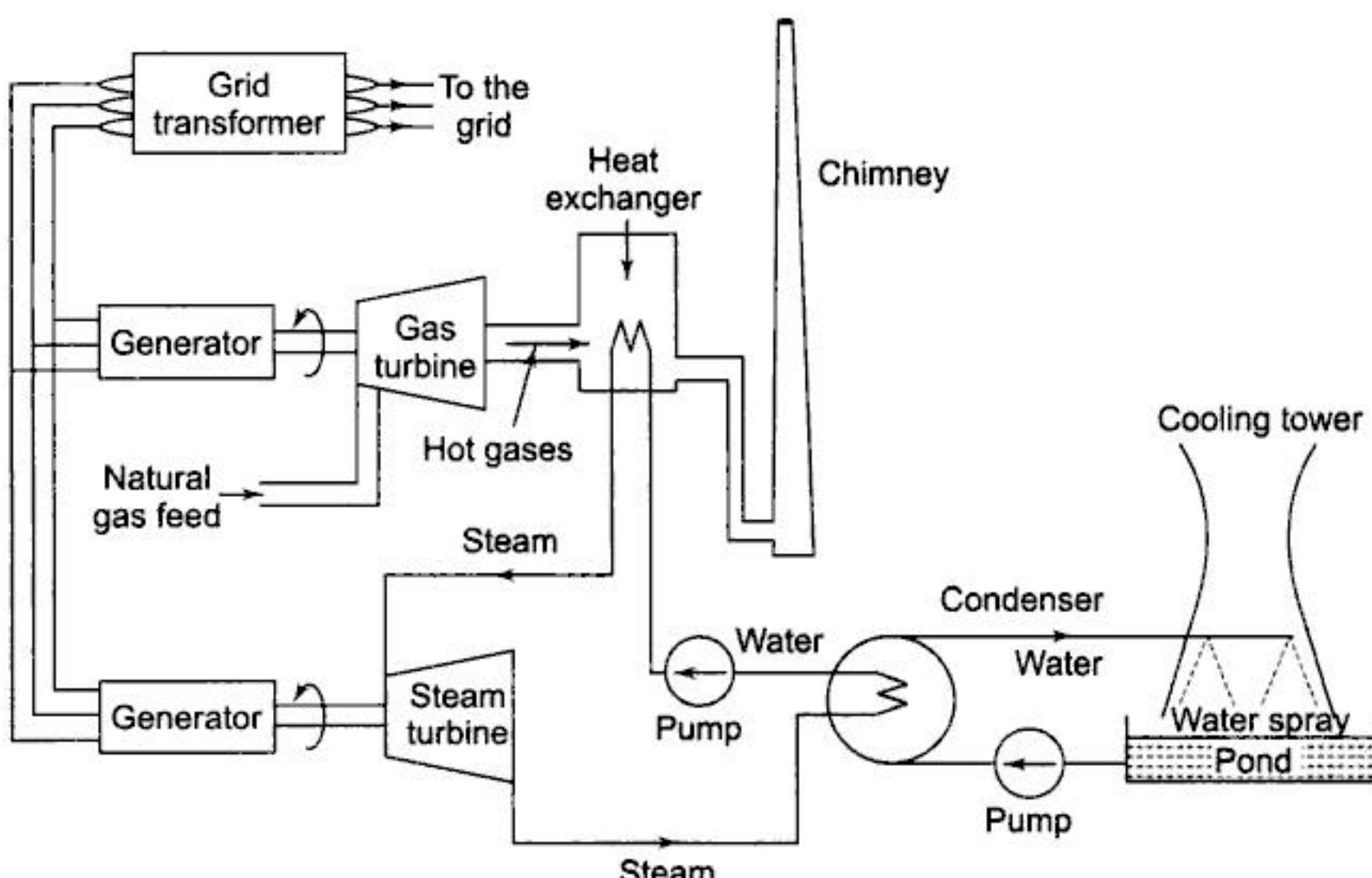
Coal-fired plants share environmental problems with some other types of fossil-fuel plants; these include “acid rain” and the “greenhouse” effect.

## Gas Turbines

With increasing availability of natural gas (methane) (recent finds in Bangladesh) primemovers based on gas turbines have been developed on the lines similar to those used in aircraft. Gas combustion generates high temperatures and pressures, so that the efficiency of the gas turbine is comparable to that of steam turbine. Additional advantage is that exhaust gas from the turbine still has sufficient heat content, which is used to raise steam to run a conventional steam turbine coupled to a generator. This is called combined-cycle gas-turbine (CCGT) plant. The schematic diagram of such a plant is drawn in Fig. 1.7.

The CCGT plant has a fast start of 2–3 min for the gas turbine and about 20 mins for the steam turbine. Local storage tanks of gas can be used in case of gas supply interruption. The unit can take up to 10% overload for short periods of time to take care of any emergency.

CCGT unit produces 55% of CO<sub>2</sub> produced by a coal/oil-fired plant. Units are now available for a fully automated operation for 24 h or to meet the peak demands.



**Fig. 1.7** CCGT power station

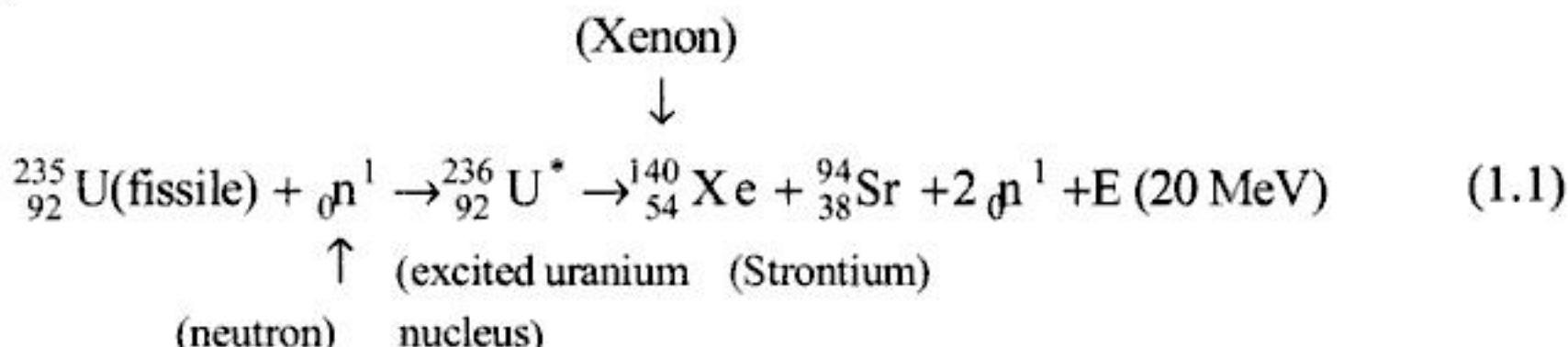
In Delhi (India) a CCGT unit of  $2 \times 110$  MW are installed at Pragati Power Plant.

There are currently many installations using gas turbines in the world with 100 MW generators. A  $6 \times 30$  MW gas turbine station has already been put up in Delhi. A gas turbine unit can also be used as synchronous compensator to help maintain flat voltage profile in the system.

## Nuclear Power Generation

## **Nuclear Reaction**

Considerable binding energy is released on breaking a large nucleus into smaller fragments. This process is called *fission*. Nucleus of uranium isotope  $^{235}\text{U}$  undergoes fission when struck by a fast moving neutron. This fission is expressed in the standard nuclear reaction as



## *Nuclear Reactor*

The fission of a nuclear material (Eq. (1.1)) is carried out in a nuclear reactor. A nuclear reactor is a very efficient source of energy as a small amount of fissile material produces large chunks of energy. For example 1 g of  $^{235}\text{U}$  releases

energy at the rate of 1 MW/day, whereas 2.6 tons of coal produces the same power in a conventional thermal plant per day (this figure is much larger for Indian coal, which contains considerable amount of dust in it).

Uranium metal extracted from the base ore consists mainly of two isotopes  $^{238}\text{U}$  (99.3% by weight) and  $^{235}\text{U}$  (0.7% by weight). Of these only  $^{235}\text{U}$  is fissile and when struck by slow moving neutrons, its nucleus splits into two fast moving neutrons and  $3 \times 10^{-11} \text{ J}$  of kinetic energy. The fast moving neutrons hit the surrounding atoms, thus producing heat before coming to rest. The neutrons travel further, hitting more atoms and producing further fissions. The number of neutrons thus multiplies and under certain critical conditions a sustainable chain reaction results. For sustainability the reactor core or moderator must slow down the moving neutrons to achieve a more effective splitting of the nuclei.

The energy given off in a reactor appears in the form of heat, which is removed by a gas or liquid coolant. The hot coolant is then used in a heat exchanger to raise steam. If the coolant is ordinary water, steam could be raised inside the reactor. Steam so raised runs a turbogenerator for producing electric energy.

### **Fuel**

Fuels used in reactors have some components of  $^{238}\text{U}$ . Natural uranium is sometimes used and although the energy density is considerably less than that for pure isotope, it is still much better than fossil fuels. The uranium used at present comes from metal-rich areas, which have limited world resources ( $\approx 2 \times 10^6$  tons). Therefore, the era of nuclear energy would be comparatively short, probably less than a century. Fortunately it is possible to manufacture certain fissile isotopes from abundant nonfissile materials like thorium by a process of conversion in a breeder reactor (details in later portion of this section). This would assure virtually an unlimited reserve of nuclear energy.

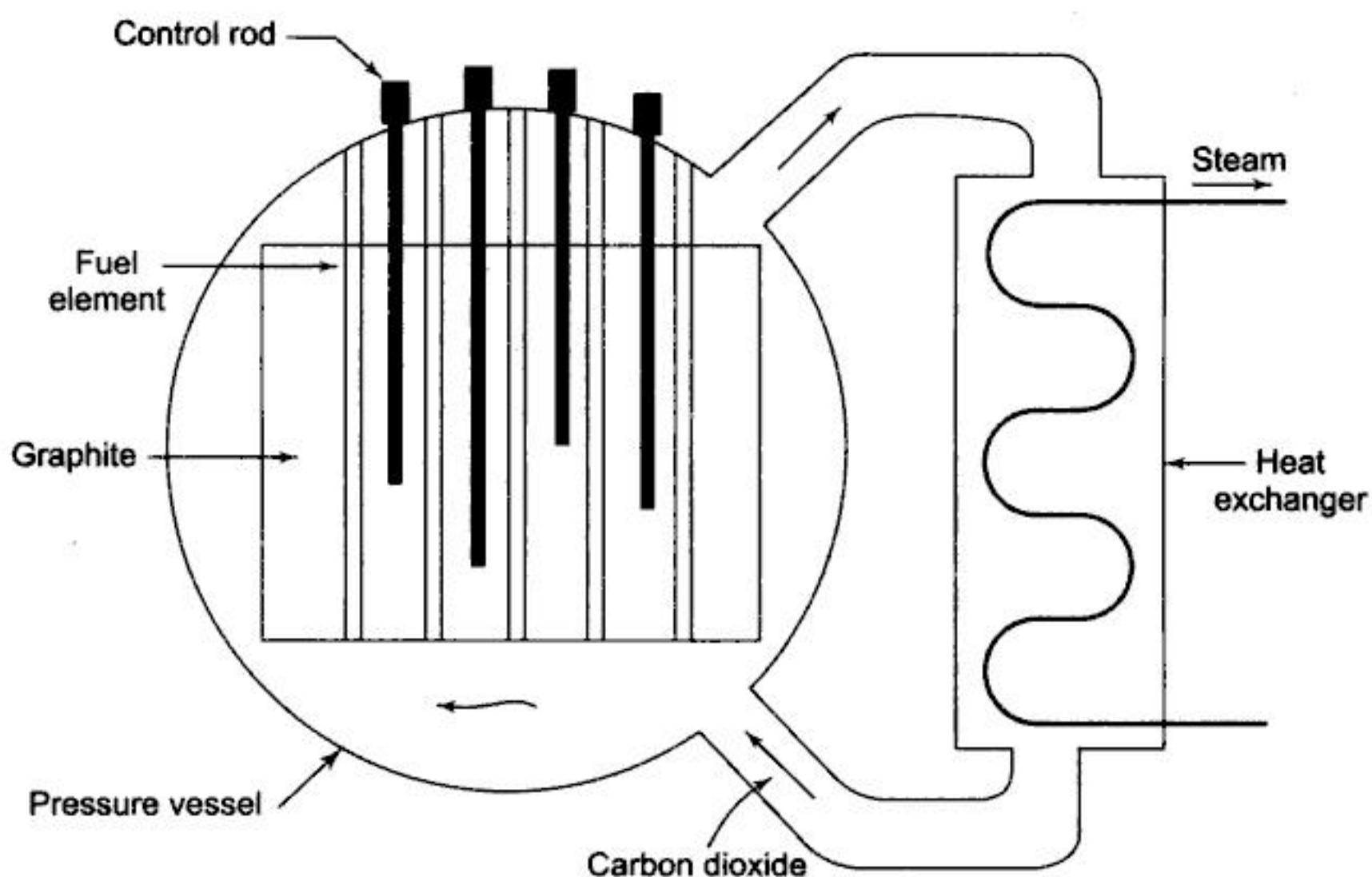
In an advanced gas-cooled reactor (AGR), whose schematic diagram is shown in Fig. 1.8, enriched uranium dioxide fuel in pellet form, encased in stainless steel cans, is used. A number of cans form a cylindrical fuel element, which is placed in a vertical hollow housing in the core. In certain reactors fuel could be in the form of rods enclosed in stainless steel.

### **Moderators**

To slow down the neutrons the reactor elements are placed inside a moderator, a substance whose nuclei absorb energy as fast moving neutrons collide with these but do not capture the neutrons. Commonly used moderators are graphite (as in AGR of Fig. 1.8), light water and heavy water. It could also be beryllium and its oxide and possibly certain organic compounds.

### **Coolants**

These remove the heat generated in the core by circulation and transfer it outside for raising steam. Common coolants are light ordinary water, heavy water, gas ( $\text{CO}_2$ ) (this is used in AGR of Fig. 1.8) and also metals like sodium or sodium-potassium alloy in liquid form.



**Fig. 1.8** Schematic view of a British Magnox type nuclear reactor

### Control Materials

In nuclear reactors, control is achieved by means of a neutron absorbing material. The control elements are commonly located in the core in the form of either rods or plates. The control rods are moved in to decrease the fission rate or neutron flux and moved out to increase it. The most commonly used neutron absorber is *boron*. This element has a very high melting point and a large cross-section for neutron absorption.

Other control materials are cadmium and an alloy of silver 15%, iridium 15% and cadmium 5%.

### Reactor Shielding

Nuclear reactors are sources of intense neutron and  $\gamma$ -radiation and, therefore, represent hazard to persons in the immediate vicinity of the reactor. Provisions for their health protection are made by surrounding the reactor core with a radiation shield, also called *biological shield*. It generally consists of a layer of concrete, about 1.8–2.5 m thick and capable of absorbing both  $\gamma$ -rays and neutrons. Part of the shield which is in immediate contact with the core heats up considerably and requires a special cooling facility to prevent it from cracking. A shield made of a 5–10 cm thick steel is located close to the core.

### Power Reactor Types

The primary purpose of a power reactor is the utilization of fission energy produced in the reactor core by converting it to a useful mechanical-electrical form. The heat generated in the fission process is used to produce steam at high temperature and pressure, which runs a turbogenerator. For raising steam a heat exchanger stage is interposed between the reactor and boiler. The choice of the heat exchange fluid (or gas) is governed by three considerations:

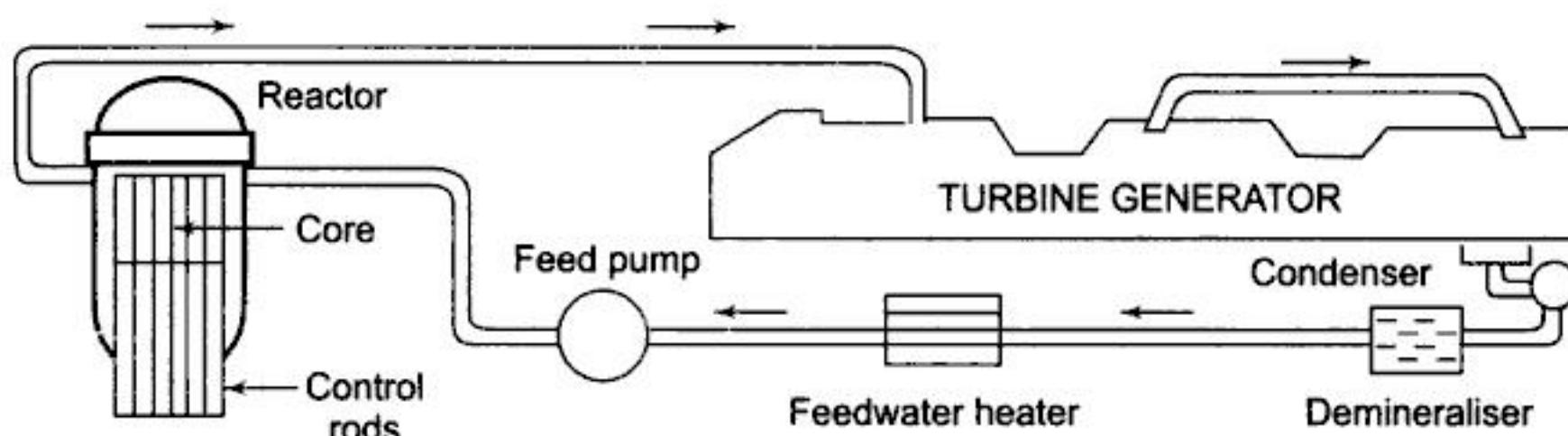
1. It must have a high thermal conductivity so as to carry away heat efficiently and give it up in a heat exchanger.
2. It must have a low neutron capture cross-section so as not to upset the reactor characteristics.
3. It must not be decomposed by intense radiation.

Now we shall discuss some of the reactor types which are in current use in various countries. Advanced gas reactor (AGR) has already been presented earlier for discussing the various components and processes involved in a reactor.

### **Boiling Water Reactor (Fig. 1.9(a))**

This type of power plant is designed to allow steam to be generated directly in the reactor core. This uses light water as moderator and coolant. Therefore, no external heat exchanger is required. Enriched uranium is used as fuel.

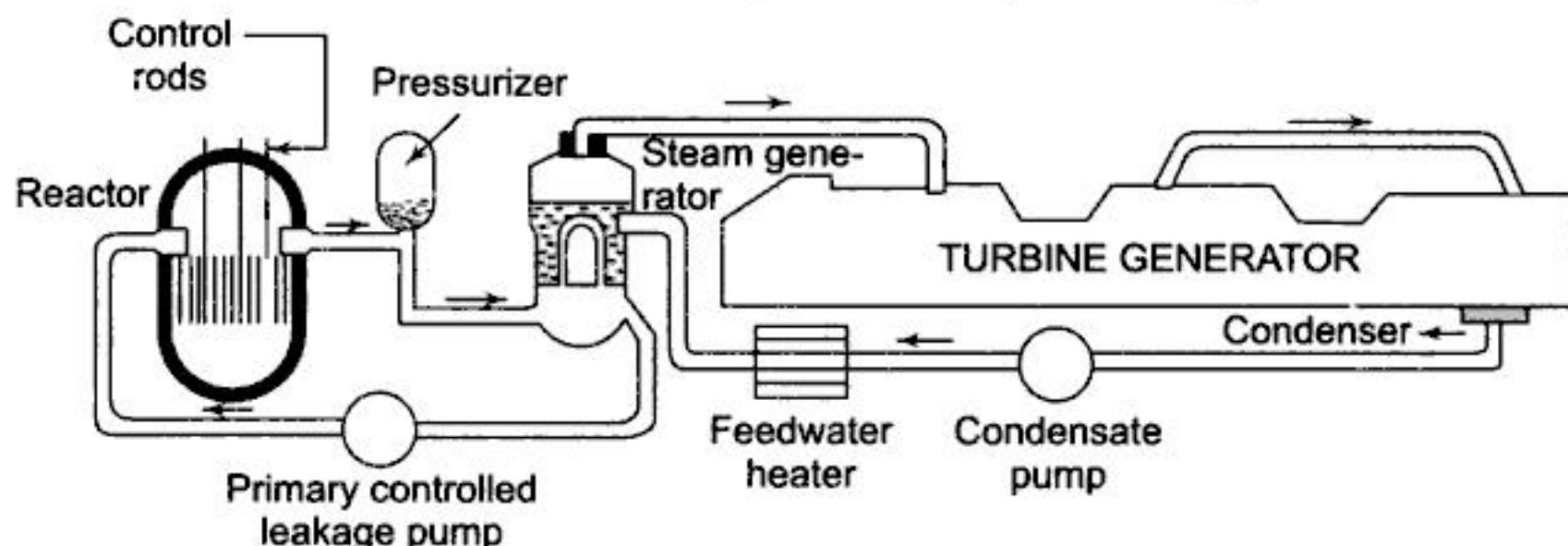
The 420 MW power station, at Tarapur (India), consists of two enriched uranium reactors of the boiling water type. These reactors were built with the help of the General Electric Company of the United States and became operational on April 1, 1969.



**Fig. 1.9(a)** Schematic diagram of a boiling water reactor (BWR)

### **Pressurized Water Reactor (Fig. 1.9(b))**

It uses slightly enriched uranium (1.4. or 2% of U<sup>235</sup>) as fuel and light water as moderator and coolant. The fuel elements are in the form of rods or plates. The core is contained in a vessel under a pressure of (6.5 to 13.8) × 10<sup>6</sup> N/m<sup>2</sup>.

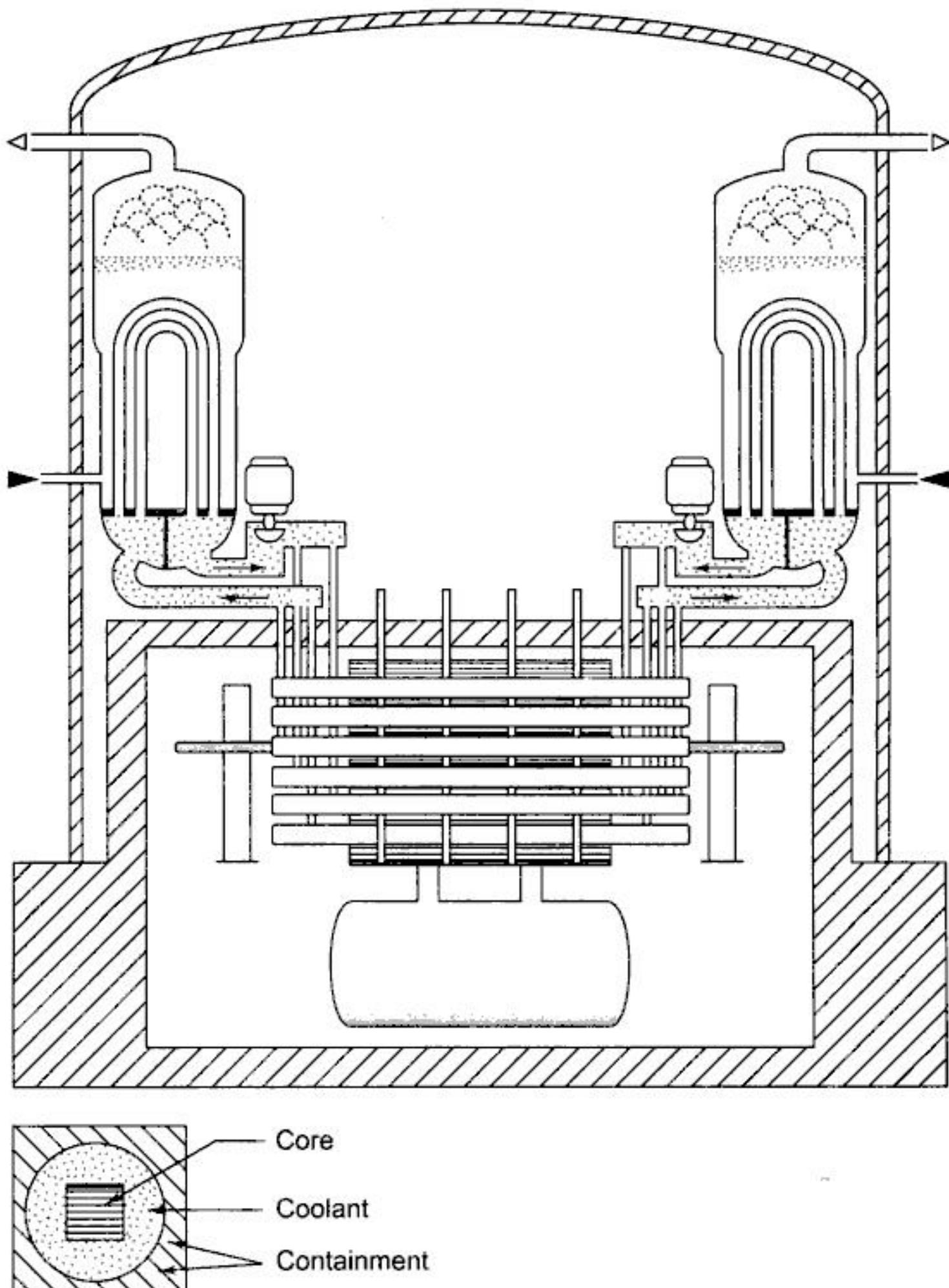


**Fig. 1.9(b)** Schematic diagram of a pressurized water reactor (PWR)

The pressurized water is circulated through the reactor core from which it removes heat. This heat is transferred to a boiler through a heat exchanger for raising steam for turbogeneration as shown in Fig. 1.9(b).

### **Heavy Water ( $D_2O$ ) Moderated Reactor (Fig. 1.10)**

It is of the pressurized water reactor type with heavy water as moderator and coolant instead of light water. The first prototype of this type of reactor is the Nuclear Power Demonstration Reactor (NPDR) called CANDU (Canada Deuterium Uranium) type reactor completed in 1962 at Canada.



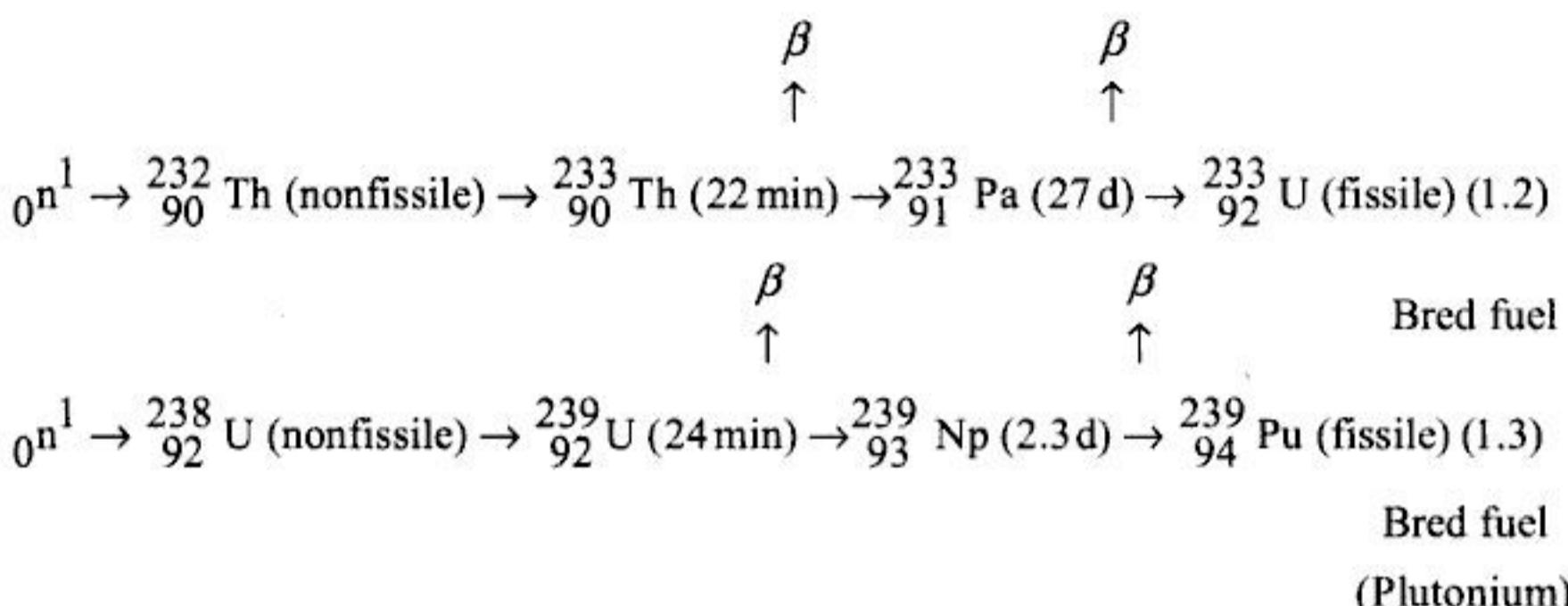
**Fig. 1.10** CANDU Reactor

The 430 MW power station near Kota at Rana Pratap Sagar in Rajasthan employs a heavy water moderated reactor using natural uranium as fuel. This

started feeding power in 1973. The other two reactors of this type are under construction, one at Kalpakkam (470 MW), about 100 km away from Chennai, and the other at Narora in Uttar Pradesh (UP), Kakrapar in Gujarat. Several other nuclear power plants will be commissioned by 2012. It is planned to raise nuclear power generator to 20,000 MW by 2020.

### **Fast Breeder Reactor (FBR)**

Such type of reactors are designed to produce more fissile material (Plutonium) than they consume (Thorium,  $U^{232}$ ). The nuclear equations for breeding are as under.



According to the above reactions there are two types of fast breeder reactors. These are:

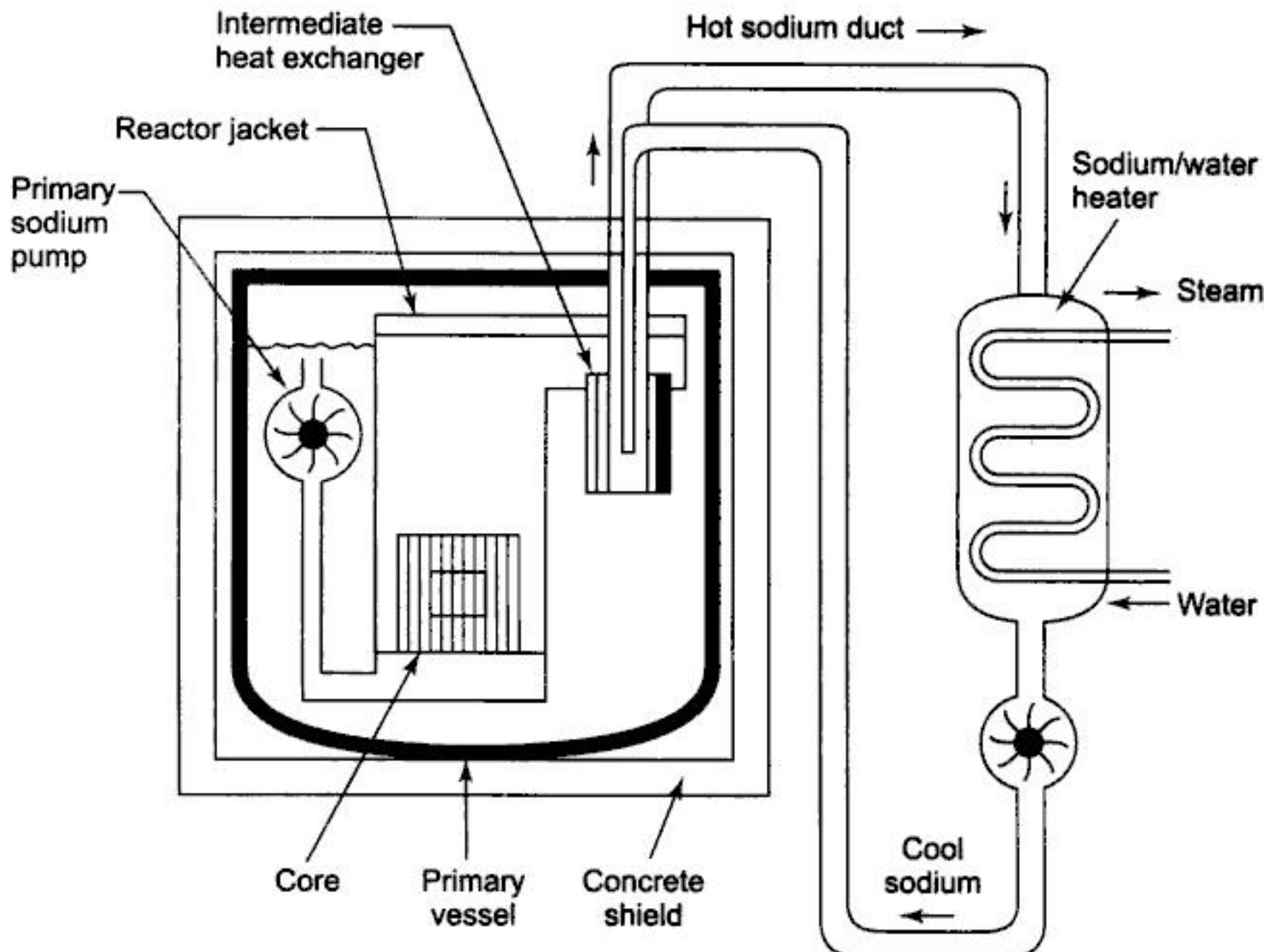
1. A blanket of  $^{232}\text{Th}$  surrounds  $^{239}\text{Pu}$  and is converted to  $^{233}\text{U}$  which is fissile (Eq. (1.2)).
2. Core 20%  $^{239}\text{Pu}$  surrounded by a blanket of 80%  $^{238}\text{U}$  (Thorium). About three neutrons are emitted when a  $^{239}\text{Pu}$  nucleus fissions. Of these one is required to sustain the reaction leaving the other two to account for breeding more  $^{239}\text{Pu}$  (Eq. 1.3).

The power density in a fast breeder reactor is considerably higher than in normal reactors. Therefore, liquid sodium which is an efficient coolant and does not moderate neutrons is used to take away heat generated in the core. Schematic diagram of an FBR reactor is shown in Fig. 1.11.

An important advantage of FBR technology, brought out through the reaction equations given above, is that it can use Thorium (as fertile material) which gets converted to  $^{233}\text{U}$ , a fissile isotope. This holds great promise for India as we have one of the world's largest deposits of Thorium—about 450,000 tons in form of sand dunes in Kerala and along the Gopalpur Chattarpur coast of Orissa.

Typical power densities ( $\text{MW/m}^3$ ) in fission reactor cores are: gas cooled 0.53, high temperature gas cooled 7.75, heavy water 18.0, boiling water 29.0, pressurized water 54.75 and fast breeder reactor 760.0.

The associated merits and problems of nuclear power plants as compared to conventional thermal plants are as follows:



**Fig. 1.11** Schematic diagram of a liquid-metal FBR

### Merits

1. A nuclear power plant is totally free of air pollution. Nuclear fuel is greener than coal.
2. It requires very little fuel in terms of volume and weight, and therefore poses no transportation problems and may be sited, independently of nuclear fuel supplies, close to load centres. However, safety considerations require that these be normally located away from populated areas.
3. It lasts longer—over 45 years as against 30 in case of coal and 15 in case of gas turbines.

### Problems

1. Nuclear reactors produce radioactive fuel waste, the disposal of which poses serious environmental hazards.
2. The rate of nuclear reaction can be lowered only by a small margin, so the load on a nuclear power plant can only be permitted to be marginally reduced below its full load value. Nuclear power stations must, therefore be reliably connected to a power network, as tripping of the lines connecting the station can be quite serious and may require shutting down of the reactor with all its consequences.

3. Because of a relatively high capital cost as against the running cost, the nuclear plant should operate continuously as a base load station. Wherever possible, it is preferable to support such a station with a pumped storage scheme discussed later (p. 33.)
4. There are risks in terms of fuel supplies and safety.

### **Safety and Environmental Considerations**

The nuclei that result from fission are called fission fragments. From the nuclear reactor there is a continuous emission of  $\beta$ - and  $\gamma$ -rays, and  $\alpha$ -particles and fission fragments. If these fission fragments cannot be retreated as a fuel element, then these become the waste products, which are highly radioactive. Some of the important waste products with their half life are as follows:

$^3_1\text{H}_2$ (Tritium)	– 12.26 years
$^{90}\text{Sr}$ (Strontium)	– 28.8 years
$^{137}\text{Cs}$ (Cesium)	– 32.2 years
$^{131}\text{I}$ (Iodine)	– 8 days
$^{85}\text{Kr}$ (Krypton)	– 10.76 years
$^{133}\text{Xe}$ (Xenon)	– 5.27 days

Waste products having a long life create serious problems. These are called high level wastes, e.g.  $^{90}\text{Sr}$ . At the moment, the wastes are concentrated in the liquid form and stored in stainless steel containers. Burying nuclear wastes deep underground currently seems to be the best long-term way to dispose them off. The location should be geologically stable, should not be earthquake prone, a type of rock that does not disintegrate in the presence of heat and radiation and not near ground water that might become contaminated. At present over 15,000 tons of spent nuclear fuel is being stored on a temporary basis in United States.

In the design and construction of reactors great care is taken to cover every contingency. Many facilities, e.g. control system, are at least duplicated and have alternative electrical supplies. In March 1979, failure in its cooling system disabled one of the reactors of Three Mile Island in Pennsylvania and a certain amount of radioactive material escaped, although a catastrophe was narrowly avoided. Then in April 1986, a severe accident destroyed a 1000 MW reactor at Chernobyl, in the then Soviet Union. Much radioactive material escaped into the atmosphere and was carried around the world by winds. Tens of thousands of people were evacuated from the reactor vicinity and hundreds of plant and rescue workers died as a result of exposure to radiation. The effects of radioactive exposure of population in neighbouring regions are still showing up in the form of various incurable diseases even in the next generation offsprings.

However, the health controls in the atomic power industry have, from the very outset, been much more rigorous than in any other industry.

## Fusion

Energy is produced in this process by combination of two light nuclei to form a single heavier one under sustained condition of extremely high temperature and high pressure for initiation. Neutron emission is not required in the process as the temperature (high) maintains the collisions of reacting nuclei.

The most promising fuels are isotopes of hydrogen known as deuterium (D) (mass 2) and tritium (T) (mass 3). The product of fusion is the helium isotope (mass 3), hydrogen, neutrons, and heat. As tritium is not a naturally occurring isotope, it is produced in the reactor shield by the interaction of the fusion neutrons and the lithium isotope of mass 6. The deuterium-deuterium fusion requires higher temperature than deuterium-tritium and the latter is more likely to be used initially.

Reserves of lithium have been estimated to be roughly equal to those of fossil fuels. Deuterium, on the other hand, is contained in sea-water of a concentration of about 34 parts per million. The potential of this energy resource is therefore vast. Total nuclear power will be around 10280 MW by 2012.

### General Remarks

The greatest danger in a fission reactor is in the case of loss of coolant in an accident. Even with the control rods (or plates) fully lowered quickly called *scram* operation, the fission does continue for sometime and its *after-heat* may cause vaporizing and dispersal of radioactive material. This possibility does not exist in fusion process as its power density is almost 1/50th that of an FBR. The radioactive waste in fusion is the radiation damage to structural materials which would require occasional renewal. These could be recycled after 50 year period compared with centuries required for  $^{90}\text{Sr}$  and  $^{137}\text{Cs}$ , the fission fragments.

Intensive international research is still proceeding to develop materials and a suitable containment method, using either magnetic fields or powerful lasers to produce the high temperatures ( $\approx 8 \times 10^7$  K) and pressure (above 1000 bar) to initiate a fusion reaction. It is unlikely that a successful fusion reactor will be available before 2020.

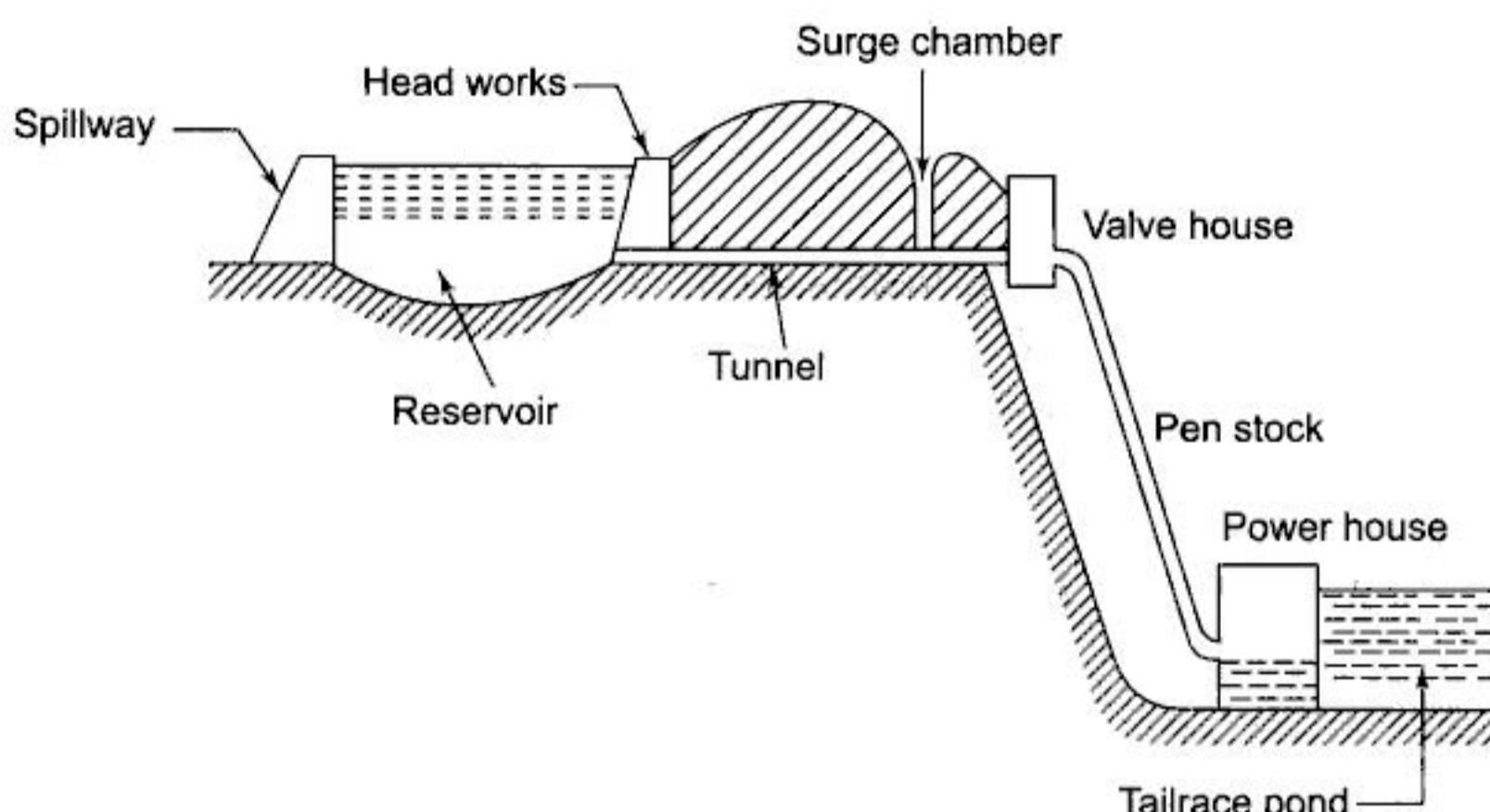
With this kind of estimated time frame for breakthrough in fusion technology, development in FBR technology and installing of power stations will continue.

France and Canada are possibly the two countries with a fairly clean record of nuclear generation. According to Indian scientists, our heavy-water based plants are most safe.

World scientists have to adopt a different reaction safety strategy-may be to discover additives to automatically inhibit reaction beyond critical rather than by mechanically inserted controlled rods, which have possibilities of several primary failure events.

## Hydro Power

The oldest and cheapest method of power generation is that of utilizing the potential energy of water. The energy is obtained almost free of running cost and is completely pollution free. Of course, it involves high capital cost because of the heavy civil engineering construction works involved. Also it requires a long gestation period of about five to eight years as compared to four to six years for steam plants. Hydroelectric stations are designed, mostly, as multipurpose projects such as river flood control, storage of drinking water, irrigation and navigation. A simple block diagram of high head hydro plant is given in Fig. 1.12. The vertical difference between the upper reservoir and the tail race pond is called the *head*.



**Fig. 1.12** A typical layout for a storage type hydro plant

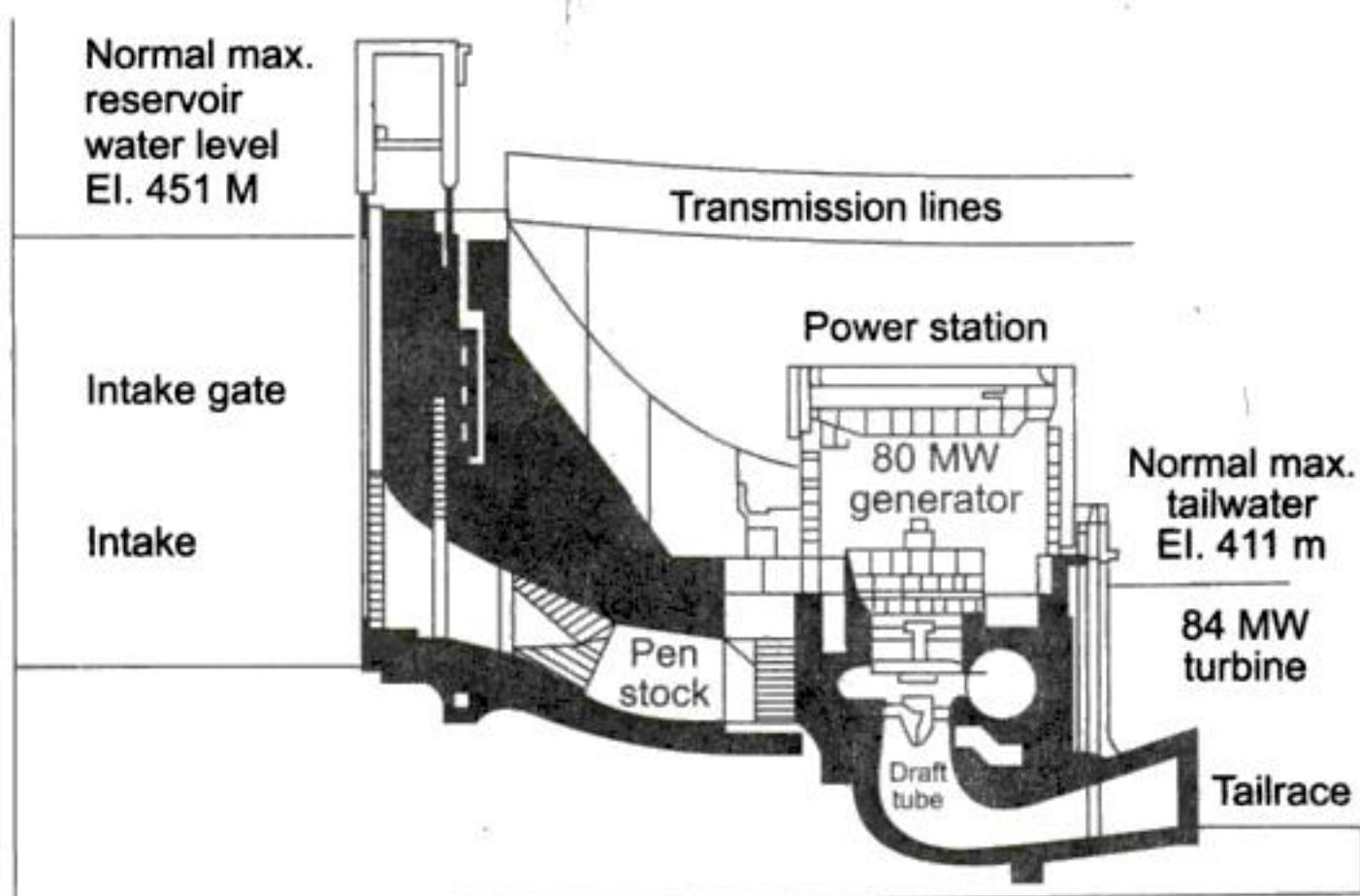
Water falling through the head gains kinetic energy which then imparts energy to the blades of the hydraulic turbine. There are three main types of hydroelectric installations. These are

1. *High head or stored*—the storage area of reservoir fills in more than 400 hectares.
2. *Medium head or pondage*—the storage fills in 200–400 hectares.
3. *Run of river*—storage (in any) fills in less than 2 h and has a 3–15 m head.

A schematic diagram for hydroelectric schemes of Type 3 is shown in Fig. 1.13.

There can be several of these turbines on a deep and wide river.

In India mini and micro hydroelectric schemes have been installed on canals wherever 1 m or so head is available. Often cascaded plants are also constructed on the same water stream where the discharge of one plant becomes the inflow of a downstream plant.



**Fig. 1.13** Run of river hydroelectric scheme—80 MW Kaplan turbine, 115.41 rpm

For the three above identified heads of water level, the kind of turbines that are employed:

1. *Pelton*: This is used for heads of 184–1840 m and consists of a bucket wheel rotor with adjustable flow nozzles.
2. *Francis*: This is used for heads of 37–490 m and is of mixed flow type.
3. *Kaplan*: This is used for run-of-river and pondage stations with heads of up to 61 m. This type has an axial-flow rotor with variable-pitch blades.

Hydroelectric plants are capable of starting quickly—almost in 5 min. The rate of taking up load on the machines is of the order of 20 MW/min. Further, no losses are incurred at standstill. Thus, hydroelectric plants are ideal for meeting peak loads. The time from start up to the actual connection to the grid can be as short as 2 min.

The power available from a hydro plant is

$$P = g \rho W H \text{ W} \quad (1.4)$$

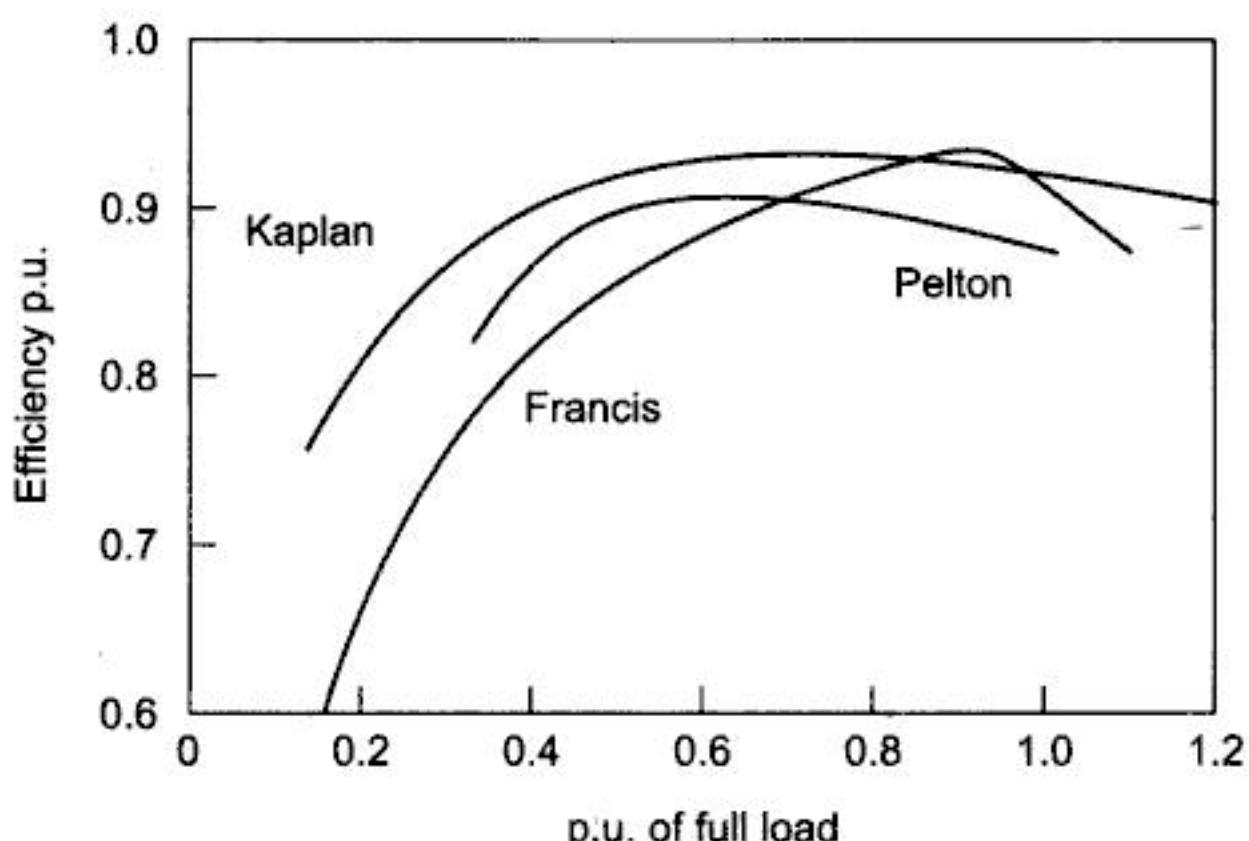
where  $W$  = discharge ( $\text{m}^3/\text{s}$ ) through the turbine,  $\rho$  = density ( $1000 \text{ kg/m}^3$ ) and  $H$  = head (m),  $g = 9.81 \text{ m/s}^2$

$$\therefore P = 9.81 W H \text{ kW} \quad (1.5)$$

Problems peculiar to hydroelectric plants which inhibit expansion are:

1. Silting—Bakra dead storage has reportedly silted fully in 30 years.
2. Seepage.
3. Ecological damage to region.
4. Displacement of human habitation from areas behind the dam which will fill up and become a lake.
5. These cannot provide base load and must be used for peak shaving and energy saving in coordination with thermal plants.

Typical efficiency curves of the three types of turbines are depicted in Fig. 1.14. As the efficiency depends upon the head, which is continuously fluctuating, water consumption in  $\text{m}^3/\text{kWh}$  is used instead of efficiency, which is related to water head.



**Fig. 1.14** Typical efficiency curves of hydraulic turbines

In certain periods when the water availability is low or when hydro-generation is not needed, it may be advantageous to run electric generators as motors from the grid, so as to act as synchronous condensers (these are overexcited). To reduce running losses, the water is pushed below the turbine runner by compressed air after closing the input valve. The runner now rotates in air and free running losses are low.

India also has a tremendous potential (5000 MW) of having large number of nano, pico, micro ( $< 1 \text{ MW}$ ), mini ( $< 1\text{--}5 \text{ MW}$ ) and small ( $< 15 \text{ MW}$ ) *hydel plants* in Himalayan region, North-East, H.P., U.P., U.K., and J.K. which must be fully exploited to generate cheap and clean power for villages situated far away from the grid power\*. At present 500 MW capacity is under construction.

### **Pumped Storage Scheme**

In areas where sufficient hydrogeneration is not available, peak load may be handled by means of pumped storage. This consists of upper and lower reservoirs and reversible turbine-generator sets, which can also be used as motor-pump sets. The upper reservoir has enough storage for about 6 h of full load generation. Such a plant acts as a conventional hydroelectric plant during the peak load periods, when production costs are the highest. The turbines are driven by water from the upper reservoir in the usual manner. During the light load period, water in the lower reservoir is pumped back into the upper one so as to be ready for use in the next cycle of the peak load period. The generators in this period change to synchronous motor action and drive the turbines which

\* Existing capacity (small hydro) is 1341 MW as on June 2001, Total estimated potential is 15000 MW.

now work as pumps. The electric power is supplied to the generator sets from the general power network or an adjoining thermal plant. The overall efficiency of the generator sets is normally as high as 60–70%. The pumped storage scheme, in fact, is analogous to the charging and discharging of a battery. It has the added advantage that the synchronous machines can be used as synchronous condensers for VAR compensation of the power network, if required. In a way from the point of view of the thermal sector of the power system, the pumped storage scheme *shaves the peaks* and fills the *troughs* of the daily load-demand curve.

Some of the existing pumped storage plants are 900 MW Srisailam in AP, 80 MW of Bhiva in MS, 400 MW Kadamparai in TN.

### **Tidal Power**

Along the shores with high tides and when a basin exists, the power in the tide can be hydroelectrically utilized. This requires a long and low dam across the basin. Two sets of turbines are located underneath the dam. As the tide comes in water flows into the basin operating one set of turbines. At low tide the water flows out of the basin operating another set of turbine.

Let tidal range from high to low be  $h$  (m) and area of water stored in the basin be  $A$  ( $\text{m}^2$ ), then the energy stored in the full basin is expressed as

$$E = \rho g A \int_0^h x dx \quad (1.5)$$

$$= \frac{1}{2} \rho g h^2 A$$

$$\begin{aligned} \text{Average power, } P &= \frac{1}{2} \rho g h^2 A / (T/2); \quad T = \text{period of tidal cycle} \\ &= 14 \text{ h } 44 \text{ min, normally} \end{aligned}$$

$$= \rho g h^2 A / T$$

A few places which have been surveyed in the world as sites for tidal power are as follows:

Passanaquoddy Bay (N. America)	5.5 m, 262 km <sup>2</sup> , 1,800 MW
San Jose (S. America)	10.7 m, 777 km <sup>2</sup> , 19,900 MW
Sever (UK)	9.8 m, 70 km <sup>2</sup> , 8,000 MW

Major sites in India where preliminary investigations have been carried out are Bhavnagar, Navalakhi (Kutch), Diamond Harbour and Ganga Sagar.

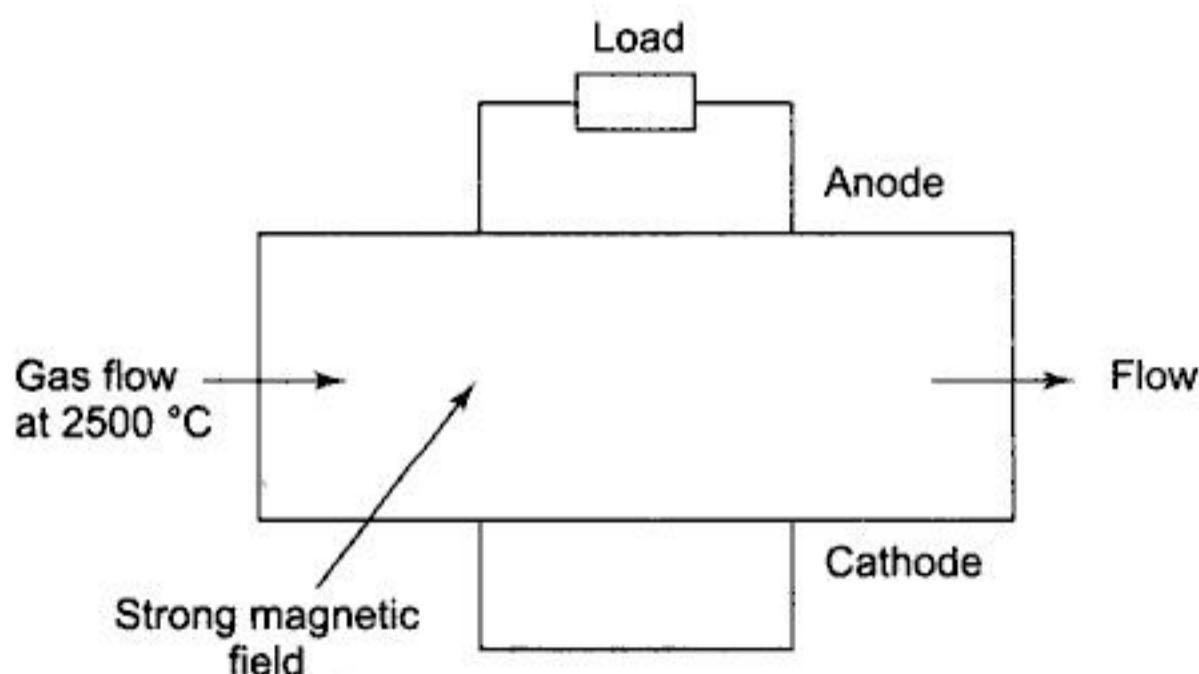
India's first Tidal Power Project is being developed by WBREDA at Durgaduani Creek in the Sunderbans delta. High tide water is stored in a reservoir and released at low tide, thus creating water flows which drive turbines that generate electricity. Total cost for 4 MW project is Rs 40 crores and will be ready by 2009.

The basin in Kandla in Gujarat has been estimated to have a capacity of 600 MW. The total potential of Indian coast is around 9000 MW, which does not compare favourably with the sites in the American continent stated above. The technical and economic difficulties still prevail.

### 1.6 MAGNETOHYDRODYNAMIC (MHD) GENERATION

In thermal generation of electric energy, the heat released by the fuel is converted to rotational mechanical energy by means of a thermocycle. The mechanical energy is then used to rotate the electric generator. Thus two stages of energy conversion are involved in which the heat to mechanical energy conversion has an inherently low efficiency. Also, the rotating machine has its associated losses and maintenance problems. In MHD technology electric energy is directly generated by the hot gases produced by the combustion of fuel without the need for mechanical moving parts.

In an MHD generator, electrically conducting gas at a very high temperature is passed in a strong magnetic field, thereby generating electricity. High temperature is needed to ionize the gas, so that it has good electrical conductivity. The conducting gas is obtained by burning a fuel and injecting a seeding material such as potassium carbonate in the products of combustion. The principle of MHD power generation is illustrated in Fig. 1.15. Electrically conducting gas as it flows is equivalent to electric current flowing in an imaginary conductor at  $90^\circ$  to the magnetic field. The result is induction of emf across an anode and cathode with current flowing through the load. About 50% efficiency can be achieved, if the MHD generator is operated in tandem with a conventional steam plant.



**Fig. 1.15** The principle of MHD power generation

Although the technological feasibility of MHD generation has been established, its economic feasibility is yet to be demonstrated. In fact with the development of CCGT systems, which are being installed in many countries, MHD development has been put on the shelf.

## 1.7 GEOTHERMAL ENERGY

The outer crust of earth contains a very large reserve of energy as sensible heat. It is estimated to be one to two orders of magnitude larger than all the energy recoverable from uranium (by fission) and thorium (by breeder reactor assuming 60–70% efficiency). Fusion as and when it becomes technologically practical would represent a large energy resource than geothermal energy.

Geothermal energy is present over the entire extent of earth's surface except that it is nearer to the surface in volcanic areas. Heat transfer from the earth's interior is by three primary means:

1. Direct heat conduction;
2. Rapid injection of ballistic magma along natural rifts penetrating deep into earth's mantle; and
3. Bubble like magma that buoys upwards towards the surface.

Rift geothermal areas in sedimentary rock basins undergo repeated injection of magma, though in small amount. Over a long period of time these processes cause massive amounts of hot water to accumulate. Examples are the Imperiod Valley of Africa. The weight of the overburden in these sedimentary basins compresses the trapped water giving rise to a geopressurized geothermal resource. These high pressures serve to increase the productivity of hot-water wells, which may be natural or drilled.

Pressure released in the hot wells causes boiling and the steam and water mixture rise upwards. This mixture is passed through steam separators, which then is used to drive low-pressure steam turbines. Corrosive effects of this wet steam, because of mineral particles in it, have been tackled by advanced metallurgy. The capital cost of these plants is 40 to 60% less than that of fossil fuel and nuclear plants, because no boiler or nuclear reactor is needed to generate steam.

Geothermal plants have proved useful for base-load power plants. These kind of plants are primarily entering the market where modest sized plants are needed with low capital cost, short construction period and life-long fuel (i.e. geothermal heat).

High air-quality standards are easily attained by geothermal plants at a minimal cost such that they have an edge over clean coal-fueled plants. Considerable research and development effort is being devoted towards geothermal plants siting, designing, fabricating, installation and operation. Efforts are also on to tap the heat potential of volcanic regions and from hot volcanic rock.

No worth mentioning effort is being made in India at present. In India, feasibility studies of a 1 MW station at Peggy valley in Ladakh are being carried out. Another geothermal field has been located at Chumantang. There are a number of hot springs in India, but the total exploitable energy potential seems to be very little.

The present installed geothermal plant capacity in the world is about 500 MW and the total estimated capacity is immense provided volcanic 'regions' heat can be utilized. Since the pressure and temperatures are low, the efficiency is even less than that of the conventional fossil-fuelled plants, but the capital costs are less and the fuel is available free of cost.

### 1.8 ENVIRONMENTAL ASPECTS OF ELECTRIC ENERGY GENERATION

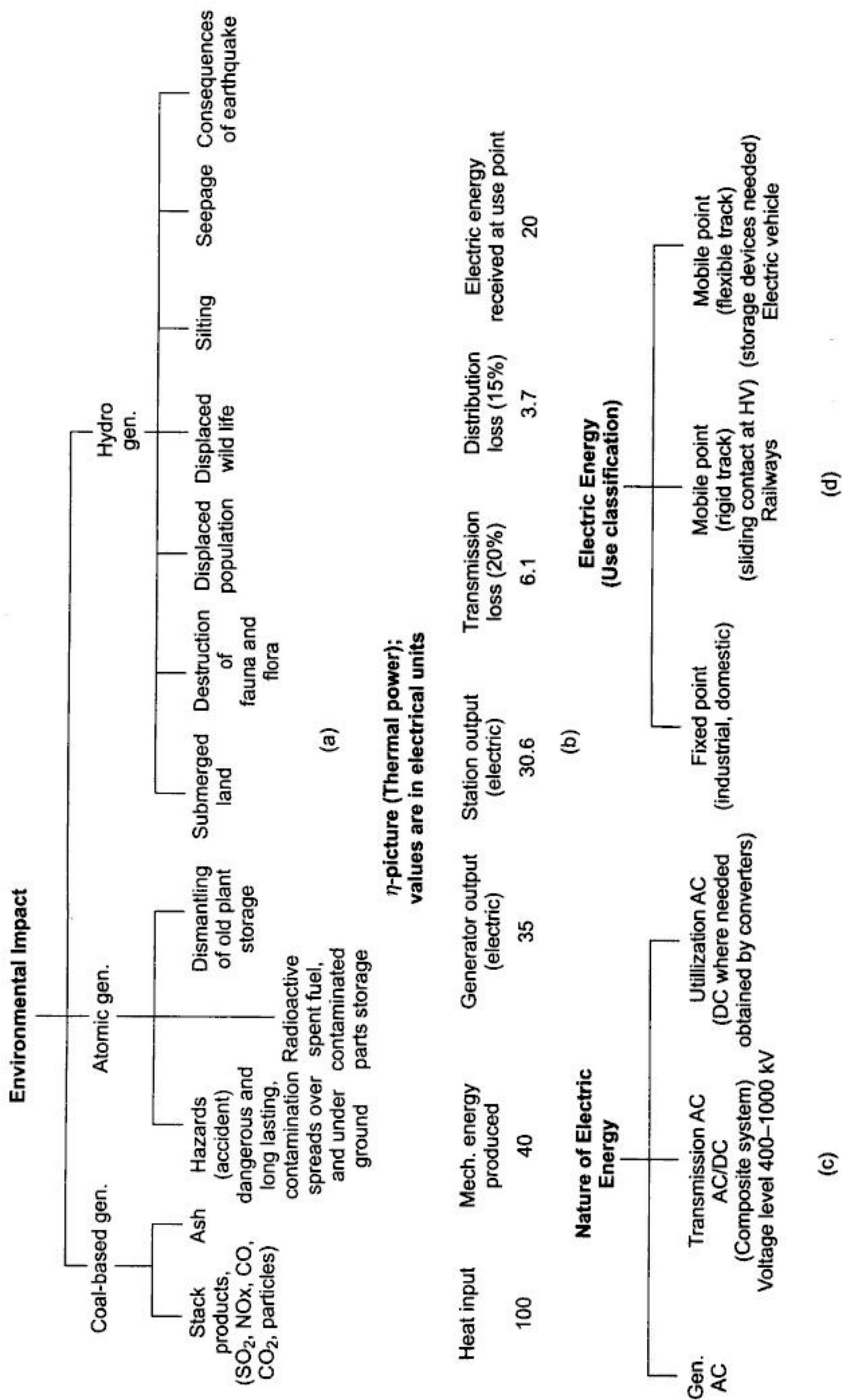
As far as environmental and health risks involved in nuclear plants of various kinds are concerned, these have already been discussed at length in Sec. 1.7. Equally the problems related to large hydroelectric plants have been dwelled upon in Sec. 1.5. Therefore, we shall now focus our attention on fossil-fuel plants including gas-based plants.

Conversion of one form of energy or another to electrical form has unwanted side effects and the pollutants generated in the process have to be disposed off. The reader may refer to Fig. 1.16, which brings out all the associated problems at a glance. Pollutants know no geographical boundary; as a result the pollution issue has become a nightmarish problem and strong national and international pressure groups have sprung up and are having a definite impact on the development of energy resources. Governmental awareness has created increasing legislation at national and international levels. The power engineers have to be fully conversant with these in their professional practice and in the survey and planning of large power projects. Lengthy, time consuming procedures at government level, PIL (public interest litigation) and demonstrative protests have delayed several projects in several countries. This has led to favouring of small size projects and redevelopment of existing sites. But with the yawning gap in electric demand and production, our country has to move forward for several large thermal, hydro and nuclear power projects.

Emphasis is being laid on conservation issues, curtailment of transmission losses, theft, subsidized power supplies and above all on *sustainable development* with *appropriate technology* wherever feasible. It has to be particularly assured that no irreversible damage is caused to the environment which would affect the living conditions of the future generations. Irreversible damages like ozone layer holes and global warming caused by increase in CO<sub>2</sub> in the atmosphere are already showing up.

#### Atmospheric Pollution

We shall treat here only pollution as caused by thermal plants using coal as feed-stock. The fossil fuel-based generating plants form the backbone of power generation in our country and also round the globe as other options (like nuclear and even hydro) have even stronger hazards associated with them. Also it should be understood that pollution in large cities like Delhi is caused more by vehicular traffic and their emission. In Delhi of course Inderprastha and Badarpur power stations contribute their share in certain areas.



**Fig. 1.16** Environmental and other aspects of electric energy production and use

Problematic pollutants in emission of coal-based generating plants are

- $\text{SO}_2$
- $\text{NO}_x$ , nitrogen oxides
- CO
- $\text{CO}_2$
- Certain hydrocarbons
- Particulates

Although the account that follows will be general, it needs to be mentioned here that Indian coal has a comparatively low sulphur content but a very high ash content, which in some coals may be as high as 53%.

A brief account of various pollutants, their likely impact and methods of abatements are presented below.

### ***Oxides of Sulphur ( $\text{SO}_2$ )***

Most of the sulphur present in the fossil fuel is oxidized to  $\text{SO}_2$  in the combustion chamber before being emitted by the chimney. In atmosphere it gets further oxidised to  $\text{H}_2\text{SO}_4$  and metallic sulphates, which are the major source of concern as these can cause acid rain, impaired visibility and damage to buildings and vegetation. Sulphate concentrations of 9–10  $\mu\text{g}/\text{m}^3$  of air aggravate asthma, lung and heart disease. It may also be noted that although sulphur does not accumulate in air, it does so in soil.

Sulphur emission can be controlled by

- use of fuel with less than 1% sulphur; generally not a feasible solution;
- use of chemical reaction to remove sulphur in the form of sulphuric acid from combustion products by limestone scrubbers or fluidized bed combustion; and
- removing sulphur from the coal by gasification or floatation processes.

It has been noticed that the byproduct sulphur could off-set the cost of sulphur recovery plant.

### ***Oxides of Nitrogen ( $\text{NO}_x$ )***

Of these Nitrogen oxide,  $\text{NO}_2$  is a major concern as a pollutant. It is soluble in water and so has adverse affect on human health as it enters the lungs on inhaling and after combining with moisture converts to nitrous and nitric acids, which damage the lungs. At levels of 25–100 parts per million,  $\text{NO}_x$  can cause acute bronchitis and pneumonia.

Emission of  $\text{NO}_x$  can be controlled by fitting advanced technology burners which can assure more complete combustion, thereby reducing these oxides from being emitted by the stack. These can also be removed from the combustion products by absorption process by certain solvents going on to the stack.

### **Oxides of Carbon ( $CO$ , $CO_2$ )**

$CO$  is a very toxic pollutant, but it gets converted to  $CO_2$  in the open atmosphere (if available) surrounding the plant. On the other hand  $CO_2$  has been identified as a major cause of global warming. It is not yet a serious problem in developing countries.

### **Hydrocarbons**

During the oxidation process in combustion chamber certain light weight hydrocarbons may be formed. The compounds are a major source of photochemical reaction that adds to depletion of ozone layer.

### **Particulates (Fly ash)**

Dust content is particularly high in the Indian coal. Particulates come out of the stack in the form of fly ash. It comprises fine particles of carbon, ash and other inert materials. In high concentrations, these cause poor visibility and respiratory diseases.

Concentration of pollutants can be reduced by the dispersal over a wider area by use of high stacks. *Precipitators* can be used to remove particles as the flue gases rise up the stack. If in the stack a vertical wire is strung in the middle and charged to a high negative potential, it emits electrons. These electrons are captured by the gas molecules thereby becoming negative ions. These ions accelerate towards the walls, get neutralized on hitting the walls and the particles drop down the walls. Precipitators have a high efficiency, upto 99% for large particles, but they have a poor performance for particles of size less than  $0.1 \mu m$  in diameter. The efficiency of precipitators is high with reasonable sulphur content in flue gases but drops for low sulphur content coals; 99% for 3% sulphur and 83% for 0.5% sulphur.

Fabric filters in form of *bag houses* also have been employed and are located before the flue gases enter the stack.

### **Thermal Pollution**

Steam from low-pressure turbine has to be liquefied in a *condenser* and reduced to lowest possible temperature to maximize the thermodynamic efficiency. The best efficiency of steam cycle practically achievable is about 40%. It means that 60% of the heat in steam at the end of cycle must be removed. This is achieved by two methods:

1. *Once through* circulation through condenser cooling tubes of sea or river water where available. This raises the temperature of water in these two sources and threatens sea and river life around in sea and downstream in river. These are serious environmental objections and many times cannot be overruled and also there may be legislation against it.

2. *Cooling towers* Cool water is circulated around the condenser tube to remove heat from the exhaust steam in order to condense it. The circulating water gets hot in the process. It is pumped to the cooling towers and is sprayed through nozzles into a rising volume of air. Some of the water evaporates providing cooling. The latent heat of water is  $2 \times 10^6$  J/kg and cooling can occur fast. But this has the disadvantage of raising the humidity to high (undesirable) levels in the surrounding areas. Of course the water evaporated must be made up in the system by adding fresh water from the source. These cooling towers are known as *wet towers*.

Closed cooling towers where condensate flows through tubes and air is blown on these tubes avoids the humidity problem but at much higher cost. In India only wet towers are being used.

### **Electromagnetic Radiation from Overhead Lines**

Biological effects of electromagnetic radiation from power lines and even cables in close proximity of buildings have recently attracted attention and have also caused some concern. Power frequency (50 to 60 Hz) and even their harmonies are not considered harmful. Investigations carried out in certain advanced countries have so far proved inconclusive. The electrical and electronics engineers, while being aware of this controversy, must know that many other environmental agents are moving around that can cause far greater harm to human health than does electromagnetic radiation.

As a piece of information it may be quoted that directly under an overhead line of 400 kV, the electric field strength is 11000 V/m and magnetic flux density (depending on current) may as much as 40  $\mu\text{T}$ . Electric field strength in the range of 10,000–15,000 V/m is considered safe.

### **Visual and Audible Impacts**

These environmental problems are caused by the following factors:

1. Right of way acquires land underneath. At present it is not a serious problem in India, but in future the problem will show up. This is futuristic.
2. Lines converging at a large substation mar the beauty of the landscape around. Underground cables as an alternative are too expensive a proposition except in congested city areas.
3. Radio frequency interference (RFI) has to be taken into account and countered by various means.
4. The phenomenon of *corona* (a sort of electric discharge around the high tension line) produces a hissing noise which is audible when habitation is in close proximity. At the towers great attention must be paid to tightness of joints, avoidance of sharp edges and use of earth screen shielding to limit audible noise to acceptable levels. (For details, see Ch.19)

5. Workers inside a power plant are subjected to various kinds of noise (particularly near the turbines) and vibration of floor. To reduce this noise to a tolerable level, foundations and vibration filters have to be designed properly and simulation studies carried out. The workers must be given regular medical examinations and sound medical advice.

## 1.9 RENEWABLE ENERGY RESOURCES

In the account that has preceded, we have concentrated mostly on those energy resources which are nonreplenishable as brought out in Fig. 1.5 (left side). These are mainly coal, oil, gas and nuclear fission. Apart from the fact that these cannot last for long, considering the galloping rate at which electricity use is rising, they have serious environmental impacts and hazards associated with electric power generation as brought out in Fig. 1.16(a). This has led to a concerted international effort in research and development of renewable energy resources. They offer viable options to address the energy security issues. India has one of the highest potential for the effective use of renewables. Special emphasis has been laid on the generation of grid quality power from them.

A major source of renewable energy is solar radiation being cyclically received by most land area of the globe. Its various manifestations are presented in Fig. 1.5 (right side) as follows:

1. Direct use;
  2. Winds on land area of globe;
  3. Potential energy of rain and snow at high altitudes, i.e. hydro energy;
  4. Biofuel.

**Gravitational pull of moon on earth** 1.Tidal energy;  
2. Wave energy

**Geothermal** It is considered renewable because the resource is unlimited.

All the above resources, other than geothermal, pass through the environment as a *energy current or flow*. Together these energy flows are called *energy flux*. The earth's habitable surface is crossed by or accessible to an average energy flux of about  $500 \text{ W/m}^2$ . If this flux can be harnessed at just about 4% efficiency, a  $10 \text{ m} \times 10 \text{ m}$  surface would contribute 2 kW of power using suitable methods. Assume that an average suburban person consumes 2 kW and a population density of 500 persons/ $\text{km}^2$ . At 2 kW per person, the total energy demand of  $1000 \text{ kW/km}^2$  could be met by using just 5% of land area for energy production. This could provide a fairly satisfactory standard of living across the globe. Realistically it is not as rosy as harnessing renewable energy is not an easy task and ridden with technological problems whose economic

solutions are yet to be found. To further complicate matters, the renewable energy flux is far from uniformly distributed round the globe.

On account of the environmental impact of harnessing hydro energy and the limitation of harnessing tidal energy, these have been treated in Sec. 1.5. Geothermal energy has also been considered alongwith thermal generation in Sec. 1.7.

We shall now study solar energy and wind energy, the methods of harnessing these and the difficulties encountered. We shall also touch up biofuel.

### **Wave Energy**

The energy content of sea waves is very high. In India, with several hundreds of kilometers of coast line, a vast source of energy is available. The power in the wave is proportional to the square of the amplitude and to the period of the motion. Therefore, the long period ( $\sim 10$  s), large amplitude ( $\sim 2$  m) waves are of considerable interest for power generation, with energy fluxes commonly averaging between 50 and 70 kW/m width of oncoming wave. Though the engineering problems associated with wave-power are formidable, the amount of energy that can be harnessed is large and development work is in progress. Sea wave power estimated potential is 20,000 MW.

### **Ocean Thermal Energy Conversion (OTEC)**

The ocean is the world's largest solar collector. Temperature difference of  $20^{\circ}\text{C}$  between warm, solar absorbing surface water and cooler 'bottom' water can occur. This can provide a continually replenished store of thermal energy which is in principle available for conversion to other energy forms. OTEC refers to the conversion of some of this thermal energy into work and thence into electricity. Estimated potential of ocean thermal power in India is 50,000 MW.

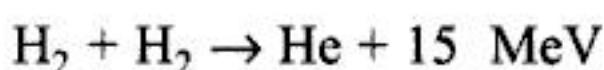
A proposed plant using sea temperature difference would be situated 25 km east of Miami (USA), where the temperature difference is  $17.5^{\circ}\text{C}$ .

## **1.10 SOLAR ENERGY AND ITS UTILIZATION**

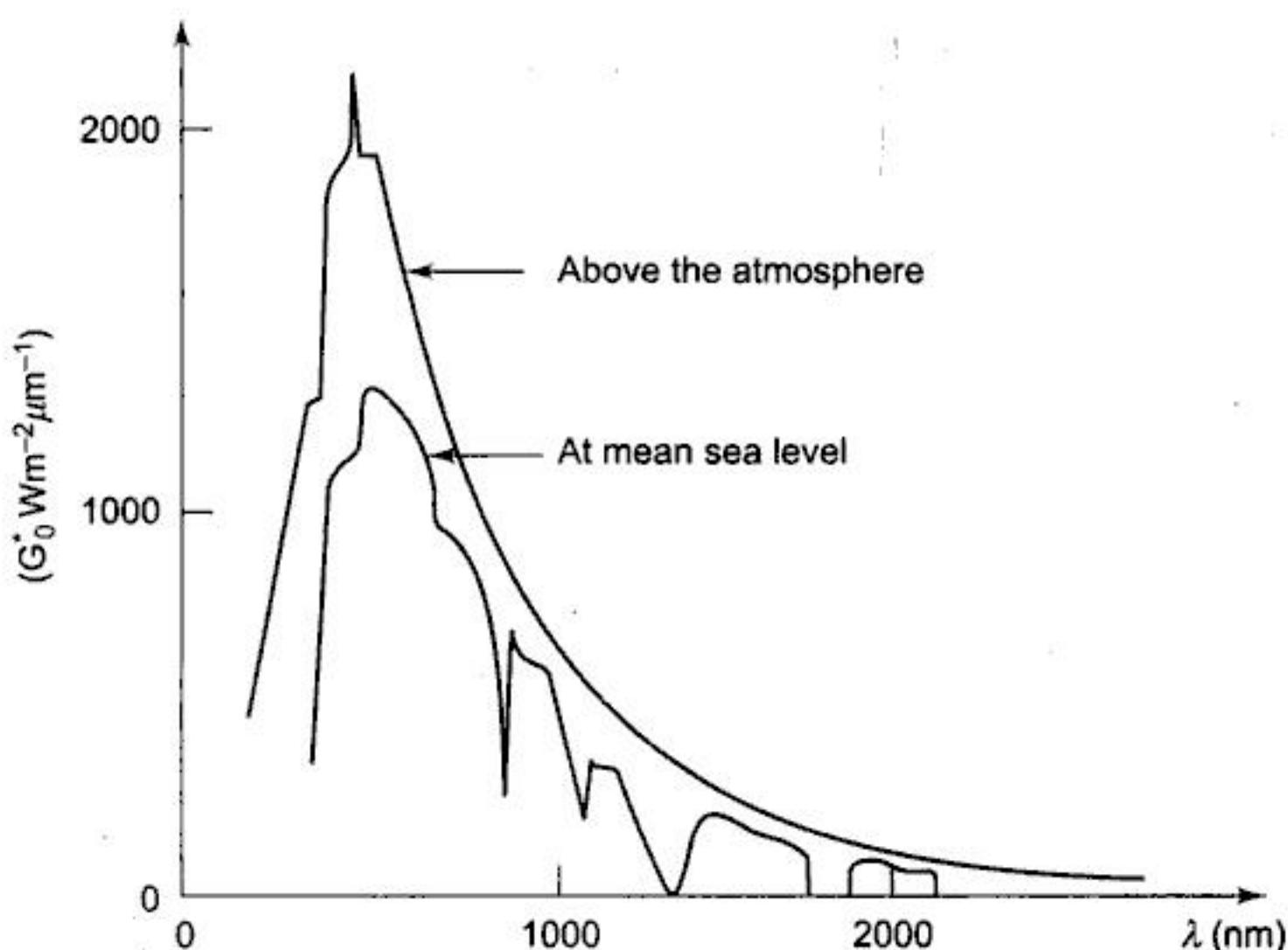
Solar energy is a free source which is not only naturally renewable but is also environment friendly and thus helps in lessening the greenhouse effects. As shall be seen in the account that follows, it can only supplement to a (very) limited extent the burgeoning need for energy across the globe. In India, with a deficient grid power and large number of sunny days across the country, solar energy as a supplement is particularly attractive.

## The Sun and Solar Energy

The sun is a spherical mass of hot gases, with a diameter of about  $1.39 \times 10^9$  m and at an average distance of  $1.5 \times 10^{11}$  m from the earth. Energy is being continuously produced in the sun through various nuclear fusion reactions, the most important one being where four protons combine to form a helium nucleus.



The mass lost in the process is converted into energy. These reactions occur in the innermost core of the sun, where the temperature is estimated to be  $(8-40) \times 10^6$  K. The various layers of differing temperatures and densities emit and absorb different wavelengths making the solar spectrum quite composite. However, the sun essentially acts as a black body having a 5800 K temperature. The spectral distribution of solar radiation at the earth's mean distance is shown in Fig. 1.17.



**Fig. 1.17** Spectral distribution of the sun's radiation

The solar constant is the radiant flux density incident on a plane normal to the sun's rays at a distance of  $1.49 \times 10^8$  km from the sun and is given by the area under the curve in Fig. 1.17. It has a value of

$$G_o^* = 1367 \text{ W/m}^2$$

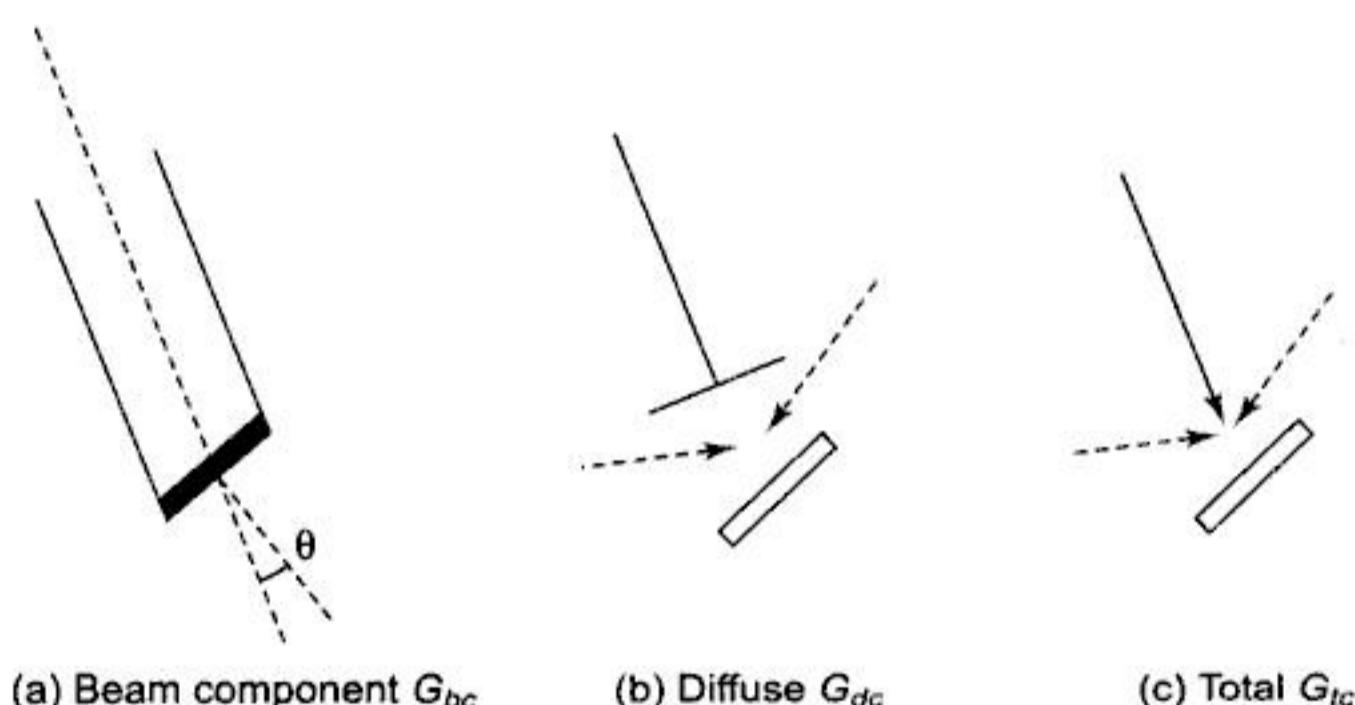
The received flux density varies by  $\pm 1.5\%$  during the day's course due to variations in the sun's output, and by about  $\pm 4\%$  over the year due to the earth's elliptic orbit. The solar spectrum can be divided into three main regions:

1. Ultraviolet region ( $\lambda < 400 \mu\text{m}$ ): 9%;
2. Visible region ( $400 \text{ nm} < \lambda < 700 \text{ nm}$ ): 45%;
3. Infrared region ( $\lambda > 700 \text{ nm}$ ): 46%.

The radiation in the wavelengths above 2500 nm are negligible.

The earth's atmosphere absorbs various components of the radiation to different levels. The short wave UV and X-ray regions are almost completely absorbed by oxygen and nitrogen gases and ions; the ozone absorbs UV rays. The atmosphere unaffected by dust or clouds acts as an open window for the visible region. Up to 20% of the IR (Infrared) radiation is absorbed by the water vapour and  $\text{CO}_2$ . The carbon dioxide concentration in the atmosphere is about 0.03% by volume and is beginning to rise with pollutants being let off into the atmosphere. The water vapour concentration can vary greatly (upto 4% by volume). Dust, water droplets and other molecules scatter the sun's radiation.

The sun's radiation at the earth's surface is composed of two components: *beam radiation* and *diffuse radiation*. Beam or direct radiation consists of radiation along the line connecting the sun and the receiver as shown in Fig. 1.18(a). Diffuse radiation is the radiation scattered by the atmosphere without any unique direction as in Fig. 1.18(b). There is also a reflected component due to terrestrial surface. Total radiation is shown in Fig. 1.18(c).



**Fig. 1.18** Components of solar radiation reaching earth

It easily follows from these figures that [21]

$$G_{bc} = G_b^* \cos \theta \quad (1.6)$$

For a horizontal surface, the relation becomes

$$G_{bh} = G_b^* \cos \theta_z \quad (1.7)$$

Here  $\theta_z$  (called the Zenith angle) is the angle of incidence of beam component of solar radiation for a horizontal surface.  $\theta$  is shown in

Fig. 1.18(a).  $G_b^*$  is intensity of beam component of normally incident solar radiation on a surface. Adding the beam of the diffuse components, we get

$$G = G_{tc} = G_{bc} + G_{dc} \quad (1.8)$$

### Variation of Insolation

Practically the earth is a sphere of radius 6400 km which rotates once in 24 h about its own axis. The axis defined by the North and South poles is shown in Fig. 1.19

Any point  $P$  on the earth's surface is determined by its latitude  $\phi$  and longitude  $\psi$ . The latitude is positive in the northern hemisphere, and negative in the southern hemisphere. The longitude is measured positive eastward from Greenwich, England. The vertical North-South plane through  $P$  is called *Local Meridional Plane*. Solar noon

at  $P$  and all places of the same longitude is defined, when the sun is included in the meridional plane. However, clocks do not necessarily show solar time as they are set to civil time common to time zones spanning  $15^\circ$  of longitude. Also the true interval between two successive solar noons is not exactly 24 h due to the elliptic orbit of the earth. The hour angle  $\omega$  is the angle by which the earth has rotated since the solar noon.

$$\omega = 15^\circ/\text{h} \times (T_{\text{solar}} - 12 \text{ h}) \quad (1.9)$$

$$\text{or } \omega = 15^\circ \text{ h} \times (T_{\text{zone}} - 12 \text{ h}) + (\psi - \psi_{\text{zone}}) \quad (1.10)$$

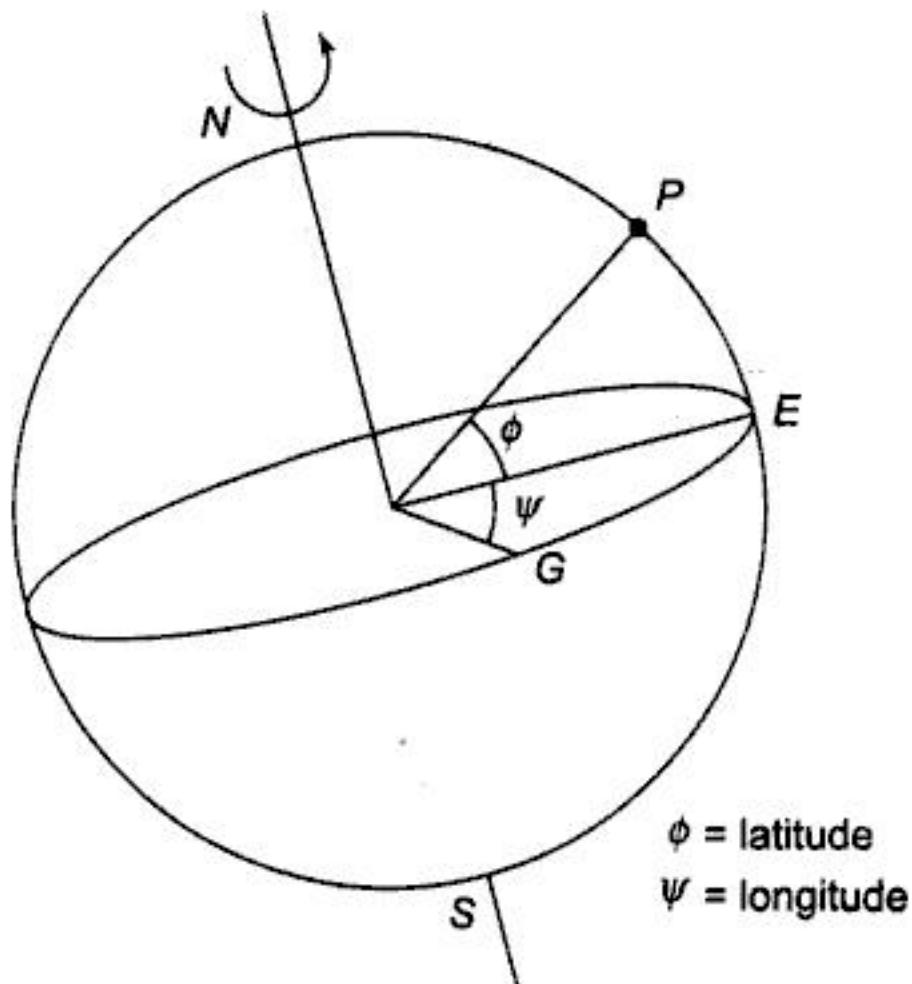
where  $T_{\text{solar}}$  is the solar time and  $T_{\text{zone}}$  is the zone time.

The earth revolves around the sun in an elliptic orbit in 365 days with its axis inclined at angle  $\delta_0 = 23.5^\circ$  to the normal to the plane of revolution around the sun.

The *declination*  $\delta$  is defined as the angle between the equatorial plane and the sun's direction. It varies from  $+23.5^\circ$  to  $-23.5^\circ$  from 21st June to 21st December—the *summer and winter solstices in the Northern Hemisphere*. It is zero on the *equinoxes*. The declination can be expressed as

$$\delta = \delta_0 \sin \left( \frac{360^\circ (284 + n)}{365} \right) \quad (1.11)$$

where  $n$  is the day of the year counted from the 1st of January.

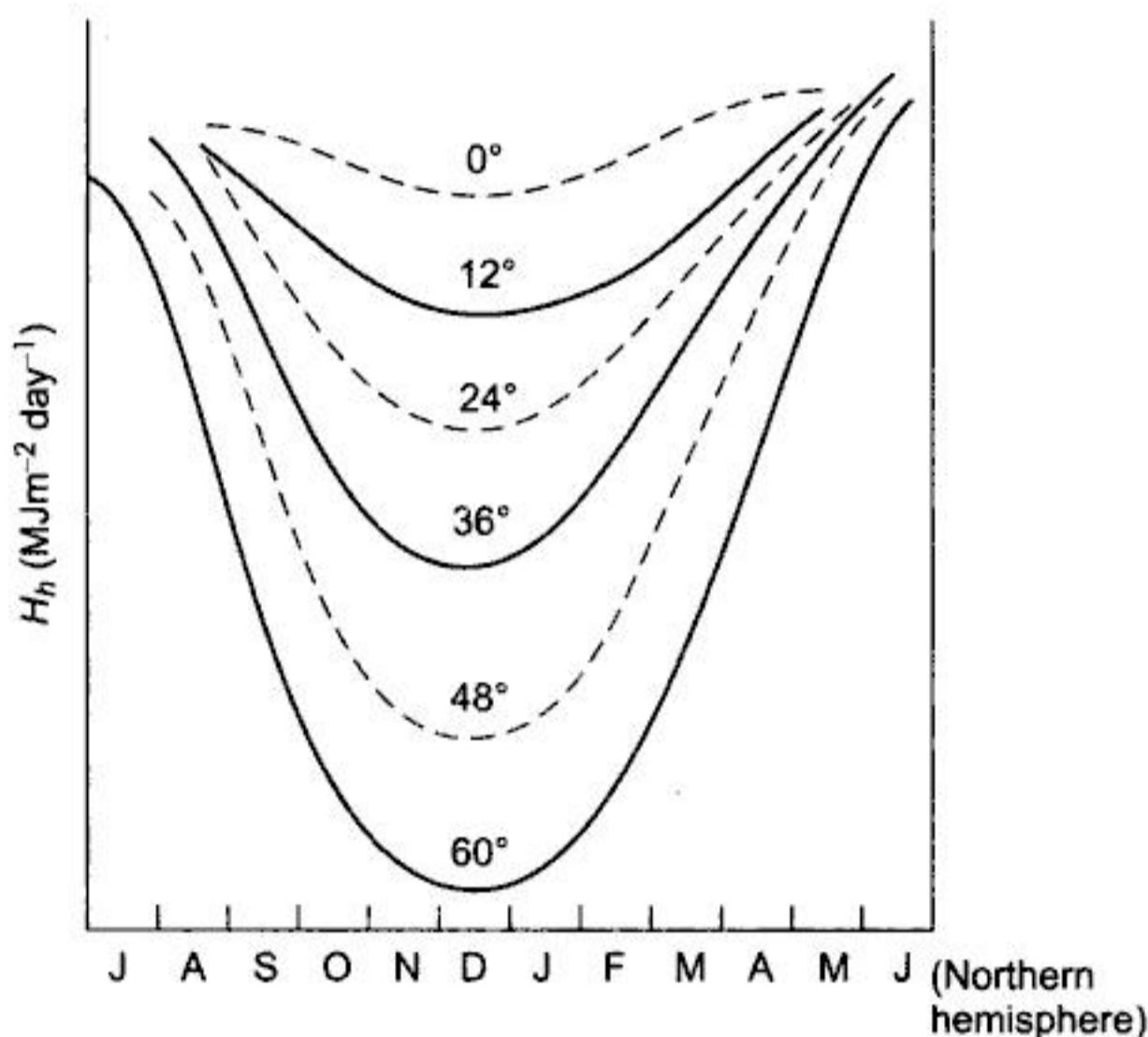


**Fig. 1.19**

The daily insolation is the total energy received from the sun per unit area in one day. The variation of daily insolation with latitude and season is shown in Fig. 1.20.

The variation arises due to three main factors:

- Variation in the length of the day;
- Orientation of the receiving surface due to the earth's declination; and
- Variation in atmospheric absorption.



**Fig. 1.20** Variation in daily insolation

### Geometry of the Collector and Solar Beam

For a tilted collector surface as in Fig. 1.21, the following angles are defined. Slope  $\beta$  is the angle between the collector surface and the horizontal surface. *Azimuth angle*  $\gamma$  is the deviation of the projection of the normal to the collector surface on a horizontal plane. In the northern hemisphere for a south facing surface or horizontal surface,  $\lambda = 0$ .  $\lambda$  is positive for surface facing West of South, and negative for surfaces facing East of South. The general relation between various angles can be shown to be

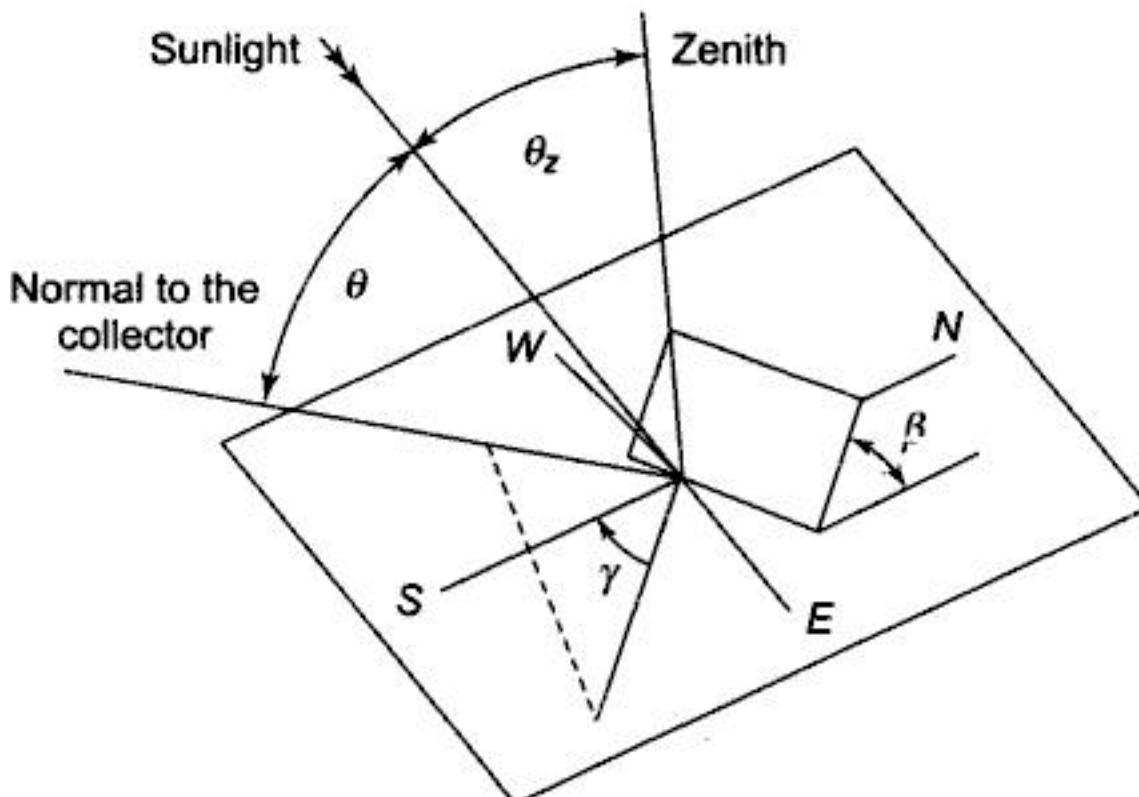
$$\cos \theta = (A - B) \sin \delta + [C \sin \omega + (D + E) \cos \omega] \cos \delta \quad (1.12)$$

where

$$A = \sin \phi \cos \beta$$

$$B = \cos \phi \sin \beta \cos \gamma$$

$$C = \sin \beta \sin \gamma$$



**Fig. 1.21** Geometry of the collector and solar beam

$$D = \cos \phi \cos \beta$$

$$E = \sin \phi \sin \beta \cos \gamma$$

$\omega$  = hour angle given by the equation

For a horizontal plane,  $\gamma = \beta = 0$ ; giving

$$\cos \theta = \sin \phi \sin \delta + \cos \phi \cos \omega \cos \delta \quad (1.13)$$

If the collector's slope equals the latitude, i.e.  $\beta = \phi$ , it will face the solar beam directly at noon. In this case:

$$\cos \theta = \cos \omega \cos \delta \quad (1.14)$$

### Optimum Orientation of the Collectors

The insolation received at the collector's plane is the sum of beam and diffuse components, i.e.,

$$H_c = \int (G_b^* \cos \theta + G_d) dt \quad (1.15)$$

To maximize the energy collected,  $\cos \theta$  should be as close to 1 as possible. This is achieved by continuous *tracking*, always maintaining  $\cos \theta$  as 1 by letting the collector directly face the solar beam. By mounting the array on a two-axis tracker, upto 40% more energy, as compared to a fixed slope collector, can be collected. But this increases complexity and results in higher capital operation and maintenance costs. Single-axis tracking is less complex, but yields a smaller gain. However, as  $\cos \theta \approx 1$  for  $\theta < 30^\circ$ , for most applications the collector can be kept with  $\beta = \phi$  and  $\gamma = 0^\circ$ . The specific tracking method to be adopted will depend on the energy demand variation. Tracking is particularly important in systems that operate under concentrated sunlight.

## Applications of Solar Energy

Solar energy finds many applications, some of these being water heating, solar drying, desalination, industrial process heating and passive/active heating of buildings. However, because of the well known advantages of electrical power, the methods of converting solar radiation into electricity have attracted the greatest attention. There are two essential ways of converting solar energy into electricity.

- (i) *Solar thermomechanical systems*: Here, the solar radiation is used to heat a working fluid which runs turbines.
- (ii) *Solar photovoltaics*: Solar photovoltaics (SPV) convert radiant energy directly into an electric current.

In both of these systems, collecting systems are used to receive the radiant energy.

These are described below:

**Flat-plate collectors** are used in low efficiency photovoltaics and low medium temperature thermal systems. In thermomechanical system the flat-plate collector acts as a heat exchanger transferring the radiant energy to a working fluid. The advantages of flat-plate collectors over concentrators are as follows:

- (i) Absorb the diffuse, direct and reflected components of the radiation;
- (ii) Comparatively easy to fabricate and cheaper; and
- (iii) Since these are usually fixed in tilt and orientation, tracking is not required—this makes them maintenance free, except for surface cleaning.

For a solar-thermal flat-plate collector the components are as follows:

- (i) A flat metallic plate painted black to absorb radiation;
- (ii) Channels attached to the plate where a working fluid removes the thermal energy; and
- (iii) Thermal insulation at the back and sides of the collector, and a glass cover to minimize thermal losses.

Flat-plate collectors are popular in water heating systems.

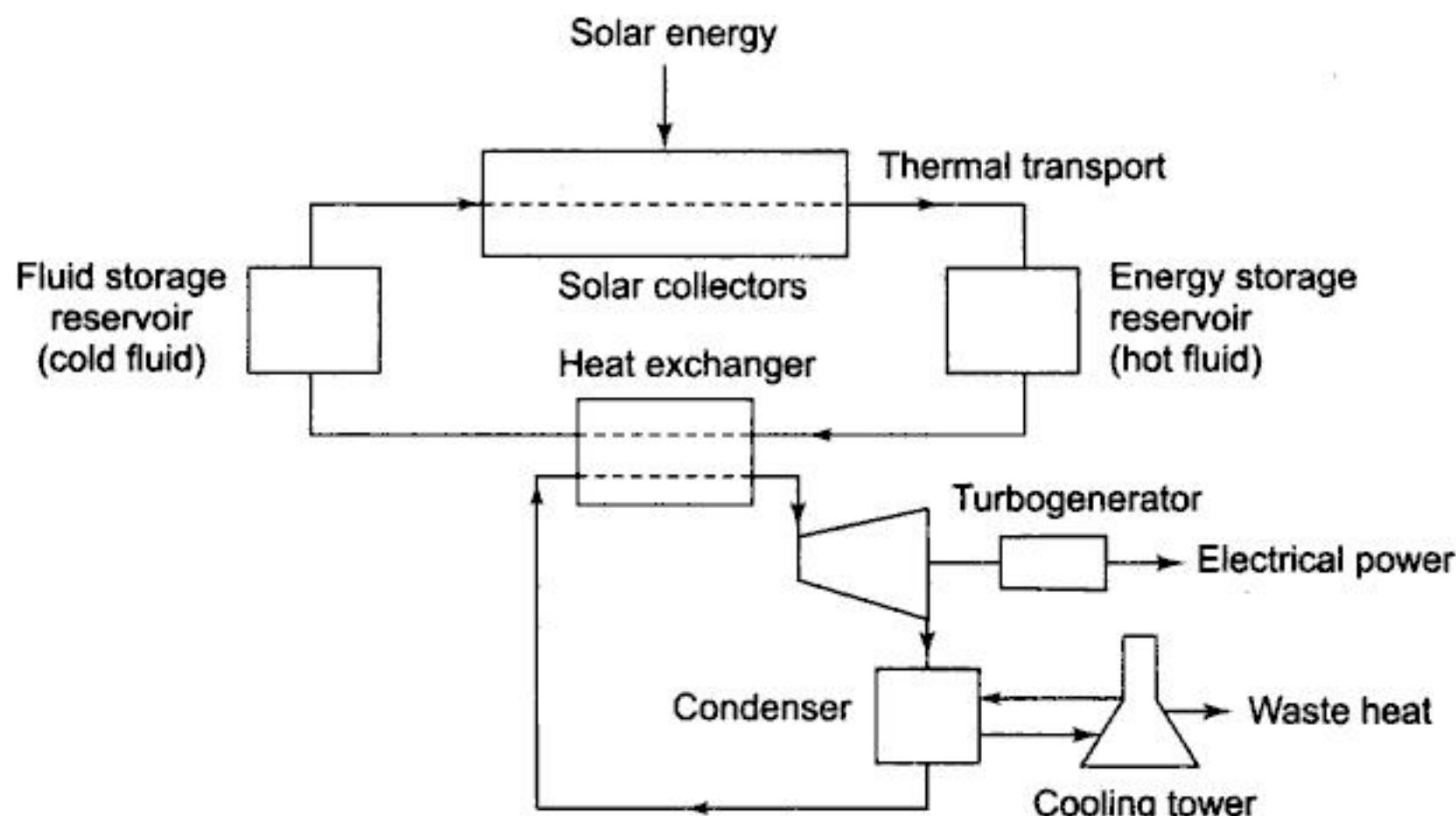
**Concentrating collectors** are used in high temperature solar thermal systems and some high efficiency photovoltaics. There are various methods of classifying solar concentrators. They may be classified as refracting or reflecting, imaging or nonimaging, and on the basis of the type of reflecting surface as parabolic, spherical or flat. High temperatures are obtained by using central tower receivers and *heliostats*.

### Solar Thermomechanical Systems

In solar thermomechanical systems, solar energy is converted to thermal energy of a working fluid. This thermal energy gets converted into shaft work

by a turbine which runs generators. Heat engines (turbine) are based on the Rankine cycle, Sterling cycle or the Brayton cycle. Usually a fossil fuel heat source is also present as standby.

A schematic flow diagram for a solar power plant operating on Rankine cycle is shown in Fig. 1.22. The maximum theoretical *thermal efficiency*, the ratio of useful work done to the heat supplied, is expressed for the Carnot cycle in terms of the temperature of the reservoirs with which it is exchanging heat.



**Fig. 1.22** Schematic diagram of a solar power plant operating on the Rankine cycle

$$\eta = 1 - \frac{T_L}{T_H} \quad (1.16)$$

where

$\eta$  = thermal efficiency of the Carnot cycle

$T_L$  = absolute temperature ( $^{\circ}\text{C} + 273$ ) of the sink

$T_H$  = absolute temperature of the source

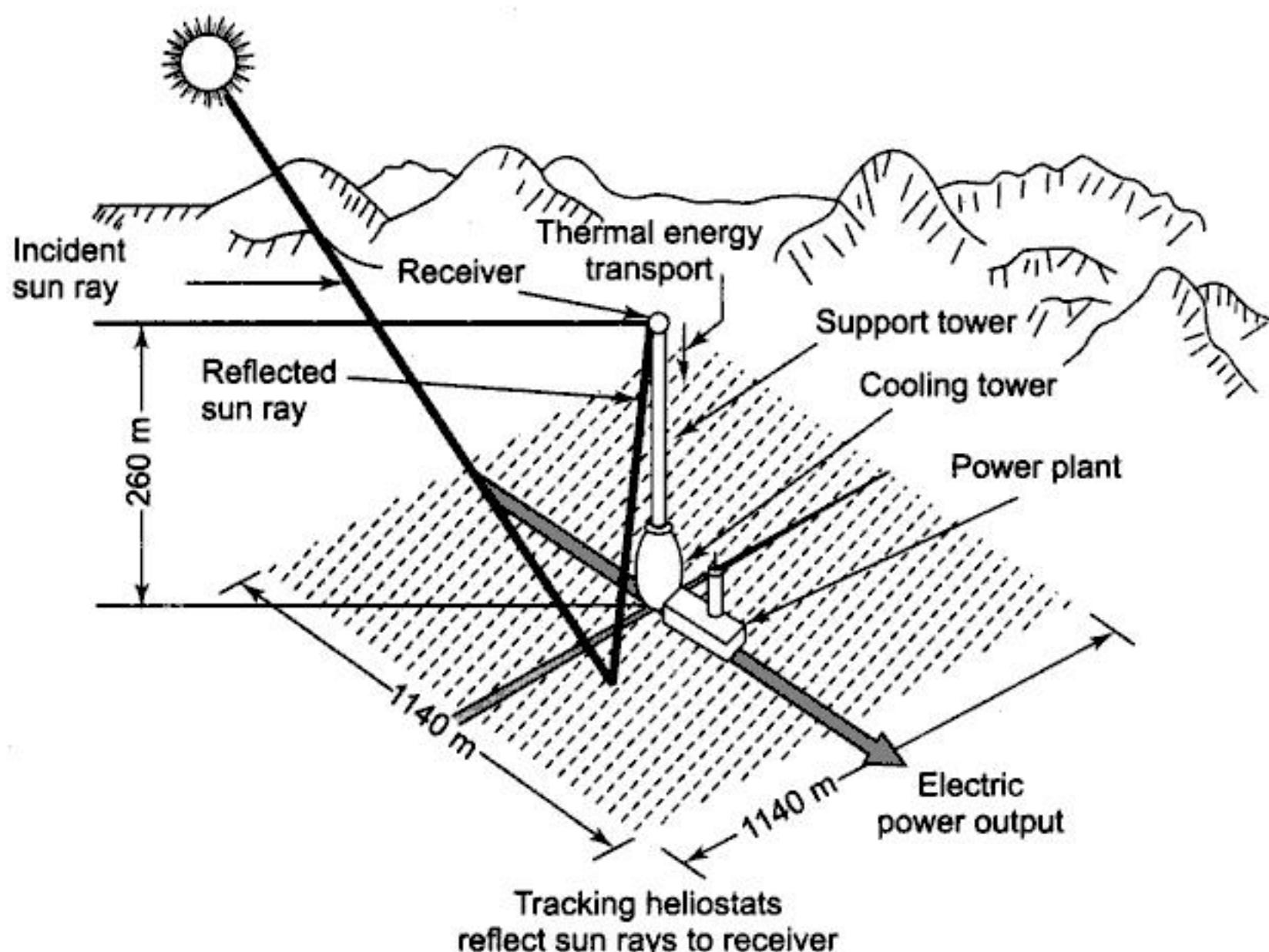
For a solar energy system collecting heat at  $121^{\circ}\text{C}$ , the maximum thermal efficiency of any heat engine using this heat and rejecting heat to atmosphere at a low temperature of  $10^{\circ}\text{C}$  is

$$\eta = 1 - \frac{273 + 10}{273 + 121} = 0.282 \text{ or } 28.2\%$$

The efficiency of a real engine will be considerably less.

For obtaining efficiencies close to those of fossil fuel based stations,  $T_H$  must be raised to the same order of value. This is achieved by installing an array of mirrors, called heliostats, tracking the sun. One proposed scheme is shown in Fig. 1.23 for major generation of electricity with reflectors (with concentration

factor of 30 or more) concentrating the sun's rays on to a single boiler for raising steam. A collector area of  $1 \text{ km}^2$  would raise 100 MW of electrical power. The cost of such a scheme at present is prohibitive.



**Fig. 1.23** Proposed scheme for a large central solar-thermal electric generation

A less attractive alternative to this scheme (because of the lower temperatures) is the use of many individual absorbers tracking the sun unidirectionally, the thermal energy being transferred by a fluid (water or liquid sodium) to a central boiler.

Solar-thermal electric systems have certain inherent disadvantages of a serious nature. These are as follows:

1. Low efficiency. Raising efficiency to acceptable value brings in prohibitive costs.
2. The efficiency of the collecting system decreases as its temperature increases, but the efficiency of the heat engine increases with temperature.
3. All solar-thermal schemes essentially require storage because of the fluctuating nature of the sun's energy, although it has been proposed that the schemes be used as pure fuel savers.
4. In general mechanical systems need great maintenance.
5. For a reliable system fossil fuel backup may be needed.

Because of these factors considerable research effort is being devoted to solar photovoltaics as a viable alternative.

## Direct Conversion of Sunlight into Electricity

### Introduction

Photovoltaic (PV) or solar cell is a semiconductor device that converts sunlight directly into electricity. Initially PV cells had very limited use, e.g., in supplying electricity to satellites in space or for meeting energy requirements of defence personnel stationed at remote areas. However, with a gradual reduction in the cost of PV cells, current international price is now between 5–10\$ per peak-watt, its use has been increasing steadily and it is projected that by the year 2010 or so its share in power generation may be around 5–10%.

A PV cell can be classified

1. in terms of materials: noncrystalline silicon, polycrystalline silicon, amorphous silicon, gallium arsenide, cadmium telluride, cadmium sulphide, indium arsenide, etc.
2. in terms of technology for fabrication single crystal bonds (or cylinders), ribbon growth, thin-film, etc.

Some of the important characteristics of various types of PV cells, measured at normal temperature ( $25^{\circ}\text{C}$ ) and under illumination level of  $100 \text{ mW/cm}^2$ , are listed in Table 1.1

**Table 1.1**

PV cell	ff*	Short-circuit current density (I <sub>sc</sub> ) (mA/cm <sup>2</sup> )	Open-circuit voltage (V <sub>oc</sub> ) (V)	Conversion efficiency (%)
Monocrystalline silicon	0.85	20–22	0.5–0.6	13–14
Polycrystalline silicon	0.85	18–20	0.5–0.6	9–12
Amorphous silicon		13–14	2.2–2.4	5–6
Gallium arsenide	0.87	—	—	20–25

\* ff is fill-factor which is defined later.

### Basic Structure of PV Cell

The basic structure of a typical PV cell is shown in Fig. 1.24(a) and (b). Various layers from top to bottom and their functions are as follows:

- Top layer is a glass cover, transparency 90–95%. Its purpose is to protect the cell from dust, moisture etc.
- The next is a transparent adhesive layer which holds the glass cover.
- Underneath the adhesive is an antireflection coating (ARC) to reduce the reflected sunlight to below 5%.
- Then follows a metallic grid (aluminium or silver) (Fig. 1.24(b)) which collects the charge carriers, generated by the cell under incidence of sunlight, for circulating to outside load.

- Under the lower side of the metallic grid lies a p-layer followed by n-layer forming a pn-junction at their interface. The thickness of the top p-layer is so chosen that enough photons cross the junction to reach the lower n-layer.
- Then follows another metallic grid in contact with the lower n-layer. This forms the second terminal of the cell.

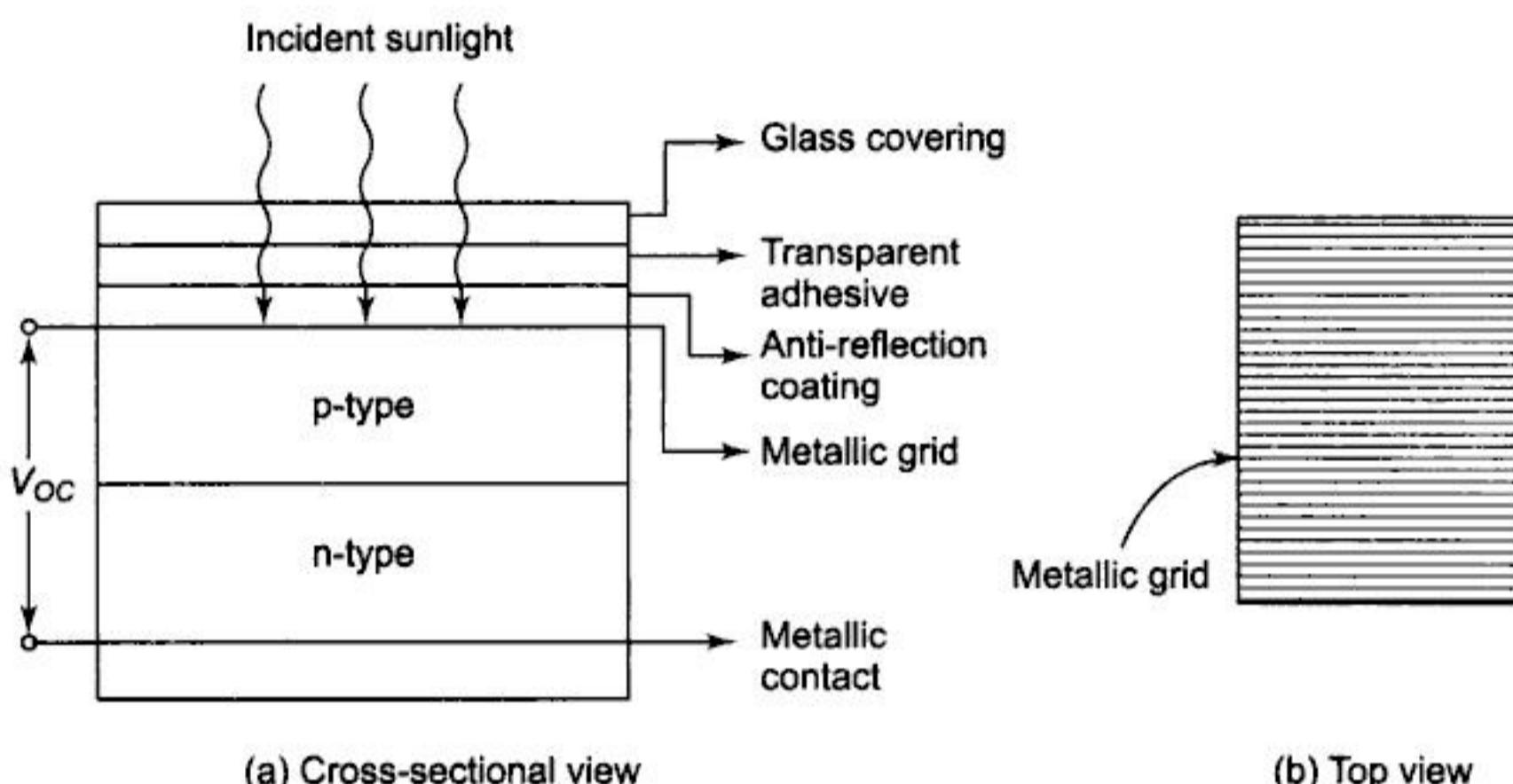


Fig. 1.24

### **Operation and Circuit Model**

The incidence of photons (sunlight) causes the generation of electron-hole pairs in both p and n-layers. Photons generated minority carriers (electrons in p-layer and holes in n-layer) freely cross the junction. This increases the minority carrier flow manyfolds. Its major component is the light generated current  $I_G$  (when load is connected across the cell terminals). There is also the thermally generated small reverse saturation current  $I_s$  (minority carrier flow in same direction as  $I_G$ ) also called *dark current* as it flows even in absence of light.  $I_G$  flows in opposite direction to  $I_D$ , the forward diode current of the junction. The cell feeds current  $I_L$  to load with a terminal voltage  $V$ .

The above operation suggests the circuit model of a PV cell as drawn in Fig. 1.25. The following Eq. (1.18) can be written from the circuit model and the well-known expression for

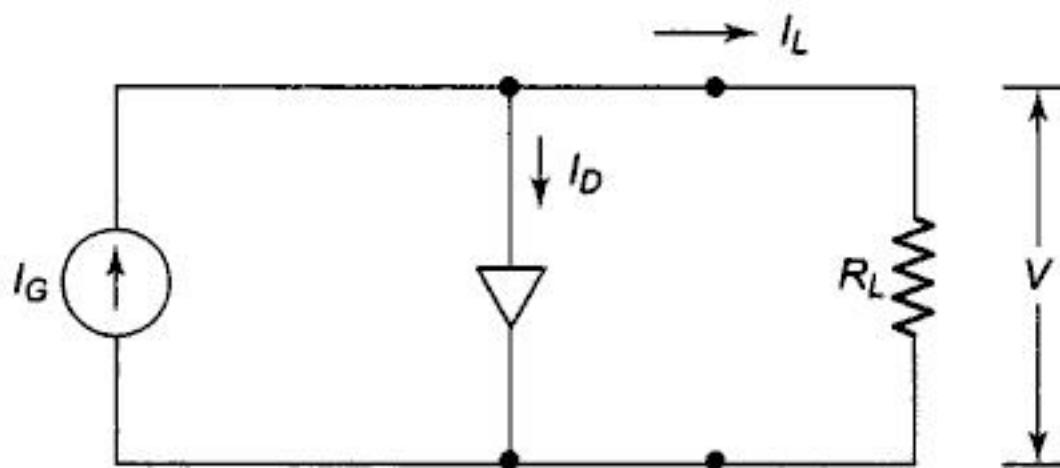
$$I_D = I_s (e^{\lambda V} - 1); \lambda = \frac{e}{kT} \quad (1.17)$$

where

$k$  = Boltzmann constant,

$e$  = electronic charge and

$T$  = cell temperature in degree K.



**Fig. 1.25** Circuit model of PV cell

$$\text{Load current } I_L = I_G - I_D$$

$$= I_G - I_s (e^{\lambda V} - 1) \quad (1.18)$$

From this equation it easily follows that

$$V_{OC} (I_L = 0) = \frac{1}{\lambda} \ln \left( \frac{I_G}{I_s} + 1 \right) \quad (1.19)$$

$$\text{and } I_{SC} (V = 0) = I_G \quad (1.20)$$

Solar radiation generated current  $I_G$  is dependent on the intensity of light. The  $I-V$  characteristics of the cell are drawn in Fig. 1.26(a) for various values of intensity of solar radiation. One typical  $I-V$  characteristic of the cell is drawn in Fig. 1.26(b). Each point on this curve belongs to a particular power output. The point  $Q$  indicated on the curve pertains to the maximum power output at which the cell should be operated. At this point.

$$P_{max} = V_{Pmax} I_{Pmax} \quad (1.21)$$

The *fill-factor (ff)* of a cell is defined as

$$ff = \frac{P_{max}}{I_{SC} V_{SC}} \quad (1.22)$$

The cell efficiency is given as

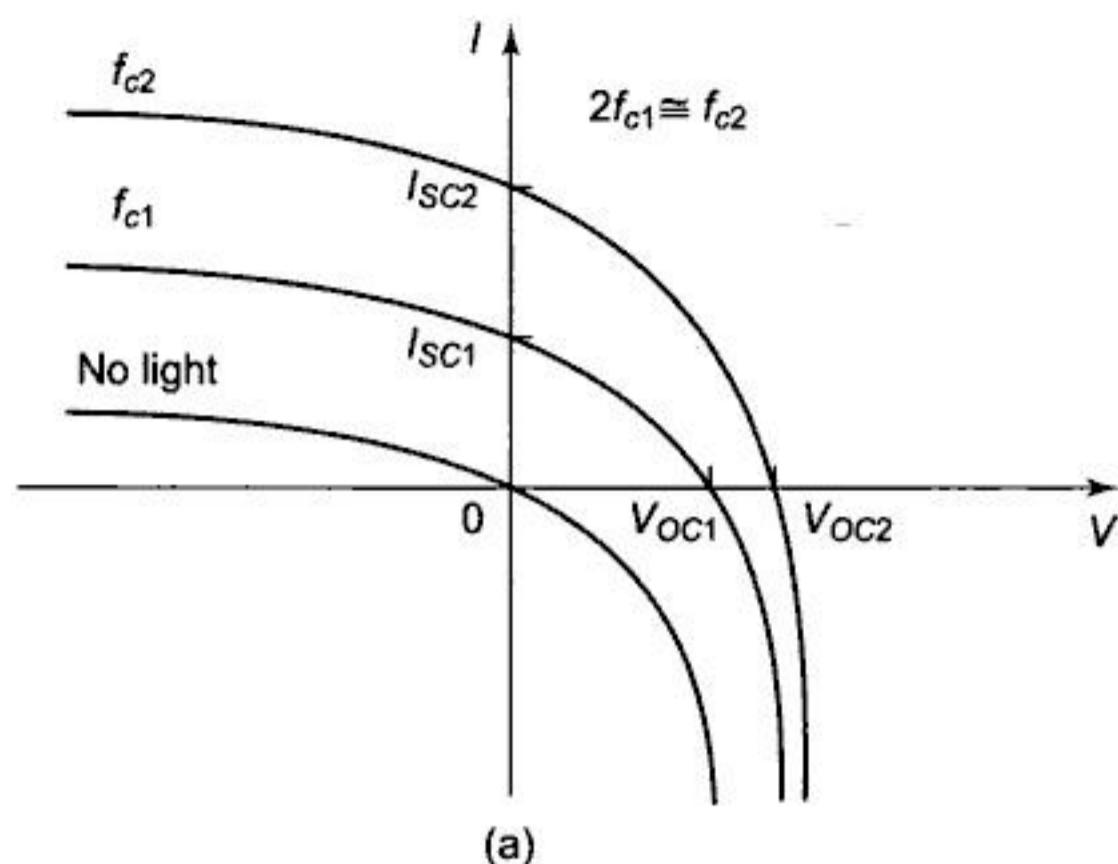
$$\eta = \frac{P_{out}}{P_{in}} \quad (1.23)$$

where  $P_{out}$  is the power delivered to load and  $P_{in}$  is the solar power incident on the cell.

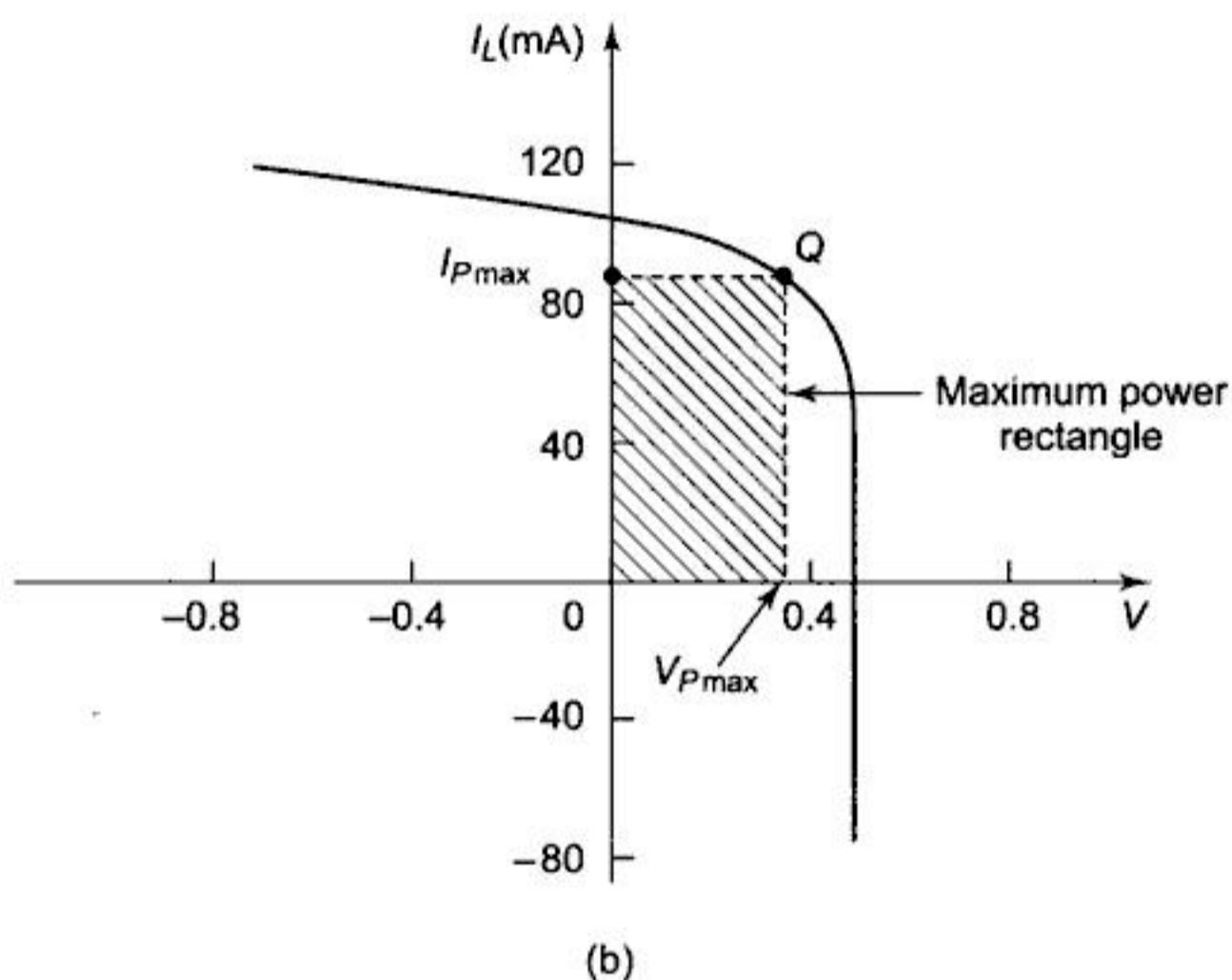
### **Effect of Temperature on Solar Cell Efficiency**

As the temperature increases, the diffusion of electrons and holes in the length of Si (or GaAs) increases causing an increase in the dark current and a decrease in  $V_{OC}$ . The overall effect causes a reduction in the efficiency of solar cell as the

temperature increases. The practical efficiency of Si solar cell is about 12% and that of GaAs solar cell is 25% at the normal temperature of 300 K. With each degree rise in temperature, the efficiency decreases by a factor of 0.0042%.



(a)

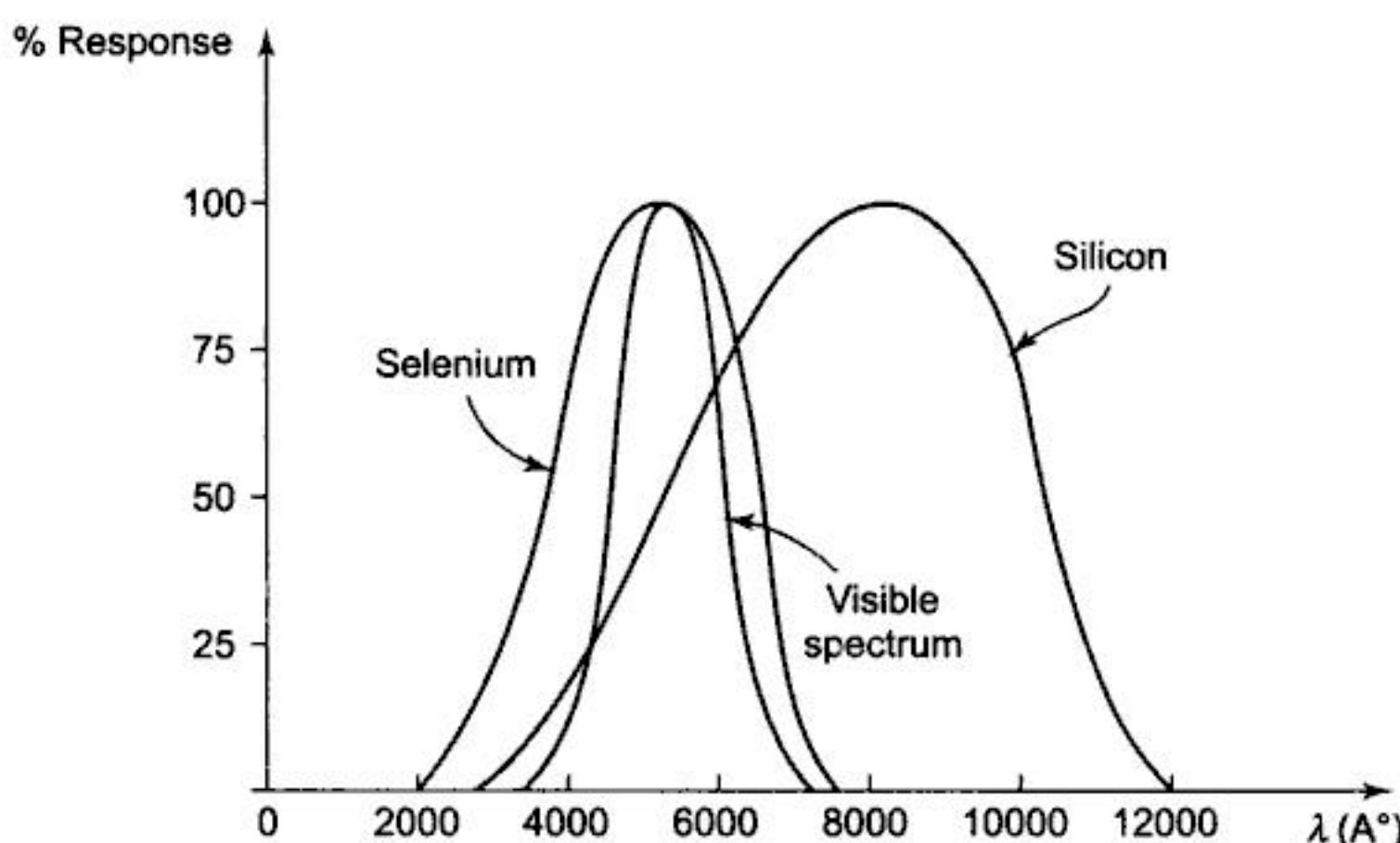


(b)

**Fig. 1.26**  $I-V$  (current–voltage) characteristics of a PV cell

### Spectral Response

It is seen from the spectral response curves of Fig. 1.27 that the Selenium cell response curve nearly matches that of the eye. Because of this fact Se cell has a widespread application in photographic equipments such as exposure meters and automatic exposure diaphragm. Silicon response also overlaps the visible spectrum but has its peak at the  $0.8 \mu\text{m}$  ( $8000 \text{ \AA}$ ) wavelength, which is in the infrared region. In general, silicon has a higher conversion efficiency and greater stability and is less subject to fatigue. It is therefore widely used for present day commercial solar cells.



**Fig. 1.27** Spectral response of Si, Se and the naked eye

### Prevalent Technologies for Fabricating Silicon PV Cell

The most commonly used methods of manufacturing silicon PV cell from purified silicon feedstock are as follows:

1. Single crystal silicon with a uniform chemical structure.
2. Polycrystalline silicon-series of crystalline structures within a PV cell.
3. Amorphous silicon with a random atomic chemical structure.

The technological details of these three types of methods for manufacturing PV cells is not within the scope of this book.

In general as the atomic structure becomes more random, less energy input and manufacturing complexity is required. However, more uniform structure means increased current collection and increased efficiency.

Most PV power uses flat-plate modules of cut and polished wafer like cells of crystalline silicon, which are now about 12% conversion efficient.

### Thin-film Technologies

There are two main reasons why thin film offers promise of significant cost reduction. These are as follows:

- (i) Thin-film cells use only a few microns of direct material, instead of tens of mills used by crystalline, polycrystalline or *ribbon silicon modules*.
- (ii) Construction of monolithic thin-film modules can be done at the same time that the cells are formatted, thus eliminating most of the cost of module fabrication. These two aspects of thin-film technology are further explained below:
  - Cadmium telluride can absorb 99% of the sun's energy in less than  $0.5 \mu\text{m}$  thickness as opposed to the 8 mill requirement for crystalline silicon.

- In conventional technologies, cells cut into individual parts are then circuited back together as discrete elements. Monolithic interconnection during cell fabrication eliminates labour and in addition produces a superior looking product because of its uniform finish.

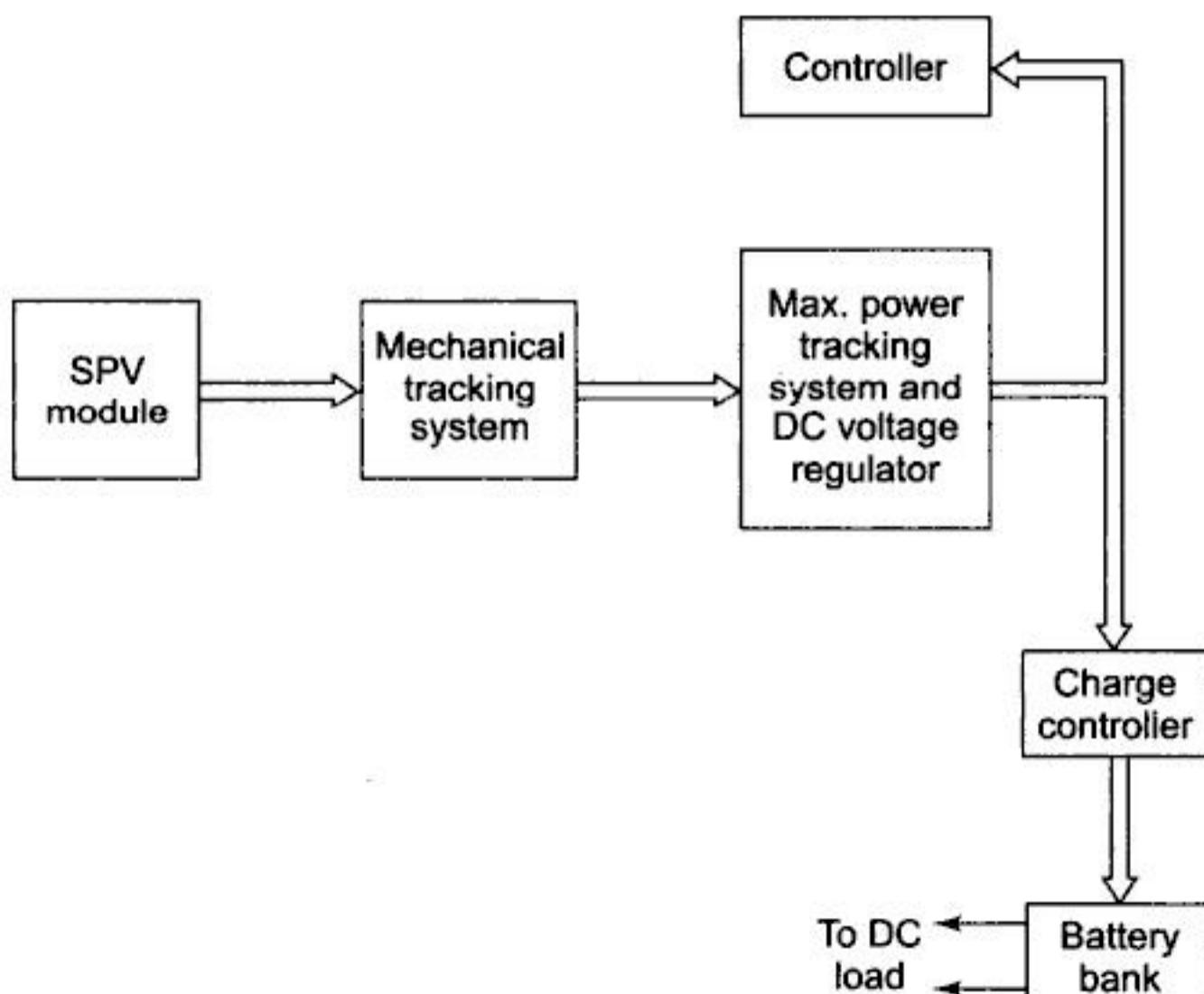
### Bulk Energy Conversion by SPV (Solar Photovoltaic) Cells

Bulk SPV power is feasible in bright, clear areas with sun most days of the year such that incident solar energy is about  $2600 \text{ kWh/m}^2$  annually.

SPV cell produces DC power which is maximum at a particular point on its  $I-V$  characteristics (which changes with sunlight received). There are three ways in which this power can be used:

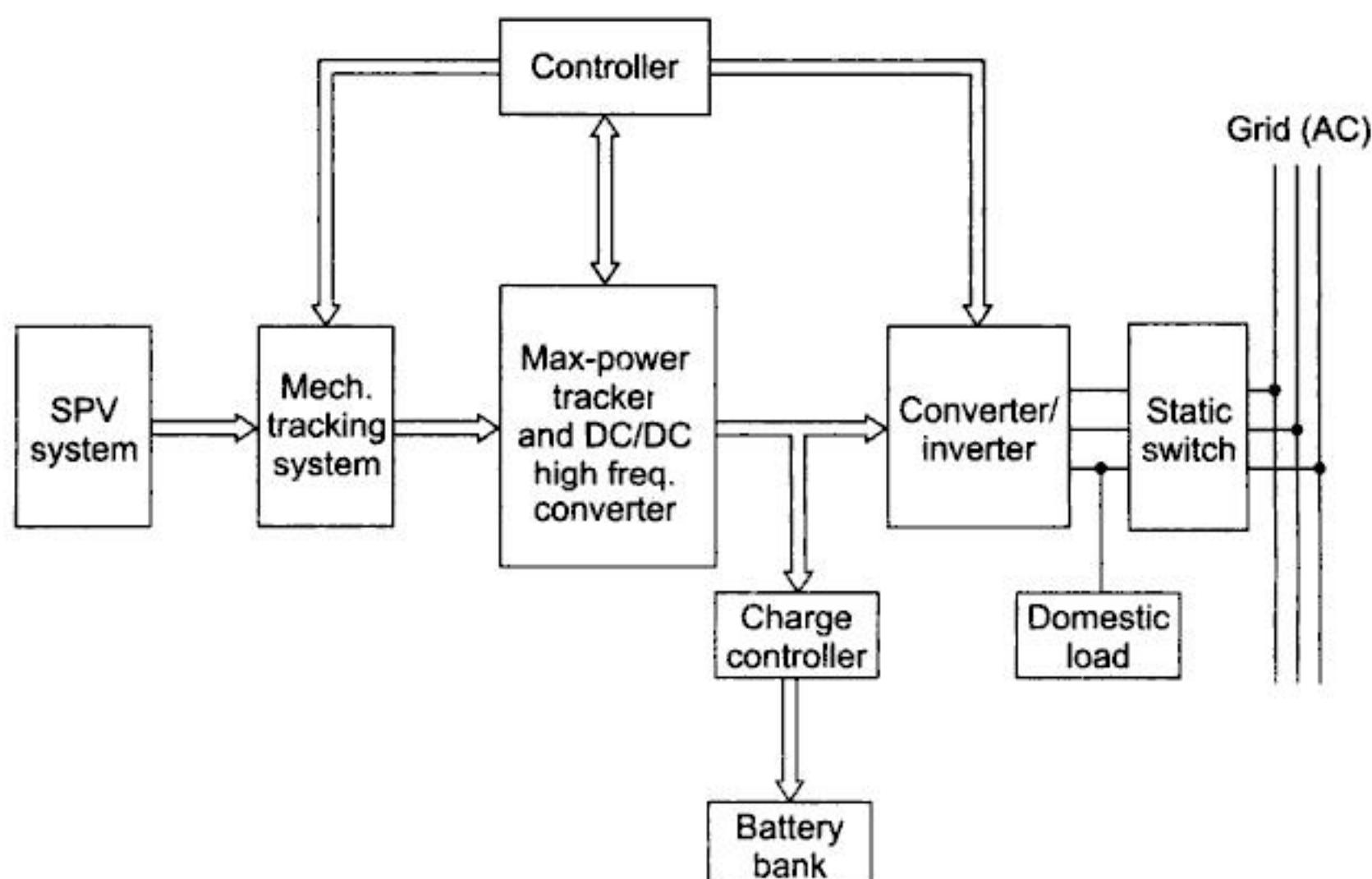
1. Storage in batteries; this kind of storage is limited in capacity and so meant for small systems. Further reconverting equipment may be required for end use in AC form.
2. PV power is suitably conditioned to AC form for grid interactive use. This is the case with bulk power production.
3. Combined storage and conditioned AC systems.

System of the first kind with battery storage and DC load is drawn in conceptual block diagram form in Fig. 1.28. Mechanical tracking system is required to orient the SPV module at an angle  $90^\circ$  to the incident radiation so as to get maximum intensity. The maximum power tracking system ensures that the load draws the maximum power from the SPV module. DC voltage regulator delivers power at rated voltage despite variation in generated voltage and power. The charge controller is meant to protect the battery bank from both overcharging as well as deep discharging.



**Fig. 1.28** SPV system for feeding DC load with battery storage

A grid interactive SPV system for domestic use is shown in the form of conceptual blocks in Fig. 1.29. Solar cells are connected in series-parallel and the voltage after conversion to AC form by solid state devices is not compatible with grid voltage (400 V at distribution load). This scheme, therefore, differs from that of Fig. 1.22 as the DC voltage has to be raised by the method of DC/DC high frequency chopping with an intervening inductor for raising the voltage. For grid interaction a converter-inverter is required so that power can flow either way depending upon the amount of solar power availability during the day. A battery via converter-inverter feeds the domestic load at night (or on a cloudy day) if the grid outage occurs.



**Fig. 1.29** Grid interactive SPV system

The process of conversion and reconversion with solid state devices like SCR (Silicon Controlled Rectifier) is called *power conditioning*. Such systems are already being used in cities in Japan, and are now available in India.

For bulk solar power systems the basic scheme will be similar except that it would directly feed power into the grid and no power need flow the other way.

As and when a breakthrough in SPV technology and sharp reduction in cost is achieved, domestic and bulk power systems will become common place. However, the intensity of solar insolation being low ( $1367 \text{ W/m}^2$ ), use of solar power requires considerable land area coverage (with shade underneath, all times, a different kind of pollution). The best estimate is that solar power will meet only 5–10% of the total electric energy need.

Total energy potential in India is  $8 \times 10^{15} \text{ kWh/yr}$ . Upto 31.12.2005, 6,00,000 solar cookers,  $55 \times 10^4 \text{ m}^2$  solar thermal collector area, 47 MW of SPV power, 270 community lights, 5,38,718 solar lanterns (PV domestic lighting units), 640 TV (solar), 54,795 PV street lights and 7002 solar PV water

pumps were installed. Village power plants (stand-alone) of 1.5 MW capacity and 1.1 MW of grid connected power plants were in operation. As per one estimate [2], solar power will overtake wind in 2040 and would become the world's overall largest source of electricity by 2050. 5000 MW grid-interactive solar power could be feasible by 2032 (MNES Annual Report 2005–06). Solar water heating systems are increasingly becoming more popular for homes, hostels, hotels and industrial and domestic purposes. Research has shown that the Gallium Arsenide (GaAs) based PV cell with multijunction device could give maximum efficiency of nearly 30% and Carbon Nano Tube (CNT) based PV cell may give upto 50% efficiency.

### 1.11 WIND POWER

A growing concern for the environmental degradation has led to the world's interest in renewable energy resources. Wind is commercially and operationally the most viable renewable energy resource.

Worldwide, five nations—Germany, USA, Denmark, Spain and India—account for 80% of the world's installed wind energy capacity total worldwide wind power installed capacity is 79,300 MW. Kinetic energy available in the wind is converted to electrical energy by using rotor, gearbox and generator. The wind turns the blades of a windmill-like machine. The rotating blades turn the shaft to which they are attached. The turning shaft typically can either power a pump or turn a generator, producing electricity. Larger blades capture more wind. As the diameter of the circle formed by the blades doubles, the power increases four times.

Wind is air set in motion by the small amount of insolation reaching the upper atmosphere of earth. Nature generates about  $1.67 \times 10^5$  kWh of wind energy annually over land area of earth and 10 times this figure over the entire globe. Wind contains kinetic energy which can easily be converted to electrical energy. Wind energy has been used in wind mills for centuries. In 1980s wind energy use received a fillip with availability of excellent wind sites and rising cost of conventionally generated electrical power. Later in 1990s interest in wind generated electrical power to displace conventional power, received further enhancement in order to reduce air pollution levels in the atmosphere. Wind is a clean power generating agent as it causes no pollution.

Power density in moving air is given by

$$P_W = KV^3 \text{ W/m}^2; \text{ Here } K = 1.3687 \times 10^{-2} \quad (1.24)$$

or

$$P_W = 0.5 \rho AV^3 \text{ W}$$

where

$\rho$  = air density ( $1201 \text{ g/m}^3$  at NTP)

$V$  = wind speed in km/h, mean air velocity (m/s)

$A$  = Swept area ( $\text{m}^2$ )

Theoretically a fraction  $16/27 = 0.5926$  of the power in the wind is recoverable. This is called *Gilbert's limit* or *Betz coefficient*. Aerodynamical efficiency for converting wind energy to mechanical energy can be reasonably assumed to be 70%. So the mechanical energy available at the rotating shaft is limited to 40% or at the most 45% of wind energy.

### Wind Characteristics

- Wind speed increases roughly as the 1/7th power of height. Typical tower heights are about 20–30 m.
- *Energy-pattern factor*: It is the ratio of actual energy in varying wind to energy calculated from the cube of mean wind speed. This factor is always greater than unity which means that energy estimates based on mean (hourly) speed are pessimistic.

### Utilization Aspects

There are three broad categories of utilization of wind energy:

1. Isolated continuous duty systems which need suitable energy storage and reconversion systems.
2. Fuel-supplement systems in conjunction with power grid or isolated conventional generating units.
3. Small rural systems which can use energy when wind is available.

Category 2 is the most predominant in use as it saves fuel and is fast growing particularly in energy deficient grids. Category 3 has application in developing countries with large isolated rural areas.

### Aeroturbine Types and Characteristics

Modern horizontal-axis aeroturbines (wind turbines) have a sophisticated blade design. They are installed on towers 20–30 m high to utilize somewhat high wind speed and also permit land use underneath. Cross-section view of a typical horizontal-axis wind turbine is shown in Fig. 1.30.

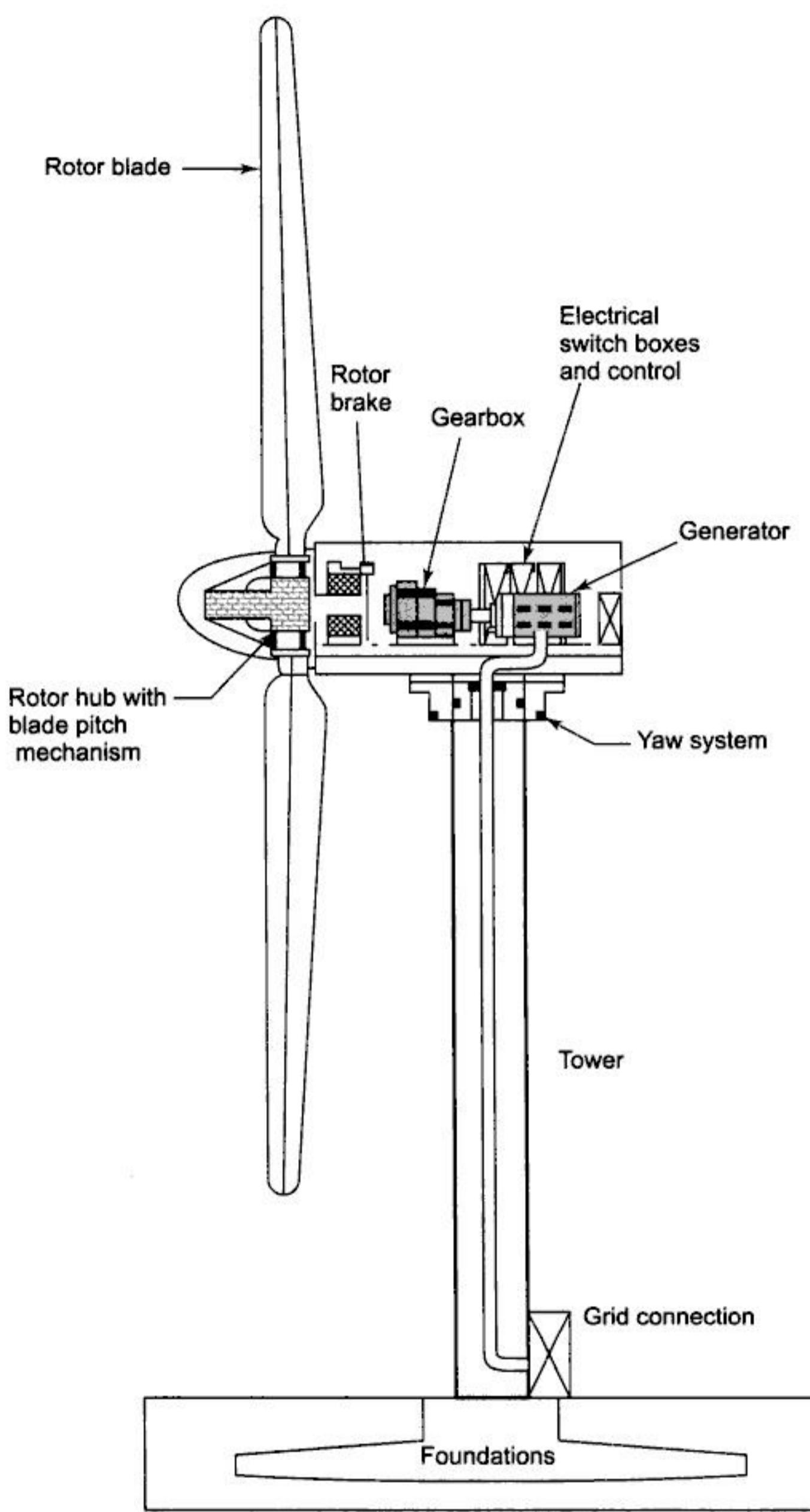
*Tip Speed*, also called *specific speed*, is by far the single most important parameter to be considered. It is defined as

$$\text{Tip speed} = \frac{\text{Peripheral speed}}{\text{wind speed}}$$

This ratio ranges from 2 to 10. Ratios less than 4 require rotor with several blades and have lower rotational speed whereas higher ratios (4 to 10) require fewer blades and have higher rotational speeds. Higher tip speed rotors have lower efficiency because of higher frictional loss. Typical blade diameters are about 20 m and rotor speeds 100–150 rpm.

### Power Coefficient

$C_p$  is defined as the fraction of wind power at the rotor shaft. It is dependent on (i) tip speed ratio and (ii) pitch angle of blades. Rather than designing for  $C_p$  (max), these factors are determined by economics.



**Fig. 1.30** Cross-sectional view of a typical horizontal-axis aeroturbine

### **Blade Arrangements**

For harnessing large power, two to three blade configurations are used. Two blade arrangement is cost effective but prone to vibrations, which disappear with three blades. No unique answer on this issue has been arrived at yet.

Modern machines have metal blades based on aircraft technology. Glass reinforced plastic has also been used successfully.

### **Vertical-axis Wind Turbines (VAWTs)**

Vertical-axis aeroturbines accept the wind from any direction and have the added advantage that the generator is located on ground. As a result the weight on tower is considerably reduced. The technology of these turbines has reached the stage where their efficiencies are comparable with those of horizontal-axis machines.

A number of vertical-axis designs have been developed and tested. We shall discuss here the one that is now commercially available—the Darrieus. The Darrieus rotor has two or more curved airfoil blades, held together at the top and the bottom. These are so positioned that they respond to wind from any direction. Physically it resembles the lower portion of an egg beater. The rotors are non-self-starting and operate at blade tip ratios of 6 to 8. These have efficiencies around 35–40%.

### **Wind to Electric Energy Conversion**

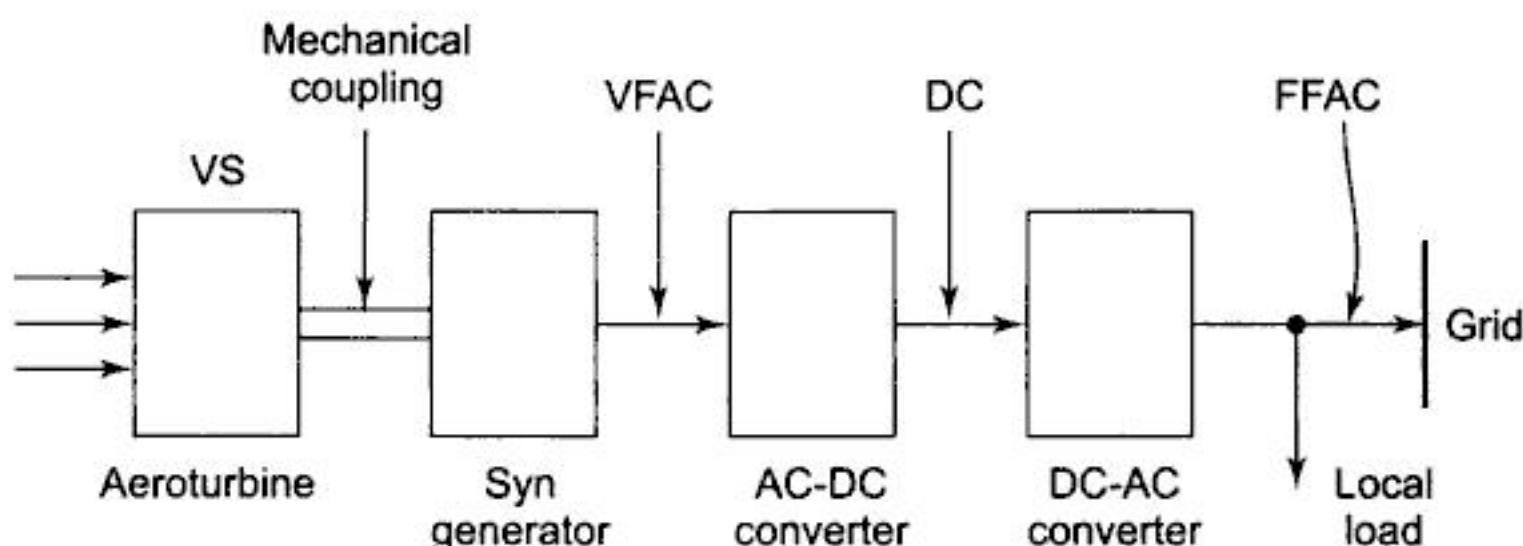
The choice of electrical system for an aeroturbine is guided by three factors:

1. *Type of electrical output:* DC, variable-frequency AC, constant-frequency AC.
2. *Aeroturbine rotational speed:* constant speed with variable blade pitch, nearly constant speed with simpler pitch-changing mechanism or variable speed with fixed pitch blades.
3. *Utilization of electrical energy output:* in conjunction with battery or other form of storage, or interconnection with power grid.

Large scale electrical energy generated from wind is expected to be fed to the power grid to displace fuel generated kWh. For this application present economics and technological developments are heavily weighted in favour of constant-speed constant-frequency (CSCF) system with alternator as the generating unit. It must be reminded here that to obtain high efficiencies, the blade pitch varying mechanism and controls have to be installed.

Wind turbines of electrical rating of 100 kW and above normally are of constant-speed type and are coupled to synchronous generators (conventional type). The turbine rated at less than 100 kW is coupled to fairly constant speed induction generators connected to grid and so operating at constant frequency drawing their excitation VARs from the grid or capacitor compensators.

With the advent of power switching technology (high power diodes and thyristors) and chip-based associated control circuitry, it has now become possible to use variable-speed constant-frequency (VSCF) systems. VSCF wind electrical systems (WES) and its associated *power conditioning* system operates as shown in Fig. 1.31.



**Fig. 1.31** Block schematic of VSCF wind electrical system; VF (variable frequency), FF (fixed frequency)

Various advantages of this kind of VSCF WES are:

1. No complex pitch changing mechanism is needed.
2. Aeroturbine always operates at maximum efficiency point (constant tip-speed ratio).
3. Extra energy in the high wind speed region of the speed-duration curve can be extracted.
4. Significant reduction in aerodynamic stresses, which are associated with constant-speed operation.

## Operation and Control of Wind Electrical Systems (WES)

To understand the operation and control of WES, let us first consider a typical wind duration curve of Fig. 1.32(a). Any point on this curve gives the number of hours in a year for which the wind speed is higher than the value corresponding to this point.

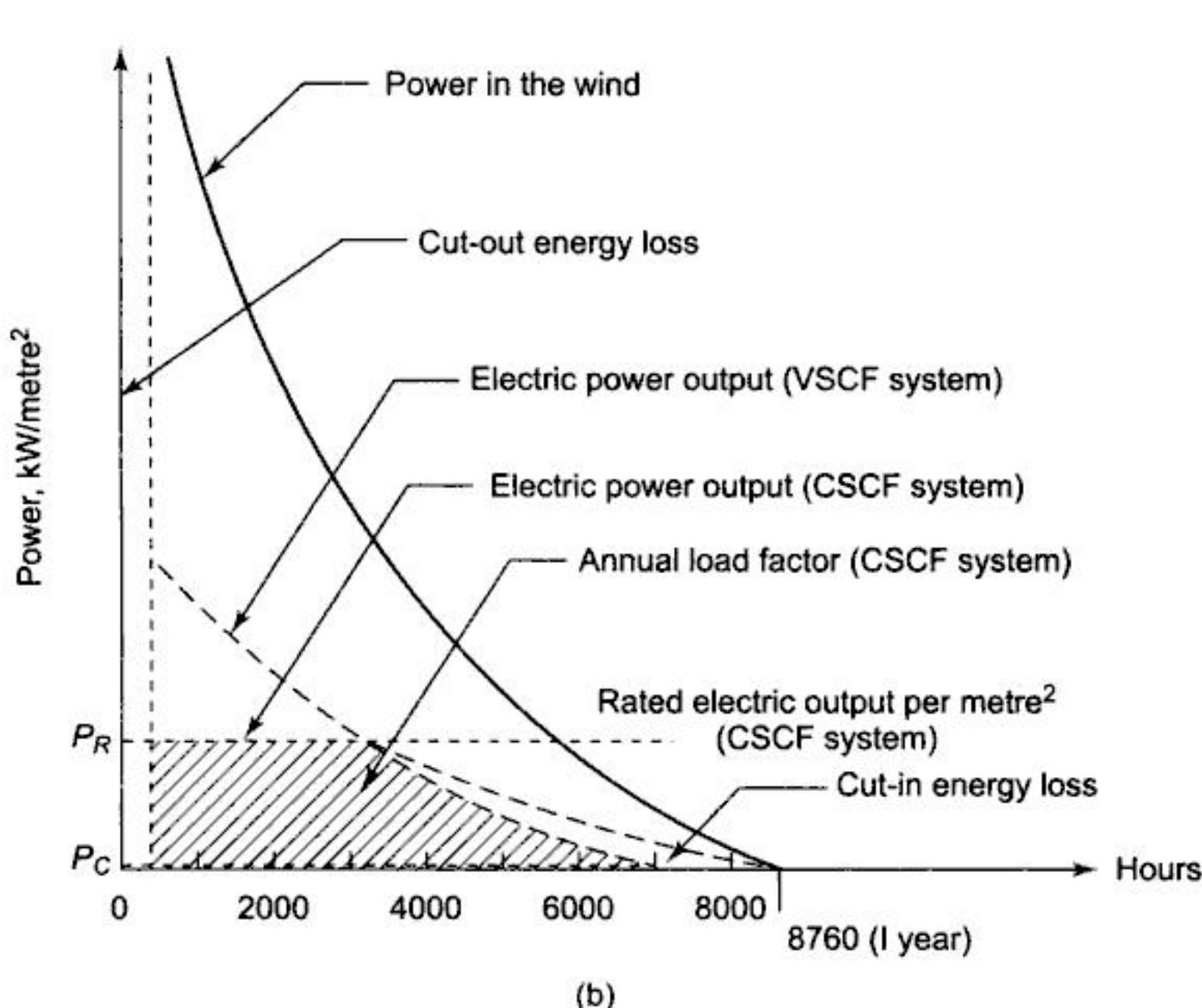
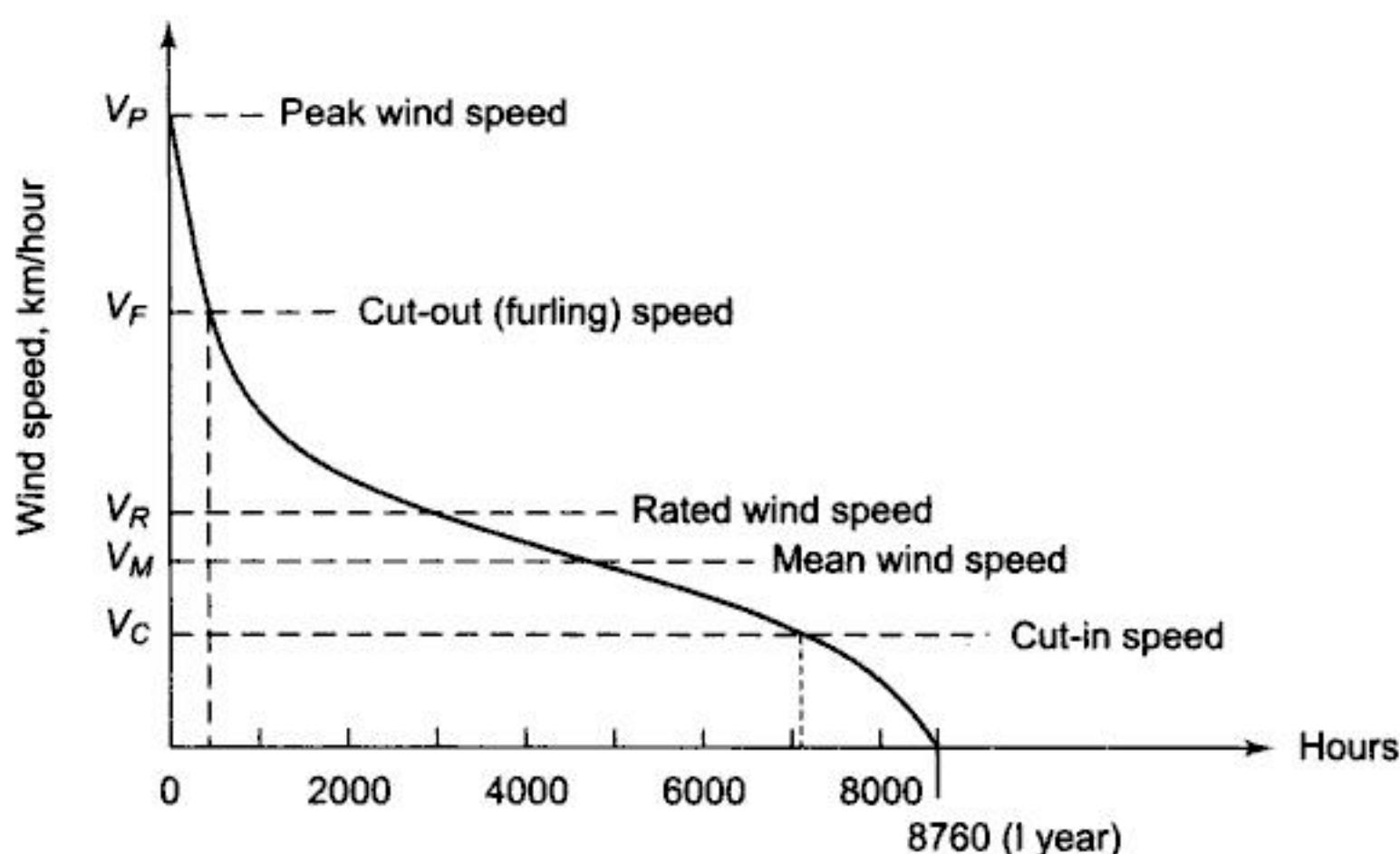
Sensors sense the wind direction and the *yaw control* (Fig. 1.30) orients the rotor to face the wind in case of horizontal-axis machines. With reference to the wind duration curve of Fig. 1.32(a) it is seen that the wind turbine begins to deliver power at the *cut-in-speed*  $V_C$  and the plant must be shut down for wind speed at the maximum safe limit called the *furling speed*  $V_F$ . Between these two limits, mechanical power output of the turbine is determined by the power coefficient  $C_p$ . The electrical power output is determined therefrom, by the coefficients  $\eta_m$  and  $\eta_g$  of the mechanical drive and the electrical generator, respectively.

In CSCF WES the conventional synchronous generator locks into the grid and maintains a constant speed irrespective of wind speed. A suitable controller senses the generating/motoring mode of operation and makes the needed pitch adjustments and other changes for a smooth operation.

As the generator output reaches the rated value, the electrical load is held constant even though the wind speed may increase beyond this value. This extra energy in the wind is allowed to be lost as indicated in Fig. 1.32(b). This is also the case for an induction generator whose speed remains substantially constant.

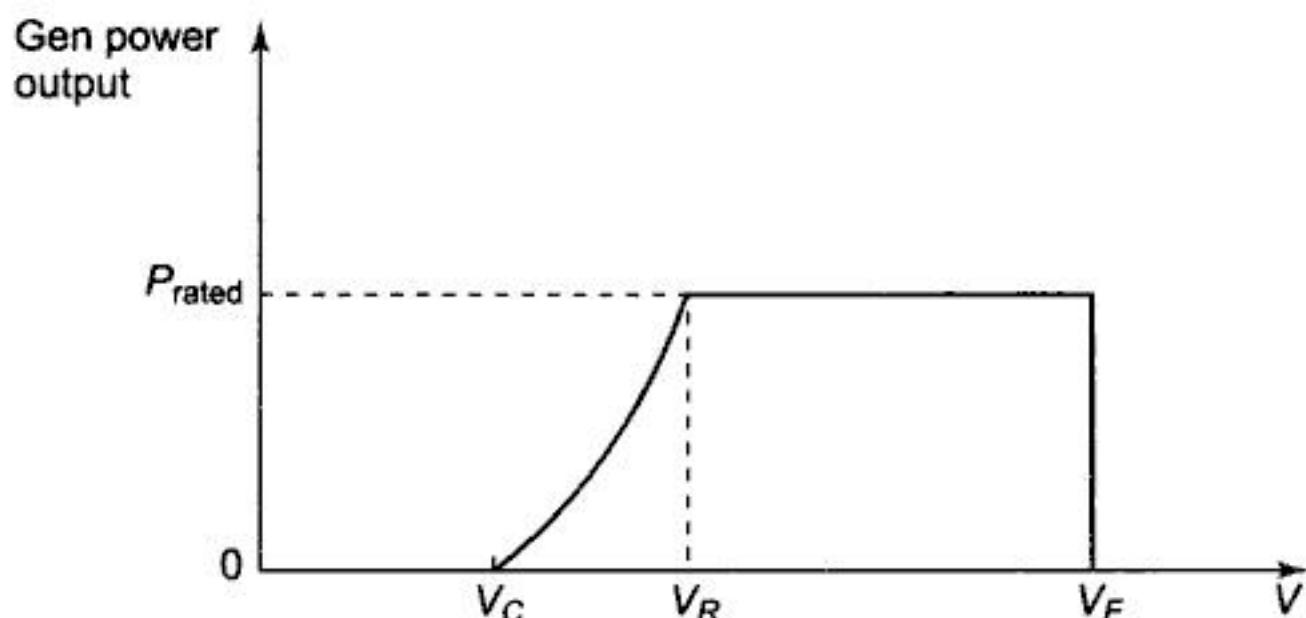
With VSCF system, suitable output controls can be installed to maintain a constant tip speed ratio and so as to keep  $C_p$  at its near maximum value. This

results in somewhat more power output throughout the operating range compared to the CSCF system as shown in Fig. 1.32(b). Also there is no need to install sophisticated pitch control systems. The extra cost entailed in the power conditioning system gets more or less balanced against the cost of extra energy output. Considerable development effort is therefore being applied to CSCF system for large rating WES. These systems are yet to be proved in the field.



**Fig. 1.32** (a) Typical wind-speed-duration curve  
(b) Power-duration curve of wind-driven generator

For CSCF WES the operating curve shown shaded in Fig. 1.32(b) is redrawn in Fig. 1.33 with speed axis reversed for clarity. The aerogenerator starts to generate power at wind speed  $V_C$ , the cut-in speed. The aerogenerator produces rated power at speed  $V_R$ . At higher wind speeds the aerogenerator speed is held constant by changing the pitch of blades (part of wind energy is being lost during this part of operation). The aerogenerator must be cut-out at  $V_F$ , the furling speed.



**Fig. 1.33** CSCF WES characteristics

Typical wind turbine rotors of 20 m diameter rotate at 100–500 rpm and are geared up to about 750 rpm to drive an eight pole induction generator excited from 400 V, 3-phase, 50 Hz rural distribution system.

Consider as an example an area with mean wind speed of 10 km/h. Power output of 300 MW is to be produced using aeroturbines of 20 m blade diameter. Let us calculate the number of aeroturbines required.

Power in the wind is given by the relationship

$$P_W = KV^2/\text{m}^2 \text{ kW}$$

$V$  = wind speed in km/h

$$K = 1.368 \times 10^{-2}$$

For the aeroturbine

$$\begin{aligned} P_{\text{aero}} (\text{mech}) &= 1.368 \times 10^{-2} \times (10)^2 \pi \left( \frac{20}{2} \right)^2 \\ &= 4290 \text{ kW} \end{aligned}$$

$$\begin{aligned} P_{\text{gen}} (\text{elect}) &= 4290 \times 0.4 = 1716 \text{ kW} \\ &\approx 1.7 \text{ MW} \end{aligned}$$

Number of aeroturbines needed

$$= \frac{300}{1.7} = 177$$

These aerogenerators will be installed in a *wind farm* with suitable  $X$ ,  $Y$  spacing so that the air turbulence of one aeroturbine on the exit side and also sideways does not affect the successive turbine.

## Wind Farm

It is seen from the example given above that to contribute significant power to the grid, several standard size wind turbines have to be employed at a site where there is a vast enough wind field, flat or in a valley. Such an arrangement is called a wind farm.

How closely can the individual wind turbines be located to each other in a wind farm? The operation of a wind turbine causes an air turbulence on its back as well as sides; the region of turbulence is called the *wake* of the turbine. Optimal location of wind turbines is such that no turbines are located in the wake of the forward and side turbines. Any turbine that lies in the wake of another has its power output reduced and over a period fatigue damage caused by stresses generated can occur specially for the yaw drive.

### Spacing Rule

A wind turbine has to be aligned perpendicular to the direction of wind. Where wind is unidirectional all day (which is rare), the spacing between turbines of a row (side ways) is  $2 D - 3 D$ ,  $D$  being the diameter of the rotor. Inter row spacing is about  $10 D$ . Normally wind is not unidirectional in which case a uniform spacing of  $5 - 7D$  is recommended. A computer software "Micropositioning" is available for this purpose.

Internal transformers and cabling will connect the aerogenerator to the grid. Each turbine will have its own control circuitry. Grid connection requires a certain sophisticated protection scheme whose purpose is

1. to isolate the wind farm in case of any internal fault in the electrical system and
2. to disconnect the wind farm if there is a fault on any section of utility network (grid).

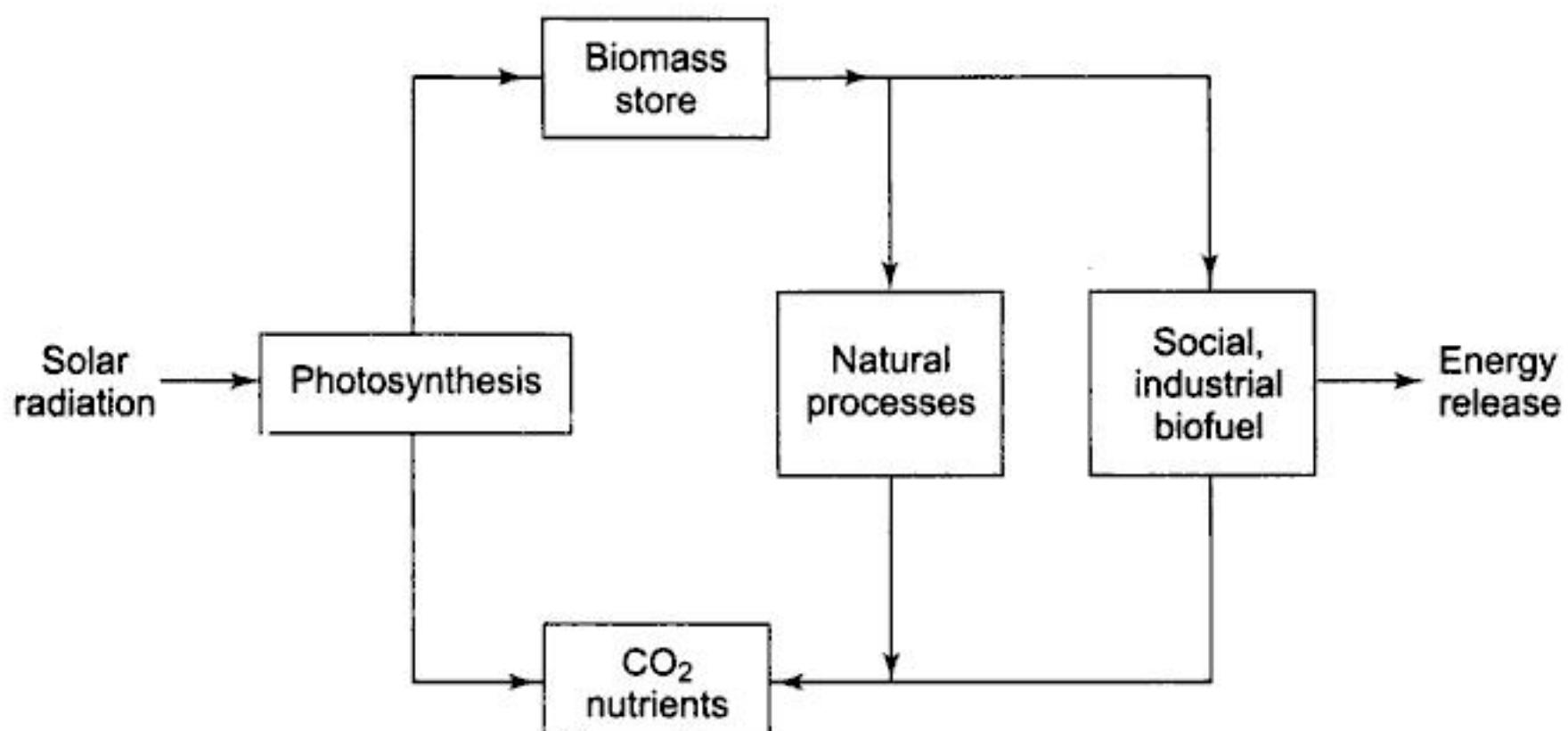
## 1.12 BIOFUELS

We shall explain the biofuels and their utilization with the help of Fig. 1.34 and explanation of certain terms.

**Biomass:** It is the material of all the plants and animals. The organic carbon part of this material reacts with oxygen in combustion and in the natural metabolic processes. The end product of these processes is mainly  $\text{CO}_2$  and heat as shown in Fig. 1.34.

**Biofuels:** The biomass can be transformed by chemical and biological processes into intermediate products like methane gas, ethanol liquid or charcoal solid.

**Agro industries:** The use of biofuels when linked carefully to natural ecological cycles (Fig. 1.34) may be nonpolluting. Such systems are called agroindustries. The well established of these industries are the sugarcane and forest product industries.



**Fig. 1.34** Biomass cycle

Biofuels can be used to produce electricity in two ways:

1. By burning in a furnace to raise steam to drive turbines or
2. By allowing fermentation in landfill sites or in special anaerobic tanks, both of which produce methane gas which can be used as fuel for household stoves and in spark ignition engines or gas turbines. The CO<sub>2</sub> produced in this process must be recycled by cultivating next crop or planting trees as CO<sub>2</sub> is absorbed by photosynthesis by plants.

Biofuels have a potential to meet about 5% of the electricity requirements of an industrialized country by exploiting all forms of these household and industrial wastes, sewerage, sledge (for digestion) and agricultural waste (cow dung, chicken litter, straw, sugarcane, etc.)

### 1.13 GENERATING RESERVE, RELIABILITY AND CERTAIN FACTORS

Electric loads present a highly fluctuating picture and is not easily amenable to statistics and probability; moreover it is a non-stationary process as its statistics are changing with time. It is, therefore, best to interpret generation and loads in terms of certain overall factors without any direct link to probability. Some of the important factors in use and their significance is presented below.

**Reserve generating capacity:** Modern generating plants are stressed to limits of temperature and pressure to reduce the overall power costs. Therefore, extra generation capacity must be installed to meet the need of scheduled *downtimes* for preventive maintenance. This reserve capacity also takes care of forced equipment outages and the possibility of the actual load exceeding the forecast, while additional generating stations are in completion phase.

The amount of reserve capacity to be provided is a subjective judgement and somewhat related to the past experience. Inadequate reserve means load outages at times and excessive reserve adds to generation costs.

**Reliability:** It is measured by the power systems ability to serve all power demands without failure over long periods of times. Various quantitative methods have been devised for estimation of reliability.

From the point of view of generation of power, reliability is added to the system by providing *spinning reserve*. In certain stations of the system some machines are kept on line but are kept only partially loaded to meet almost instantaneously any contingency of loss of a generator feeding the load. The amount of spinning reserve is also based on generator outage statistics and subjective judgement.

Reliability is considerably increased by system interconnection or grid formation and also transmission line redundancy. A cost has to be borne for a reliable system. In India with generation and line capacity shortages reliability is not a meaningful term.

**Availability (operational):** It is the percentage of the time a unit is available to produce power whether needed by the system or not. It is indeed a measure of overall unit reliability.

**Capacity Factor:** It is defined as

$$\text{Annual capacity factor} = \frac{\text{actual annual generation (MWh)}}{\text{maximum rating (MW)} \times 8760 \text{ h}}$$

It is always lower than operational availability because of the need to provide spinning reserve and variations in hourly load.

**Maximum load:** The average load over half hour of maximum output.

$$\text{Annual Load Factor: Annual LF} = \frac{\text{Total annual load (MWh)}}{\text{annual peak load (MW)} \times 8760 \text{ h}}$$

LF varies with the type of load, being poor for lighting load (about 12%), and high for industrial load (80–90%).

**Diversity Factor:** It is already introduced earlier in Sec. 1.3. It is defined again as

$$DF = \frac{\sum \text{individual maximum demand of consumers}}{\text{maximum load on the system}}$$

This factor is more than unity. It is high for domestic load. It can be made high by adjustment of timing and kind operation in each shift in industry by providing incentives.

Diversity factor also has same meaning at HV buses where loads are fed to different time zones in a large country. Although India uses one civil time, there is half an hour of diversity between the eastern and western region.

Consider four loads which are constant at  $L_{\max}$  for 6 h duration and zero for rest of the time. Now we calculate LF and DF for two imaginary cases.

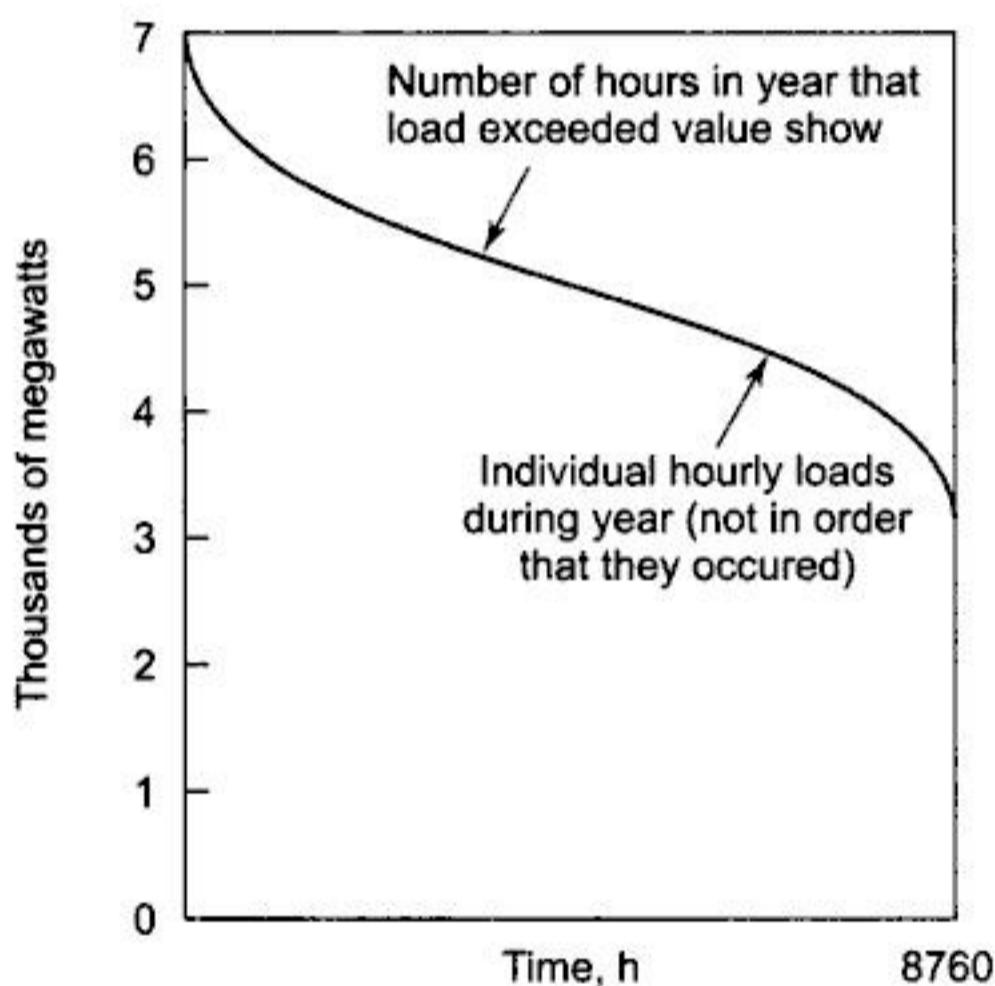
**Case 1.** Loads occur one after another around 24 h of day. Then

$$LF = \frac{L_{\max} \times 24}{L_{\max} \times 24} = 1; DF = 4$$

**Case 2.** All the loads occur at the same time. Then the total load is  $4 L_{\max}$  (the capacity of supply point). Then

$$LF = \frac{4L_{\max} \times 6}{4L_{\max} \times 24} = 0.25; DF = 1$$

**Local Variations:** For computational purpose the yearly data can be plotted in the form of a *load-duration* curve as shown in Fig. 1.35.



**Fig. 1.35** Annual load–duration curve

## Generating Capacity Mix

For economically meeting the cyclically varying load the generating capacity can be divided into three basic types (all types may not be available in a small power system):

1. Base-load capacity
2. Intermediate-load-range capacity
3. Peaking capacity

*Base-load capacity* runs at full rating continuously round the year (except for preventive maintenance when such capacity kept in spare is brought in). These are large units, which exploit the *economy of scale*, with all the fuel-economizing features built in; see Fig. 1.6(b).

Nuclear units have very high capital cost and very low fuel cost and thus are base-load units. Also not much underloading is permitted in these units.

For large hydroelectric dams, throughout the year, the basin is more or less full; thus, hydro unit could serve the base load purpose.

*Intermediate-load-range capacity* is employed to pick up the load when it rises above the base value. As this capacity does not run round the year, these units may be less efficient than base units.

*Peaking capacity* is run to take up the peak load of the day and the season. Since their annual output is not very high, high efficiency like in base-load unit is not a necessity. Gas turbines, small hydro units and pumped storage units are most suitable for peaking load as these are *quick start*.

These aspects of the three types of capacity are tabulated in Table 1.2

**Table 1.2**

Designation capacity	Capital cost	Fuel cost	Typical annual LF%
Base load	High	Low	65–75
Intermediate load	Intermediate	Intermediate	30–40
Peaking load	Low	High	5–15

**Interconnection and Pooling:** As our country is moving towards national grid formation and more private power generating companies are coming up, which can feed power in the grid, certain inherent saving and advantages result. These are as follows:

1. *Economy interchange*, which is permitted by low fuel cost stations near the coal bearing regions.
2. Each pool region needs less reserve capacity (both standby and spinning) as the reserve capacity gets pooled by interconnection.
3. *Diversity interchange* between various regions make the countrywise LF higher.

The above mentioned interchanges are possible only when strong power transporting lines link the regions into a grid.

## Load Management

The procedure to modify the shape of load curve so that the load factor (LF) is raised for operational economy have been mentioned earlier here and there. These procedures and methods are as follows:

1. Offering tariff incentives in light load periods to fill in the troughs.
2. Peaking shaving by
  - (i) pumped storage (hydro) and
  - (ii) CCGT use at peak load.
3. In advanced countries radio-controlled means (Ripple control) are employed to cut-off comfort conditioning at the peak load hour and then to switch them on.

As mentioned earlier by various 'load management' schemes, it is possible to shift demand away from peak hours (Sec. 1.1). Remote timer controlled on/off switches help achieve adjustment of electric use by a consumer [30]. Most of the potential for load control lies in the domestic sector. Power companies are now planning the introduction of system-wide load management schemes.

Future power systems would include a transmission mix of AC and DC. Future controllers would be more and more microprocessor based, which can be modified or upgraded without requiring hardware changes, and without bringing the entire system down. While one controller is in action the duplicate controller is there as a 'hot standby' in case of sudden need.

It is by now clear that HVDC transmission is already a reliable, efficient and cost-effective alternative to HVAC for many applications. Currently a great deal of effort is being devoted to further research and development in solid state technology. This gives hope that HVDC converters and multiterminal DC (MTDC) systems will play an even greater role in the power systems of the 21st century.

#### **1.14 ENERGY STORAGE**

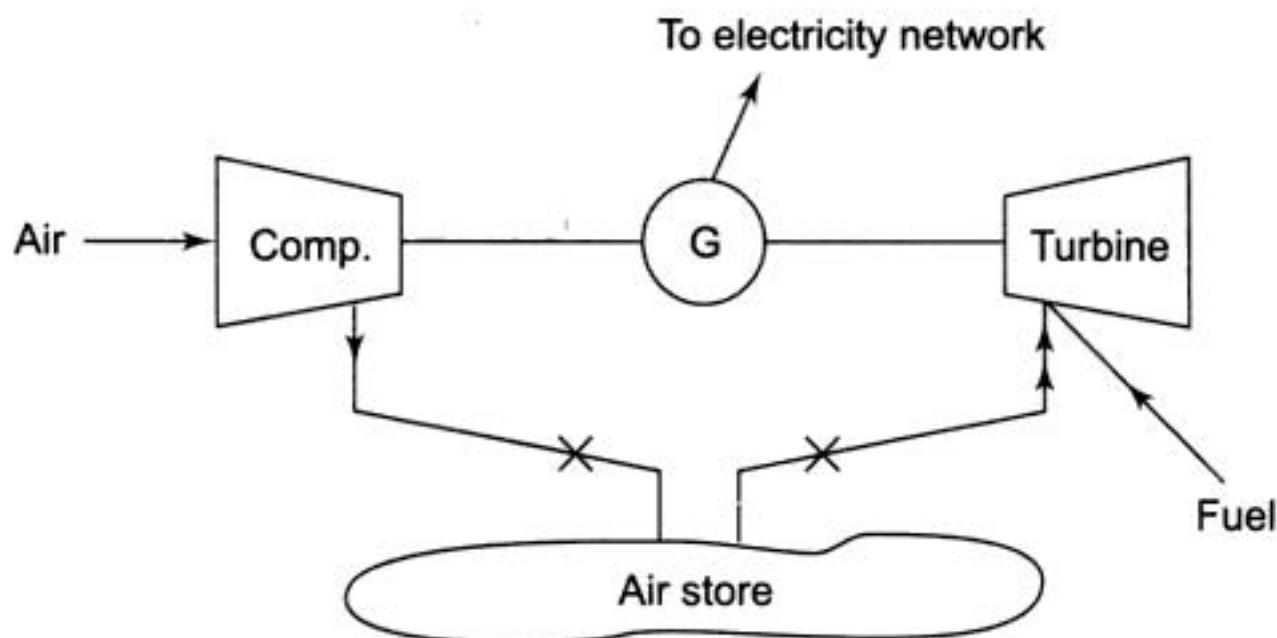
Because of tremendous difficulties in storage of electricity it has to be constantly generated, transmitted and utilized. Large scale storage of energy, which can be quickly converted to electrical form, can help fast changing loads. This would help to ease operation and make the overall system economical as large capacity need not be kept on line to take up short duration load surges. The options available as follows:

- pumped storage (see Sec. 1.5.3)
- compressed air storage
- heat storage
- hydrogen gas storage
- batteries
- fly wheels, superconducting coils; of doubtful promise

Most important of these is the pumped storage which has been dealt at length in Sec. 1.5.3.

#### **Compressed Air Storage**

Compressed air can be stored in natural underground caverns or old mines. The energy stored equals the volume of air multiplied by pressure. At the time of need this air can be mixed with gas fuel to run a gas turbine as shown in Fig. 1.36. Gas fuel combustion efficiency is thereby doubled compared to normal operation. A disadvantage of the scheme is that much of the energy used in compressing air appears in the form of heat and gets wasted; temperature of air is 450°C at 20 bar pressure.



**Fig. 1.36** Compressed air storage and use

## Heat Storage

No large scale storage of heat has been found to be feasible. Water with good specific and latent heat has been suggested. Liquid sodium is another candidate. (It is used in FBRs for heat transfer). In a generating station, boilers can be kept ready on full steam for the turbine to pick up fast rising load. Boiler steam when not in use can heat feed-water for boilers in the station.

## Secondary Batteries

These have possible use in local fluctuating loads, electric vehicles and as backup for wind and solar power (see Fig. 1.28). Considerable research and development effort is being devoted by international laboratories for secondary batteries. The present status is as in Table 1.3

**Table 1.3**

Battery	Energy density	Operating temperature (where high)	Remarks
Lead-acid cell (popular)	15 Wh/kg (low)	—	—
Nickel-Cadmium cell	40 Wh/kg	—	—
Sodium-sulphur cell	200 Wh/kg	300°C	Sodium electrolyte, liquid electrodes

A 3 MW battery storage plant has been installed in Berlin for frequency control in emergencies.

## Hydrogen Energy Systems

Hydrogen can be used as a medium for energy transmission and storage. Transmission of natural gas via a network is well established. India is setting up a national gas grid; Hazira-Jagdishpur pipeline is one link of the grid being developed. The energy transmission capacity of a gas pipeline is high compared to electric energy transmission via HV lines.

Calorific value (cv) of hydrogen gas =  $12 \times 10^6 \text{ J m}^{-3}$  (ATP)

Power transmitted = flow rate (volume)  $\times$  cv (at working pressure)

For long gas pipelines pressure drop is compensated by booster compressor stations.

A typical gas system: Pipe of internal diameter = 0.914 m

Pressure = 68 atm

Gas velocity = 7 m/s

Power transferred = 12 GW (gigawatts)

Using gas, electric power can be generated by CCGT near the load centres.

A 1 m diameter pipe carrying hydrogen gas is equivalent to 4–400 kV, 3-phase transmission lines. The major advantage that hydrogen gas has is that it can be stored. The disadvantage being it is produced by electrolysis of water. An alternative method, under development, is to use heat from a nuclear station to 'crack' water for releasing hydrogen at a temperature of about 3000°C. Solar energy is also being used for water splitting to generate hydrogen which can be converted back to electricity by means of fuel cells. Also the use of hydrogen as fuel for aircraft and automobiles could encourage its large scale production, storage and distribution.

Hydrogen may prove to be a wonder element. It is non-polluting, safe and sustainable. It is most lauded alternative transportation fuel after biofuels. However, system efficiency, system cost and safety related aspects are yet to be addressed for H<sub>2</sub> based technology. A fuel cell run car is available in US and Japan. An H<sub>2</sub> and fuel cell facility is available at Solar Energy Centre, Gurgaon. Today price is Rs 150/W. Alcohol mixed petrol is tried successfully in some countries such as Brazil. Class I cities generate  $27 \times 10^6$  tons of municipal waste,  $4400 \times 10^3 \text{ m}^3$  of sewage per year. This can be converted to energy. Estimated potential from urban, municipal and industrial waste to energy is 25000 MW or 50 million units/year of grid power. With the largest cattle population in the world of some 262 million, India holds tremendous potential for biogas development. India has 135 million hectares of wasteland (poor/semidesert) ideal for Jatropha plantation for production of **biodiesel**. It is nontoxic, 100% natural and biodegradable supplement for diesel.

India has total offshore gas hydrate resources of 1894 trillion m<sup>3</sup> which is 1900 times the country's current gas reserves. Even if 1% of the estimated gas hydrate reserves is tapped, our energy requirement for the coming decades can be met.

World-over hydrogen is produced from:

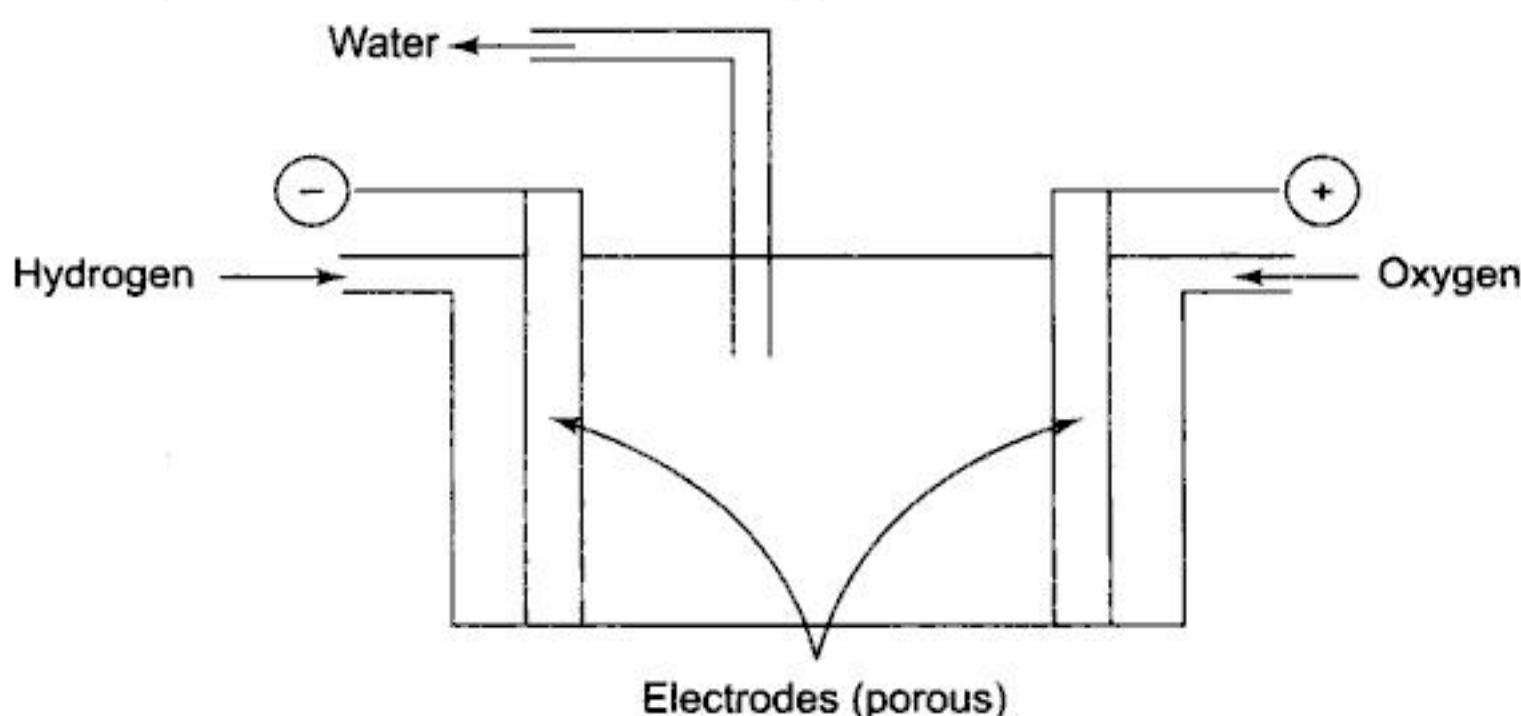
steam reformation of natural gas	(48%)
partial oxidation of oil	(30%)
gasification of coal	(18%)
electrolysis of water	(4%)

In India, about 2.8 MMT produced annually (fertilizer industry, petroleum refineries) for captive consumption. About 0.4 MMT by-product hydrogen

available from chlor-alkali industries. It is used in automobiles and power generation using IC engines and fuel cells. H<sub>2</sub> is also produced through renewable energy sources and can be stored in many ways such as hydrides, carbon nano-tubes and nano-fibres.

### Fuel Cell

A fuel cell converts chemical energy to electrical form by electrochemical reaction with no intermediate combustion cycle. One electrode is continuously supplied with fuel (H<sub>2</sub>) and the other with an oxidant (usually oxygen). A simple form of a fuel cell is shown in Fig. 1.37 where the fuel is hydrogen, which diffuses through a porous metal (nickel) electrode. This electrode has a catalyst deposited around the pores, which aids the absorption of H<sub>2</sub> on its surface. In this process hydrogen ions react with hydroxyl ions in the electrolyte to form water (2H<sub>2</sub> + O<sub>2</sub> → 2H<sub>2</sub>O). The cell has a theoretical emf of 1.2 V at 25°C. Other fuels are CO (1.33 V) and methanol (1.21 V) both at 25°C. Conversion efficiencies of practical cell are about 80%. The major use of the cell is in conjunction with hydrogen-oxygen system.



**Fig. 1.37** Hydrogen-oxygen fuel cell

Intense R&D effort is on for various types of cells for power generation. Most successful of these is the phosphoric cell, which uses methane as fuel and operates at about 200–300°C. It has been constructed to produce 200 kW of electric power plus 200 kW heat energy with an overall efficiency of 80%. Higher temperatures give still higher efficiency. The main reason why fuel cells are not in wide use is its high cost. Global electricity generating capacity from fuel cells will grow from just 75 MW in 2001 to 15,000 MW by 2010. R&D projects on PAFC, PEMFC, DMFC, DEFC and SOFC are supported by MNES, and other funding agencies and organisations such as NTPC, IITs, BHEL, CECRI, Karaikudi. PEMFC is considered suitable for use in automobiles and also for decentralized power generation (few kW). Other FCs for higher scale (MW scale). Fuel cell based 3 kW UPS is also developed. Fuel cell-battery hybrid electric vehicle is also available.

### 1.15 ENERGY CONSERVATION

We shall restrict our discussion only to energy conservation associated with generation, transmission, distribution and utilization of electric energy. Some of the issues have been touched upon in previous sections. Various important conservation methods have been brought on the right most side of Fig. 1.5.

#### Generation

Ideal heat engine has Carnot cycle efficiency. To bring the practical efficiency as close to Carnot as possible. Rankine cycle is used by heating steam to the highest temperature (and pressure) permitted by economically feasible boiler and piping materials. This is done by *superheating* steam in upper part of the boiler. Other steps that are taken to improve the overall efficiency of the turbine set as brought out in Fig. 1.6(b) are as follows:

1. LP steam from IP and HP turbines is reheated in the boiler before feeding it to LP turbines.
2. Some of the LP steam from IP turbine and also LP turbine is used to heat feedwater to boiler.
3. Hot water from generator cooler heated by LP steam from LP turbines constitutes a part of feedwater to boiler.
4. Cooled water from cooling towers is circulated through condenser tubes to reduce the condenser pressure to the lowest possible value. This lowers the back pressure of LP ends of LP turbines.
5. Heating feedwater by the heat stored in flue gases from the combustion chamber forms the first stage of feedwater heating process.

#### Cogeneration

Process steam is used for generation before or after. For details see Sec. 1.3.

**CCGT:** Gas turbine combined with steam turbine is employed for peak load shaving. This is more efficient than normal steam turbine and has a quick automated start and shut down. It improves the load factor of the steam station. For details refer to Sec. 1.3.

**T & D (transmission and distribution) Loss:** Losses in transmission and distribution should be kept low while designing these lines. Of course this has to be matched against the cost factor. In any case this loss should not exceed 20%. For transporting large chunks of power over long distances, HVDC option must be considered as this method has lower transmission loss; see Ch. 20.

#### Energy Storage

It can play an important role where there is time or rate mismatch between supply and demand of energy. This has been discussed in Sec. 1.14. Pumped storage (hydro) scheme has been considered in Sec. 1.5.3.

#### Industry

In India corporate sector is required by law, to include in their annual report, the measures taken for energy conservation.

Steps for energy conservation:

- Keep an energy audit. This will put a finger on the places and items where there is wasteful use.
- Use of energy efficient electric drives. No oversize motors as these would run at low power factor and efficiency.
- Use of regenerative braking particularly the motors which require frequent start/stop operation. In regeneration, energy stored in mechanically moving parts of the machinery being driven and the rotor of the motor is fed back electrically to the mains.
- Use of high efficiency motors, which are now becoming available.

## **Building—Industrial, Commercial and Domestic**

### ***Heating***

Electric space heating is out of question. Space heating in areas, where needed, is done by steam boiler and piping. Electric geysers are commonly used for heating water for bath in severe winter. Its thermostat should be set at the lowest acceptable temperature. In India where most areas have large number of sunny days, hot water for bath and kitchen by solar water heaters is becoming common for commercial buildings, hostels and even hospitals. For cloudy days in winter some electric support system may be necessary for essential use.

### ***Cooling***

Chilled water system saves electricity for space cooling. Air conditioning for offices and individual rooms in houses should be set at temperature of 27°C and 60% relative humidity. In dry areas in hot weather a contrivance called ‘water cooler’ is quite common now. The humidity and temperature are both uncontrolled except manual switching ON/OFF. In large cities in Northern India all these appliances plus refrigerators constitute more load than lighting. At present there is no other natural alternative to cool high rise buildings.

### ***Lighting***

Buildings should be designed to bring in natural light but in summer heat access has to be almost eliminated. A combination of fluorescent tube and electric lamp gives acceptable and efficient lighting for homes and even offices. In commercial buildings and street lighting, energy efficient devices as CFL (Compact Fluorescent Lamp) should be used. Their initial high cost is made up by the reduced electricity bill.

There is a lot of wasteful use of lighting prevalent in commercial buildings where the lights are kept ‘on’ even when not needed or the room(s) is not being used. As in corporate sector commercial buildings, energy audit must be made a requirement in educational institutions and other users where large areas are lighted. Public has to be educated that 20 units of electricity at user end require 100 units of heat input at generation end.

In India where vast regions are deficient in electric supply and are subjected to long hours of power (load) shedding (mostly random), the use of small diesel/petrol generators and inverters are very common in commercial and domestic use. These are highly wasteful energy devices for the following reasons:

- Small diesel/petrol engines (1–2 kVA) are highly inefficient. So it is a national waste. Further these cause serious noise pollution.
- Inverters are storage battery based. When the line power is ON, they draw a heavy charging current over and above the normal load. When a large number of consumers use these inverters, power load is considerably increased when the power is ON. This in fact adds to line outage. Further, pollution is caused inside the house by battery fumes.

This situation needs to be rectified speedily. By proper planned maintenance the downtime of existing large stations can be cut down. Further, HVAC and transmission lines must be installed to evacuate the excess power available in Eastern region of the country. Power Grid Corporation has already been set up for this purpose. These actions will also improve the load factor of most power stations, which would indirectly contribute to energy conservation.

### **Losses**

There is now a movement towards estimating and monitoring AT&C (aggregate technical and commercial) loss in the country. GOI has adopted AT&C as a measure of commercial efficiency in all distribution reform programmes.

### **1.16 GROWTH OF POWER SYSTEMS IN INDIA**

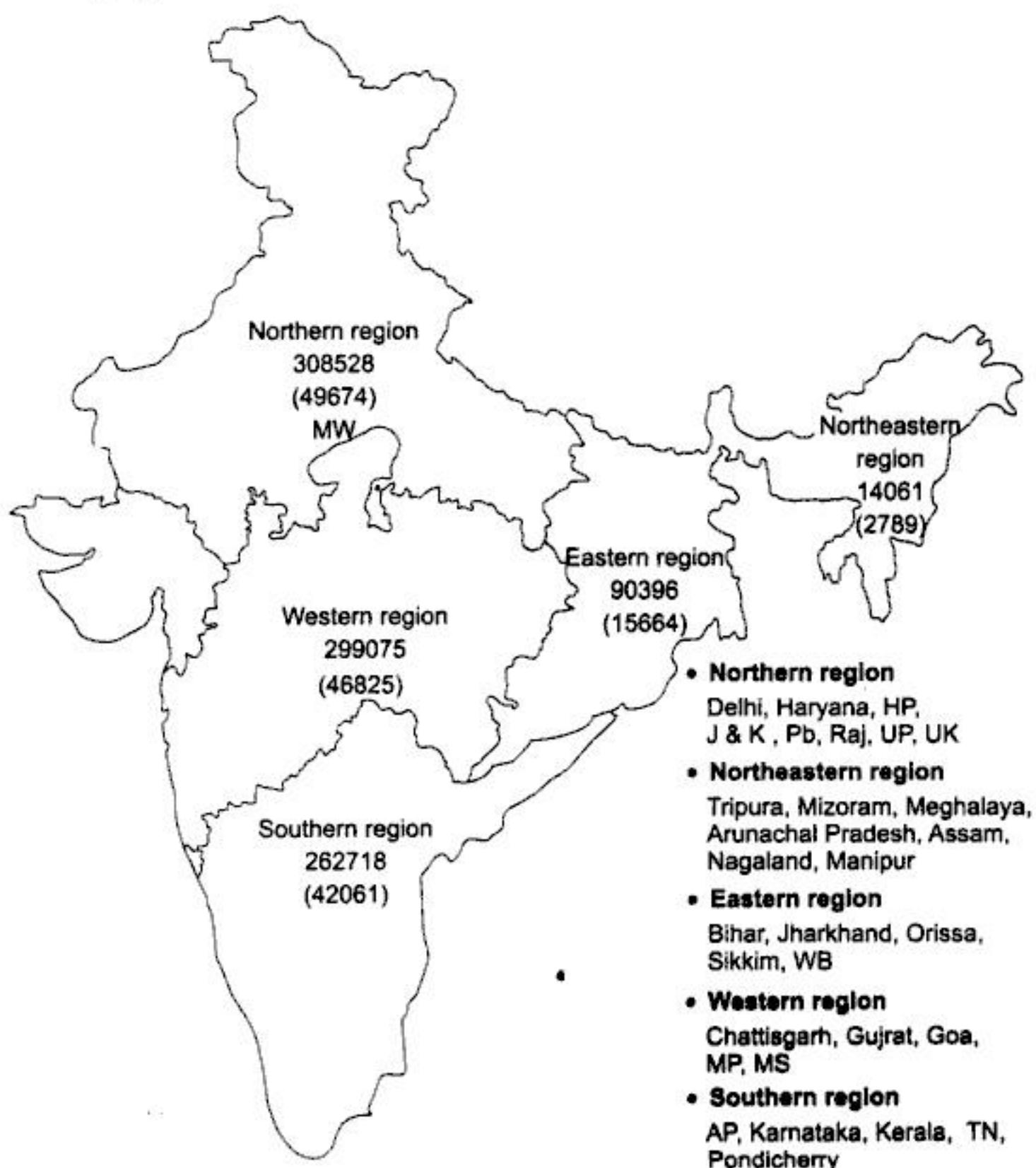
India is fairly rich in natural resources like coal and lignite; while some oil reserves have been discovered so far, intense exploration is being undertaken in various regions of the country. India has immense water power resources also of which only around 25% have so far been utilised, i.e., only 25,000 MW has so far been commissioned up to the end of 9th plan. As per a recent report of the CEA (Central Electricity Authority), the total potential of hydro power is 84,040 MW at 60% load factor. As regards nuclear power, India is deficient in uranium, but has rich deposits of thorium which can be utilised at a future date in fast breeder reactors. Since independence, the country has made tremendous progress in the development of electric energy and today it has the largest system among the developing countries.

When India attained independence, the installed capacity was as low as 1360 MW in the early stages of the growth of power system, the major portion of generation was through thermal stations, but due to economical reasons, hydro development received attention in areas like Kerala, Tamil Nadu, Uttar Pradesh and Punjab.

In the beginning of the First Five Year Plan (1951–56), the total installed capacity was around 2300 MW (560 MW hydro, 1004 MW thermal, 149 MW through oil stations and 587 MW through non-utilities). For transporting this

power to the load centres, transmission lines of up to 110 kV voltage level were constructed.

The emphasis during the Second Plan (1956–61) was on the development of basic and heavy industries and thus there was a need to step up power generation. The total installed capacity which was around 3420 MW at the end of the First Five Year Plan became 5700 MW at the end of the Second Five Year Plan in 1962, the introduction of 230 kV transmission voltage came up in Tamil Nadu and Punjab. During this Plan, totally about 1009 circuit kilometres were energized. In 1965–66, the total installed capacity was increased to 10,170 MW. During the Third Five Year Plan (1961–66) transmission growth took place very rapidly, with a nine-fold expansion in voltage level below 66 kV. Emphasis was on rural electrification. A significant development in this phase was the emergence of an inter-state grid system. The country was divided into five regions, each with a regional electricity board, to promote integrated operation of the constituent power systems. Figure 1.38 (a) shows these five regions of the country with projected energy requirement and peak load in the year 2011–12 [19].



**Fig. 1.38(a)** Map of India showing five regional projected energy requirement in MWh and peak load in MW for year 2011–12

During the Fourth Five Year Plan, India started generating nuclear power. At the Tarapur Nuclear Plant  $2 \times 210$  MW units were commissioned in April-May 1969. This station uses two boiling water reactors of American design. By August 1972, the first unit of 220 MW of the Rajasthan Atomic Power Project, Kota (Rajasthan), was added to the nuclear generating capability. The total generating capacity at Kota is 430 MW with nuclear reactors of Canadian design which use natural uranium as fuel and heavy water as a moderator and coolant. The third nuclear power station of  $2 \times 235$  MW has been commissioned at Kalpakkam (Tamil Nadu). This is the first nuclear station to be completely designed, engineered and constructed by Indian scientists and engineers. A reactor research centre has been set up near the Madras Atomic Power Station to carry out study in fast breeder reactor technology. The fourth nuclear power plant has been set up at Narora in Uttar Pradesh. It has two units of 235 MW each. The fifth is in Kaiga in Karnataka and sixth in Gujarat near Surat, Kakrapar (440 MW). Several other nuclear power plants will be commissioned by 2012.

The growth of generating capacity so far and future projection for 2011–2012 A.D. are given in Table 1.4.

**Table 1.4** Growth of installed capacity in India (in MW)

Year	Hydro	Nuclear	Thermal	Diesel	Total
1970–71	6383	420	7503	398	14704
1978–79	11378	890	16372	—	28640
1984–85	14271	1095	27074	—	42240
2000–01	25141	2720	71060	≈2700 MW renewable	101630
2005–06	33941	3900	84149	6190	128180

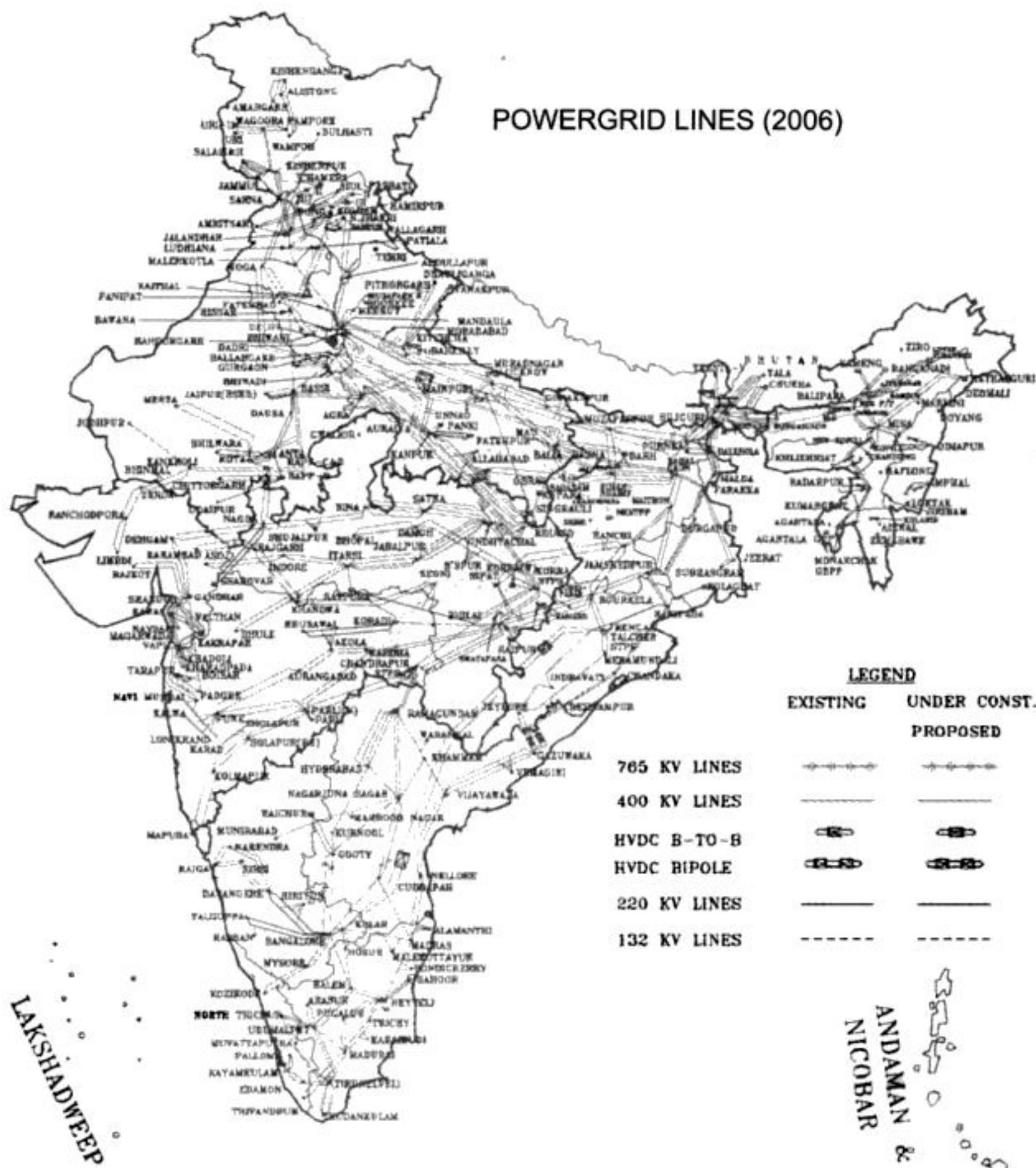
Pattern of utilization of electrical energy in 1997–98 was: Domestic 20.69%, commercial 6.91%, irrigation 30.54%, industry 35.22% and others is 6.65%. It is expected to remain more or less same in 2004–05.

To be self-sufficient in power, BHEL has plants spread out all over the country and these turn out an entire range of power equipment, viz. turbo sets, hydro sets, turbines for nuclear plants, high pressure boilers, power transformers, switch gears, etc. Each plant specializes in a range of equipment. BHEL's first 500 MW turbo-generator was commissioned at Singrauli. Today BHEL is considered one of the major power plant equipment manufacturers in the world. Figure 1.38(b) shows main powergrid lines as on 2006.

### 1.17 Deregulation

For over one hundred years, the electric power industry worldwide operated as a *regulated* industry. In any area there was only one company or government agency (mostly state-owned) that produced, transmitted, distributed and sold electric power and services. Deregulation as a concept came in early 1990s. It brought in changes designed to encourage competition.

## POWER MAP OF INDIA



**Fig. 1.38 (b)** Showing main powergrid lines as on 2006

*Restructuring* involves disassembly of the power industry and reassembly into another form or functional organization. *Privatization* started sale by a government of its state-owned electric utility assets, and operating economy, to private companies. In some cases, deregulation was driven by privatization needs. The state wants to sell its electric utility investment and change the rules (deregulation) to make the electric industry more palatable for potential investors, thus raising the price it could expect from the sale. *Open access* is nothing but a common way for a government to encourage competition in the electric industry and tackle monopoly. The consumer is assured of good quality power supply at competitive price.

The structure for deregulation is evolved in terms of Genco (Generation Company), Transco (Transmission Company) and ISO (Independent System Operator). It is expected that the optimal bidding will help Genco to maximize its payoffs. The consumers are given choice to buy energy from different retail energy suppliers who in turn buy the energy from Genco in a power market (independent power producer, IPP).

The restructuring of the electricity supply industry that normally accompanies the introduction of competition provides a fertile ground for the growth of embedded generation, i.e. generation that is connected to the distribution system rather than to the transmission system.

The earliest reforms in power industries were initiated in Chile. They were followed by England, the USA, etc. Now India is also implementing the restructuring. Lot of research is needed to clearly understand the power system operation under deregulation. The focus of research is now shifting towards finding the optimal bidding methods which take into account local optimal dispatch, revenue adequacy and market uncertainties.

India has now enacted the Electricity Regulatory Commission's Act, 1998 and the Electricity (Laws) Amendment Act, 1998. These laws enable setting up of Central Electricity Regulatory Commission (CERC) at central level and State Electricity Regulatory Commissions (SERC) at state level. The main purpose of CERC is to promote efficiency, economy and competition in bulk electricity supply. Orissa, Haryana, Andhra Pradesh, etc. have started the process of restructuring the power sector in their respective states.

Four important terms connected with deregulated power systems are defined below:

1. **Open Access:** Open access is the non-discriminatory provision for the use of transmission lines or distribution system or associated facilities with such lines or system by any licensee or consumer or a person engaged in generation in accordance with the regulations specified by the Appropriate Commission. It will promote competition and, in turn, lead to availability of cheaper power.
2. **Wheeling:** Wheeling is the operation whereby the distribution system and associated facilities of a transmission or distribution licensee are used by another person for the conveyance of electricity on payment of charges to be determined by the Appropriate Commission.
3. **Energy Banking:** Energy banking is a process under which the Captive Power Plant (CPP) or a co-generator supplies power to the grid not with the intention of selling it to either a third party or to a licensee, but with the intention of exercising his eligibility to draw back this power from the grid at a prescribed time during next financial year, after deduction of banking charges.
4. **Unbundling/Corporatization:** Many state electricity boards have either been unbundled or corporatized. Distribution business is privatized in some states such as Delhi and Orissa.

### 1.18 DISTRIBUTED AND DISPERSED GENERATION

Distributed Generation (DG) entails using many small generators of 2–50 MW output, installed at various strategic points throughout the area, so that each provides power to a small number of consumers nearby. These may be solar, mini/micro hydel or wind turbine units, highly efficient gas turbines, small combined cycle plants, micro-turbines since these are the most economical choices.

Dispersed generation refers to use of still smaller generating units, of less than 500 kW output and often sized to serve individual homes or businesses. Micro gas turbines, fuel cells, diesel, and small wind and solar PV generators make up this category. The beauty is these are modular and relocatable power generating technologies.

Dispersed generation has been used for decades as an emergency backup power source. Most of these units are used only for reliability reinforcement. Nowadays inverters are being increasingly used in domestic sector as an emergency supply during black outs.

The distributed/dispersed generators can be stand alone/autonomous or grid connected depending upon the requirement.

At the time of writing this (2007) there still is and will probably always be some economy of scale favouring large generators. But the margin of economy decreased considerably in last 10 years [23]. Even if the power itself costs a bit more than central station power, there is no need of transmission lines, and perhaps a reduced need for distribution equipment as well. Another major advantage of dispersed generation is its modularity, portability and relocatability. Dispersed generators also include two new types of fossil fuel units—fuel cells and microgas turbines.

The main challenge today is to upgrade the existing technologies and to promote development, demonstration, scaling up and commercialization of new and emerging technologies for widespread adaptation. In the rural sector main thrust areas are biomass briquetting, biomass-based cogeneration, etc. In solar PV (Photovoltaic), large size solar cells/modules based on crystalline silicon thin films need to be developed. Solar cells efficiency is to be improved to 15% to be of use at commercial level. Other areas are development of high efficiency inverters. Urban and industrial wastes are used for various energy applications including power generation which was around 35 MW in 2005.

However, recently there has been a considerable revival in connecting generation to the distribution network and this has come to be known as *embedded* or dispersed generation. The term ‘embedded generation’ comes from the concept of generation embedded in the distribution network while ‘dispersed generation’ is used to distinguish it from central generation. The two terms can be used interchangeably.

There are already 35 million improved chulhas. If growing energy needs in the rural areas are met by decentralized and hybrid energy systems (distributed/dispersed generation), this can stem growing migration of rural population to urban areas in search of better living conditions. Thus, India will be able to

achieve a smooth transition from fossil fuel economy to sustainable renewable-energy based economy and bring “Energy for all” and ‘Energy for ever’ era for equitable, environment-friendly, and sustainable development.

### **1.19 POWER SYSTEM ENGINEERS AND POWER SYSTEM STUDIES**

The power system engineer of the first decade of the twenty-first century has to face a variety of challenging tasks, which he can meet only by keeping abreast of the recent scientific advances and the latest techniques. On the planning side, he or she has to make decisions on how much electricity to generate—where, when, and by using what fuel. He has to be involved in construction tasks of great magnitude both in generation and transmission. He has to solve the problems of planning and coordinated operation of a vast and complex power network, so as to achieve a high degree of economy and reliability. In a country like India, he has to additionally face the perennial problem of power shortages and to evolve strategies for energy conservation and load management.

For planning the operation, improvement and expansion of a power system, a power system engineer needs load flow studies, short circuit studies, and stability studies. He has to know the principles of economic load despatch and load frequency control. All these problems are dealt within the next few chapters after some basic concepts in the theory of transmission lines are discussed. The solutions to these problems and the enormous contribution made by digital computers to solve the planning and operational problems of power systems is also investigated.

### **1.20 USE OF COMPUTERS AND MICROPROCESSORS**

The first methods for solving various power system problems were AC and DC network analyzers developed in early 1930s. AC analyzers were used for load flow and stability studies whereas DC were preferred for short-circuit studies.

Analogue computers were developed in 1940s and were used in conjunction with AC network analyzer to solve various problems for off-line studies. In 1950s many analogue devices were developed to control the on-line functions such as generation control, frequency and tie-line control.

The 1950s also saw the advent of digital computers which were first used to solve a load flow problem in 1956. Power system studies by computers gave greater flexibility, accuracy, speed and economy. Till 1970s, there was a widespread use of computers in system analysis. With the entry of microprocessors in the arena, now, besides main frame computers, mini, micro and personal computers are all increasingly being used to carry out various power system studies and solve power system problems for off-line and on-line applications.

Off-line applications include research, routine evaluation of system performance and data assimilation and retrieval. It is mainly used for planning and analyzing some new aspects of the system. On-line and real time applications include data-logging and the monitoring of the system state.

A large central computer is used in central load despatch centres for economic and secure control of large integrated systems. Microprocessors and computers installed in generating stations control various local processes such as starting up of a generator from the cold state, etc. Table 1.5 depicts the time scale of various hierarchical control problems to be solved by computers/microprocessors. Some of these problems are tackled in this book.

**Table 1.5**

<i>Time scale</i>	<i>Control problems</i>
Milliseconds excitation control	Relaying and system voltage control and
2 s–5 min	AGC (Automatic generation control)
10 min–few hours — do —	ED (Economic despatch) Security analysis
few hours–1 week	UC (Unit commitment)
1 month–6 months	Maintenance scheduling
1 year–10 years	System planning (modification/extension)

### **1.21 PROBLEMS FACING INDIAN POWER INDUSTRY AND ITS CHOICES**

The electricity requirements of India have grown tremendously and the demand has been running ahead of supply. Electricity generation and transmission processes in India are very inefficient in comparison with those of some developed countries. As per one estimate, in India generating capacity is utilized on an average for 3600 hours out of 8760 hours in a year, while in Japan it is used for 5100 hours. If the utilization factor could be increased, it should be possible to avoid power cuts. The transmission loss in 1997–98 on a national basis was 23.68% consisting of both technical losses in transmission lines and transformers, and also non-technical losses caused by energy thefts and meters not being read correctly. It should be possible to achieve considerable saving by reducing this loss to 15% by the end of the Tenth Five Year Plan by using well known ways and means and by adopting sound commercial practices. Further, every attempt should be made to improve system load factors by flattening the load curve by giving proper tariff incentives and taking other administrative measures. As per the Central Electricity Authority's (CEA) sixteenth annual power survey of India report, the all India load factor up to 1998–99 was of the order of 78%. In future it is likely to be 71%. By 2001, 5.07 lakh of villages (86%) have been electrified and 117 lakh of pumpsets have been energized.

Assuming a very modest average annual energy growth of 5%, India's electrical energy requirement in the year 2010 will be enormously high. A difficult and challenging task of planning, engineering and constructing new

power stations is imminent to meet this situation. The government has built several super thermal stations such as at Singrauli (Uttar Pradesh), Farakka (West Bengal), Korba (Madhya Pradesh), Ramagundam (Andhra Pradesh) and Neyveli (Tamil Nadu), Chandrapur (Maharashtra) all in coal mining areas, each with a capacity in the range of 2000 MW\*. Many more super thermal plants would be built in future. Intensive work must be conducted on boiler furnaces to burn coal with high ash content. National Thermal Power Corporation (NTPC) is in charge of these large scale generation projects.

Hydro power will continue to remain cheaper than the other types for the next decade. As mentioned earlier, India has so far developed only around 18% of its estimated total hydro potential of 89,000 MW. The utilization of this perennial source of energy would involve massive investments in dams, channels and generation-transmission system. The Central Electricity Authority, the Planning Commission and the Ministry of Power are coordinating to work out a perspective plan to develop all hydroelectric sources by the end of this century to be executed by the National Hydro Power Corporation (NHPC). NTPC has also started recently development of hydro and nuclear power plants.

Nuclear energy assumes special significance in energy planning in India. Because of limited coal reserves and its poor quality, India has no choice but to keep going on with its nuclear energy plans. According to the Atomic Energy Commission, India's nuclear power generation will increase to 10,000 MW by year 2010. Everything seems to be set for a take off in nuclear power production using the country's thorium reserves in breeder reactors.

In India, concerted efforts to develop solar energy and other non-conventional sources of energy need to be emphasized, so that the growing demand can be met and depleting fossil fuel resources may be conserved. To meet the energy requirement, it is expected that the coal production will have to be increased to more than 450 million tonnes in 2004–2005 as compared to 180 million tonnes in 1988.

A number of 400 kV lines are operating successfully since 1980s as mentioned already. This was the first step in working towards a national grid. There is a need in future to go in for even higher voltages (800 kV). It is expected that by the year 2011–12, 5400 circuit km of 800 kV lines and 48,000 circuit km of 400 kV lines would be in operation. Also lines may be series and shunt compensated to carry huge blocks of power with greater stability. There is a need for constructing HVDC (High Voltage DC) links in the country since DC lines can carry considerably more power at the same voltage and require fewer conductors. A 400 kV Singrauli–Vindhyachal of 500 MW capacity first HVDC back-to-back scheme has been commissioned by NPTC (National Power Transmission Corporation) followed by first point-to-point bulk

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\* NTPC has also built seven gas-based combined cycle power stations such as Anta and Auraiya.

EHVDC transmission of 1500 MW at  $\pm 500$  kV over a distance of 915 km from Rihand to Delhi. Power Grid recently commissioned on 14 Feb. 2003 a 2000 MW Talcher-Kolar  $\pm 500$  kV HVDC bipole transmission system thus enabling excess power from East to flow to South. 7000 ckt km of  $\pm 500$  kV HVDC line is expected by 2011–12.

At the time of writing, the whole energy scenario is so clouded with uncertainty that it would be unwise to try any quantitative predictions for the future. However, certain trends that will decide the future developments of electric power industry are clear.

Generally, unit size will go further up from 500 MW. A higher voltage (1200 kV) will come eventually at the transmission level. There is little chance for six-phase transmission becoming popular though there are few such lines in USA. More of HVDC lines will come in operation. As population has already touched the 1000 million mark in India, we may see a trend to go toward underground transmission in urban areas.

Public sector investment in power has increased from Rs 2600 million in the First Plan to Rs 2,42,330 million in the Seventh Plan (1985–90). Shortfall in the Sixth Plan has been around 26%. There have been serious power shortages and generation and availability of power in turn have lagged too much from the industrial, agricultural and domestic requirements. Huge amounts of funds (of the order of Rs 18,93,200 million) will be required if we have to achieve power surplus position by the time we reach the terminal year to the XI Plan (2011–2012). Otherwise achieving a target of 975 billion units of electric power will remain an utopian dream.

Power grid is planning creation of transmission highways to conserve Right-of-way. Strong national grid is being developed in phased manner. In 2001 the interregional capacity was 5000 MW. It is expected that by 2011–12, it will be 30,000 MW. Huge investment is planned to the tune of US\$ 20 billion in the coming decade. Present figures for HVDC is 3136 circuit km, 800 kV is 950 circuit km, 400 kV is 45,500 circuit km and 220/132 kV is 2,15,000 circuit km. State-of-the art technologies which are, being used in India currently are HVDC bipole, HVDC back-to-back, SVC (Static Var Compensator), FACTS (Flexible AC Transmissions) devices etc. Improved O and M (Operation and Maintenance) technologies which are being used today are hotline maintenance, emergency restoration system, thermovision scanning, etc. 24 hours of supply of good-quality power would help small industries in rural areas. It will also facilitate delivery of modern health care, education, and application of information and communication technologies.

Because of power shortages, many of the industries, particularly power-intensive ones, have installed their own captive power plants.\* Currently 20%

\* Captive diesel plants (and small diesel sets for commercial and domestic uses) are very uneconomical from a national point of view. Apart from being lower efficiency plants they use diesel which should be conserved for transportation sector.

of electricity generated in India comes from the captive power plants and this is bound to go up in the future. Consortium of industrial consumers should be encouraged to put up coal-based captive plants. Import should be liberalized to support this activity. Now alternative fuels are increasingly being used for surface transportation. Battery operated vehicles (electric cars) are being used.

With the ever increasing complexity and growth of power networks and their economic and integrated operation, several central/regional automatic load despatch centres with real time computer control have been established. In very near future it is envisaged that using SCADA (Supervisory Control and Data Acquisition) etc. will be possible to achieve nationwide on-line monitoring and real time control of power system. It may also be pointed out that this book will also help in training and preparing the large number of professionals trained in computer aided power system operation and control that would be required to handle vast expansion planned in power system in the coming decades.

## ANNEXURE 1.1

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### 1. The Indian Electricity Rules; [25]

The Indian Electricity Act, 1910 deals with the provisions relating to supply and use of electrical energy and the rights and obligations of persons licensed under Part II of that Act to supply energy. Under section 36A of the Act, a Board called the Central Electricity Board is constituted to exercise the powers conferred by section 37. In exercise of the powers conferred under that section, the Central Electricity Board framed the Indian Electricity Rules, 1956 for the whole or any part of the territories to which the Act extends, to regulate the generation, transmission, supply and use of energy, and generally to carry out the purposes and objects of the Act.

## ANNEXURE 1.2

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**Table 1.6** Installed generation capacity (As on 31.03.2006)  
(Source CEA, New Delhi)

		Thermal	MW	Coal 68519
			82411	Gas 12690
			(66%)	Diesel 1202
Wind	4434	Nuclear	3360 (3%)	
Small Hydro	777	Hydro	32326 (26%)	
Biomass/waste	980	Green	6191 (5%)	
		Total	124288	

**Table 1.7** Growth in transmission system (2006)

132 kV	120000 ckm
220 kV	120000 ckm
400 kV	60000 ckm
HVDC bi-pole lines 5000 MW	
HVDC back to back 3000 MW	
765 kV 400 kV op 1150 ckm	
(Kishenpur-Moga, Anpara-Unnao Tehri-Meerut)	

**Table 1.8** The landmark events in Indian power sector

1948	Electricity (supply) Act
1950–60	Growth of State Grid Systems
1962	First 220 kV voltage level
1964	Constitution of Regional Electricity Boards
65–73	Interconnecting State Grids to form Regional Grids
75	Central PSUs in Generation and Transmission
77	First 400 kV voltage level
80–88	Growth of Regional Grids-400 kV back-bone
89	HVDC back to back
89	Formation of Power Grid Corporation of India
90	First HVDC bi-pole line
90	Generation of electricity opened to Private Sector
98	Electricity Regulatory Commission Act
98	765 kV Transmission line (initially charged at 400 kV)
2003	Electricity Act 2003-open access in Transmission
05	The Rajiv Gandhi scheme of Rural Electricity Infrastructure and Household Electrification
5–6	National Electricity Plan
06	Synchronization of NR with ER-NER-WR
07	765 kV Transmission
10–11	800 kV HVDC bi-pole line

**Table 1.9** Cumulative physical achievements as on 31.12.2005

S.No.	Sources/Systems	Cumulative Achievement
<b>I. Power From Renewables</b>		
<b>A. Grid-interactive renewable power</b>		
1.	Solar Photovoltaic Power	2.74 MW
2.	Wind Power	4433.90 MW
3.	Small Hydro Power (up to 25 MW)	1747.98 MW
4.	Biomass Power	376.53 MW
5.	Bagasse Cogeneration	491.00 MW
6.	Biomass Gasifier	1.00 MW
7.	Energy Recovery from Waste	34.95 MW
	Sub Total (A)	<b>7088.10 MW</b>
<b>B. Distributed renewable power</b>		
8.	Biomass Gasifier	69.87 MW
9.	Energy Recovery from Waste	11.03 MW
	Sub Total (B)	<b>80.90 MW</b>
	<b>Total (A + B)</b>	<b>7169.00 MW</b>

(contd.)

	<b>II. Remote Village Electrification</b>	2195 villages 594 hamlets
	<b>III. Decentralized Energy Systems</b>	
10.	Family Type Biogas Plants	38 lakh
11.	Community/Institutional/Night-soil-based biogas plants	3902 nos.
12.	Improved Chulha	3.52 crore
13.	Solar Photovoltaic Programme	
	i. Solar Street Lighting System	54795 nos.
	ii. Home Lighting System	342607 nos.
	iii. Solar Lantern	538718 nos.
	iv. Solar Power Plants	1566 kW <sub>p</sub>
14	Solar Thermal Programme	
	i. Solar Water Heating Systems	1.5 million sq.m. co
	ii. Box solar cookers	5.99 lakh
	iii. Concentrating dish cookers	2000 nos.
	iv. Community solar cookers	12 nos.
15.	Wind Pumps	1082 nos.
16.	Aero-generator/Hybrid Systems	410 kW
17.	Solar Photovoltaic Pumps	7002 nos.

MW = Megawatt; kW = kilowatt; kW<sub>p</sub> = kilowatt peak; sq.m. = square meter

## Additional Solved Examples

**Example 1.6** If the average flow during the period of interest is  $575 \text{ m}^3/\text{s}$  and head is 100 m, find the power that can be developed per cubic meter per second if the efficiency of the hydraulic turbine and electric generator together is 90%.

*Solution*

$$\begin{aligned} P &= 9.81 \eta \rho W H \times 10^{-6} \text{ MW} \\ &= 9.81 \times 0.9 \times 1000 \times 575 \times 100 \times 10^{-6} \text{ MW} \\ &= 98.1 \times 0.9 \times 5.75 = 508 \text{ MW} \end{aligned}$$

**Example 1.7** How much power can be extracted from a 5 m/s wind striking a wind mill whose blades have a radius of 3 m? Assume that the efficiency of the turbine is 40%.

*Solution*

$$\text{Power} = \frac{1}{2} C_p \rho A V^3$$

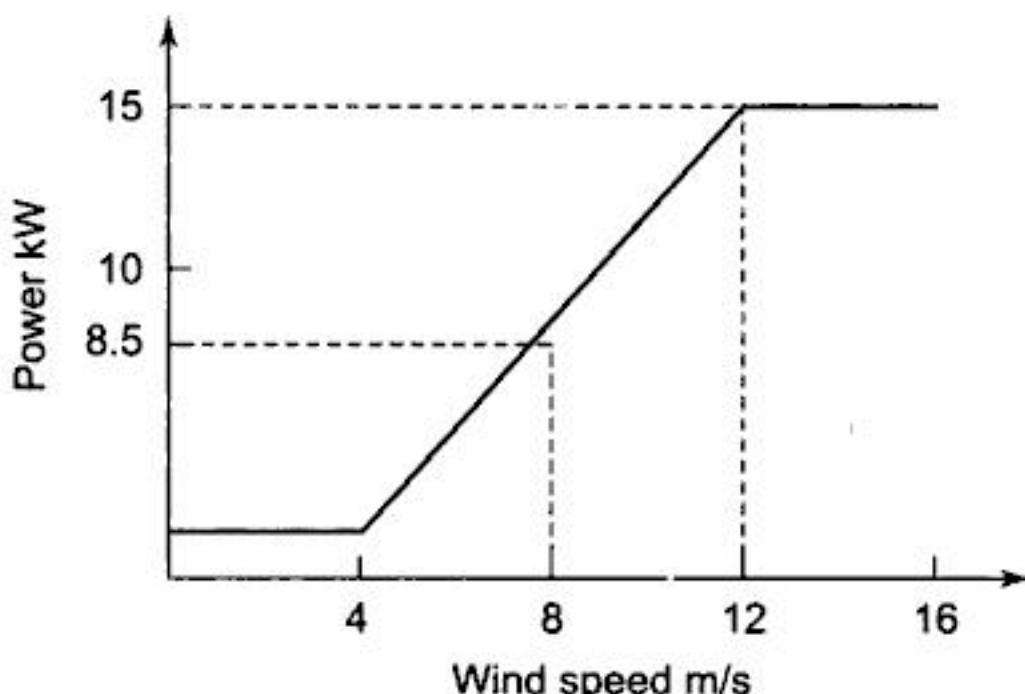
$C_p = 0.4$  = power coefficient gives the maximum amount of wind power that can be converted into mechanical power by wind turbine

$$A = \pi r^2 = 3.14 \times 3^2 = 28.26 \text{ m}^2$$

$$V = 5 \text{ m/s} \quad \rho = 1.24 \text{ kg/m}^3$$

$$\therefore \text{Power} = \frac{1}{2} \times 0.4 \times 1.24 \times 28.26 \times 5^3 \times 10^{-3} = 0.876 \text{ kW}$$

**Example 1.8** A wind generator whose power curve is shown in Fig. 1.39 has a blade diameter of 10 m. Find the net efficiency of the Wind Energy Converting System at a wind speed of 5 m/s.



**Fig. 1.39**

**Solution**

$$A = \pi r^2 = 3.14 \times 5^2$$

$$V = 8 \text{ m/s}$$

$$\rho = 1.24 \text{ kg/m}^3$$

$$P = 1/2 \rho AV^3$$

$$P_{\text{input}} = 1/2 \times 1.24 \times 3.14 \times 5^2 \times 8^3 \times 10^{-3} = 24.9 \text{ kW}$$

8 m/s wind velocity

$$P = 8.5 \text{ kW}$$

$$\eta = 8.5/24.9 \times 100 = 34.14\%$$

**Example 1.9** Calculate the maximum power by a solar cell at an intensity of 200 W/m<sup>2</sup>. Given  $V_{OC} = 0.24$  V,  $I_{SC} = -9$  mA,  $V_{max} = 0.14$  V and  $I_{max} = -6$  mA. Also calculate the cell efficiency if the area is 4 cm<sup>2</sup>.

**Solution**

Solar cell maximum power  $P_{max} = I_{max} V_{max}$

$$P_{max} (\text{output}) = -6 \times 10^{-3} \times 0.14 = -0.84 \text{ mW}$$

$$= -0.84 \times 10^{-3} \text{ W}$$

$$P_{\text{input}} = \text{Intensity} \times \text{area}$$

$$= 200 \times 4 \times 10^{-4} \text{ W}$$

$$\text{cell } \eta = \frac{0.84 \times 10^{-3}}{200 \times 4 \times 10^{-4}} \times 100 = 1.05\%$$

## Problems

- 1.1 Maximum demand of a generating station is 200 MW, a load factor is 70%. The plant capacity factor and plant use factor are 50% and 70%, respectively. Determine
  - (a) daily energy produced
  - (b) installed capacity of plant
  - (c) the reserve capacity of plant
  - (d) maximum energy that can be produced daily if the plant is running all the time
  - (e) the minimum energy that could be produced daily if the plant is running at full load
  - (f) utilization factor
- 1.2 Load factor of a consumer is 40% and the monthly consumption is 500 kWh. If the rate of electricity is Rs 200 per kW of maximum demand plus Rs 2.00 per kWh. Find
  - (a) the monthly bill and the average cost per kWh.
  - (b) the overall cost per kWh if the consumption is increased by 20% with the same load factor.
  - (c) The overall cost per kWh if the consumption remains the same but the load factor.

## References

### Books

1. Nagrath, I.J. and D.P. Kothari, *Electric Machines*, Tata McGraw-Hill, New Delhi, 3rd edn, 2004.
2. Kothari, D. P., R. Ranjan and K.C. Singhal, *Renewable Energy Sources and Technology*, PHI, 2007.
3. Kashkari, C., *Energy Resources, Demand and Conservation with Special Reference to India*, Tata McGraw-Hill, New Delhi, 1975.
4. Kandpal, T. C. and H.P. Garg, *Financial Evaluation of Renewable Technologies*, Macmillian India Ltd, New Delhi, 2003.
5. Sullivan, R.L., *Power System Planning*, McGraw-Hill, New York, 1977.
6. Skrotzki, B.G.A. and W.A. Vopat, *Power Station Engineering and Economy*, McGraw-Hill, New York, 1960.
7. Car, T.H., *Electric Power Stations*, vols I and II, Chapman and Hall, London, 1944.
8. Central Electricity Generating Board, *Modern Power Station Practice*, 2nd edn, Pergamon, 1976.
9. Golding, E.W., *The Generation of Electricity by Wind Power*, Chapman and Hall, London, 1976.
10. El-Wanil, M. M., *Powerplant Technology*, McGraw-Hill, New York, 1984.
11. Bennet, D.J., *The Elements of Nuclear Power*, Longman, 1972.

12. Casazza, J. and F. Delea, *Understanding Electric Power Systems: An Overview of the Technology and Marketplace*, IEEE Press and Wiley Interscience, 2003.
13. Steinberg, M.J. and T.H. Smith, *Economy-loading of Power Plants and Electric Systems*, Wiley, New York, 1943.
14. Power System Planning and Operations: Future Problems and Research Needs, EPRI EL-377-SR, February 1977.
15. Tvidell, J.W. and A.D. Weir, *Renewable Energy Resources*, Tailor & Francis London, 2nd edn, 2006.
16. Mahalanabis, A.K., D.P. Kothari and S.I. Ahson, *Computer Aided Power System Analysis and Control*, Tata McGraw-Hill, New Delhi, 1988.
17. Robert Noyes (Ed.), *Cogeneration of Steam and Electric Power*, Noyes Dali Corp., USA, 1978.
18. Rustebakke, H. M. (Ed.) *Electric Utility Systems and Practices*, 4th edn, Wiley New York, Aug. 1983.
19. CEA 12, *Annual Survey of Power Report*, Aug. 1985; 14th Report, March 1991; 16th Electric Power Survey of India, Sept. 2000.
20. Kothari, D.P. and D.K. Sharma (Eds), *Energy Engineering: Theory and Practice*, S. Chand, 2000.
21. Kothari, D.P. and I.J. Nagrath, *Basic Electrical Engineering*, Tata McGraw-Hill, New Delhi, 2nd edn, 2002. (Ch. 15).
22. Wehenkel, L.A., *Automatic Learning Techniques in Power Systems*, Kluwer, Norwell, MA: 1997.
23. Philipson, L. and H. Lee Willis, *Understanding Electric Utilities and Deregulation*, Marcel Dekker Inc, NY, 1999.
24. Mohan Muna Singhe, *Economics of Power Systems Reliability and Planning*, John Hopkins University Press, 1980.
25. *The Indian Electricity Rules*, 1956, Commercial Law Publishers, Delhi, 2005.
26. Nick Jenkins *et al.*, *Embedded Generation*, IEE, UK, 2000.
27. Ned Mohan, *First Course on Power Systems*, MNPER, Minneapolis, 2006.

## Papers

28. Kusko, A., A Prediction of Power System Development, 1968–2030, *IEEE Spectrum*, April, 1968, 75.
29. Fink, L. and K. Carlsen, *Operating under Stress and Strain*, *IEEE Spectrum*, Mar. 1978.
30. Talukdar, S.N., *et al.*, Methods for Assessing Energy Management Options, *IEEE Trans.*, Jan. 1981, PAS-100, no. 1, 273.
31. Morgen, M.G. and S.N. Talukdar, Electric Power Load Management: Some Technological, Economic, Regularity and Social Issues, *Proc. IEEE*, Feb. 1979, vol. 67, no. 2, 241.
32. Sachdev, M.S., Load Forecasting—Bibliography, *IEEE Trans.*, PAS-96, 1977, 697.
33. Sporn, P., *Our Environment—Options on the Way into the Future*, *ibid*, May 1977, 49.
34. Kothari, D.P., Energy Problems Facing the Third World, *Seminar to the Bio-Physics Workshop*, 8 Oct. 1986, Trieste, Italy.
35. Kothari, D.P., *Energy System Planning and Energy Conservation*, Presented at XXIV National Convention of IIIE, New Delhi, Feb. 1982.

36. Kothari, D.P., et al., Minimization of Air Pollution due to Thermal Plants, *J.I.E.* (India), Feb. 1977, 57 : 65.
37. Kothari, D.P. and J. Nanda, Power Supply Scenario in India 'Retrospects and Prospects', *Proc. NPC Cong., on Captive Power Generation*, New Delhi, Mar. 1986.
38. *National Solar Energy Convention*, Organized by SESI, 1–3 Dec. 1988, Hyderabad.
39. Kothari, D.P., Mini and Micro Hydropower Systems in India, invited chapter in the book, *Energy Resources and Technology*, Scientific Publishers, 1992, pp. 147–158.
40. *Power Line*, vol. 5. no. 9, June 2001.
41. United Nations: *Electricity Costs and Tariffs: A General Study*, 1972.
42. Shikha, T.S. Bhatti and D.P. Kothari, Wind as an Eco-friendly Energy Source to meet the Electricity Needs of SAARC Region", *Proc. Int. Conf. (ICME 2001)*, BUET, Dhaka, Bangladesh, Dec. 2001, pp. 11–16.
43. Bansal, R.C., D.P. Kothari and T.S. Bhatti, On Some of the Design Aspects of Wind Energy Conversion Systems", *Int. J. Energy Conversion and Management*, vol. 43, 16, Nov. 2002, pp. 2175–2187.
44. Kothari, D.P. and A. Arora, Fuel Cells in Transportation-Beyond Batteries, *Proc. Nat. Conf. on Transportation Systems*, IIT Delhi, April 2002, pp. 173–176.
45. Saxena, Anshu and D.P. Kothari, et al., "Analysis of Multimedia and Hypermedia for Computer Simulation and Growth., EJEISA, UK, vol. 3, 1 Sept. 2001, pp. 14–28.
46. Bansal, R.C., T.S. Bhatti and D.P. Kothari, A Bibliographical Survey on Induction Generators for Application of Non-conventional Energy Systems', *IEEE Trans. on Energy Conversions*, vol. 18, Sept. 2003, pp. 433–439.
47. Kolhe, M., J.C. Joshi and D.P. Kothari, 'LOLP of stand-alone solar PV system', *Int. Journal of Energy Technology and Policy*, vol. 1, no. 3, 2003, pp. 315–323.
48. Shikha, T.S. Bhatti and D.P. Kothari, Vertical Axis Wind Rotor with Concentration by Convergent Nozzles, *Wind Engg.*, vol. 27, no. 6, 2003, pp. 555–559.
49. Shikha, T.S. Bhatti and D.P. Kothari, On Some Aspects of Technological Development of Wind Turbines, *Energy Engg.*, vol. 129, no. 3, Dec. 2003, pp. 69–80.
50. Shikha, T.S. Bhatti, and D.P. Kothari, 'Wind Energy Conversion Systems as a Distributed Source of Generation', *Energy Engg.*, vol. 129, no. 3, Dec. 2003, pp. 81–95.
51. Shikha, T.S. Bhatti and D.P. Kothari, 'Indian Scenario of Wind Energy: Problems and Solutions', *Energy Sources*, vol. 26, no. 9, July 2004, pp. 811–819.
52. Kolhe, M., J.C. Joshi and D.P. Kothari, 'Performance Analysis of a Directly Coupled PV Water Pumping Systems', *IEEE Trans., on Energy Conversion*, vol. 19, no. 3, Sept. 2004, pp. 613–618.
53. Shikha, T.S. Bhatti and D.P. Kothari, 'The Power Coefficient of Windmills in Ideal Conditions', *Int. J. of Global Energy Issues*, vol. 21, no. 3, 2004, pp. 236–242.

54. Bansal, R.C., T.S. Bhatti and D.P. Kothari, 'Automatic Reactive Power Control of Isolated Wind-Diesel Hybrid Power Systems for Variable Wind Speed/Slip', *Electric Power Components and Systems*, vol. 32, 2004, pp. 901–912.
55. Shikha, T.S. Bhatti, and D.P. Kothari, 'Wind Energy in India: Shifting Paradigms and Challenges Ahead', *Energy Engg.*, vol. 130, no. 3, Dec. 2004. pp. 67–80.
56. Shikha, T.S. Bhatti and D.P. Kothari, 'New Horizons for Offshore Wind Energy: Shifting Paradigms and Challenges', *Energy Sources*, vol. 27, no. 4, March 2005, pp. 349–360.
57. Shikha, T.S. Bhatti and D.P. Kothari, 'Development of Vertical and Horizontal Axis Wind Turbines: A Review,' *Int J. of Wind Engg.*, vol. 29, no. 3, May 2005, pp. 287–300.
58. Shikha, T.S. Bhatti and D.P. Kothari, 'A Review of Wind Resource Assessment Technology', *Energy Engg.*, vol. 132, no. 1, April 2006, pp. 8–14.
59. Kaushik, S.C., S. Ramesh and D.P. Kothari, 'Energy Conservation Studies in Buildings,' *Presented at PCRA Conf*, New Delhi, May 19, 2005.
60. Goyal, H., T.S. Bhatti and D.P. Kothari, 'A Novel Technique proposed for automatic control of small Hydro Power Plants,' *Special Issue of Int. J. of Global Energy Issues*, vol. 24, 2005, pp. 29–46.
61. Bansal, R.C., T.S. Bhatti, D.P. Kothari and S. Bhat, 'Reactive Power Control of Wind-diesel-microhydro Power Systems Using Matlab/Simulink', *Int. J. of Global Energy Issues*, vol. 24, no. 1, 2005, pp. 86–99.
62. Goyal, H., M. Hanmandlu and D.P. Kothari, 'A New Optimal Flow Control Approach for Automatic Control of Small Hydro Power Plants', *J.I.E. (I)*, vol. 87, May 2006, pp. 1–5.

U.S. Department of Energy ([www.eia.doe.gov](http://www.eia.doe.gov)).  
[www.epa.gov/globalwarming/kids/greenhouse.html](http://www.epa.gov/globalwarming/kids/greenhouse.html).  
2004 Environment Report, [www.xcellenergy.com](http://www.xcellenergy.com).

## Chapter 2

# Inductance and Resistance of Transmission Lines

### 2.1 INTRODUCTION

The four parameters which affect the performance of a transmission line as an element of a power system are inductance, capacitance, resistance and conductance. Shunt conductance, which is normally due to leakage over line insulators, is almost always neglected in overhead transmission lines. This chapter deals with the series line parameters, i.e. inductance and resistance. These parameters are uniformly distributed along the line and they together form the series impedance of the line.

Inductance is by far the most dominant line parameter from a power system engineer's viewpoint. As we shall see in later chapters, it is the inductive reactance which limits the transmission capacity of a line.

### 2.2 DEFINITION OF INDUCTANCE

Voltage induced in a circuit is given by

$$e = \frac{d\psi}{dt} \text{ V} \quad (2.1)$$

where  $\psi$  represents the flux linkages of the circuit in weber-turns (Wb-T). This can be written in the form

$$e = \frac{d\psi}{di} \cdot \frac{di}{dt} = L \frac{di}{dt} \text{ V} \quad (2.2)$$

where  $L = \frac{d\psi}{di}$  is defined as the inductance of the circuit in henry, which in general may be a function of  $i$ . In a linear magnetic circuit, i.e., a circuit with constant permeability, flux linkages vary linearly with current such that the inductance is constant given by

$$L = \frac{\psi}{i} \text{ H}$$

or

$$\psi = Li \text{ Wb-T} \quad (2.3)$$

If the current is alternating, the above equation can be written as

$$\lambda = LI \quad (2.4)$$

where  $\lambda$  and  $I$  are the rms values of flux linkages and current respectively. These are of course in phase.

Replacing  $\frac{d}{dt}$  in Eq. (2.1) by  $j\omega$ , we get the steady state AC voltage drop due to alternating flux linkages as

$$V = j\omega LI = j\omega\lambda \text{ V} \quad (2.5)$$

On similar lines, the mutual inductance between two circuits is defined as the flux linkages of one circuit due to current in another, i.e.,

$$M_{12} = \frac{\lambda_{12}}{I_2} \text{ H} \quad (2.6)$$

The voltage drop in circuit 1 due to current in circuit 2 is

$$V_1 = j\omega M_{12} I_2 = j\omega\lambda_{12} \text{ V} \quad (2.7)$$

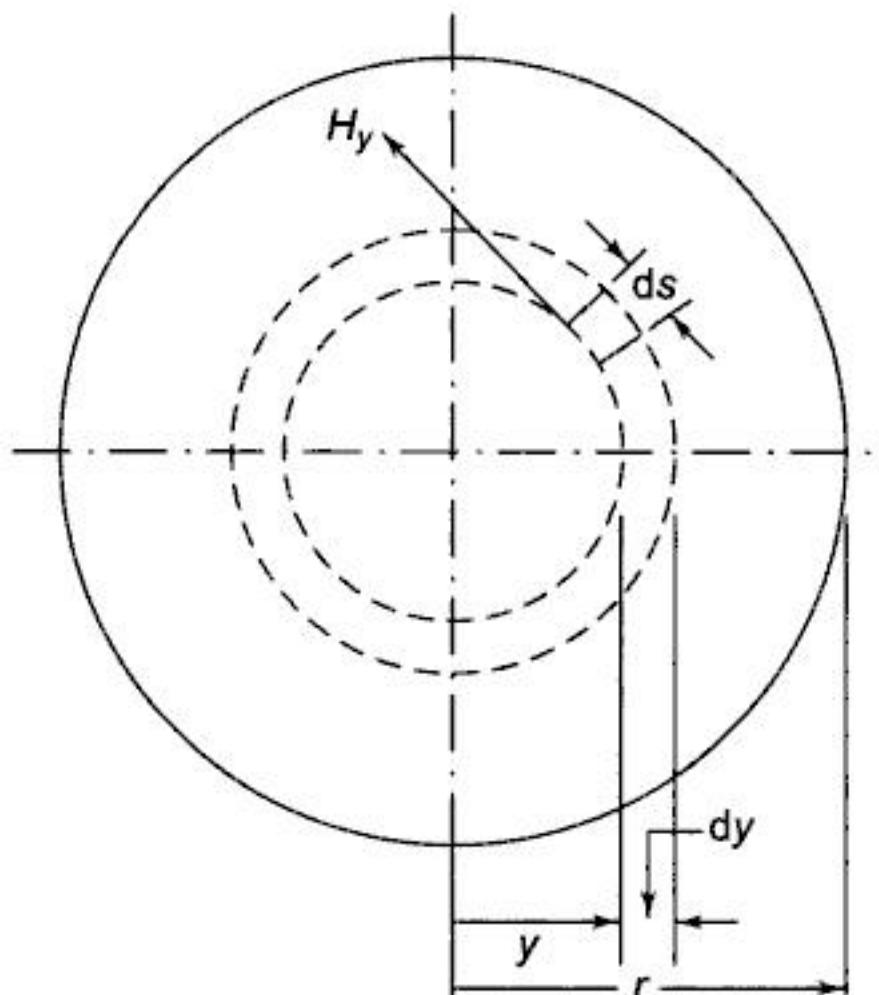
The concept of mutual inductance is required while considering the coupling between parallel lines and the influence of power lines on telephone lines.

### **2.3 FLUX LINKAGES OF AN ISOLATED CURRENT-CARRYING CONDUCTOR**

Transmission lines are composed of parallel conductors which, for all practical purposes, can be considered as infinitely long. Let us first develop expressions for flux linkages of a long isolated current-carrying cylindrical conductor with return path lying at infinity. This system forms a single-turn circuit, flux linking which is in the form of circular lines concentric to the conductor. The total flux can be divided into two parts, that which is internal to the conductor and the flux external to the conductor. Such a division is helpful as the internal flux progressively links a smaller amount of current as we proceed inwards towards the centre of the conductor, while the external flux always links the total current inside the conductor.

#### **Flux Linkages due to Internal Flux**

Figure 2.1 shows the cross-sectional view of a long cylindrical conductor carrying current  $I$ .



**Fig. 2.1** Flux linkages due to internal flux (cross-sectional view)

The mmf round a concentric closed circular path of radius  $y$  internal to the conductor as shown in the figure is

$$\oint H_y \cdot ds I_y \quad (\text{Ampere's law}) \quad (2.8)$$

where

$H_y$  = magnetic field intensity (AT/m)

$I_y$  = current enclosed (A)

By symmetry,  $H_y$  is constant and is in the direction of  $ds$  all along the circular path. Therefore, from Eq. (2.8), we have

$$2\pi y H_y = I_y \quad (2.9)$$

Assuming uniform current density\*

$$I_y = \left( \frac{\pi y^2}{\pi r^2} \right) I = \left( \frac{y^2}{r^2} \right) I \quad (2.10)$$

From Eqs (2.9) and (2.10), we obtain

$$H_y = \frac{yI}{2\pi r^2} \text{ AT/m} \quad (2.11)$$

The flux density  $B_y$ ,  $y$  metres from the centre of the conductors, is

$$B_y = \mu H_y = \frac{\mu y I}{2\pi r^2} \text{ Wb/m}^2 \quad (2.12)$$

where  $\mu$  is the permeability of the conductor.

\* For power frequency of 50 Hz, it is quite reasonable to assume uniform current density. The effect of non-uniform current density is considered later in this chapter while treating resistance.

Consider now an infinitesimal tubular element of thickness  $dy$  and length 1m. The flux in the tubular element  $d\phi = B_y dy$  webers links the fractional turn ( $I_y/I = y^2/r^2$ ) resulting in flux linkages of

$$d\lambda = \left( \frac{y^2}{r^2} \right) d\phi = \left( \frac{y^2}{r^2} \right) \frac{\mu y I}{2\pi r^2} dy \quad \text{Wb-T/m} \quad (2.13)$$

Integrating, we get the total internal flux linkages as

$$\lambda_{int} = \int_0^r \frac{\mu I}{2\pi r^4} y^3 dy = \frac{\mu I}{8\pi} \quad \text{Wb-T/m} \quad (2.14)$$

For a relative permeability  $\mu_r = 1$  (non-magnetic conductor),  $\mu = 4\pi \times 10^{-7}$  H/m; therefore

$$\lambda_{int} = \frac{I}{2} \times 10^{-7} \quad \text{Wb-T/m} \quad (2.15)$$

and

$$L_{int} = \frac{1}{2} \times 10^{-7} \quad \text{H/m} \quad (2.16)$$

### Flux Linkages due to Flux between Two Points External to Conductor

Figure 2.2 shows two points  $P_1$  and  $P_2$  at distances  $D_1$  and  $D_2$  from a conductor which carries a current of  $I$  amperes. As the conductor is far removed from the return current path, the magnetic field external to the conductor is concentric circles around the conductor and therefore all the flux between  $P_1$  and  $P_2$  lies within the concentric cylindrical surfaces passing through  $P_1$  and  $P_2$ .

Magnetic field intensity at distance  $y$  from the conductor is

$$H_y = \frac{I}{2\pi y} \quad \text{AT/m}$$

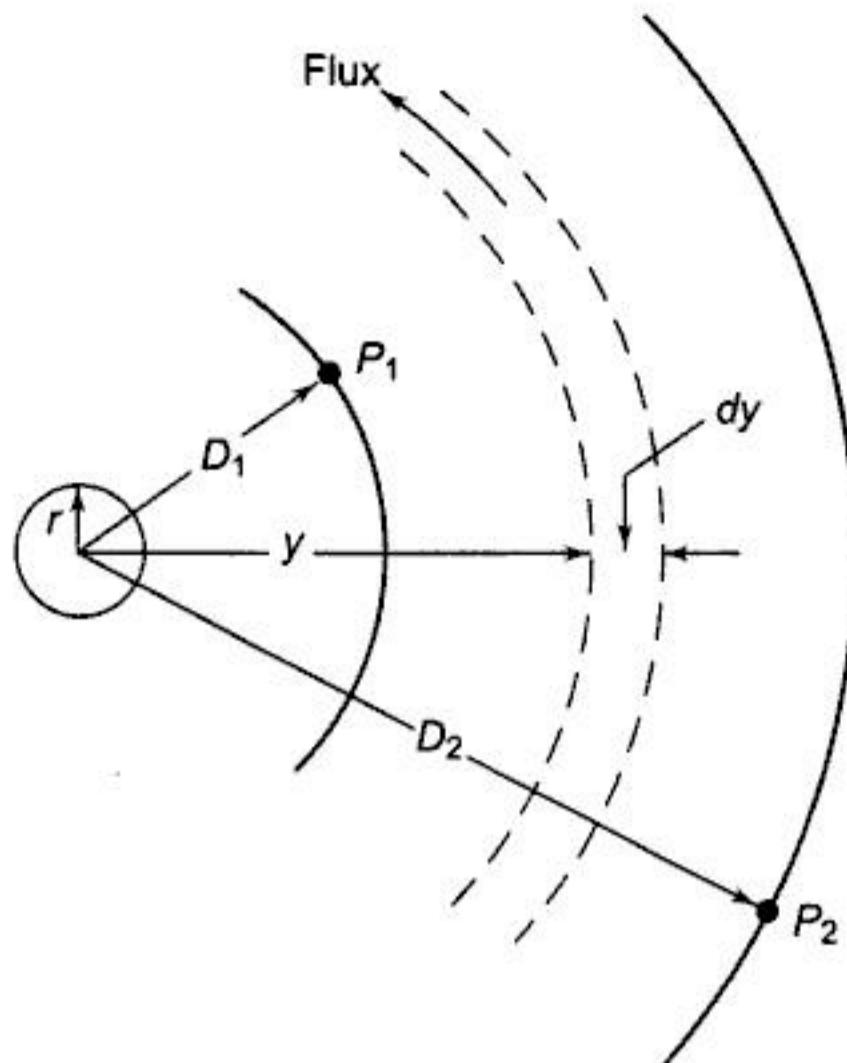
The flux  $d\phi$  contained in the tubular element of thickness  $dy$  is

$$d\phi = \frac{\mu I}{2\pi y} dy \quad \text{Wb/m length of conductor}$$

The flux  $d\phi$  being external to the conductor links all the current in the conductor which together with the return conductor at infinity forms a single return, such that its flux linkages are given by

$$d\lambda = 1 \times d\phi = \frac{\mu I}{2\pi y} dy$$

Therefore, the total flux linkages of the conductor due to flux between points  $P_1$  and  $P_2$  is



**Fig. 2.2** Flux linkages due to flux between external points  $P_1, P_2$

$$\lambda_{12} = \int_{D_1}^{D_2} \frac{\mu I}{2\pi y} dy = \frac{\mu}{2\pi} I \ln \frac{D_2}{D_1} \text{ Wb-T/m}$$

where  $\ln$  stands for natural logarithm\*.

Since  $\mu_r = 1$ ,  $\mu = 4\pi \times 10^{-7}$

$$\therefore \lambda_{12} = 2 \times 10^{-7} I \ln \frac{D_2}{D_1} \text{ Wb/m} \quad (2.17)$$

The inductance of the conductor contributed by the flux included between points  $P_1$  and  $P_2$  is then

$$L_{12} = 2 \times 10^{-7} \ln \frac{D_2}{D_1} \text{ H/m} \quad (2.18)$$

or

$$L_{12} = 0.461 \log \frac{D_2}{D_1} \text{ mH/km} \quad (2.19)$$

### Flux Linkages due to Flux up to an External Point

Let the external point be at distance  $D$  from the centre of the conductor. Flux linkages of the conductor due to external flux (from the surface of the conductor up to the external point) is obtained from Eq. (2.17) by substituting  $D_1 = r$  and  $D_2 = D$ , i.e.,

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\* Throughout the book  $\ln$  denotes natural logarithm (base  $e$ ), while  $\log$  denotes logarithm to base 10.

$$\lambda_{\text{ext}} = 2 \times 10^{-7} I \ln \frac{D}{r} \quad (2.20)$$

Total flux linkages of the conductor due to internal and external flux are

$$\begin{aligned}\lambda &= \lambda_{\text{int}} + \lambda_{\text{ext}} \\ &= \frac{I}{2} \times 10^{-7} + 2 \times 10^{-7} I \ln \frac{D}{r} \\ &= 2 \times 10^{-7} I \left( \frac{1}{4} + \ln \frac{D}{r} \right) \\ &= 2 \times 10^{-7} I \ln \frac{D}{re^{-1/4}}\end{aligned}$$

Let

$$r' = re^{-1/4} = 0.7788r$$

$$\therefore \lambda = 2 \times 10^{-7} I \ln \frac{D}{r'} \text{ Wb-T/m} \quad (2.21a)$$

Inductance of the conductor due to flux upto an external point is therefore

$$L = 2 \times 10^{-7} \ln \frac{D}{r'} \text{ H/m} \quad (2.21b)$$

Here  $r'$  can be regarded as the radius of a fictitious conductor with no internal inductance but the same total inductance as the actual conductor.

## 2.4 INDUCTANCE OF A SINGLE-PHASE TWO-WIRE LINE

Consider a simple two-wire line composed of solid round conductors carrying currents  $I_1$  and  $I_2$  as shown in Fig. 2.3. In a single-phase line,

$$I_1 + I_2 = 0$$

or

$$I_2 = -I_1$$

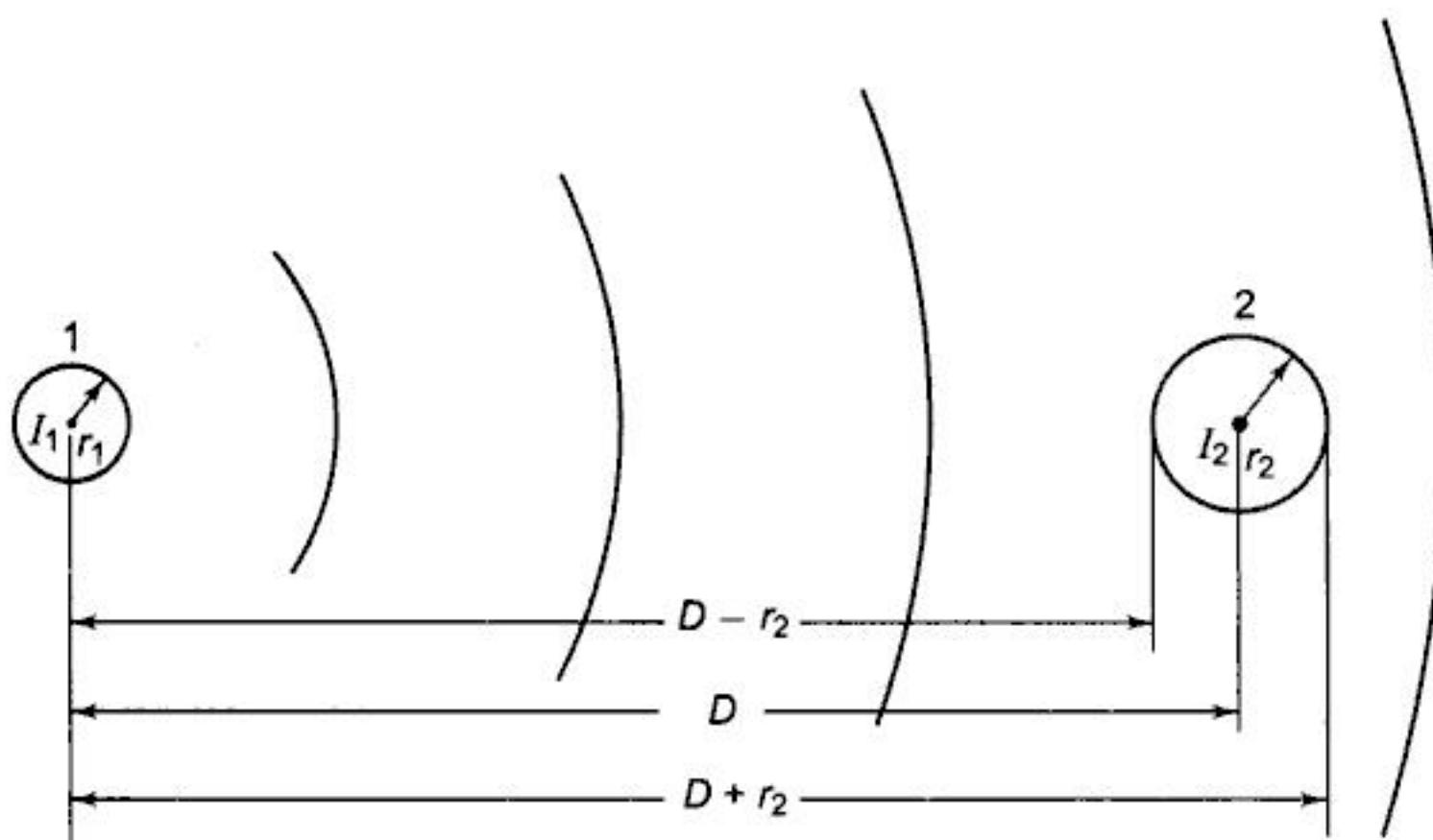
It is important to note that the effect of earth's presence on magnetic field geometry\* is insignificant. This is so because the relative permeability of earth is about the same as that of air and its electrical conductivity is relatively small.

To start with, let us consider the flux linkages of the circuit caused by current in conductor 1 only. We make three observations in regard to these flux linkages:

1. External flux from  $r_1$  to  $(D - r_2)$  links all the current  $I_1$  in conductor 1.

---

\* The electric field geometry will, however, be very much affected as we shall see later while dealing with capacitance.



**Fig.2.3** Single-phase two-wire line and the magnetic field due to current in conductor 1 only

2. External flux from  $(D - r_2)$  to  $(D + r_2)$  links a current whose magnitude progressively reduces from  $I_1$  to zero along this distance, because of the effect of negative current flowing in conductor 2.
3. Flux beyond  $(D + r_2)$  links a net current of zero.

For calculating the total inductance due to current in conductor 1, a simplifying assumption will now be made. If  $D$  is much greater than  $r_1$  and  $r_2$  (which is normally the case for overhead lines), it can be assumed that the flux from  $(D - r_2)$  to the centre of conductor 2 links all the current  $I_1$  and the flux from the centre of conductor 2 to  $(D + r_2)$  links zero current\*.

Based on the above assumption, the flux linkages of the circuit caused by current in conductor 1 as per Eq. (2.21a) are

$$\lambda_1 = 2 \times 10^{-7} I_1 \ln \frac{D}{r'_1} \quad (2.22a)$$

The inductance of the conductor due to current in conductor 1 only is then

$$L_1 = 2 \times 10^{-7} \ln \frac{D}{r'_1} \quad (2.22b)$$

Similarly, the inductance of the circuit due to current in conductor 2 is

$$L_2 = 2 \times 10^{-7} \ln \frac{D}{r'_2} \quad (2.23)$$

Using the superposition theorem, the flux linkages and likewise the inductances of the circuit caused by current in each conductor considered separately may be added to obtain the total circuit inductance. Therefore, for the complete circuit

\* Kimbark [10] has shown that the results based on this assumption are fairly accurate even when  $D$  is not much larger than  $r_1$  and  $r_2$ .

$$L = L_1 + L_2 = 4 \times 10^{-7} \ln \frac{D}{\sqrt{r'_1 r'_2}} \text{ H/m} \quad (2.24)$$

If  $r'_1 = r'_2 = r'$ ; then

$$L = 4 \times 10^{-7} \ln D/r' \text{ H/m} \quad (2.25a)$$

$$L = 0.921 \log D/r' \text{ mH/km} \quad (2.25b)$$

Transmission lines are infinitely long compared to  $D$  in practical situations and therefore the end effects in the above derivation have been neglected.

## 2.5 CONDUCTOR TYPES

So far we have considered transmission lines consisting of single solid cylindrical conductors for forward and return paths. To provide the necessary flexibility for stringing, conductors used in practice are always stranded except for very small cross-sectional areas. Stranded conductors are composed of strands of wire, electrically in parallel, with alternate layers spiralled in opposite direction to prevent unwinding. The total number of strands ( $N$ ) in concentrically stranded cables with total annular space filled with strands of uniform diameter ( $d$ ) is given by

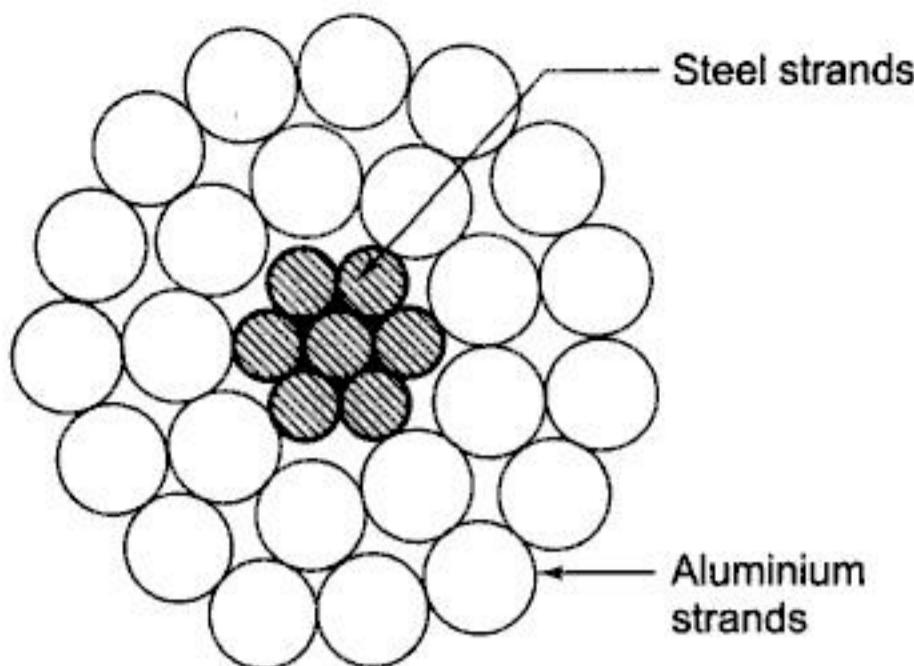
$$N = 3x^2 - 3x + 1 \quad (2.26a)$$

where  $x$  is the number of layers wherein the single central strand is counted as the first layer. The overall diameter ( $D$ ) of a stranded conductor is

$$D = (2x - 1)d \quad (2.26b)$$

Aluminium is now the most commonly employed conductor material. It has the advantages of being cheaper and lighter than copper though with less conductivity and tensile strength. Low density and low conductivity result in larger overall conductor diameter, which offers another incidental advantage in high voltage lines. Increased diameter results in reduced electrical stress at conductor surface for a given voltage so that the line is *corona free*. The low tensile strength of aluminium conductors is made up by providing central strands of high tensile strength steel. Such a conductor is known as aluminium conductor steel reinforced (ACSR) and is most commonly used in overhead transmission lines. Figure 2.4 shows the cross-sectional view of an ACSR conductor with 24 strands of aluminium and 7 strands of steel.

In extra high voltage (EHV) transmission line, *expanded* ACSR conductors are used. These are provided with paper or hessian between various layers of strands so as to increase the overall conductor diameter in an attempt to reduce electrical stress at conductor surface and prevent corona. The most effective way of constructing *corona-free* EHV lines (Ch.19) is to provide several conductors per phase in suitable geometrical configuration. These are known as *bundled conductors* and are a common practice now for EHV lines.



**Fig. 2.4** Cross-sectional view of ACSR-7 steel strands, 24 aluminium strands

## 2.6 FLUX LINKAGES OF ONE CONDUCTOR IN A GROUP

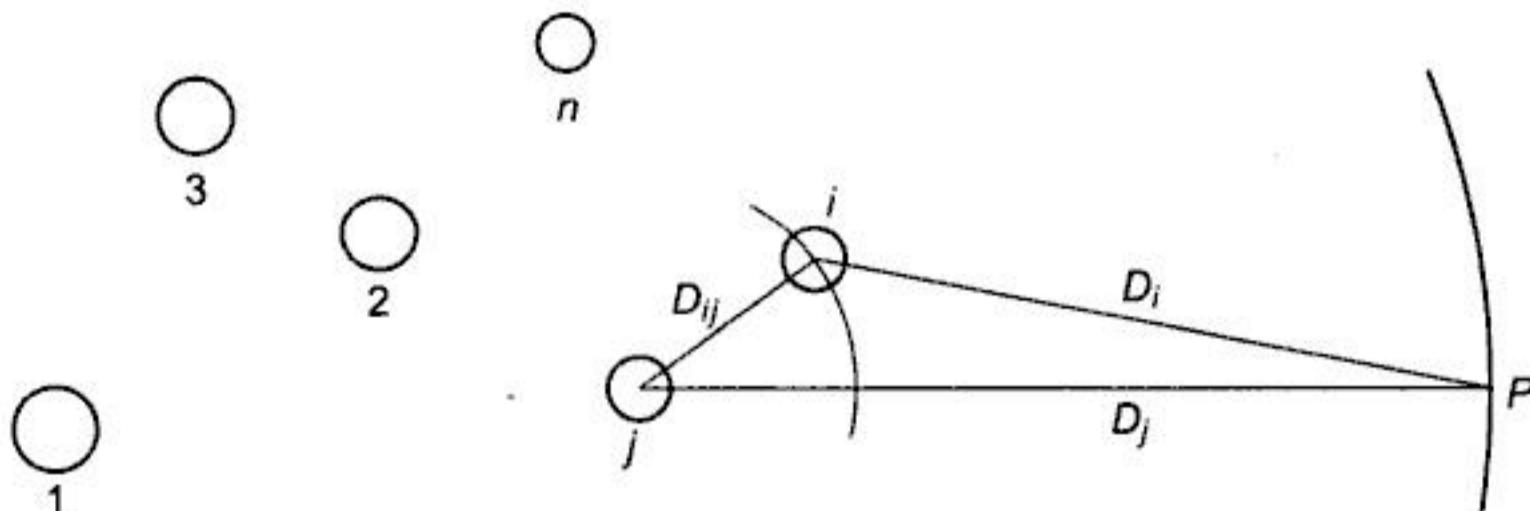
As shown in Fig. 2.5, consider a group of  $n$  parallel round conductors carrying phasor currents  $I_1, I_2, \dots, I_n$  whose sum equals zero. Distances of these conductors from a remote point  $P$  are indicated as  $D_1, D_2, \dots, D_n$ . Let us obtain an expression for the total flux linkages of the  $i$ th conductor of the group considering flux upto the point  $P$  only.

The flux linkages of  $i$ th conductor due to its own current  $I_i$  (self linkages) are given by [see Eq. (2.21)]

$$\lambda_{ii} = 2 \times 10^{-7} I_i \ln \frac{D_i}{r'_i} \text{ Wb-T/m} \quad (2.27)$$

The flux linkages of conductor  $i$  due to current in conductor  $j$  [refer to Eq. (2.17)] is

$$\lambda_{ij} = 2 \times 10^{-7} I_j \ln \frac{D_j}{D_{ij}} \text{ Wb-T/m} \quad (2.28)$$



**Fig. 2.5** Arbitrary group of  $n$  parallel round conductors carrying currents

where  $D_{ij}$  is the distance of  $i$ th conductor from  $j$ th conductor carrying current  $I_j$ . From Eq. (2.27) and by repeated use of Eq. (2.28), the total flux linkages of conductor  $i$  due to flux upto point  $P$  are

$$\lambda_i = \lambda_{i1} + \lambda_{i2} + \dots + \lambda_{in} + \dots + \lambda_{in}$$

$$= 2 \times 10^{-7} \left( I_1 \ln \frac{D_1}{D_{i1}} + I_2 \ln \frac{D_2}{D_{i2}} + \dots + I_i \ln \frac{D_i}{r'_i} + \dots + I_n \ln \frac{D_n}{D_{in}} \right)$$

The above equation can be reorganized as

$$\lambda_i = 2 \times 10^{-7} \left[ \left( I_1 \ln \frac{1}{D_{i1}} + I_2 \ln \frac{1}{D_{i2}} + \dots + I_i \ln \frac{1}{r'_i} + \dots + I_n \ln \frac{1}{D_{in}} \right) \right.$$

$$\left. + (I_1 \ln D_1 + I_2 \ln D_2 + \dots + I_i \ln D_i + \dots + I_n \ln D_n) \right] \quad (2.29)$$

But,  $I_n = -(I_1 + I_2 + \dots + I_{n-1})$ .

Substituting for  $I_n$  in the second term of Eq. (2.29) and simplifying, we have

$$\lambda_i = 2 \times 10^{-7} \left[ \left( I_1 \ln \frac{1}{D_{i1}} + I_2 \ln \frac{1}{D_{i2}} + \dots + I_i \ln \frac{1}{r'_i} + \dots + I_n \ln \frac{1}{D_{in}} \right) \right.$$

$$\left. + \left( I_1 \ln \frac{D_1}{D_n} + I_2 \ln \frac{D_2}{D_n} + \dots + I_i \ln \frac{D_i}{D_n} + \dots + I_{n-1} \ln \frac{D_{n-1}}{D_n} \right) \right]$$

In order to account for total flux linkages of conductor  $i$ , let the point  $P$  now recede to infinity. The terms such as  $\ln D_1/D_n$ , etc. approach  $\ln 1 = 0$ . Also for the sake of symmetry, denoting  $r'_i$  as  $D_{ii}$ , we have

$$\lambda_i = 2 \times 10^{-7} \left( I_1 \ln \frac{1}{D_{i1}} + I_2 \ln \frac{1}{D_{i2}} + \dots + I_i \ln \frac{1}{D_{ii}} \right.$$

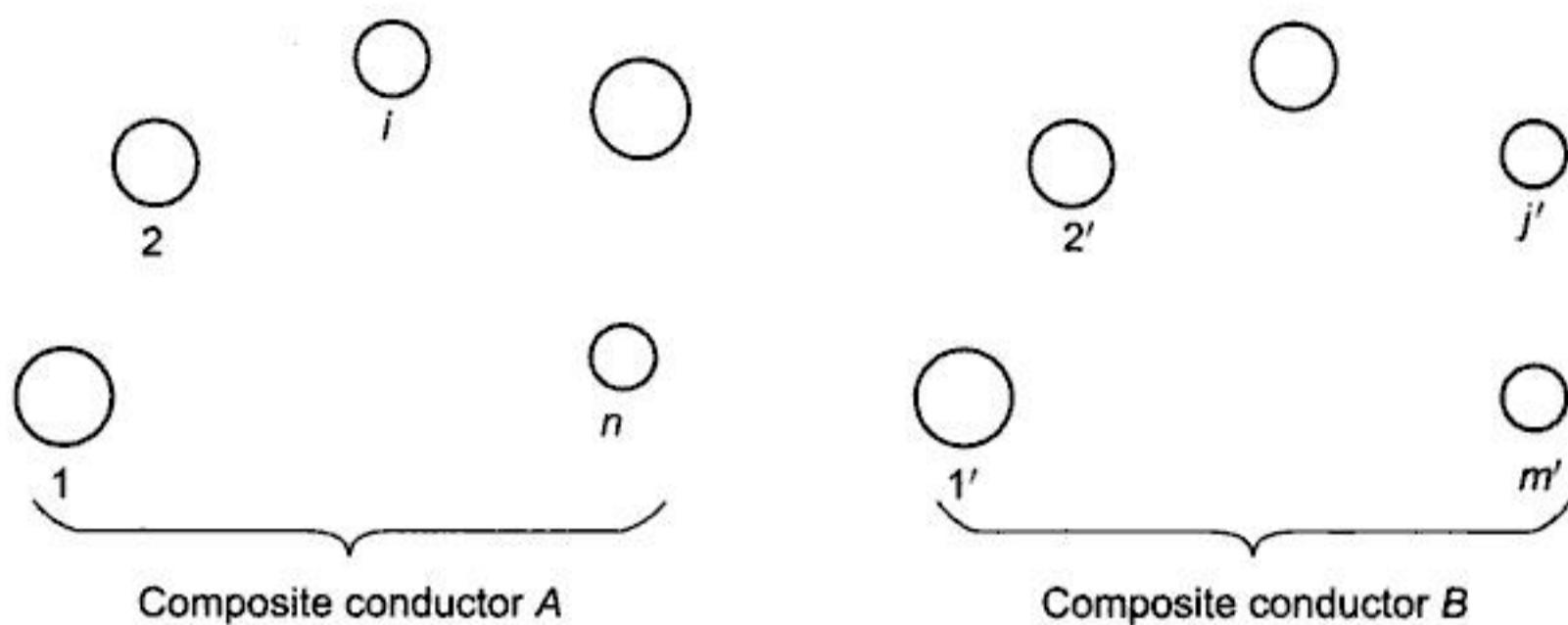
$$\left. + \dots + I_n \ln \frac{1}{D_{in}} \right) \text{ Wb-T/m} \quad (2.30)$$

## 2.7 INDUCTANCE OF COMPOSITE CONDUCTOR LINES

We are now ready to study the inductance of transmission lines composed of composite conductors. Figure 2.6 shows such a single-phase line comprising composite conductors  $A$  and  $B$  with  $A$  having  $n$  parallel filaments and  $B$  having  $m'$  parallel filaments. Though the inductance of each filament will be somewhat different (their resistances will be equal if conductor diameters are chosen to be uniform), it is sufficiently accurate to assume that the current is equally divided among the filaments of each composite conductor. Thus, each filament of  $A$  is taken to carry a current  $I/n$ , while each filament of conductor  $B$  carries the return current of  $-I/m'$ .

Applying Eq. (2.30) to filament  $i$  of conductor  $A$ , we obtain its flux linkages as

$$\lambda_i = 2 \times 10^{-7} \frac{I}{n} \left( \ln \frac{1}{D_{i1}} + \ln \frac{1}{D_{i2}} + \dots + \ln \frac{1}{D_{ii}} + \dots + \ln \frac{1}{D_{in}} \right)$$



**Fig. 2.6** Single-phase line consisting of two composite conductors

$$\begin{aligned} & -2 \times 10^{-7} \frac{I}{m'} \left( \ln \frac{1}{D_{i1'}} + \ln \frac{1}{D_{i2'}} + \dots + \ln \frac{1}{D_{im'}} \right) \\ & = 2 \times 10^{-7} I \ln \frac{(D_{i1'} D_{i2'} \dots D_{im'})^{1/m'}}{(D_{i1} D_{i2} \dots D_{in})^{1/n}} \text{ Wb-T/m} \end{aligned}$$

The inductance of filament  $i$  is then

$$L_i = \frac{\lambda_i}{I/n} = 2n \times 10^{-7} \ln \frac{(D_{i1'} \dots D_{ij'} \dots D_{im'})^{1/m'}}{(D_{i1} \dots D_{ii} \dots D_{in})^{1/n}} \text{ H/m} \quad (2.31)$$

The average inductance of the filaments of composite conductor  $A$  is

$$L_{\text{avg}} = \frac{L_1 + L_2 + L_3 + \dots + L_n}{n}$$

Since conductor  $A$  is composed of  $n$  filaments electrically in parallel, its inductance is

$$L_A = \frac{L_{\text{avg}}}{n} = \frac{L_1 + L_2 + \dots + L_n}{n^2} \quad (2.32)$$

Using the expression for filament inductance from Eq. (2.31) in Eq. (2.32), we obtain

$$L_A = 2 \times 10^{-7} \ln \frac{[(D_{11'} \dots D_{1j'} \dots D_{1m'}) \dots (D_{i1'} \dots D_{ij'} \dots D_{im'}) \dots (D_{nl'} \dots D_{nj'} \dots D_{nm'})]^{1/m'n}}{[(D_{11} \dots D_{1j} \dots D_{1n}) \dots (D_{il} \dots D_{ii} \dots D_{in}) \dots (D_{nl} \dots D_{ni} \dots D_{nn})]^{1/n^2}} \text{ H/m} \quad (2.33)$$

The numerator of the argument of the logarithm in Eq. (2.33) is the  $m'$ nth root of the  $m'n$  terms, which are the products of all possible mutual distances from the  $n$  filaments of conductor  $A$  to  $m'$  filaments of conductor  $B$ . It is called *mutual geometric mean distance* (mutual GMD) between conductors  $A$  and  $B$  and is abbreviated as  $D_m$ . Similarly, the denominator of the argument of the logarithm in Eq. (2.33) is the  $n^2$ th root of  $n^2$  product terms ( $n$  sets of  $n$  product terms each). Each set of  $n$  product term pertains to a filament and consists of  $r'(D_{ii})$  for that filament and  $(n - 1)$  distances from that filament to every other filament in conductor  $A$ . The denominator is defined as the *self geometric mean distance* (self GMD) of conductor  $A$ , and is abbreviated as  $D_{sA}$ . Sometimes, self GMD is also called *geometric mean radius* (GMR).

In terms of the above symbols, we can write Eq. (2.33) as

$$L_A = 2 \times 10^{-7} \ln \frac{D_m}{D_{sA}} \text{ H/m} \quad (2.34a)$$

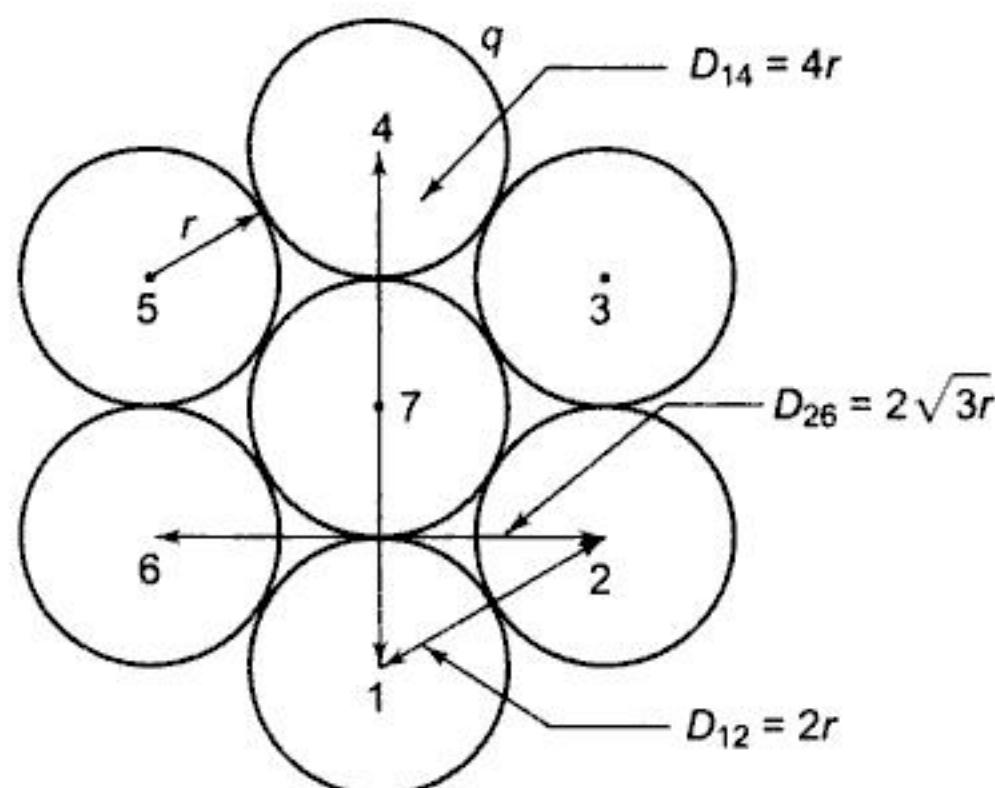
$$= 0.461 \log \frac{D_m}{D_{sA}} \text{ mH/km} \quad (2.34b)$$

Note the similarity of the above relation with Eq. (2.22b), which gives the inductance of one conductor of a single-phase line for the special case of two solid, round conductors. In Eq. (2.22b)  $r_1'$  is the self GMD of a single conductor and  $D$  is the mutual GMD of two single conductors.

The inductance of the composite conductor  $B$  is determined in a similar manner, and the total inductance of the line is

$$L = L_A + L_B \quad (2.35)$$

**Example 2.1** A conductor is composed of seven identical copper strands, each having a radius  $r$ , as shown in Fig. 2.7. Find the self GMD of the conductor.



**Fig. 2.7** Cross-section of a seven-strand conductor

**Solution**

The self GMD of the seven strand conductor is the 49th root of the 49 distances. Thus

$$D_s = ((r')^7(D_{12}^2 D_{26}^2 D_{14} D_{17})^6(2r)^6)^{1/49}$$

Substituting the values of various distances,

$$D_s = ((0.7788r)^7(2^2 r^2 \times 3 \times 2^2 r^2 \times 2^2 r \times 2r \times 2r)^6)^{1/49}$$

or

$$D_s = \frac{2r(3(0.7788))^{1/7}}{6^{1/49}} = 2.177r$$

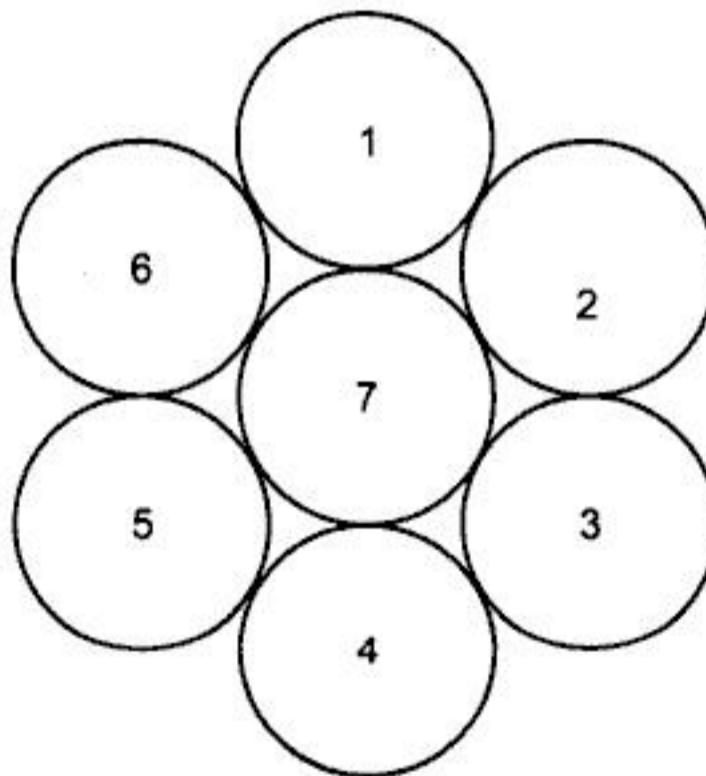
**Example 2.2** The outside diameter of the single layer of aluminium strands of an ACSR conductor shown in Fig. 2.8 is 5.04 cm. The diameter of each strand is 1.68 cm. Determine the 50 Hz reactance at 1 m spacing; neglect the effect of the central strand of steel and advance reasons for the same.

**Solution**

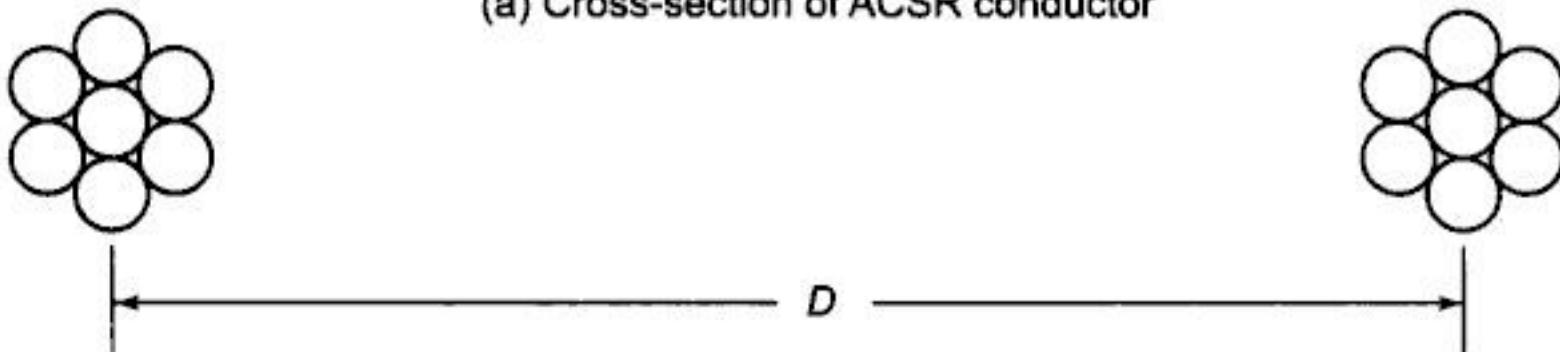
The conductivity of steel being much poorer than that of aluminium and the internal inductance of steel strands being  $\mu$ -times that of aluminium strands, the current conducted by the central strands of steel can be assumed to be zero.

$$\text{Diameter of steel strand} = 5.04 - 2 \times 1.68 = 1.68 \text{ cm}$$

Thus, all strands are of the same diameter, say  $d$ . For the arrangement of strands as given in Fig. 2.8(a),



(a) Cross-section of ACSR conductor



(b) Line composed of two ACSR conductors

**Fig. 2.8**

$$D_{12} = D_{16} = d$$

$$D_{13} = D_{15} = \sqrt{3}d$$

$$D_{14} = 2d$$

$$D_s = \left( \left[ \left( \frac{d'}{2} \right) d^2 (\sqrt{3}d)^2 (\sqrt{2}d) \right]^6 \right)^{1/36}$$

Substituting  $d' = 0.7788d$  and simplifying

$$D_s = 1.155d = 1.155 \times 1.68 = 1.93 \text{ cm}$$

$$D_m \approx D \quad \text{since } D \gg d$$

Now, the inductance of each conductor is

$$L = 0.461 \log \frac{100}{1.93} = 0.789 \text{ mH/km}$$

$$\text{Loop inductance} = 2 \times 0.789 = 1.578 \text{ mH/km}$$

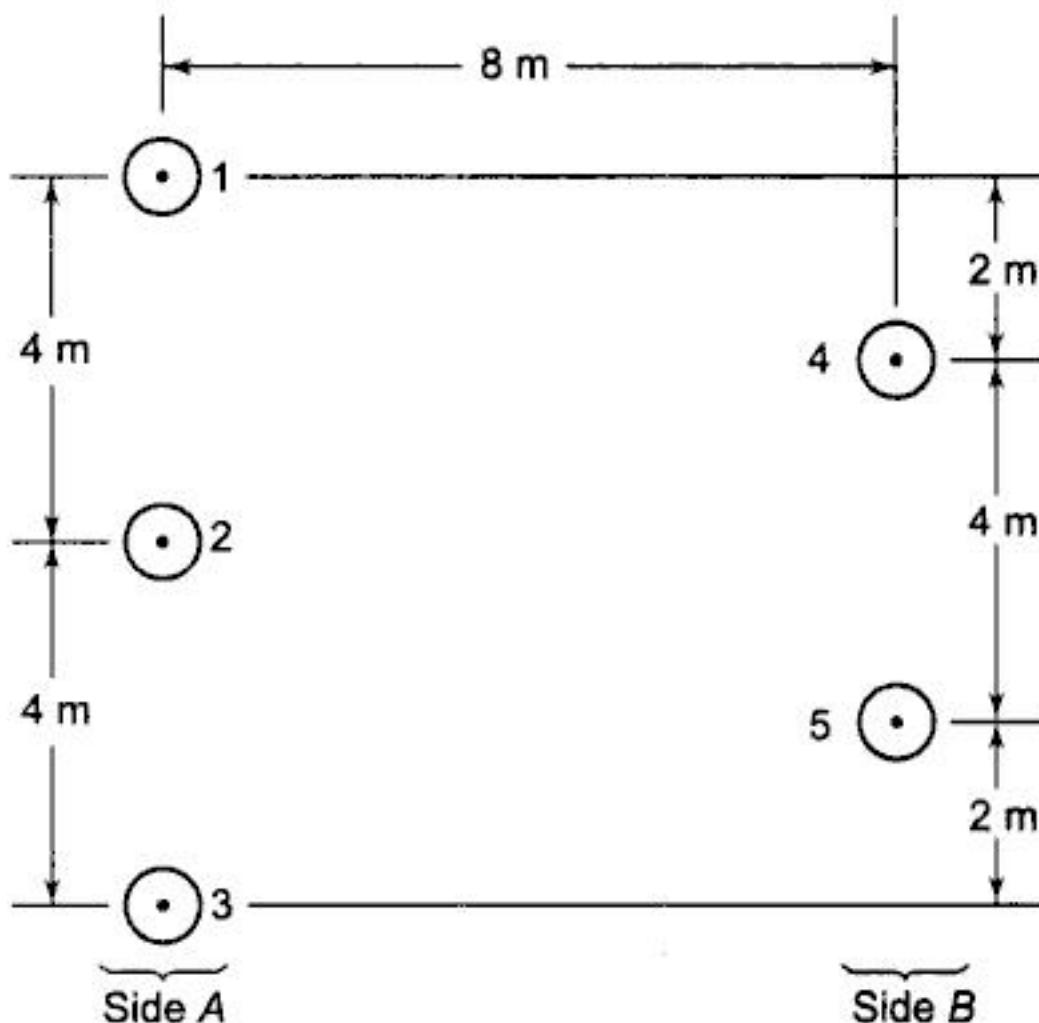
$$\text{Loop reactance} = 1.578 \times 314 \times 10^{-3} = 0.495 \text{ ohms/km}$$

**Example 2.3** The arrangement of conductors of a single-phase transmission line is shown in Fig. 2.9, wherein the forward circuit is composed of three solid wires 2.5 mm in radius and the return circuit of two-wires of radius 5 mm placed symmetrically with respect to the forward circuit. Find the inductance of each side of the line and that of the complete line.

**Solution**

The mutual GMD between sides *A* and *B* is

$$D_m = ((D_{14}D_{15}) (D_{24}D_{25}) (D_{34}D_{35}))^{1/6}$$



**Fig. 2.9** Arrangement of conductors for Example 2.3

From the figure it is obvious that

$$D_{14} = D_{24} = D_{25} = D_{35} = \sqrt{68} \text{ m}$$

$$D_{15} = D_{34} = 10 \text{ m}$$

$$D_m = (68^2 \times 100)^{1/6} = 8.8 \text{ m}$$

The self GMD for side A is

$$D_{sA} = ((D_{11}D_{12}D_{13})(D_{21}D_{22}D_{23})(D_{31}D_{32}D_{33}))^{1/9}$$

Here,

$$D_{11} = D_{22} = D_{33} = 2.5 \times 10^{-3} \times 0.7788 \text{ m}$$

Substituting the values of various interdistances and self distances in  $D_{sA}$ , we get

$$\begin{aligned} D_{sA} &= ((2.5 \times 10^{-3} \times 0.7788)^3 \times 4^4 \times 8^2)^{1/9} \\ &= 0.367 \text{ m} \end{aligned}$$

Similarly,

$$\begin{aligned} D_{sB} &= ((5 \times 10^{-3} \times 0.7788)^2 \times 4^2)^{1/4} \\ &= 0.125 \text{ m} \end{aligned}$$

Substituting the values of  $D_m$ ,  $D_{sA}$  and  $D_{sB}$  in Eq. (2.25b), we get the various inductances as

$$L_A = 0.461 \log \frac{8.8}{0.367} = 0.635 \text{ mH/km}$$

$$L_B = 0.461 \log \frac{8.8}{0.125} = 0.85 \text{ mH/km}$$

$$L = L_A + L_B = 1.485 \text{ mH/km}$$

If the conductors in this problem are each composed of seven identical strands as in Example 2.1, the problem can be solved by writing the conductor self distances as

$$D_{ii} = 2.177r_i$$

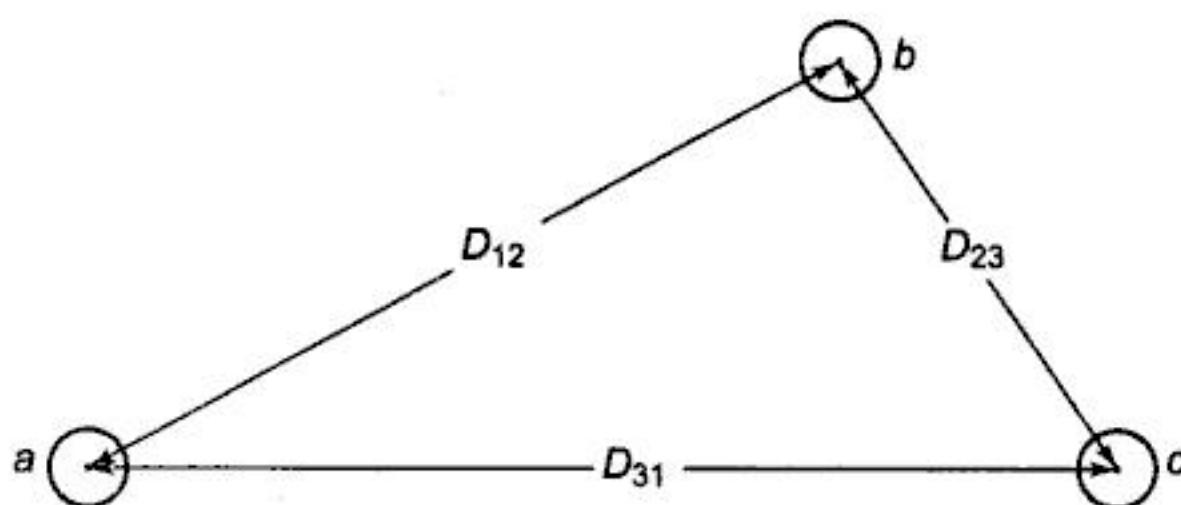
where  $r_i$  is the strand radius.

## 2.8 INDUCTANCE OF THREE-PHASE LINES

So far we have considered only single-phase lines. The basic equations developed can, however, be easily adapted to the calculation of the inductance of three-phase lines. Figure 2.10 shows the conductors of a three-phase line with unsymmetrical spacing.

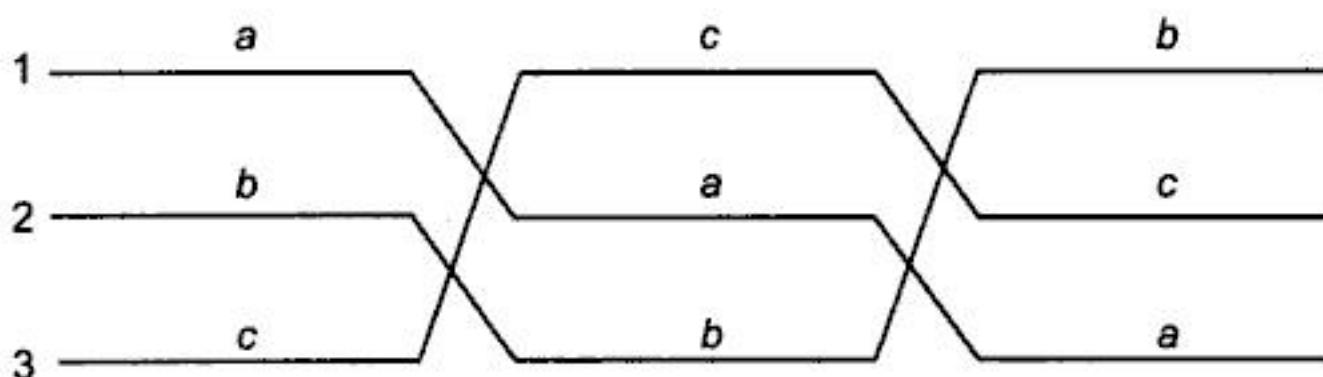
Assume that there is no neutral wire, so that

$$I_a + I_b + I_c = 0$$



**Fig. 2.10** Cross-sectional view of a three-phase line with unsymmetrical spacing

Unsymmetrical spacing causes the flux linkages and therefore the inductance of each phase to be different resulting in unbalanced receiving-end voltages even when sending-end voltages and line currents are balanced. Also voltages will be induced in adjacent communication lines even when line currents are balanced. This problem is tackled by exchanging the positions of the conductors at regular intervals along the line such that each conductor occupies the original position of every other conductor over an equal distance. Such an exchange of conductor positions is called *transposition*. A complete transposition cycle is shown in Fig. 2.11. This arrangement causes each conductor to have the same average inductance over the transposition cycle. Over the length of one transposition cycle, the total flux linkages and hence net voltage induced in a nearby telephone line is zero.



**Fig. 2.11** A complete transposition cycle

To find the average inductance of each conductor of a transposed line, the flux linkages of the conductor are found for each position it occupies in the transposed cycle. Applying Eq. (2.30) to conductor *a* of Fig. 2.11, for section 1 of the transposition cycle wherein *a* is in position 1, *b* is in position 2 and *c* is in position 3, we get

$$\lambda_{a1} = 2 \times 10^{-7} \left( I_a \ln \frac{1}{r_a'} + I_b \ln \frac{1}{D_{12}} + I_c \ln \frac{1}{D_{31}} \right) \text{ Wb-T/m}$$

For the second section

$$\lambda_{a2} = 2 \times 10^{-7} \left( I_a \ln \frac{1}{r_a'} + I_b \ln \frac{1}{D_{23}} + I_c \ln \frac{1}{D_{12}} \right) \text{ Wb-T/m}$$

For the third section

$$\lambda_{a3} = 2 \times 10^{-7} \left( I_a \ln \frac{1}{r_a} + I_b \ln \frac{1}{D_{13}} + I_c \ln \frac{1}{D_{23}} \right) \text{ Wb-T/m}$$

Average flux linkages of conductor  $a$  are

$$\begin{aligned} \lambda_a &= \frac{\lambda_{a1} + \lambda_{a2} + \lambda_{a3}}{3} \\ &= 2 \times 10^{-7} \left( I_a \ln \frac{1}{r_a} + I_b \ln \frac{1}{(D_{12}D_{23}D_{31})^{1/3}} + I_c \ln \frac{1}{(D_{12}D_{23}D_{31})^{1/3}} \right) \end{aligned}$$

But,  $I_b + I_c = -I_a$ ; hence

$$\lambda_a = 2 \times 10^{-7} I_a \ln \frac{(D_{12}D_{23}D_{31})^{1/3}}{r_a}$$

Let

$$D_{eq} = (D_{12}D_{23}D_{31})^{1/3} = \text{equivalent equilateral spacing}$$

Then

$$L_a = 2 \times 10^{-7} \ln \frac{D_{eq}}{r_a} = 2 \times 10^{-7} \ln \frac{D_{eq}}{r_a} \text{ H/m} \quad (2.36)$$

This is the same relation as Eq. (2.34a) where  $D_m = D_{eq}$ , the mutual GMD between the three-phase conductors. If  $r_a = r_b = r_c$ , we have

$$L_a = L_b = L_c$$

It is not the present practice to transpose the power lines at regular intervals. However, an interchange in the position of the conductors is made at switching stations to balance the inductance of the phases. For all practical purposes the dissymmetry can be neglected and the inductance of an untransposed line can be taken equal to that of a transposed line.

If the spacing is equilateral, then

$$D_{eq} = D$$

and

$$L_a = 2 \times 10^{-7} \ln \frac{D}{r_a} \text{ H/m} \quad (2.37)$$

If  $r_a = r_b = r_c$ , it follows from Eq. (2.37) that

$$L_a = L_b = L_c$$

**Example 2.4** Show that over the length of one transposition cycle of a power line, the total flux linkages of a nearby telephone line are zero, for balanced three-phase currents.



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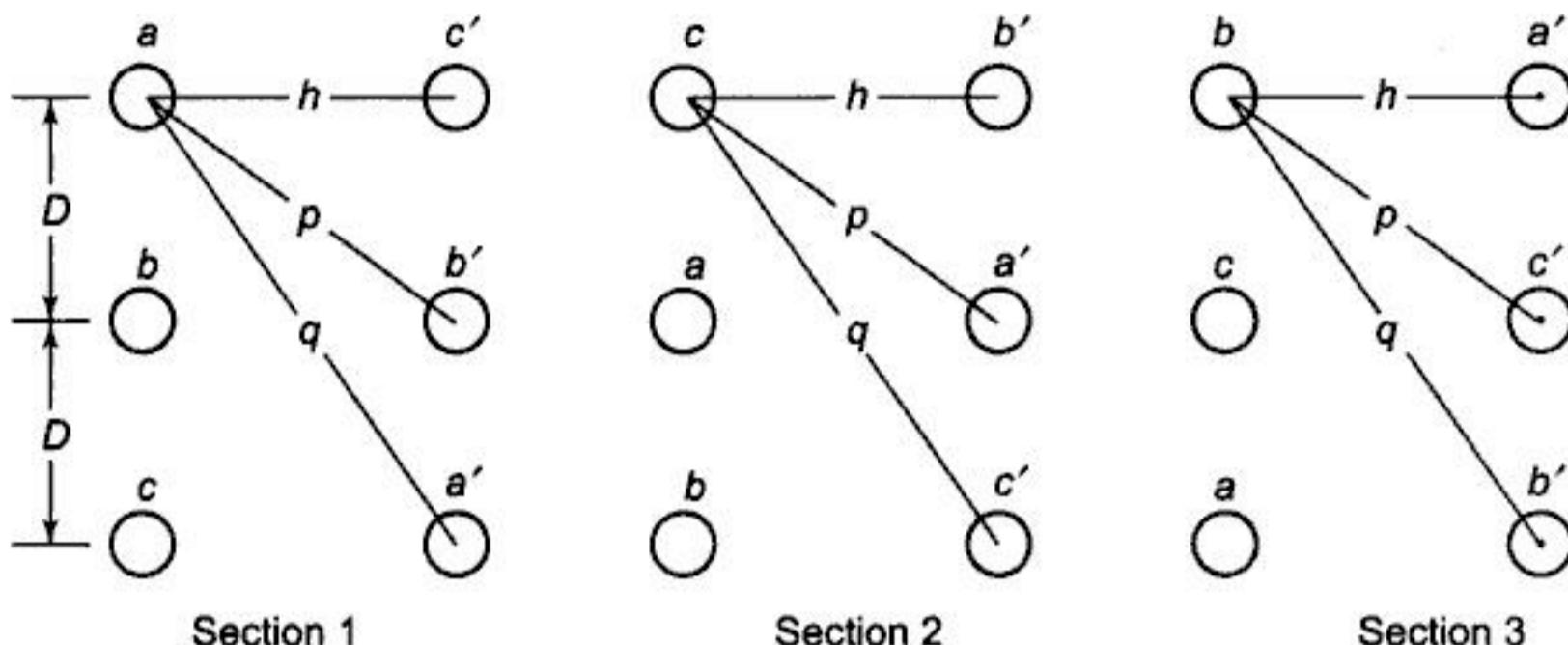


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Figure 2.15 shows the three sections of the transposition cycle of two parallel circuit three-phase lines with vertical spacing (it is a very commonly used configuration).



**Fig. 2.15**

It may be noted here that conductors  $a$  and  $a'$  in parallel compose phase  $a$  and similarly  $b$  and  $b'$  compose phase  $b$  and  $c$  and  $c'$  compose phase  $c$ . In order to achieve high  $D_s$ , the conductors of two phases are placed diametrically opposite to each other and those of the third phase are horizontally opposite to each other. (The reader can try other configurations to verify that these will lead to low  $D_s$ ). Applying the method of GMD, the equivalent equilateral spacing is

$$D_{eq} = (D_{ab} D_{bc} D_{ca})^{1/3} \quad (2.42)$$

where  $D_{ab}$  = mutual GMD between phases  $a$  and  $b$  in section 1 of the transposition cycle  
 $= (D_p D_p)^{1/4} = (D_p)^{1/2}$

$D_{bc}$  = mutual GMD between phases  $b$  and  $c$  in section 1 of the transposition cycle  
 $= (D_p)^{1/2}$

$D_{ca}$  = mutual GMD between phases  $c$  and  $a$  in section 1 of the transposition cycle  
 $= (2D_h)^{1/2}$

$$\text{Hence } D_{eq} = 2^{1/6} D^{1/2} p^{1/3} h^{1/6} \quad (2.43)$$

It may be noted here that  $D_{eq}$  remains the same in each section of the transposition cycle, as the conductors of each parallel circuit rotate cyclically, so do  $D_{ab}$ ,  $D_{bc}$  and  $D_{ca}$ . The reader is advised to verify this for sections 2 and 3 of the transposition cycle in Fig. 2.15.

Self GMD in section 1 of phase  $a$  (i.e., conductors  $a$  and  $a'$ ) is

$$D_{sa} = (r'q r'q)^{1/4} = (r'q)^{1/2}$$

Self GMD of phases  $b$  and  $c$  in section 1 are respectively

$$D_{sb} = (r'hr'h)^{1/4} = (r'h)^{1/2}$$

$$D_{sc} = (r'qr'q)^{1/4} = (r'q)^{1/2}$$

$$\therefore \text{Equivalent self GMD, } D_s = (D_{sa}D_{sb}D_{sc})^{1/3}$$

$$= (r')^{1/2}q^{1/3}h^{1/6} \quad (2.44)$$

Because of the cyclic rotation of conductors of each parallel circuit over the transposition cycle,  $D_s$  also remains the same in each transposition section. The reader should verify this for sections 2 and 3 in Fig. 2.15.

The inductance per phase is

$$L = 2 \times 10^{-7} \ln \frac{D_{eq}}{D_s}$$

$$= 2 \times 10^{-7} \ln \frac{2^{1/6} D^{1/2} p^{1/3} h^{1/6}}{(r')^{1/2} q^{1/3} h^{1/6}}$$

$$= 2 \times 10^{-7} \ln \left( 2^{1/6} \left( \frac{D}{r'} \right)^{1/2} \left( \frac{p}{q} \right)^{1/3} \right) \text{H/phase/m} \quad (2.45)$$

The self inductance of each circuit is given by

$$L_s = 2 \times 10^{-7} \ln \frac{(2)^{1/3} D}{r'}$$

Equation (2.45) can now be written as

$$L = \frac{1}{2} \left[ 2 \times 10^{-7} \ln \frac{(2)^{1/3} D}{r'} + 2 \times 10^{-7} \ln \left( \frac{p}{q} \right)^{2/3} \right] \quad (2.46)$$

$$= \frac{1}{2} (L_s + M)$$

where  $M$  is the mutual inductance between the two circuits, i.e.

$$M = 2 \times 10^{-7} \ln \left( \frac{p}{q} \right)^{2/3}$$

This is a well known result for the two coupled circuits connected in parallel (at similar polarity ends).

If  $h \gg D$ ,  $\left( \frac{p}{q} \right) \rightarrow 1$  and  $M \rightarrow 0$ , i.e. the mutual impedance between the circuits becomes zero. Under this condition,

$$L = 1 \times 10^{-7} \ln \frac{3\sqrt{2} D}{r'} \quad (2.47)$$



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For the example in hand, the equivalent line will have  $d = 7$  m and conductor diameter (for same total cross-sectional area) as  $\sqrt{2} \times 1.725$  cm.

$$X_L = 314 \times 0.461 \times 10^{-3} \log \frac{(7 \times 7 \times 14)^{1/3}}{0.7788 \times \sqrt{2} \times 1.725 \times 10^{-3}}$$

$$= 0.531 \text{ ohm/km}$$

This is 76.41% higher than the corresponding value for a bundled conductor line. As already pointed out, lower reactance of a bundled conductor line increases its transmission capacity.

## 2.11 RESISTANCE

Though the contribution of line resistance to series line impedance can be neglected in most cases, it is the main source of line power loss. Thus while considering transmission line economy, the presence of line resistance must be considered.

The effective AC resistance is given by

$$R = \frac{\text{average power loss in conductor in watts}}{I^2} \text{ ohms} \quad (2.48)$$

where  $I$  is the rms current in the conductor in amperes. Ohmic or DC resistance is given by the formula

$$R_O = \frac{\rho l}{A} \text{ ohms} \quad (2.49)$$

where  $\rho$  = resistivity of the conductor, ohm-m

$l$  = length, m

$A$  = cross-sectional area,  $\text{m}^2$

The effective resistance given by Eq. (2.48) is equal to the DC resistance of the conductor given by Eq. (2.49) only if the current distribution is uniform throughout the conductor.

For small changes in temperature, the resistance increases with temperature in accordance with the relationship

$$R_t = R (1 + \alpha_0 t) \quad (2.50)$$

where  $R$  = resistance at temperature  $0^\circ\text{C}$

$\alpha_0$  = temperature coefficient of the conductor at  $0^\circ\text{C}$

Equation (2.50) can be used to find the resistance  $R_{t_2}$  at a temperature  $t_2$ , if resistance  $R_{t_1}$  at temperature  $t_1$  is known

$$\frac{R_{t_2}}{R_{t_1}} = \frac{1/\alpha_0 + t_2}{1/\alpha_0 + t_1} \quad (2.51)$$

## 2.12 SKIN EFFECT AND PROXIMITY EFFECT

The distribution of current throughout the cross-section of a conductor is uniform only when DC is passing through it. On the contrary when AC is flowing through a conductor, the current is non-uniformly distributed over the cross-section in a manner that the current density is higher at the surface of the conductor compared to the current density at its centre. This effect becomes more pronounced as frequency is increased. This phenomenon is called *skin effect*. It causes larger power loss for a given rms AC than the loss when the same value of DC is flowing through the conductor. Consequently, the effective conductor resistance is more for AC than for DC. A qualitative explanation of the phenomenon is given below.

Imagine a solid round conductor (a round shape is considered for convenience only) to be composed of annular filaments of equal cross-sectional area. The flux linking the filaments progressively decreases as we move towards the outer filaments for the simple reason that the flux inside a filament does not link it. The inductive reactance of the imaginary filaments therefore decreases outwards with the result that the outer filaments conduct more AC than the inner filaments (filaments being parallel). With the increase of frequency the non-uniformity of inductive reactance of the filaments becomes more pronounced, so also the non-uniformity of current distribution. For large solid conductors the skin effect is quite significant even at 50 Hz. The analytical study of skin effect requires the use of Bessel's functions and is beyond the scope of this book.

Apart from the skin effect, non-uniformity of current distribution is also caused by *proximity effect*. Consider a two-wire line as shown in Fig. 2.18. Each line conductor can be divided into sections of equal cross-sectional area (say three sections). Pairs  $aa'$ ,  $bb'$  and  $cc'$  can form three loops in parallel. The flux linking loop  $aa'$  (and therefore its inductance) is the least and it increases somewhat for loops  $bb'$  and  $cc'$ . Thus the density of AC flowing through the conductors is highest at the inner edges ( $aa'$ ) of the conductors and is the least at the outer edges ( $cc'$ ). This type of non-uniform AC current distribution becomes more pronounced as the distance between conductors is reduced. Like skin effect, the non-uniformity of current distribution caused by proximity effect also increases the effective conductor resistance. For normal spacing of overhead lines, this effect is always of negligible order. However, for underground cables where conductors are located close to each other, proximity effect causes an appreciable increase in effective conductor resistance.



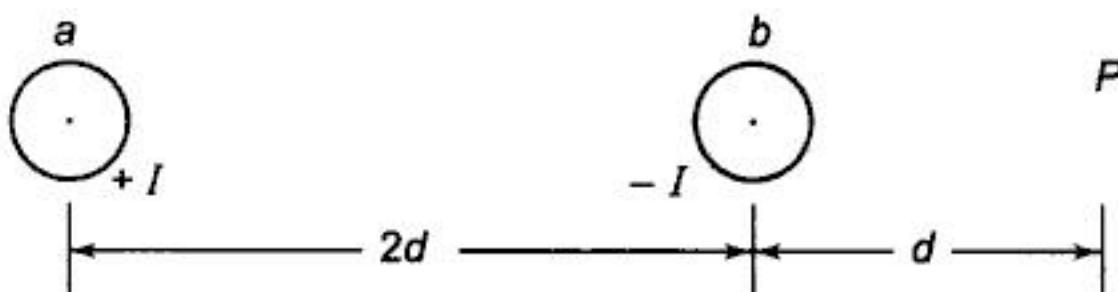
**Fig. 2.18**



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each sheath has a mean diameter of 7.5 cm, estimate the longitudinal voltage induced per km of sheath when the circuit carries a current of 800 A.

- 2.5 Two long parallel conductors carry currents of  $+I$  and  $-I$ . What is the magnetic field intensity at a point  $P$ , shown in Fig. P-2.5?

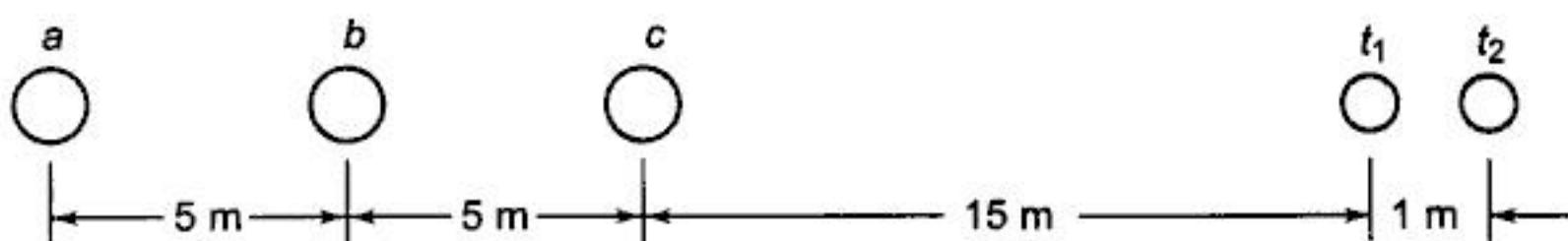


**Fig. P-2.5**

- 2.6 Two three-phase lines connected in parallel have self-reactances of  $X_1$  and  $X_2$ . If the mutual reactance between them is  $X_{12}$ , what is the effective reactance between the two ends of the line?

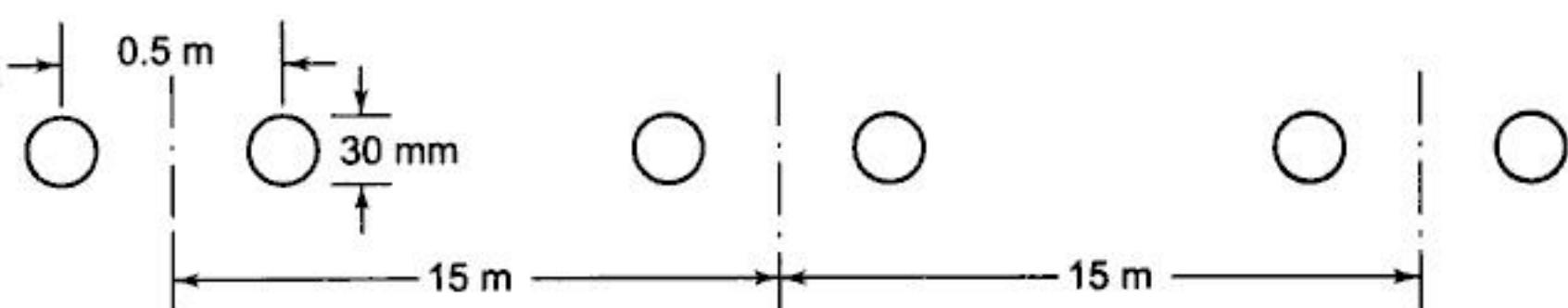
- 2.7 A single-phase 50 Hz power line is supported on a horizontal cross-arm. The spacing between conductors is 2.5 m. A telephone line is also supported on a horizontal cross-arm in the same horizontal plane as the power line. The conductors of the telephone line are of solid copper spaced 0.6 m between centres. The distance between the nearest conductors of the two lines is 20 m. Find the mutual inductance between the circuits and the voltage per kilometre induced in the telephone line for 150 A current flowing over the power line.

- 2.8 A telephone line runs parallel to an untransposed three-phase transmission line, as shown in Fig. P-2.8. The power line carries balanced current of 400 A per phase. Find the mutual inductance between the circuits and calculate the 50 Hz voltage induced in the telephone line per km.



**Fig. P-2.8** Telephone line parallel to a power line

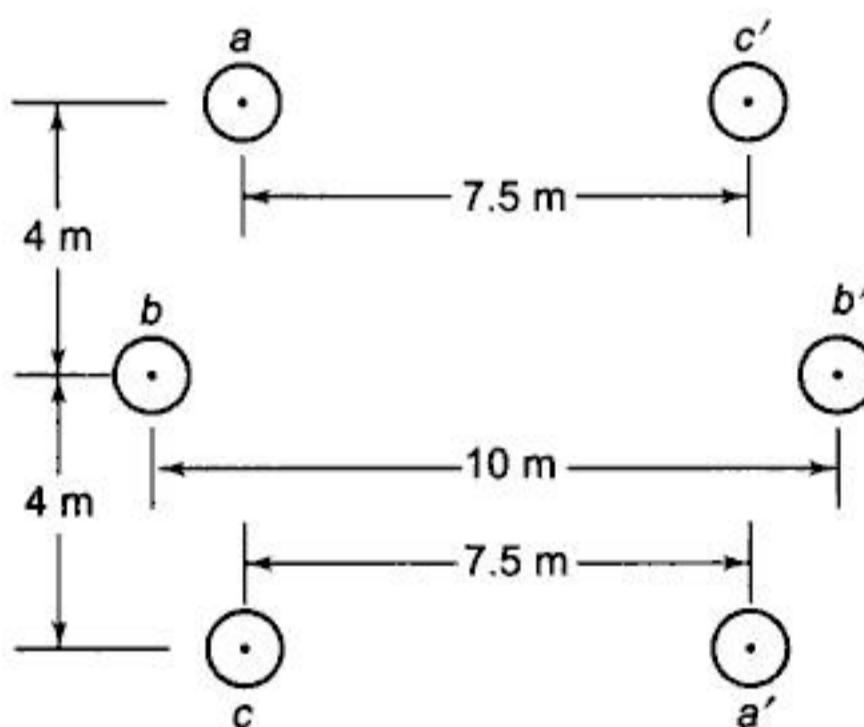
- 2.9 A 500 kV line has a bundling arrangement of two conductors per phase as shown in Fig. P-2.9.



**Fig. P-2.9** 500 kV, three-phase bundled conductor line

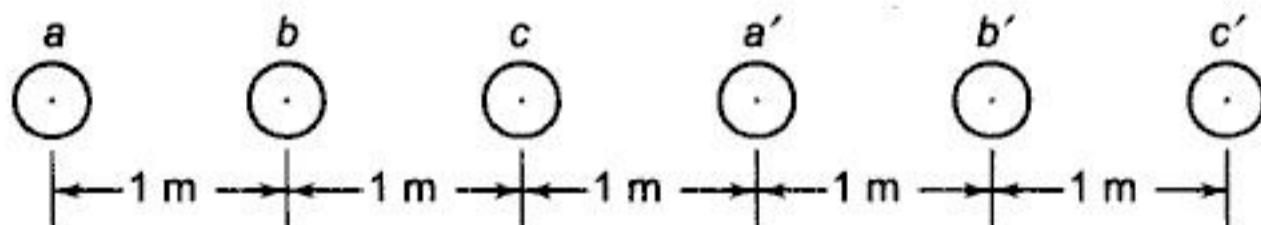
Compute the reactance per phase of this line at 50 Hz. Each conductor carries 50% of the phase current. Assume full transposition.

- 2.10 An overhead line 50 kms in length is to be constructed of conductors 2.56 cm in diameter, for single-phase transmission. The line reactance must not exceed 31.4 ohms. Find the maximum permissible spacing.
- 2.11 In Fig. P-2.11 which depicts two three-phase circuits on a steel tower there is symmetry about both the horizontal and vertical centre lines. Let each three-phase circuit be transposed by replacing  $a$  by  $b$  and then by  $c$ , so that the reactances of the three-phases are equal and the GMD method of reactance calculations can be used. Each circuit remains on its own side of the tower. Let the self GMD of a single conductor be 1 cm. Conductors  $a$  and  $a'$  and other corresponding phase conductors are connected in parallel. Find the reactance per phase of the system.



**Fig. P-2.11**

- 2.12 A double-circuit three-phase line is shown in Fig. P-2.12. The conductors  $a, a'$ ;  $b, b'$  and  $c, c'$  belong to the same phase respectively. The radius of each conductor is 1.5 cm. Find the inductance of the double-circuit line in mH/km/phase.



**Fig. P-2.12** Arrangement of conductors for a double-circuit three-phase line

- 2.13 A three-phase line with equilateral spacing of 3 m is to be rebuilt with horizontal spacing ( $D_{13} = D_{12} = D_{23}$ ). The conductors are to be fully transposed. Find the spacing between adjacent conductors such that the new line has the same inductance as the original line.
- 2.14 Find the self GMD of three arrangements of bundled conductors shown in Fig. 2.16 in terms of the total cross-sectional area  $A$  of conductors (same in each case) and the distance  $d$  between them.

## References

### Books

1. *Electrical Transmission and Distribution Book*, Westinghouse Electric and Manufacturing Co., East Pittsburgh, Pennsylvania, 1964.
2. Waddicor, H., *Principles of Electric Power Transmission*, 5th edn, Chapman and Hall, London, 1964.
3. Kothari, D.P. and I.J. Nagrath, *Electric Machines*, 3rd edn, Tata McGraw-Hill, New Delhi, 2003.
4. Stevenson, W.D., *Elements of Power System Analysis*, 4th edn, McGraw-Hill, New York, 1982.
5. *EHV Transmission Line Reference Book*, Edison Electric Institute, 1968.
6. The Aluminium Association, *Aluminium Electrical Conductor Handbook*, New York, 1971.
7. Woodruff, L.F., *Principles of Electric Power Transmission*, Wiley, New York, 1947.
8. Gross, C.A., *Power System Analysis*, Wiley, New York, 1979.
9. Weedy, B.M. and B.J. Cory, *Electric Power Systems*, 4th edn, Wiley, New York, 1998.
10. Kimbark, E.W., *Electrical Transmission of Power and Signals*, Wiley, New York, 1949.

### Paper

11. Reichman, J., 'Bundled Conductor Voltage Gradient Calculations,' *AIEE Trans.*, 1959, pt III, 78 : 598.

## Chapter 3

# Capacitance of Transmission Lines

### 3.1 INTRODUCTION

The capacitance together with conductance forms the shunt admittance of a transmission line. As mentioned earlier the conductance is the result of leakage over the surface of insulators and is of negligible order. When an alternating voltage is applied to the line, the line capacitance draws a leading sinusoidal current called the *charging current* which is drawn even when the line is open circuited at the far end. The line capacitance being proportional to its length, the charging current is negligible for lines less than 100 km long. For longer lines the capacitance becomes increasingly important and has to be accounted for.

### 3.2 ELECTRIC FIELD OF A LONG STRAIGHT CONDUCTOR

Imagine an infinitely long straight conductor far removed from other conductors (including earth) carrying a uniform charge of  $q$  coulomb/metre length. By symmetry, the equipotential surfaces will be concentric cylinders, while the lines of electrostatic stress will be radial. The electric field intensity at a distance  $y$  from the axis of the conductor is

$$\epsilon = \frac{q}{2\pi ky} \text{ V/m}$$

where  $k$  is the permittivity\* of the medium.

As shown in Fig. 3.1, consider two points  $P_1$  and  $P_2$  located at distances  $D_1$  and  $D_2$  respectively from the conductor axis. The potential difference  $V_{12}$  (between  $P_1$  and  $P_2$ ) is given by

$$V_{12} = \oint \epsilon dy = \oint \frac{q}{2\pi ky} dy \text{ V}$$

\* In SI units the permittivity of free space is  $k_0 = 8.85 \times 10^{-12}$  F/m. Relative permittivity for air is  $k_r = k/k_0 = 1$ .



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The potential difference between any two conductors of the group can then be obtained by adding the contributions of the individual charged conductors by repeated application of Eq. (3.1). So, the potential difference between conductors  $a$  and  $b$  (voltage drop from  $a$  to  $b$ ) is

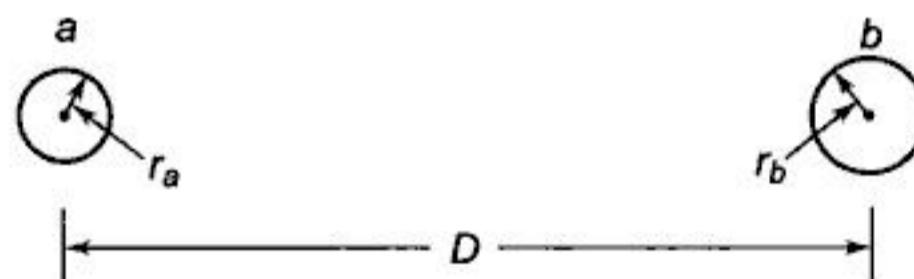
$$V_{ab} = \frac{1}{2\pi k} \left( q_a \ln \frac{D_{ab}}{r_a} + q_b \ln \frac{r_b}{D_{ba}} + q_c \ln \frac{D_{cb}}{D_{ca}} + \dots q_n \ln \frac{D_{nb}}{D_{na}} \right) \text{ V} \quad (3.2)$$

Each term in Eq. (3.2) is the potential drop from  $a$  to  $b$  caused by charge on one of the conductors of the group. Expressions on similar lines could be written for voltage drop between any two conductors of the group.

If the charges vary sinusoidally, so do the voltages (this is the case for AC transmission line), the expression of Eq. (3.2) still applies with charges/metre length and voltages regarded as phasor quantities. Equation (3.2) is thus valid for instantaneous quantities and for sinusoidal quantities as well, wherein all charges and voltages are phasors.

### 3.4 CAPACITANCE OF A TWO-WIRE LINE

Consider a two-wire line shown in Fig. 3.3 excited from a single-phase source. The line develops equal and opposite sinusoidal charges on the two conductors which can be represented as phasors  $q_a$  and  $q_b$  so that  $q_a = -q_b$ .



**Fig. 3.3** Cross-sectional view of a two-wire line

The potential difference  $V_{ab}$  can be written in terms of the contributions made by  $q_a$  and  $q_b$  by use of Eq. (3.2) with associated assumptions (i.e.  $D/r$  is large and ground is far away). Thus,

$$V_{ab} = \frac{1}{2\pi k} \left( q_a \ln \frac{D}{r_a} + q_b \ln \frac{r_b}{D} \right) \quad (3.3)$$

Since  $q_a = -q_b$ , we have

$$V_{ab} = \frac{q_a}{2\pi k} \ln \frac{D^2}{r_a r_b}$$

The line capacitance  $C_{ab}$  is then

$$C_{ab} = \frac{q_a}{V_{ab}} = \frac{\pi k}{\ln(D/(r_a r_b)^{1/2})} \text{ F/m length of line} \quad (3.4a)$$

or

$$C_{ab} = \frac{0.0121}{\log(D/(r_a r_b)^{1/2})} \mu\text{F}/\text{km} \quad (3.4\text{b})$$

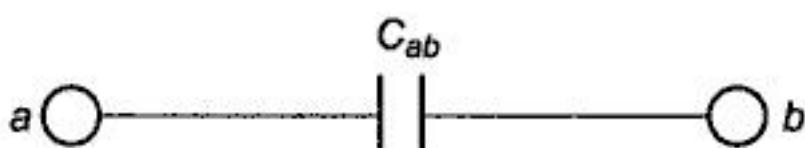
If

$$r_a = r_b = r,$$

$$C_{ab} = \frac{0.0121}{\log(D/r)} \mu\text{F}/\text{km} \quad (3.4\text{c})$$

The associated line charging current is

$$I_c = j\omega C_{ab} V_{ab} \text{ A/km} \quad (3.5)$$



(a) Line-to-line capacitance



(b) Line-to-neutral capacitance

**Fig. 3.4**

As shown in Figs 3.4(a) and (b) the line-to-line capacitance can be equivalently considered as two equal capacitances in series. The voltage across the lines divides equally between the capacitances such that the neutral point  $n$  is at the ground potential. The capacitance of each line to neutral is then given by

$$C_n = C_{an} = C_{bn} = 2C_{ab} = \frac{0.0242}{\log(D/r)} \mu\text{F}/\text{km} \quad (3.6)$$

The assumptions inherent in the above derivation are:

- (i) The charge on the surface of each conductor is assumed to be uniformly distributed, but this is strictly not correct.

If non-uniformity of charge distribution is taken into account, then

$$C_n = \frac{0.0242}{\log\left(\frac{D}{2r} + \left(\frac{D^2}{4r^2} - 1\right)^{1/2}\right)} \mu\text{F}/\text{km} \quad (3.7)$$

If  $D/2r \gg 1$ , the above expression reduces to that of Eq. (3.6) and the error caused by the assumption of uniform charge distribution is negligible.

- (ii) The cross-section of both the conductors is assumed to be circular, while in actual practice stranded conductors are used. The use of the radius of the circumscribing circle for a stranded conductor causes insignificant error.



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The capacitance of line to neutral immediately follows as

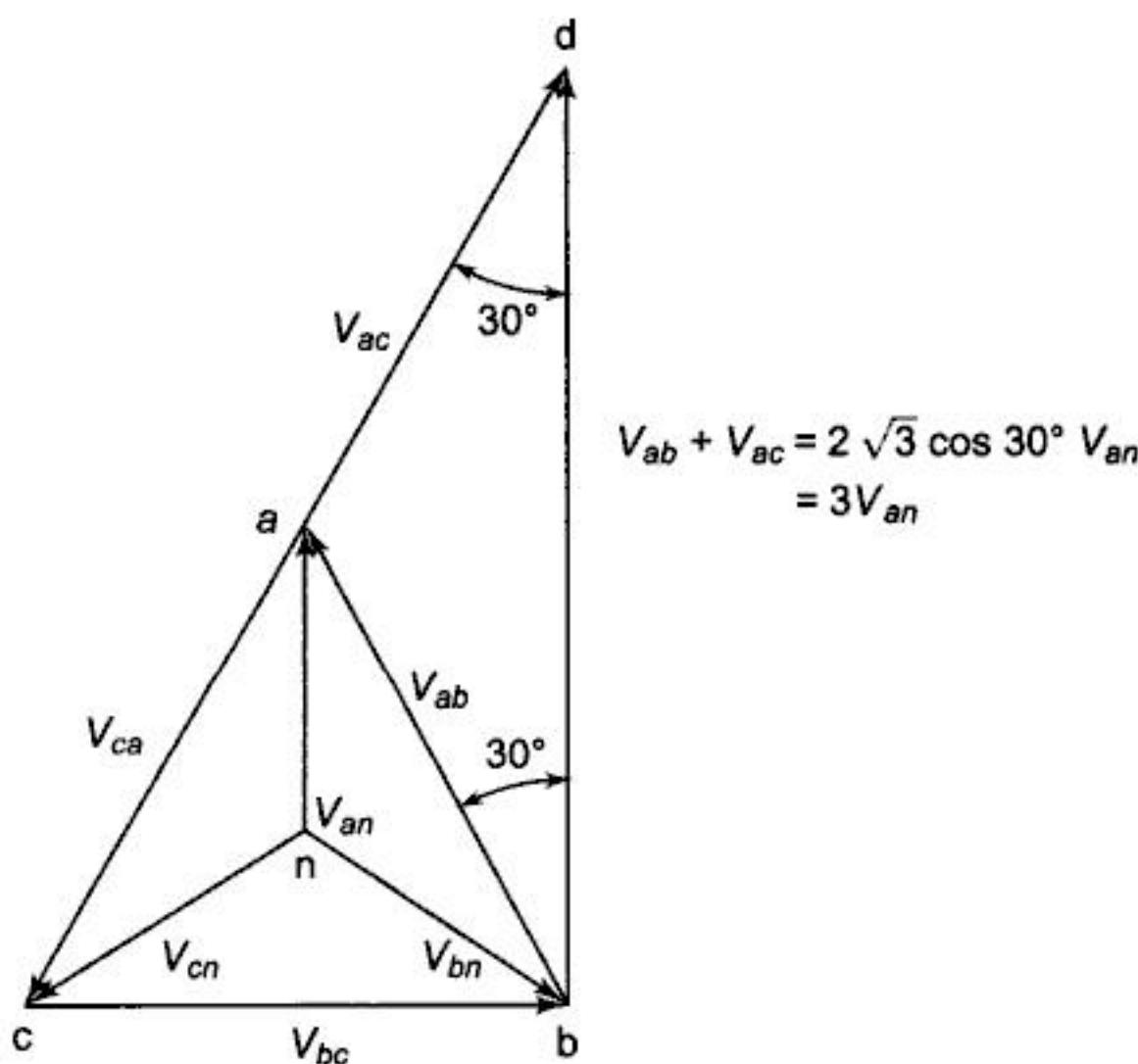
$$C_n = \frac{q_a}{V_{an}} = \frac{2\pi k}{\ln(D/r)} \quad (3.14a)$$

For air medium ( $k_r = 1$ ),

$$C_n = \frac{0.0242}{\log(D/r)} \mu\text{F/km} \quad (3.14b)$$

The line charging current of phase  $a$  is

$$I_a \text{ (line charging)} = j\omega C_n V_{an} \quad (3.15)$$



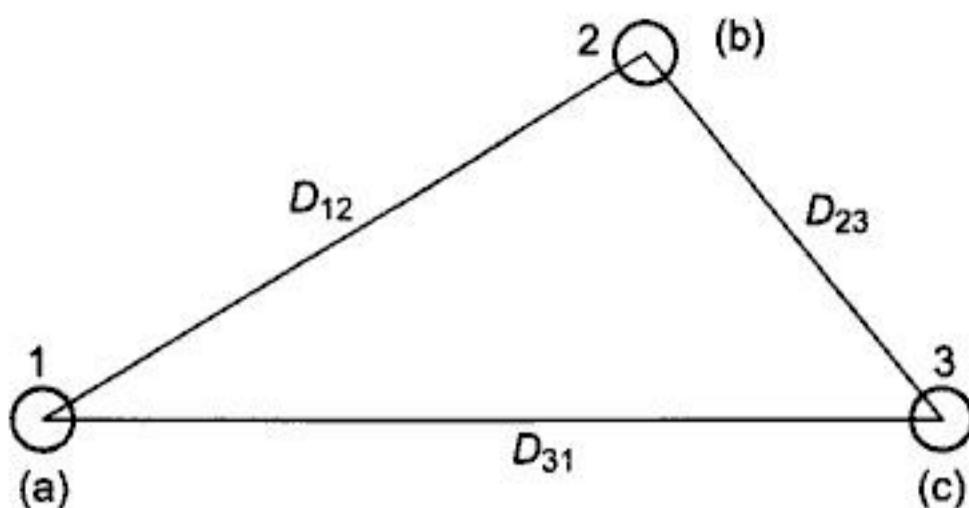
**Fig. 3.6** Phasor diagram of balanced three-phase voltages

### 3.6 CAPACITANCE OF A THREE-PHASE LINE WITH UNSYMMETRICAL SPACING

Figure 3.7 shows the three identical conductors of radius  $r$  of a three-phase line with unsymmetrical spacing. It is assumed that the line is fully transposed. As the conductors are rotated cyclically in the three sections of the transposition cycle, correspondingly three expressions can be written for  $V_{ab}$ . These expressions are:

For the first section of the transposition cycle

$$V_{ab} = \frac{1}{2\pi k} \left( q_{a1} \ln \frac{D_{12}}{r} + q_{b1} \ln \frac{r}{D_{12}} + q_{c1} \ln \frac{D_{23}}{D_{31}} \right) \quad (3.16a)$$



**Fig. 3.7** Cross-section of a three-phase line with asymmetrical spacing (fully transposed)

For the second section of the transposition cycle

$$V_{ab} = \frac{1}{2\pi k} \left( q_{a2} \ln \frac{D_{23}}{r} + q_{b2} \ln \frac{r}{D_{23}} + q_{c2} \ln \frac{D_{31}}{D_{12}} \right) \quad (3.16b)$$

For the third section of the transposition cycle

$$V_{ab} = \frac{1}{2\pi k} \left( q_{a3} \ln \frac{D_{31}}{r} + q_{b3} \ln \frac{r}{D_{31}} + q_{c3} \ln \frac{D_{12}}{D_{23}} \right) \quad (3.16c)$$

If the voltage drop along the line is neglected,  $V_{ab}$  is the same in each transposition cycle. On similar lines three such equations can be written for  $V_{bc} = V_{ab} \angle -120^\circ$ . Three more equations can be written equating to zero the summation of all line charges in each section of the transposition cycle. From these nine (independent) equations, it is possible to determine the nine unknown charges. The rigorous solution though possible is too involved.

With the usual spacing of conductors sufficient accuracy is obtained by assuming

$$q_{a1} = q_{a2} = q_{a3} = q_a; q_{b1} = q_{b2} = q_{b3} = q_b; q_{c1} = q_{c2} = q_{c3} = q_c \quad (3.17)$$

This assumption of equal charges/unit length of a line in the three sections of the transposition cycle requires, on the other hand, three different values of  $V_{ab}$  designated as  $V_{ab1}$ ,  $V_{ab2}$  and  $V_{ab3}$  in the three sections. The solution can be considerably simplified by taking  $V_{ab}$  as the average of these three voltages, i.e.

$$V_{ab} (\text{avg}) = \frac{1}{3} (V_{ab1} + V_{ab2} + V_{ab3})$$

or

$$V_{ab} = \frac{1}{6\pi k} \left[ q_a \ln \left( \frac{D_{12} D_{23} D_{31}}{r^3} \right) + q_b \ln \left( \frac{r^3}{D_{12} D_{23} D_{31}} \right) + q_c \ln \left( \frac{D_{12} D_{23} D_{31}}{D_{12} D_{23} D_{31}} \right) \right]$$



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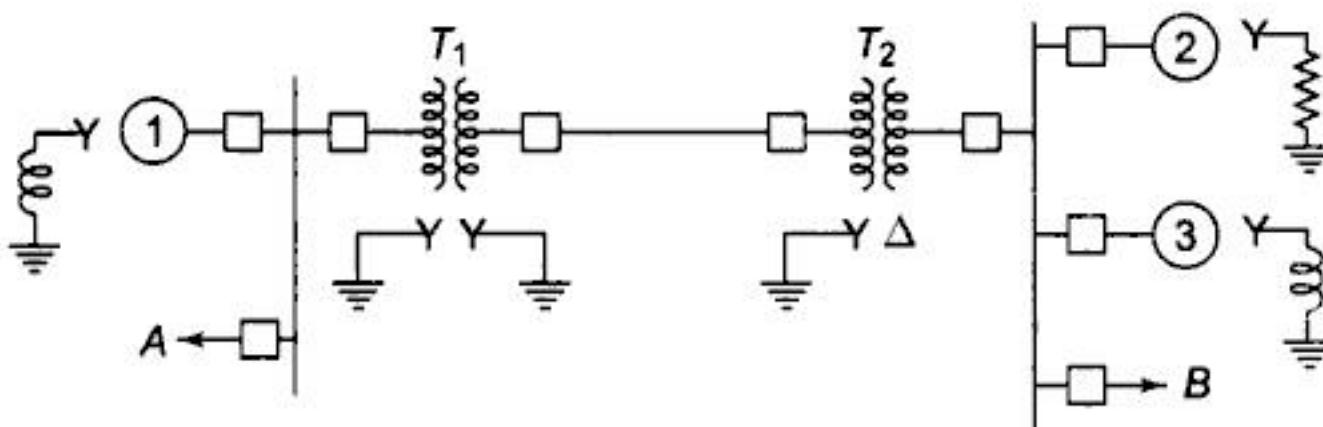


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shown depending on the information required in a system study, e.g. circuit breakers need not be shown in a load flow study but are a must for a protection study. Power system networks are represented by one-line diagrams using suitable symbols for generators, motors, transformers and loads. It is a convenient practical way of network representation rather than drawing the actual three-phase diagram which may indeed be quite cumbersome and confusing for a practical size power network. Generator and transformer connections—star, delta, and neutral grounding are indicated by symbols drawn by the side of the representation of these elements. Circuit breakers are represented as rectangular blocks. Figure 4.5 shows the one-line diagram of a simple power system. The reactance data of the elements are given below the diagram.



**Fig. 4.5** One-line representation of a simple power system

Generator No. 1	30 MVA,	10.5 kV,	$X'' = 1.6 \text{ ohms}$
Generator No. 2	15 MVA,	6.6 kV,	$X'' = 1.2 \text{ ohms}$
Generator No. 3	25 MVA,	6.6 kV,	$X'' = 0.56 \text{ ohms}$
Transformer $T_1$ (3 phase)	15 MVA,	33/11 kV,	$X = 15.2 \text{ ohms per phase on high tension side}$
Transformer $T_2$ (3 phase)	15 MVA,	33/6.2 kV,	$X = 16 \text{ ohms per phase on high tension side}$
Transmission line	20.5 ohms/phase		
Load A	40 MW,	11 kV,	0.9 lagging power factor
Load B	40 MW,	6.6 kV,	0.85 lagging power factor

*Note:* Generators are specified in three-phase MVA, line-to-line voltage and per phase reactance (equivalent star). Transformers are specified in three-phase MVA, line-to-line transformation ratio, and per phase (equivalent star) impedance on one side. Loads are specified in three-phase MW, line-to-line voltage and power factor.

The impedance diagram on single-phase basis for use under balanced operating conditions can be easily drawn from the one-line diagram. For the system of Fig. 4.5 the impedance diagram is drawn in Fig. 4.6. Single-phase transformer equivalents are shown as ideal transformers with transformer impedances indicated on the appropriate side. Magnetizing reactances of the transformers have been neglected. This is a fairly good approximation for most power system studies. The generators are represented as voltage sources with series resistance and inductive reactance (synchronous machine model will be discussed in Sec. 4.6). The transmission line is represented by a  $\pi$ -model (to be



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$$\frac{V_{1B}}{V_{2B}} = \frac{1}{a} \quad (4.11a)$$

Therefore,  $\frac{V_{1B}}{V_{2B}} = a$  (as  $(VA)_B$  is common) (4.11b)

$$Z_{1B} = \frac{V_{1B}}{I_{1B}}, Z_{2B} = \frac{V_{2B}}{I_{2B}} \quad (4.11c)$$

From Fig. 4.7(a), we can write

$$V_2 = (V_1 - I_1 Z_p) a - I_2 Z_s \quad (4.12)$$

We shall convert Eq. (4.12) into per unit form

$$V_2(\text{pu}) V_{2B} = [V_1(\text{pu}) V_{1B} - I_1(\text{pu}) I_{1B} Z_p(\text{pu}) Z_{1B}] \\ a - I_2(\text{pu}) I_{2B} Z_s(\text{pu}) Z_{2B}$$

Dividing by  $V_{2B}$  throughout and using base relations (4.11a, b, c), we get

$$V_2(\text{pu}) = V_1(\text{pu}) - I_1(\text{pu}) Z_p(\text{pu}) - I_2(\text{pu}) Z_s(\text{pu}) \quad (4.13)$$

Now

$$\frac{I_1}{I_2} = \frac{I_{1B}}{I_{2B}} = a$$

or

$$\frac{I_1}{I_{1B}} = \frac{I_2}{I_{2B}}$$

$$\therefore I_1(\text{pu}) = I_2(\text{pu}) = I(\text{pu})$$

Equation (4.13) can therefore be written as

$$V_2(\text{pu}) = V_1(\text{pu}) - I(\text{pu}) Z(\text{pu}) \quad (4.14)$$

where

$$Z(\text{pu}) = Z_p(\text{pu}) + Z_s(\text{pu})$$

Equation (4.14) can be represented by the simple equivalent circuit of Fig. 4.7(b) which does not require an ideal transformer. Considerable simplification has therefore been achieved by the per unit method with a common voltampere base and voltage bases on the two sides in the ratio of transformation.

$Z(\text{pu})$  can be determined directly from the equivalent impedance on primary or secondary side of a transformer by using the appropriate impedance base.

On primary side:

$$Z_1 = Z_p + Z_s/a^2$$

$$Z_1(\text{pu}) = \frac{Z_1}{Z_{1B}} = \frac{Z_p}{Z_{1B}} + \frac{Z_s}{Z_{1B}} \times \frac{1}{a^2}$$

But

$$a^2 Z_{1B} = Z_{2B}$$



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Here

$S$  = complex power (VA, kVA, MVA)

$|S|$  = apparent power (VA, kVA, MVA); it signifies rating of equipments (generators, transformers)

$P = |V| |I| \cos \theta$  = real (active) power (watts, kW, MW)

$Q = |V| |I| \sin \theta$  = reactive power

= voltamperes reactive (VAR)

= kilovoltamperes reactive (kVAR)

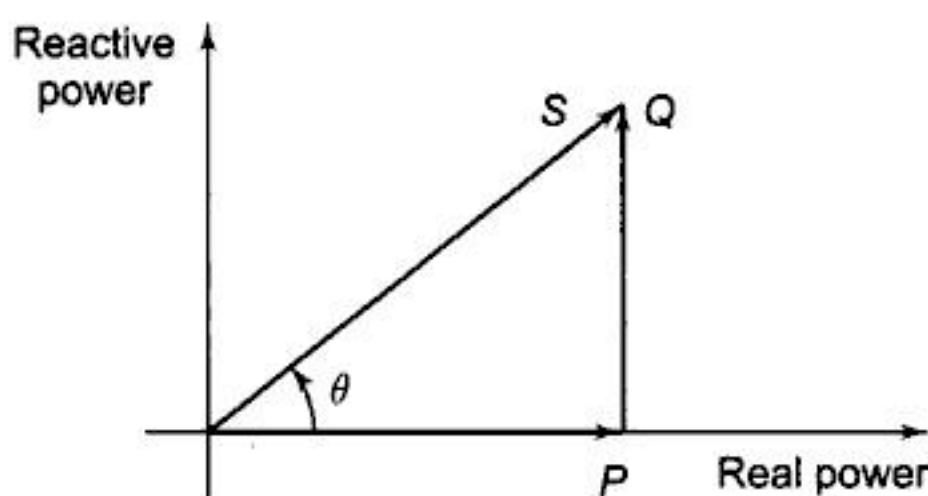
= megavoltamperes reactive (MVAR)

It immediately follows from Eq. (4.17) that  $Q$ , the reactive power, is positive for lagging current (lagging power factor load) and negative for leading current (leading power factor load). With the direction of current indicated in Fig. 4.9,  $S = P + jQ$  is supplied by the source and is absorbed by the load.

Equation (4.17) can be represented by the phasor diagram of Fig. 4.10 where

$$\theta = \tan^{-1} \frac{Q}{P} = \text{positive for lagging current} \quad (4.18)$$

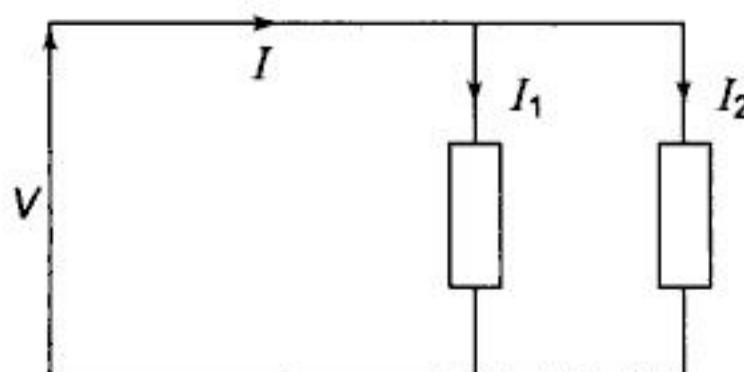
= negative for leading current



**Fig. 4.10** Phasor representation of complex power (lagging pf load)

If two (or more) loads are in parallel as in Fig. 4.11

$$\begin{aligned} S &= VI^* = V(I_1^* + I_2^*) \\ &= VI_1^* + VI_2^* \\ &= S_1 + S_2 = (P_1 + P_2) + j(Q_1 + Q_2) \end{aligned} \quad (4.19)$$



**Fig. 4.11** Two loads in parallel



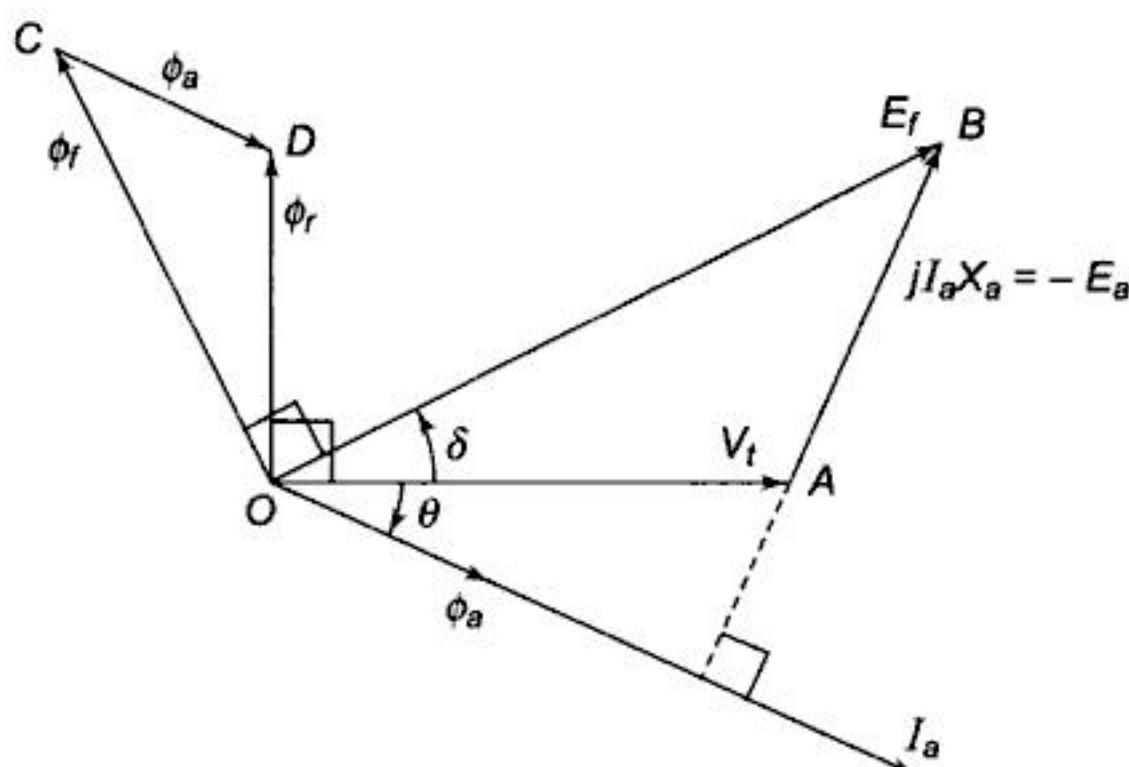
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**Fig. 4.15** Phasor diagram of synchronous generator

Here

$\theta$  = power factor angle

$\delta$  = angle by which  $E_f$  leads  $V_t$ , called *load angle* or *torque angle*

We shall see in Sec. 5.10 that  $\delta$  mainly determines the power delivered by the generator and the magnitude of  $E_f$  (i.e. excitation) determines the VARs delivered by it.

Because of the assumed linearity of the magnetic circuit, voltage phasors  $E_f$ ,  $E_a$  and  $V_t$  are proportional to flux phasors  $\phi_f$ ,  $\phi_a$  and  $\phi_r$ , respectively; further, voltage phasors lag  $90^\circ$  behind flux phasors. It therefore easily follows from Fig. 4.15 that phasor  $AB = -E_a$  is proportional to  $\phi_a$  (and therefore  $I_a$ ) and is  $90^\circ$  leading  $\phi_a$  (or  $I_a$ ). With the direction of phasor  $AB$  indicated on the diagram

$$AB = jI_a X_a$$

where  $X_a$  is the constant of proportionality.

In terms of the above definition of  $X_a$ , we can directly write the following expression for voltages without the need of invoking flux phasors.

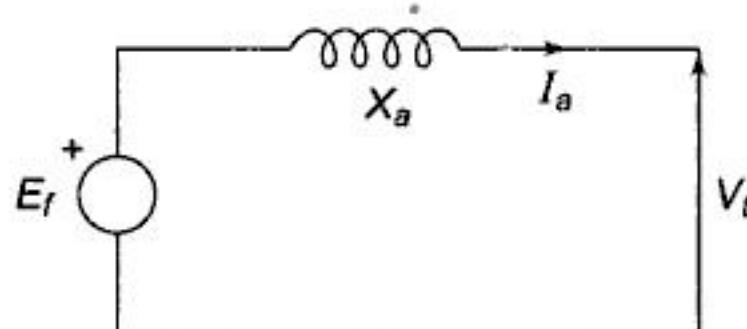
$$V_t = E_f - jI_a X_a \quad (4.24)$$

where

$E_f$  = voltage induced by field flux  $\phi_f$  alone  
= no load emf

The circuit model of Eq. (4.24) is drawn in Fig. 4.16 wherein  $X_a$  is interpreted as inductive reactance which accounts for the effect of armature reaction thereby avoiding the need of resorting to addition of fluxes [Eq.(4.23)].

The circuit of Fig. 4.16 can be easily modified to include



**Fig. 4.16**

Circuit model of round rotor synchronous generator (resistance and leakage reactance neglected)



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From the above discussion we can draw the general conclusion that a synchronous machine (generating or motoring) while operating at constant power supplies positive reactive power into the bus bar (or draws negative reactive power from the bus bar) when overexcited. An underexcited machine, on the other hand, feeds negative reactive power into the bus bar (or draws positive reactive power from the bus bar).

Consider now the power delivered by a synchronous generator to an infinite bus. From Fig. 4.19 this power is

$$P = |V_t| |I_a| \cos \theta$$

The above expression can be written in a more useful form from the phasor geometry. From Fig. 4.19

$$\frac{|E_f|}{\sin(90^\circ + \theta)} = \frac{|I_a| X_s}{\sin \delta}$$

or

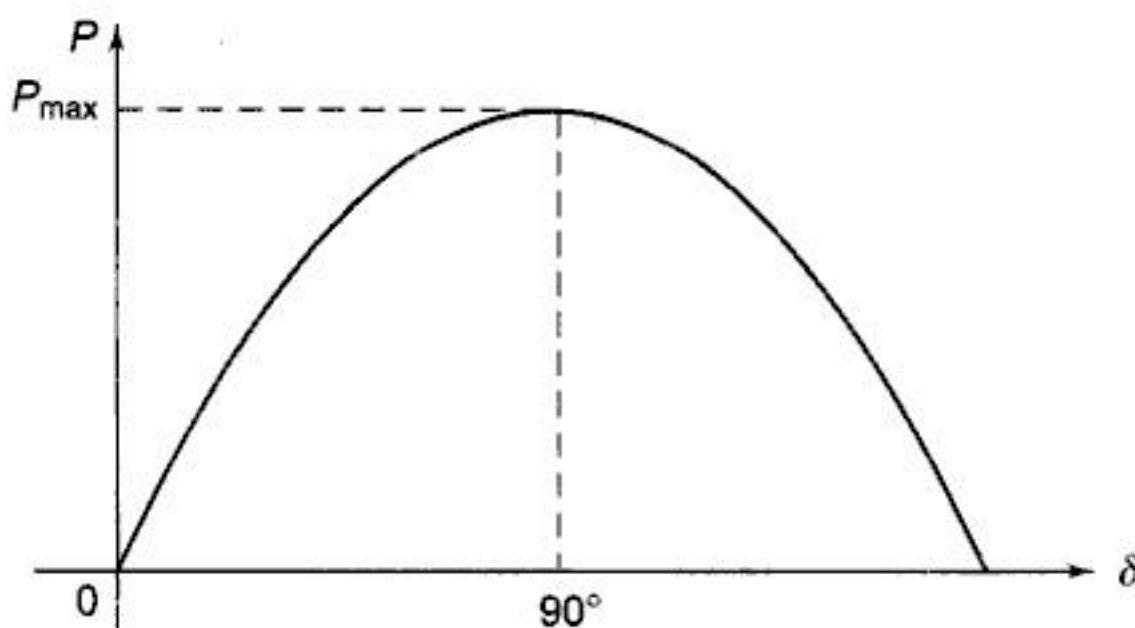
$$|I_a| \cos \theta = \frac{|E_f|}{X_s} \sin \delta \quad (4.27)$$

$$\therefore P = \frac{|E_f| |V_t|}{X_s} \sin \delta \quad (4.28)$$

The plot of  $P$  versus  $\delta$ , shown in Fig. 4.25, is called the *power angle curve*. The maximum power that can be delivered occurs at  $\delta = 90^\circ$  and is given by

$$P_{\max} = \frac{|E_f| |V_t|}{X_s} \quad (4.29)$$

For  $P > P_{\max}$  or for  $\delta > 90^\circ$  the generator will have stability problem. This problem (the stability) will be discussed at length in Ch. 12.



**Fig. 4.25** Power angle curve of a synchronous generator



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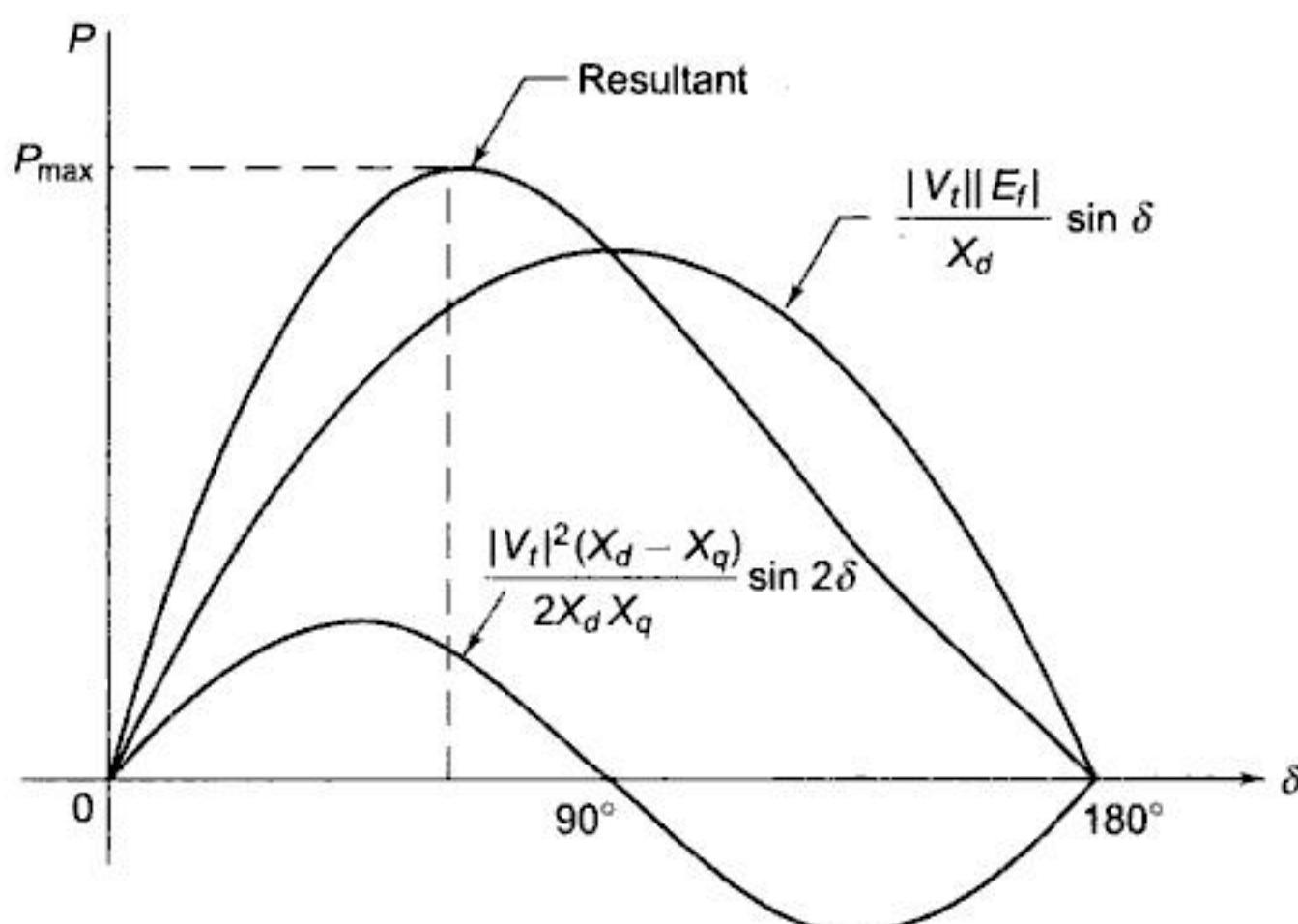
1. Draw  $V_t$  and  $I_a$  at angle  $\theta$ .
2. Draw  $I_a R_a$ . Draw  $CQ = jI_a X_d$  ( $\perp$  to  $I_a$ ).
3. Make  $|CP| = |I_a| X_q$  and draw the line  $OP$  which gives the direction of  $E_f$  phasor.
4. Draw a  $\perp$  from  $Q$  to the extended line  $OP$  such that  $OA = E_f$ .

It can be shown by the above theory that the power output of a salient pole generator is given by

$$P = \frac{|V_t||E_f|}{X_d} \sin \delta + \frac{|V_t|^2 (X_d - X_q)}{2X_d X_q} \sin 2\delta \quad (4.31)$$

The first term is the same as for a round rotor machine with  $X_s = X_d$  and constitutes the major part in power transfer. The second term is quite small (about 10–20%) compared to the first term and is known as *reluctance power*.

$P$  versus  $\delta$  is plotted in Fig. 4.31. It is noticed that the maximum power output occurs at  $\delta < 90^\circ$  (about  $70^\circ$ ). Further  $\frac{dP}{d\delta}$  (change in power per unit change in power angle for small changes in power angle), called the *synchronizing power coefficient*, in the operating region ( $\delta < 70^\circ$ ) is larger in a salient pole machine than in a round rotor machine.



**Fig. 4.31** Power angle curve for salient pole generator

In this book we shall neglect the effect of saliency and take

$$X_s = X_d$$

in all types of power system studies considered.

During a machine transient, the direct axis reactance changes with time acquiring the following distinct values during the complete transient.



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#### 4.10 REPRESENTATION OF LOADS

Load drawn by consumers is the toughest parameter to assess scientifically. The magnitude of the load, in fact, changes continuously so that the load forecasting problem is truly a statistical one. The loads are generally composed of industrial and domestic components. An industrial load consists mainly of large three-phase induction motors with sufficient load constancy and predictable duty cycle, whereas the domestic load mainly consists of lighting, heating and many single-phase devices used in a random way by householders. The design and operation of power systems both economically and electrically are greatly influenced by the nature and magnitude of loads.

In representation of loads for various system studies such as load flow and stability studies, it is essential to know the variation of real and reactive power with variation of voltage. Normally in such studies the load is of composite nature with both industrial and domestic components. A typical composition of load at a bus may be

Induction motors	55 – 75%
Synchronous motors	5 – 15%
Lighting and heating	20 – 30%

Though it is always better to consider the P – V and Q – V characteristics of each of these loads for simulation, the analytic treatment would be very cumbersome and complicated. In most of the analytical work one of the following three ways of load representation is used.

##### (i) Constant Power Representation

This is used in load flow studies. Both the specified MW and MVAR are taken to be constant.

##### (ii) Constant Current Representation

Here the load current is given by Eq. (4.17), i.e.

$$I = \frac{P - jQ}{V^*} = |I| \angle (\delta - \theta)$$

where  $V = |V| \angle \delta$  and  $\theta = \tan^{-1} Q/P$  is the power factor angle. It is known as constant current representation because the magnitude of current is regarded as constant in the study.

##### (iii) Constant Impedance Representation

This is quite often used in stability studies. The load specified in MW and MVAR at nominal voltage is used to compute the load impedance (Eq. (4.22b)). Thus

$$Z = \frac{V}{I} = \frac{VV^*}{P - jQ} = \frac{|V|^2}{P - jQ} = \frac{1}{Y}$$

which then is regarded as constant throughout the study.



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2. Van E. Mablekos, *Electric Machine Theory for Power Engineers*, Harper and Raw, New York, 1980.
3. DelToro, V., *Electric Machines and Power Systems*, Prentice-Hall, Englewood Cliffs, N.J., 1985.
4. Kothari, D.P. and I.J. Nagrath, *Theory and Problems of Electric Machines*, 2nd edn, Tata McGraw-Hill, New Delhi, 2002.
5. Ned Mohan, *First course on Power Systems*, MNPERE, Minneapolis, 2006.

## Paper

6. IEEE Committee Report, "The Effect of Frequency and Voltage on Power System Load", *Presented at IEEE Winter Power Meeting*, New York, 1966.

# **Chapter 5**

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# **Characteristics and Performance of Power Transmission Lines**

## **5.1 INTRODUCTION**

This chapter deals primarily with the characteristics and performance of transmission lines. A problem of major importance in power systems is the flow of load over transmission lines such that the voltage at various nodes is maintained within specified limits. While this general interconnected system problem will be dealt with in Ch. 6, attention is presently focussed on performance of a single transmission line so as to give the reader a clear understanding of the principle involved.

Transmission lines are normally operated with a balanced three-phase load; the analysis can therefore proceed on a per phase basis. A transmission line on a per phase basis can be regarded as a two-port network, wherein the sending-end voltage  $V_S$  and current  $I_S$  are related to the receiving-end voltage  $V_R$  and current  $I_R$  through *ABCD* constants\* as

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \quad (5.1)$$

Also the following identity holds for *ABCD* constants:

$$AD - BC = 1 \quad (5.2)$$

These constants can be determined easily for short- and medium-length lines by suitable approximations lumping the line impedance and shunt admittance. For long lines exact analysis has to be carried out by considering the distribution of resistance, inductance and capacitance parameters and the *ABCD* constants of the line are determined therefrom.

Equations for power flow on a line and receiving- and sending-end circle diagrams will also be developed in this chapter so that various types of end conditions can be handled.

---

\* Refer to Appendix B

The following nomenclature has been adopted in this chapter:

$z$  = series impedance/unit length/phase

$y$  = shunt admittance/unit length/phase to neutral

$r$  = resistance/unit length/phase

$L$  = inductance/unit length/phase

$C$  = capacitance/unit length/phase to neutral

$l$  = transmission line length

$Z = zl$  = total series impedance/phase

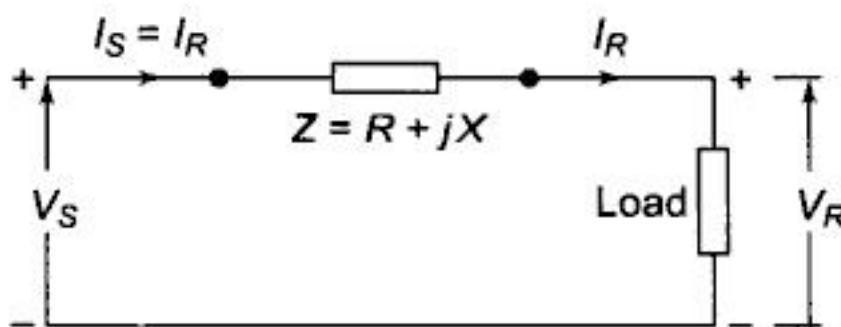
$Y = yl$  = total shunt admittance/phase to neutral

Subscript  $S$  stands for a sending-end quantity

Subscript  $R$  stands for a receiving-end quantity

## 5.2 SHORT TRANSMISSION LINE

For short lines of length 100 km or less, the total 50 Hz shunt admittance\* ( $j\omega Cl$ ) is small enough to be negligible resulting in the simple equivalent circuit of Fig. 5.1.



**Fig. 5.1** Equivalent circuit of a short line

This being a simple series circuit, the relationship between sending-end receiving-end voltages and currents can be immediately written as:

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \quad (5.3)$$

The phasor diagram for the short line is shown in Fig. 5.2 for the lagging current case. From this figure we can write

$$\begin{aligned} |V_S| &= [(|V_R| \cos \phi_R + |I| R)^2 + (|V_R| \sin \phi_R + |I| X)^2]^{1/2} \\ |V_S| &= [|V_R|^2 + |I|^2 (R^2 + X^2) + 2|V_R| |I| (R \cos \phi_R + X \sin \phi_R)]^{1/2} \end{aligned} \quad (5.4)$$

$$= |V_R| \left[ 1 + \frac{2|I|R}{|V_R|} \cos \phi_R + \frac{2|I|X}{|V_R|} \sin \phi_R + \frac{|I|^2 (R^2 + X^2)}{|V_R|^2} \right]^{1/2}$$

\* For overhead transmission lines, shunt admittance is mainly capacitive susceptance ( $j\omega Cl$ ) as the line conductance (also called *leakance*) is always negligible.

The last term is usually of negligible order.

$$\therefore |V_S| \approx |V_R| \left[ 1 + \frac{2|I|R}{|V_R|} \cos \phi_R + \frac{2|I|X}{|V_R|} \sin \phi_R \right]^{1/2}$$

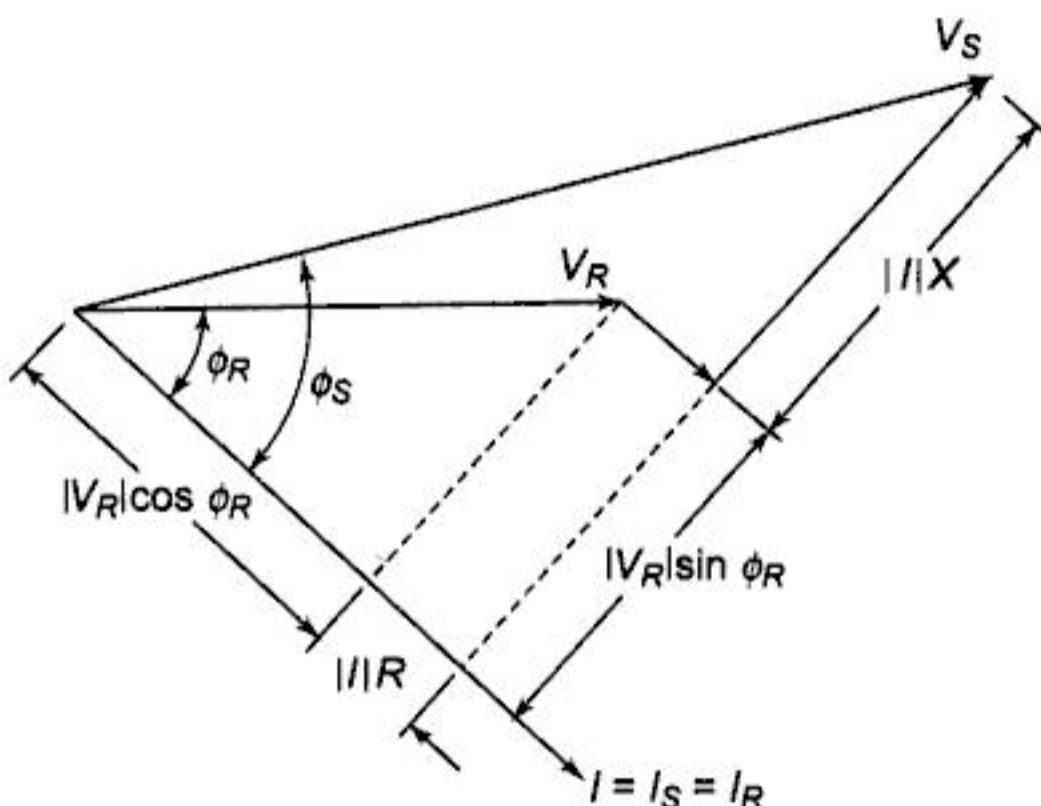
Expanding binomially and retaining first order terms, we get

$$|V_S| \approx |V_R| \left[ 1 + \frac{|I|R}{|V_R|} \cos \phi_R + \frac{|I|X}{|V_R|} \sin \phi_R \right]$$

or

$$|V_S| \approx |V_R| + |I| (R \cos \phi_R + X \sin \phi_R) \quad (5.5)$$

The above equation is quite accurate for the normal load range.



**Fig. 5.2** Phasor diagram of a short line for lagging current

## Voltage Regulation

Voltage regulation of a transmission line is defined as the rise in voltage at the receiving-end, expressed as percentage of full load voltage, when full load at a specified power factor is thrown off, i.e.

$$\text{Per cent regulation} = \frac{|V_{R0}| - |V_{RL}|}{|V_{RL}|} \times 100 \quad (5.6)$$

where

$|V_{R0}|$  = magnitude of no load receiving-end voltage

$|V_{RL}|$  = magnitude of full load receiving-end voltage  
(at a specified power factor)

For short line,  $|V_{R0}| = |V_S|$ ,  $|V_{RL}| = |V_R|$

$$\therefore \text{Per cent regulation} = \frac{|V_S| - |V_R|}{|V_R|}$$

$$= \frac{|I|R \cos \phi_R + |I|X \sin \phi_R}{|V_R|} \times 100 \quad (5.7)$$

In the above derivation,  $\phi_R$  has been considered positive for a lagging load. It will be negative for a leading load.

$$\text{Per cent regulation} = \frac{|I|R \cos \phi_R - |I|X \sin \phi_R}{|V_R|} \times 100 \quad (5.8)$$

Voltage regulation becomes negative (i.e. load voltage is more than no load voltage), when in Eq. (5.8)

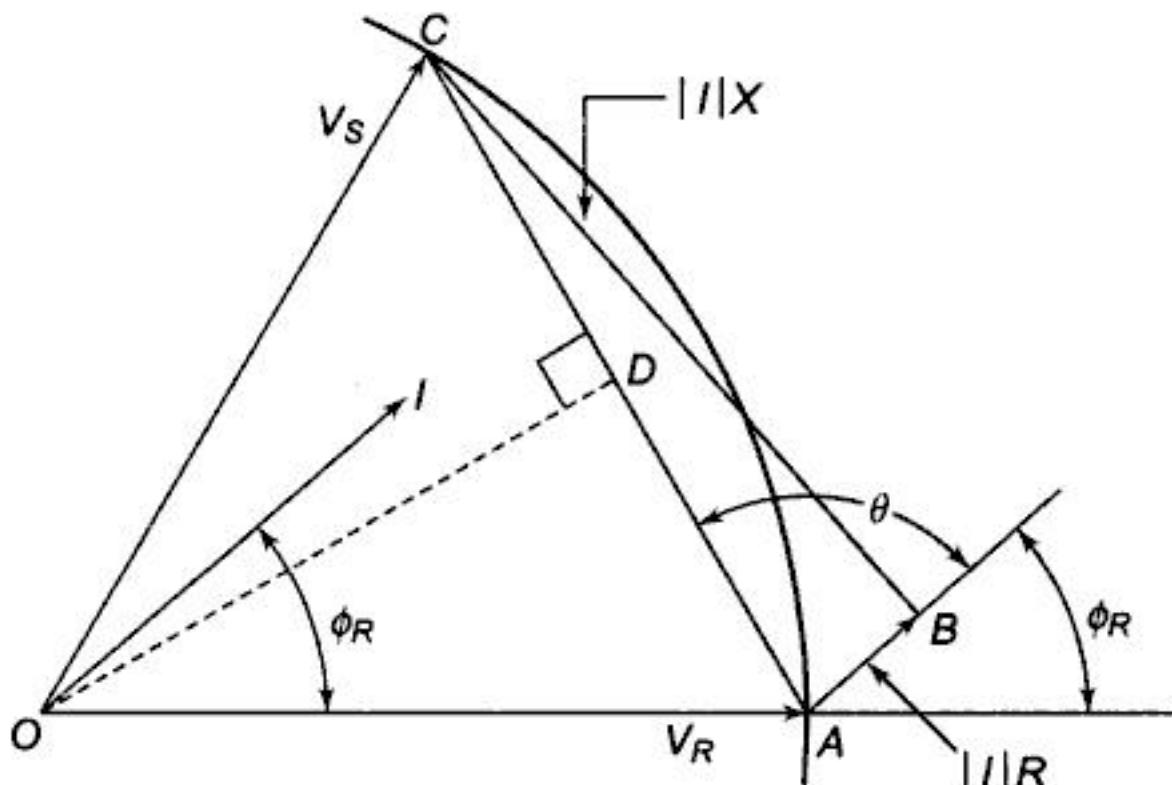
$$X \sin \phi_R > R \cos \phi_R, \text{ or } \tan \phi_R (\text{leading}) > \frac{R}{X}$$

It also follows from Eq. (5.8) that for zero voltage regulation

$$\begin{aligned} \tan \phi_R &= \frac{R}{X} = \cot \theta \\ \text{i.e., } \phi_R (\text{leading}) &= \frac{\pi}{2} - \theta \end{aligned} \quad (5.9)$$

where  $\theta$  is the angle of the transmission line impedance. This is, however, an approximate condition. The exact condition for zero regulation is determined as follows.

Figure 5.3 shows the phasor diagram under conditions of zero voltage regulation, i.e.



**Fig. 5.3** Phasor diagram under zero regulation condition

$$|V_S| = |V_R|$$

$$\text{or } OC = OA$$

$$\sin \angle AOD = \frac{AD}{OA} = \frac{AC/2}{|V_R|} = \frac{|I||Z|}{2|V_R|}$$

or

$$\angle AOD = \sin^{-1} \frac{|I||Z|}{2|V_R|}$$

It follows from the geometry of angles at  $A$ , that for zero voltage regulation,

$$\phi_R \text{ (leading)} = \left( \frac{\pi}{2} - \theta + \sin^{-1} \frac{|I||Z|}{2|V_R|} \right) \quad (5.10)$$

From the above discussion it is seen that the voltage regulation of a line is heavily dependent upon load power factor. Voltage regulation improves (decreases) as the power factor of a lagging load is increased and it becomes zero at a leading power factor given by Eq. (5.10).

**Example 5.1** A single-phase 50 Hz generator supplies an inductive load of 5,000 kW at a power factor of 0.707 lagging by means of an overhead transmission line 20 km long. The line resistance and inductance are 0.0195 ohm and 0.63 mH per km. The voltage at the receiving-end is required to be kept constant at 10 kV.

Find (a) the sending-end voltage and voltage regulation of the line; (b) the value of the capacitors to be placed in parallel with the load such that the regulation is reduced to 50% of that obtained in part (a); and (c) compare the transmission efficiency in parts (a) and (b).

### Solution

The line constants are

$$R = 0.0195 \times 20 = 0.39 \Omega$$

$$X = 314 \times 0.63 \times 10^{-3} \times 20 = 3.96 \Omega$$

(a) This is the case of a short line with  $I = I_R = I_S$  given by

$$|I| = \frac{5,000}{10 \times 0.707} = 707 \text{ A}$$

From Eq. (5.5),

$$\begin{aligned} |V_S| &\simeq |V_R| + |I|(R \cos \phi_R + X \sin \phi_R) \\ &= 10,000 + 707(0.39 \times 0.707 + 3.96 \times 0.707) \text{ V} \\ &= 12.175 \text{ kV} \end{aligned}$$

$$\text{Voltage regulation} = \frac{12.175 - 10}{10} \times 100 = 21.75\%$$

(b) Voltage regulation desired =  $\frac{21.75}{2} = 10.9\%$

$$\therefore \frac{|V_S| - 10}{10} = 0.109$$

or

$$\text{new value of } |V_S| = 11.09 \text{ kV}$$

Figure 5.4 shows the equivalent circuit of the line with a capacitive reactance placed in parallel with the load.

Assuming  $\cos \phi_R$  now to be the power factor of load and capacitive reactance taken together, we can write

$$(11.09 - 10) \times 10^3 = |I_R| (R \cos \phi_R + X \sin \phi_R) \quad (\text{i})$$

Since the capacitance does not draw any real power, we have

$$|I_R| = \frac{5,000}{10 \times \cos \phi_R} \quad (\text{ii})$$

Solving Eqs (i) and (ii), we get

$$\cos \phi_R = 0.911 \text{ lagging}$$

and

$$|I_R| = 549 \text{ A}$$

Now

$$\begin{aligned} I_C &= I_R - I \\ &= 549(0.911 - j0.412) - 707(0.707 - j0.707) \\ &= 0.29 + j273.7 \end{aligned}$$

Note that the real part of 0.29 appears due to the approximation in (i). Ignoring it, we have

$$I_C = j273.7 \text{ A}$$

$$\therefore X_C = \frac{1}{314 \times C} = \left| \frac{V_R}{I_C} \right| = \frac{10 \times 1000}{273.7}$$

or

$$C = 87 \mu\text{F}$$

(c) Efficiency of transmission;

$$\eta = \frac{\text{output}}{\text{output} + \text{loss}}$$

Case (a)

$$\eta = \frac{5,000}{5,000 + (707)^2 \times 0.39 \times 10^{-3}} = 96.2\%$$

Case (b)

$$\eta = \frac{5,000}{5,000 + (549)^2 \times 0.39 \times 10^{-3}} = 97.7\%$$

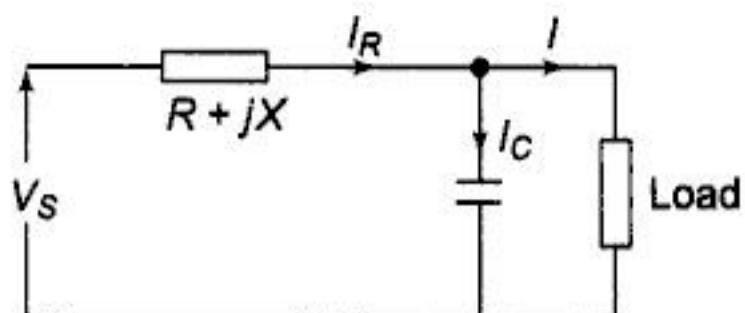
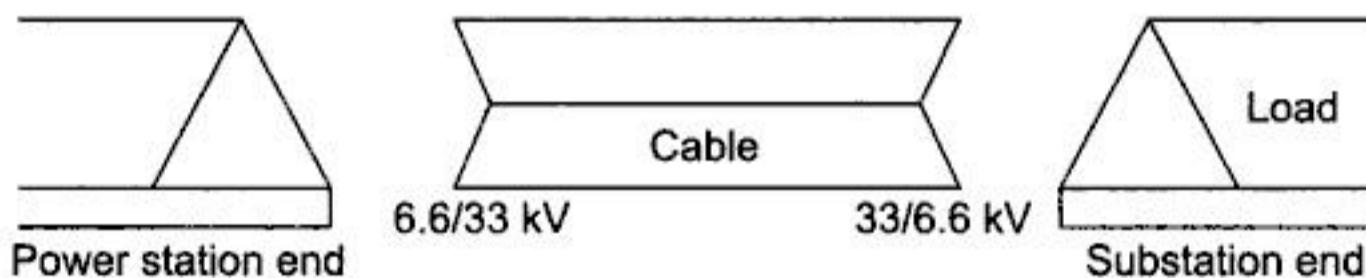


Fig. 5.4

It is to be noted that by placing a capacitor in parallel with the load, the receiving-end power factor improves (from 0.707 lag to 0.911 lag), the line current reduces (from 707 A to 549 A), the line voltage regulation decreases (one half the previous value) and the transmission efficiency improves (from 96.2 to 97.7%). Adding capacitors in parallel with load is a powerful method of improving the performance of a transmission system and will be discussed further towards the end of this chapter.

**Example 5.2** A substation as shown in Fig. 5.5 receives 5 MVA at 6 kV, 0.85 lagging power factor on the low voltage side of a transformer from a power station through a cable having per phase resistance and reactance of 8 and 2.5 ohms, respectively. Identical 6.6/33 kV transformers are installed at each end of the line. The 6.6 kV side of the transformers is delta connected while the 33 kV side is star connected. The resistance and reactance of the star connected windings are 0.5 and 3.75 ohms, respectively and for the delta connected windings are 0.06 and 0.36 ohms. What is the voltage at the bus at the power station end?



**Fig. 5.5**

### Solution

It is convenient here to employ the per unit method. Let us choose,

$$\text{Base MVA} = 5$$

$$\begin{aligned}\text{Base kV} &= 6.6 \text{ on low voltage side} \\ &= 33 \text{ on high voltage side}\end{aligned}$$

$$\text{Cable impedance} = (8 + j2.5) \Omega/\text{phase}$$

$$= \frac{(8 + j2.5) \times 5}{(33)^2} = (0.037 + j0.0115) \text{ pu}$$

Equivalent star impedance of 6.6 kV winding of the transformer

$$= \frac{1}{3} (0.06 + j0.36) = (0.02 + j0.12) \Omega/\text{phase}$$

Per unit transformer impedance,

$$Z_T = \frac{(0.02 + j0.12) \times 5}{(6.6)^2} + \frac{(0.5 + j3.75) \times 5}{(33)^2}$$

$$= (0.0046 + j0.030) \text{ pu}$$

$$\begin{aligned}\text{Total series impedance} &= (0.037 + j0.0115) + 2(0.0046 + j0.030) \\ &= (0.046 + j0.072) \text{ pu}\end{aligned}$$

Given: Load MVA = 1 pu

$$\text{Load voltage} = \frac{6}{6.6} = 0.91 \text{ pu}$$

$$\therefore \text{Load current} = \frac{1}{0.91} = 1.1 \text{ pu}$$

Using Eq. (5.5), we get

$$\begin{aligned}|V_S| &= 0.91 + 1.1(0.046 \times 0.85 + 0.072 \times 0.527) \\ &= 0.995 \text{ pu} \\ &= 0.995 \times 6.6 = 6.57 \text{ kV (line-to-line)}\end{aligned}$$

**Example 5.3** Input to a single-phase short line shown in Fig. 5.6 is 2,000 kW at 0.8 lagging power factor. The line has a series impedance of  $(0.4 + j0.4)$  ohms. If the load voltage is 3 kV, find the load and receiving-end power factor. Also find the supply voltage.

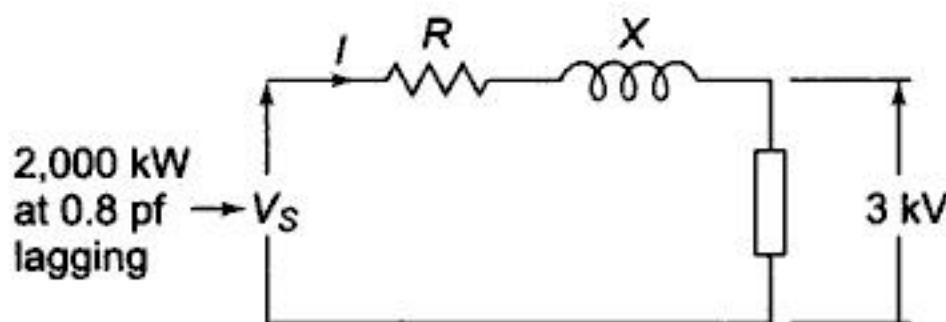


Fig. 5.6

### Solution

It is a problem with mixed-end conditions—load voltage and input power are specified. The exact solution is outlined below:

$$\begin{aligned}\text{Sending-end active/reactive power} &= \text{receiving-end active/reactive power} \\ &\quad + \text{active/reactive line losses}\end{aligned}$$

For active power

$$|V_S| |I| \cos \phi_S = |V_R| |I| \cos \phi_R + |I|^2 R \quad (\text{i})$$

For reactive power

$$|V_S| |I| \sin \phi_S = |V_R| |I| \sin \phi_R + |I|^2 X \quad (\text{ii})$$

Squaring (i) and (ii), adding and simplifying, we get

$$\begin{aligned}|V_S|^2 |I|^2 &= |V_R|^2 |I|^2 + 2|V_R| |I|^2 (|I|R \cos \phi_R \\ &\quad + |I|X \sin \phi_R) + |I|^4 (R^2 + X^2) \quad (\text{iii})\end{aligned}$$

*Note:* This, in fact, is the same as Eq. (5.4) if  $|I|^2$  is cancelled throughout. For the numerical values given

$$|Z|^2 = (R^2 + X^2) = 0.32$$

$$|V_S| |I| = \frac{2,000 \times 10^3}{0.8} = 2,500 \times 10^3$$

$$|V_S| |I| \cos \phi_S = 2,000 \times 10^3$$

$$|V_S| |I| \sin \phi_S = 2,500 \times 10^3 \times 0.6 = 1,500 \times 10^3$$

From Eqs (i) and (ii), we get

$$|I| \cos \phi_R = \frac{2,000 \times 10^3 - 0.4 |I|^2}{3,000} \quad (\text{iv})$$

$$|I| \sin \phi_R = \frac{1,500 \times 10^3 - 0.4 |I|^2}{3,000} \quad (\text{v})$$

Substituting all the known values in Eq. (iii), we have

$$(2,500 \times 10^3)^2 = (3,000)^2 |I|^2 + 2 \times 3,000 |I|^2 \left[ 0.4 \times \frac{2,000 \times 10^3 - 0.4 |I|^2}{3,000} \right. \\ \left. + 0.4 \times \frac{1,500 \times 10^3 - 0.4 |I|^2}{3,000} \right] + 0.32 |I|^4$$

Simplifying, we get

$$0.32 |I|^4 - 11.8 \times 10^6 |I|^2 + 6.25 \times 10^{12} = 0$$

which upon solution yields

$$|I| = 725 \text{ A}$$

Substituting for  $|I|$  in Eq. (iv), we get

$$\cos \phi_R = 0.82$$

$$\therefore \text{Load } P_R = |V_R| |I| \cos \phi_R = 3,000 \times 725 \times 0.82 \\ = 1,790 \text{ kW}$$

Now

$$|V_S| = |I| \cos \phi_S = 2,000$$

$$\therefore |V_S| = \frac{2,000}{725 \times 0.8} = 3.44 \text{ kV}$$

### 5.3 MEDIUM TRANSMISSION LINE

For lines more than 100 km long, charging currents due to shunt admittance cannot be neglected. For lines in range 100 km to 250 km length, it is sufficiently accurate to lump all the line admittance at the receiving-end resulting in the equivalent diagram shown in Fig. 5.7.

Starting from fundamental circuit equations, it is fairly straightforward to write the transmission line equations in the *ABCD* constant form given below:

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 + YZ & Z \\ Y & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \quad (5.11)$$

#### Nominal-T Representation

If all the shunt capacitance is lumped at the middle of the line, it leads to the nominal-*T* circuit shown in Fig. 5.8.

For the nominal-*T* circuit, the following circuit equations can be written:

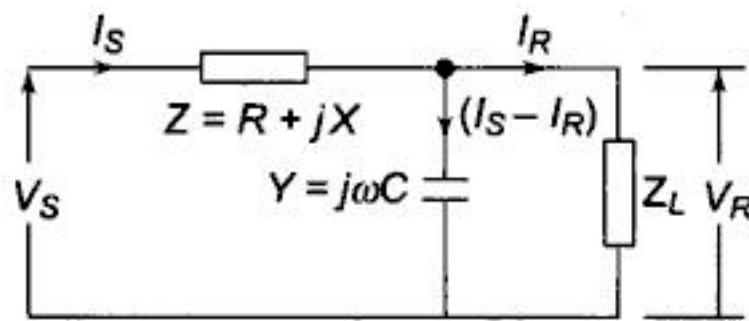
$$\begin{aligned} V_C &= V_R + I_R(Z/2) \\ I_S &= I_R + V_C Y \\ &= I_R + YV_R + I_R(Z/2)Y \\ V_S &= V_C + I_S(Z/2) \end{aligned}$$

Substituting for  $V_C$  and  $I_S$  in the last equation, we get

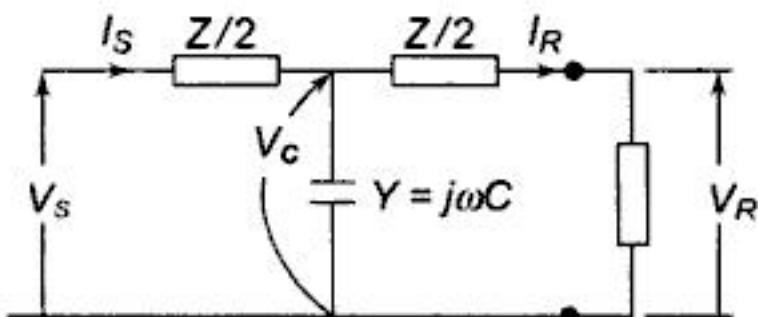
$$\begin{aligned} V_S &= V_R + I_R(Z/2) + (Z/2) \left[ I_R \left( 1 + \frac{ZY}{2} \right) + YV_R \right] \\ &= V_R \left( 1 + \frac{ZY}{2} \right) + I_R Z \left( 1 + \frac{YZ}{4} \right) \end{aligned}$$

Rearranging the results, we get the following equations:

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \left( 1 + \frac{1}{2} ZY \right) & Z \left( 1 + \frac{1}{4} YZ \right) \\ Y & \left( 1 + \frac{1}{2} YZ \right) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \quad (5.12)$$



**Fig. 5.7** Medium line, localized load-end capacitance

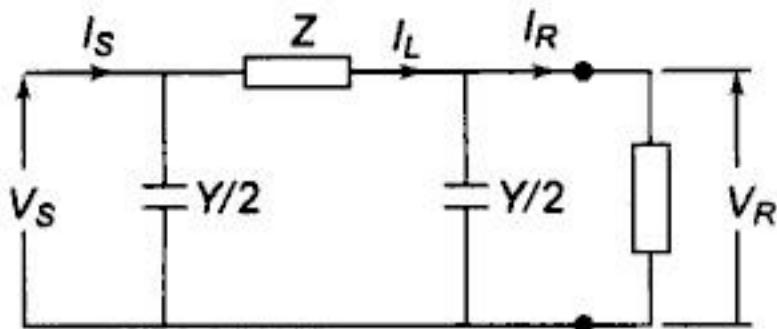


**Fig. 5.8** Medium line, nominal-*T* representation

### Nominal- $\pi$ Representation

In this method the total line capacitance is divided into two equal parts which are lumped at the sending- and receiving-ends resulting in the nominal- $\pi$  representation as shown in Fig. 5.9.

From Fig. 5.9, we have



**Fig. 5.9** Medium line, nominal- $\pi$  representation

$$I_S = I_R + \frac{1}{2} V_R Y + \frac{1}{2} V_s Y$$

$$V_s = V_R + (I_R + \frac{1}{2} V_R Y) Z = V_R \left( 1 + \frac{1}{2} YZ \right) + I_R Z$$

$$\begin{aligned} \therefore I_S &= I_R + \frac{1}{2} V_R Y + \frac{1}{2} Y \left[ V_R \left( 1 + \frac{1}{2} YZ \right) + I_R Z \right] \\ &= V_R Y \left( 1 + \frac{1}{4} YZ \right) + I_R \left( 1 + \frac{1}{2} YZ \right) \end{aligned}$$

Finally, we have

$$\begin{bmatrix} V_s \\ I_S \end{bmatrix} = \begin{bmatrix} \left( 1 + \frac{1}{2} YZ \right) & Z \\ Y \left( 1 + \frac{1}{4} YZ \right) & \left( 1 + \frac{1}{2} YZ \right) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \quad (5.13)$$

It should be noted that nominal- $T$  and nominal- $\pi$  with the above constants are not equivalent to each other. The reader should verify this fact by applying star-delta transformation to either one.

**Example 5.4** Using the nominal- $\pi$  method, find the sending-end voltage and voltage regulation of a 250 km, three-phase, 50 Hz, transmission line delivering 25 MVA at 0.8 lagging power factor to a balanced load at 132 kV. The line conductors are spaced equilaterally 3 m apart. The conductor resistance is 0.11 ohm/km and its effective diameter is 1.6 cm. Neglect leakance.

**Solution**

Now,

$$L = 0.461 \log \frac{D}{r'} = 0.461 \log \frac{300}{0.7788 \times 0.8} = 1.24 \text{ mH/km}$$

$$C = \frac{0.0242}{\log D/r} = \frac{0.0242}{\log \frac{300}{0.8}} = 0.0094 \mu\text{F/km}$$

$$R = 0.11 \times 250 = 27.5 \Omega$$

$$X = 2\pi fL = 2\pi \times 50 \times 1.24 \times 10^{-3} \times 250 = 97.4 \Omega$$

$$Z = R + jX = 27.5 + j97.4 = 101.2 \angle 74.2^\circ \Omega$$

$$\begin{aligned} Y &= j\omega Cl = 314 \times 0.0094 \times 10^{-6} \times 250 \angle 90^\circ \\ &= 7.38 \times 10^{-4} \angle 90^\circ \Omega \end{aligned}$$

$$I_R = \frac{25 \times 1,000}{\sqrt{3} \times 132} \angle -36.9^\circ = 109.3 \angle -36.9^\circ \text{ A}$$

$$V_R (\text{per phase}) = (132/(\sqrt{3})) \angle 0^\circ = 76.2 \angle 0^\circ \text{ kV}$$

$$\begin{aligned} V_S &= \left( 1 + \frac{1}{2} YZ \right) V_R + ZI_R \\ &= \left( 1 + \frac{1}{2} \times 7.38 \times 10^{-4} \angle 90^\circ \times 101.2 \angle 74.2^\circ \right) \times 76.2 \\ &\quad + 101.2 \angle 74.2^\circ \times 109.3 \times 10^{-3} \angle -36.9^\circ \\ &= 76.2 + 2.85 \angle 164.2^\circ + 11.06 \angle 37.3^\circ \\ &= 82.26 + j7.48 = 82.6 \angle 5.2^\circ \end{aligned}$$

$$\therefore |V_S| (\text{line}) = 82.6 \times \sqrt{3} = 143 \text{ kV}$$

$$1 + \frac{1}{2} YZ = 1 + 0.0374 \angle 164.2^\circ = 0.964 + j0.01$$

$$|V_{R0}| (\text{line no load}) = \frac{143}{\left| 1 + \frac{1}{2} YZ \right|} = \frac{143}{0.964} = 148.3 \text{ kV}$$

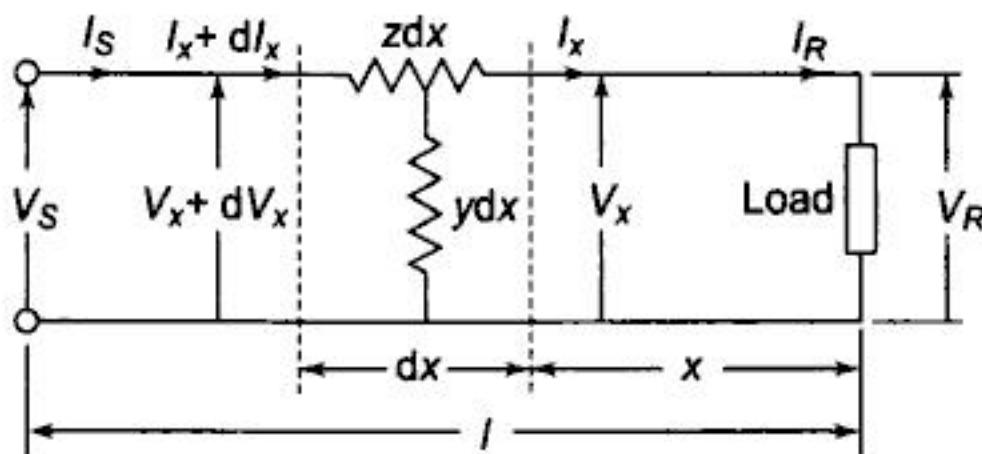
$$\therefore \text{Voltage regulation} = \frac{148.3 - 132}{132} \times 100 = 12.3\%$$

## 5.4 THE LONG TRANSMISSION LINE—RIGOROUS SOLUTION

For lines over 250 km, the fact that the parameters of a line are not lumped but distributed uniformly throughout its length, must be considered.

Figure 5.10 shows one phase and the neutral return (of zero impedance) of a transmission line. Let  $dx$  be an elemental section of the line at a distance  $x$  from the receiving-end having a series impedance  $z dx$  and a shunt admittance  $y dx$ . The rise in voltage\* to neutral over the elemental section in the direction of increasing  $x$  is  $dV_x$ . We can write the following differential relationships across the elemental section:

\* Here  $V_x$  is the complex expression of the rms voltage, whose magnitude and phase vary with distance along the line.



**Fig. 5.10** Schematic diagram of a long line

$$dV_x = I_x z \, dx \quad \text{or} \quad \frac{dV_x}{dx} = zI_x \quad (5.14)$$

$$dI_x = V_x y \, dx \quad \text{or} \quad \frac{dI_x}{dx} = yV_x \quad (5.15)$$

It may be noticed that the kind of connection (e.g.  $T$  or  $\pi$ ) assumed for the elemental section, does not affect these first order differential relations.

Differentiating Eq. (5.14) with respect to  $x$ , we obtain

$$\frac{d^2 V_x}{dx^2} = \frac{dI_x}{dx} z$$

Substituting the value of  $\frac{dI_x}{dx}$  from Eq. (5.15), we get

$$\frac{d^2 V_x}{dx^2} = yzV_x \quad (5.16)$$

This is a linear differential equation whose general solution can be written as follows:

$$V_x = C_1 e^{\gamma x} + C_2 e^{-\gamma x} \quad (5.17)$$

where

$$\gamma = \sqrt{yz} \quad (5.18)$$

and  $C_1$  and  $C_2$  are arbitrary constants to be evaluated.

Differentiating Eq. (5.17) with respect to  $x$ ,

$$\frac{dV_x}{dx} = C_1 \gamma e^{\gamma x} - C_2 \gamma e^{-\gamma x} = zI_x$$

$$I_x = \frac{C_1}{Z_c} e^{\gamma x} - \frac{C_2}{Z_c} e^{-\gamma x} \quad (5.19)$$

where

$$Z_c = \left( \frac{z}{\gamma} \right)^{1/2} \quad (5.20)$$

The constants  $C_1$  and  $C_2$  may be evaluated by using the end conditions, i.e. when  $x = 0$ ,  $V_x = V_R$  and  $I_x = I_R$ . Substituting these values in Eqs (5.17) and (5.19) gives

$$\begin{aligned} V_R &= C_1 + C_2 \\ I_R &= \frac{1}{Z_c} (C_1 - C_2) \end{aligned}$$

which upon solving yield

$$\begin{aligned} C_1 &= \frac{1}{2} (V_R + Z_c I_R) \\ C_2 &= \frac{1}{2} (V_R - Z_c I_R) \end{aligned}$$

With  $C_1$  and  $C_2$  as determined above, Eqs (5.17) and (5.19) yield the solution for  $V_x$  and  $I_x$  as

$$V_x = \left( \frac{V_R + Z_c I_R}{2} \right) e^{\gamma x} + \left( \frac{V_R - Z_c I_R}{2} \right) e^{-\gamma x} \quad (5.21)$$

$$I_x = \left( \frac{V_R/Z_c + I_R}{2} \right) e^{\gamma x} - \left( \frac{V_R/Z_c - I_R}{2} \right) e^{-\gamma x}$$

Here  $Z_c$  is called the *characteristic impedance* of the line and  $\gamma$  is called the *propagation constant*.

Knowing  $V_R$ ,  $I_R$  and the parameters of the line, using Eq. (5.21) complex number rms values of  $V_x$  and  $I_x$  at any distance  $x$  along the line can be easily found out.

A more convenient form of expression for voltage and current is obtained by introducing hyperbolic functions. Rearranging Eq. (5.21), we get

$$V_x = V_R \left( \frac{e^{\gamma x} + e^{-\gamma x}}{2} \right) + I_R Z_c \left( \frac{e^{\gamma x} - e^{-\gamma x}}{2} \right)$$

$$I_x = V_R \frac{1}{Z_c} \left( \frac{e^{\gamma x} - e^{-\gamma x}}{2} \right) + I_R \left( \frac{e^{\gamma x} + e^{-\gamma x}}{2} \right)$$

These can be rewritten after introducing hyperbolic functions, as

$$V_x = V_R \cosh \gamma x + I_R Z_c \sinh \gamma x \quad (5.22)$$

$$I_x = I_R \cosh \gamma x + V_R \frac{1}{Z_c} \sinh \gamma x$$

when  $x = l$ ,  $V_x = V_S$ ,  $I_x = I_S$

$$\therefore \begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \cosh \gamma l & Z_c \sinh \gamma l \\ \frac{1}{Z_c} \sinh \gamma l & \cosh \gamma l \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \quad (5.23)$$

Here

$$A = D = \cosh \gamma l$$

$$B = Z_c \sinh \gamma l \quad (5.24)$$

$$C = \frac{1}{Z_c} \sinh \gamma l$$

In case  $[V_S \ I_S]$  is known,  $[V_R \ I_R]$  can be easily found by inverting Eq. (5.23). Thus

$$\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \begin{bmatrix} V_S \\ I_S \end{bmatrix} \quad (5.25)$$

### Evaluation of ABCD Constants

The  $ABCD$  constants of a long line can be evaluated from the results given in Eq. (5.24). It must be noted that  $\gamma = \sqrt{yz}$  is in general a complex number and can be expressed as

$$\gamma = \alpha + j\beta \quad (5.26)$$

The hyperbolic function of complex numbers involved in evaluating  $ABCD$  constants can be computed by any one of the three methods given below:

#### Method 1

$$\begin{aligned} \cosh(\alpha l + j\beta l) &= \cosh \alpha l \cos \beta l + j \sinh \alpha l \sin \beta l \\ \sinh(\alpha l + j\beta l) &= \sinh \alpha l \cos \beta l + j \cosh \alpha l \sin \beta l \end{aligned} \quad (5.27)$$

Note that sinh, cosh, sin and cos of real numbers as in Eq. (5.27) can be looked up in standard tables.

#### Method 2

$$\cosh \gamma l = 1 + \frac{\gamma^2 l^2}{2!} + \frac{\gamma^4 l^4}{4!} + \dots \approx \left(1 + \frac{YZ}{2}\right) \quad (5.28a)$$

$$\sinh \gamma l = \gamma l + \frac{\gamma^3 l^3}{3!} + \frac{\gamma^5 l^5}{5!} + \dots \approx \sqrt{YZ} \left(1 + \frac{YZ}{6}\right)$$

This series converges rapidly for values of  $\gamma l$  usually encountered for power lines and can be conveniently approximated as above. The corresponding expressions for  $ABCD$  constants are

$$\begin{aligned} A = D &\approx 1 + \frac{YZ}{2} \\ B &\approx Z \left( 1 + \frac{YZ}{6} \right) \\ C &\approx Y \left( 1 + \frac{YZ}{6} \right) \end{aligned} \quad (5.28b)$$

The above approximation is computationally convenient and quite accurate for lines up to 400/500 km.

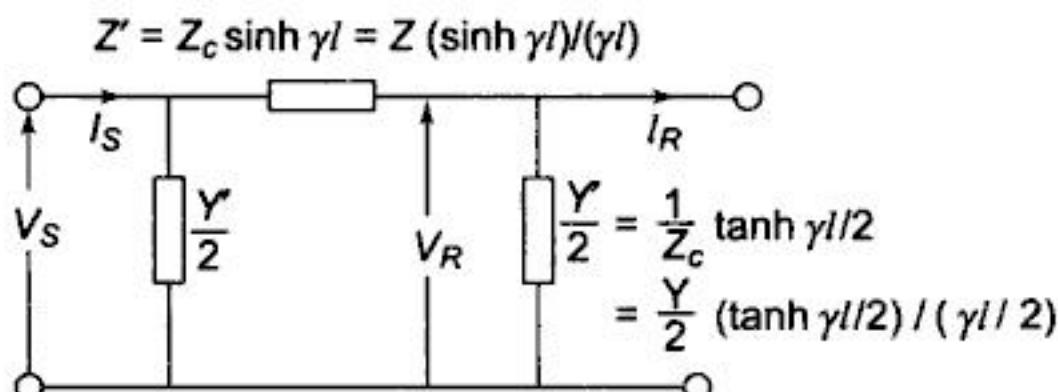
### Method 3

$$\cosh(\alpha l + j\beta l) = \frac{e^{\alpha l} e^{j\beta l} + e^{-\alpha l} e^{-j\beta l}}{2} = \frac{1}{2} (e^{\alpha l} \angle \beta l + e^{-\alpha l} \angle -\beta l) \quad (5.29)$$

$$\sinh(\alpha l + j\beta l) = \frac{e^{\alpha l} e^{j\beta l} - e^{-\alpha l} e^{-j\beta l}}{2} = \frac{1}{2} (e^{\alpha l} \angle \beta l - e^{-\alpha l} \angle -\beta l)$$

## 5.5 THE EQUIVALENT CIRCUIT OF A LONG LINE

So far as the end conditions are concerned, the exact equivalent circuit of a transmission line can be established in the form of a  $T$ - or  $\pi$ -network.



**Fig. 5.11** Equivalent- $\pi$  network of a transmission line

The parameters of the equivalent network are easily obtained by comparing the performance equations of a  $\pi$ -network and a transmission line in terms of end quantities.

For a  $\pi$ -network shown in Fig. 5.11 [refer to Eq. (5.13)].

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} \left( 1 + \frac{1}{2} Y' Z' \right) & Z' \\ Y' \left( 1 + \frac{1}{4} Y' Z' \right) & \left( 1 + \frac{1}{2} Y' Z' \right) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \quad (5.30)$$

According to exact solution of a long line [refer to Eq. (5.23)].

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \cosh \gamma l & Z_c \sinh \gamma l \\ \frac{1}{Z_c} \sinh \gamma l & \cosh \gamma l \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \quad (5.31)$$

For exact equivalence, we must have

$$Z' = Z_c \sinh \gamma l \quad (5.32)$$

$$1 + \frac{1}{2} Y' Z' = \cosh \gamma l \quad (5.33)$$

From Eq. (5.32)

$$Z' = \sqrt{\frac{z}{y}} \sinh \gamma l = zl \frac{\sinh \gamma l}{l \sqrt{yz}} = Z \left( \frac{\sinh \gamma l}{\gamma l} \right) \quad (5.34)$$

Thus  $\frac{\sinh \gamma l}{\gamma l}$  is the factor by which the series impedance of the nominal- $\pi$

must be multiplied to obtain the  $Z'$  parameter of the equivalent- $\pi$ .

Substituting  $Z'$  from Eq. (5.32) in Eq. (5.33), we get

$$1 + \frac{1}{2} Y' Z_c \sinh \gamma l = \cosh \gamma l$$

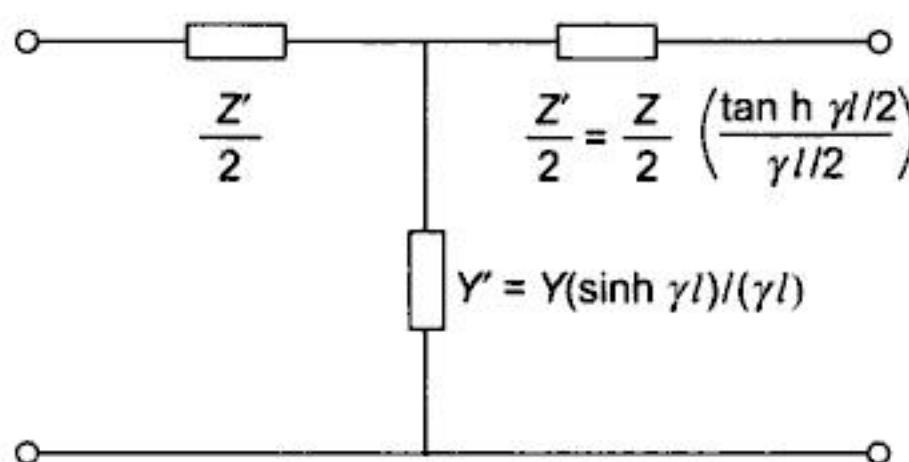
$$\begin{aligned} \therefore \frac{1}{2} Y' &= \frac{1}{Z_c} \left( \frac{\cosh \gamma l - 1}{\sinh \gamma l} \right) \\ &= \frac{1}{Z_c} \tanh \frac{\gamma l}{2} = \sqrt{\frac{y}{z}} \tanh \frac{\gamma l}{2} \\ &= \frac{yl}{2} \left( \frac{\tanh \gamma l/2}{\gamma l/2} \right) \end{aligned}$$

$$\text{or} \quad \frac{1}{2} Y' = \frac{Y}{2} \left( \frac{\tanh \gamma l/2}{\gamma l/2} \right) \quad (5.35)$$

Thus  $\left( \frac{\tanh \gamma l/2}{\gamma l/2} \right)$  is the factor by which the shunt admittance arm of the nominal- $\pi$  must be multiplied to obtain the shunt parameter ( $Y'/2$ ) of the equivalent- $\pi$ .

Note that  $Y' \left( 1 + \frac{1}{2} Y' Z' \right) = \frac{1}{Z_c} \sinh \gamma l$  is a consistent equation in terms of the above values of  $Y'$  and  $Z'$ .

For a line of medium length  $\frac{\tanh \gamma l/2}{\gamma l/2} \approx 1$  and  $\frac{\sinh \gamma l}{\gamma l} \approx 1$  so that the equivalent- $\pi$  network reduces to that of nominal- $\pi$ . The equivalent- $T$  network is shown in Fig. 5.12.



**Fig. 5.12** Equivalent- $T$  network of a transmission line

Equivalent- $T$  network parameters of a transmission line are obtained on similar lines.

As we shall see in Ch. 6 equivalent- $\pi$  (or nominal- $\pi$ ) network is easily adopted to load flow studies and is, therefore, universally employed.

**Example 5.5** A 50 Hz transmission line 300 km long has a total series impedance of  $40 + j125$  ohms and a total shunt admittance of  $10^{-3}$  mho. The receiving-end load is 50 MW at 220 kV with 0.8 lagging power factor. Find the sending-end voltage, current, power and power factor using

- (a) short line approximation,
- (b) nominal- $\pi$  method,
- (c) exact transmission line equation [Eq. (5.27)],
- (d) approximation [Eq. (5.28b)].

Compare the results and comment.

**Solution**

$$Z = 40 + j125 = 131.2 \angle 72.3^\circ \Omega$$

$$Y = 10^{-3} \angle 90^\circ \Omega$$

The receiving-end load is 50 MW at 220 kV, 0.8 pf lagging.

$$\therefore I_R = \frac{50}{\sqrt{3} \times 220 \times 0.8} \angle -36.9^\circ = 0.164 \angle -36.9^\circ \text{ kA}$$

$$V_R = \frac{220}{\sqrt{3}} \angle 0^\circ = 127 \angle 0^\circ \text{ kV}$$

(a) Short line approximation:

From Eq. (5.3),

$$\begin{aligned} V_S &= 127 + 0.164 \angle -36.9^\circ \times 131.2 \angle 72.3^\circ \\ &= 145 \angle 4.9^\circ \end{aligned}$$

$$|V_S|_{\text{line}} = 251.2 \text{ kV}$$

$$I_S = I_R = 0.164 \angle -36.9^\circ \text{ kA}$$

Sending-end power factor =  $\cos(4.9^\circ + 36.9^\circ = 41.8^\circ)$

$$= 0.745 \text{ lagging}$$

$$\begin{aligned} \text{Sending-end power} &= \sqrt{3} \times 251.2 \times 0.164 \times 0.745 \\ &= 53.2 \text{ MW} \end{aligned}$$

(b) Nominal- $\pi$  method:

$$A = D = 1 + \frac{1}{2} YZ = 1 + \frac{1}{2} \times 10^{-3} \angle 90^\circ \times 131.2 \angle 72.3^\circ$$

$$= 1 + 0.0656 \angle 162.3^\circ = 0.938 \angle 1.2^\circ$$

$$B = Z = 131.2 \angle 72.3^\circ$$

$$C = Y \left( 1 + \frac{1}{4} YZ \right) = Y + \frac{1}{4} Y^2 Z$$

$$= 0.001 \angle 90^\circ + \frac{1}{4} \times 10^{-6} \angle 180^\circ \times 131.2 \angle 72.3^\circ$$

$$= 0.001 \angle 90^\circ$$

$$V_S = 0.938 \angle 1.2^\circ \times 127 + 131.2 \angle 72.3^\circ \times 0.164 \angle -36.9^\circ$$

$$= 119.1 \angle 1.2^\circ + 21.5 \angle 35.4^\circ = 137.4 \angle 6.2^\circ$$

$$|V_S|_{\text{line}} = 238 \text{ kV}$$

$$I_S = 0.001 \angle 90^\circ \times 127 + 0.938 \angle 1.2^\circ \times 0.164 \angle -36.9^\circ$$

$$= 0.127 \angle 90^\circ + 0.154 \angle -35.7^\circ = 0.13 \angle 16.5^\circ$$

Sending-end pf =  $\cos(16.5^\circ - 6.2^\circ) = 0.984$  leading

Sending-end power =  $\sqrt{3} \times 238 \times 0.13 \times 0.984$

$$= 52.7 \text{ MW}$$

(c) Exact transmission line equations (Eq. 5.29).

$$\gamma l = \alpha l + j\beta l = \sqrt{YZ}$$

$$= \sqrt{10^{-3} \angle 90^\circ \times 131.2 \angle 72.3^\circ}$$

$$= 0.0554 + j0.3577$$

$$= 0.362 \angle 81.2^\circ$$

$$\cosh(\alpha l + j\beta l) = \frac{1}{2}(e^{\alpha l} \angle \beta l + e^{-\alpha l} \angle -\beta l)$$

$$\beta l = 0.3577 \text{ (radians)} = \angle 20.49^\circ$$

$$e^{0.0554} \angle (20.49^\circ) = 1.057 \angle 20.49^\circ = 0.99 + j0.37$$

$$e^{-0.0554} \angle -20.49^\circ = 0.946 \angle -20.49^\circ = 0.886 - j0.331$$

$$\therefore \cosh \gamma l = 0.938 + j0.02 = 0.938 \angle 1.2^\circ$$

$$\sinh \gamma l = 0.052 + j0.35 = 0.354 \angle 81.5^\circ$$

$$Z_c = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{131.2 \angle 72.3^\circ}{10^{-3} \angle 90^\circ}} = 362.21 \angle -8.85^\circ$$

$$A = D = \cosh \gamma l = 0.938 \angle 1.2^\circ$$

$$B = Z_c \sinh \gamma l = 362.21 \angle -8.85^\circ \times 0.354 \angle 81.5^\circ \\ = 128.2 \angle 72.65^\circ$$

Now

$$V_S = 0.938 \angle 1.2^\circ \times 127 \angle 0^\circ + 128.2 \angle 72.65^\circ \times 0.164 \angle -36.9^\circ \\ = 119.13 \angle 1.2^\circ + 21.03 \angle 35.75^\circ \\ = 136.97 \angle 6.2^\circ \text{ kV}$$

$$|V_S|_{\text{line}} = 237.23 \text{ kV}$$

$$C = \frac{1}{Z_c} \sinh \gamma l = \frac{1}{362.21 \angle -8.85^\circ} \times 0.354 \angle 81.5^\circ \\ = 9.77 \times 10^{-4} \angle 90.4^\circ$$

$$I_S = 9.77 \times 10^{-4} \angle 90.4^\circ \times 127 + 0.938 \angle 1.2^\circ \times 0.164 \angle -36.9^\circ \\ = 0.124 \angle 90.4^\circ + 0.154 \angle -35.7^\circ \\ = 0.1286 \angle 15.3^\circ \text{ kA}$$

Sending-end pf =  $\cos(15.3^\circ - 6.2^\circ = 9.1^\circ) = 0.987$  leading

$$\text{Sending-end power} = \sqrt{3} \times 237.23 \times 0.1286 \times 0.987 \\ = 52.15 \text{ MW}$$

(d) Approximation (5.28b):

$$A = D = 1 + \frac{1}{2} YZ$$

$$= 0.938 \angle 1.2^\circ \text{ (already calculated in part (b))}$$

$$B = Z \left( 1 + \frac{YZ}{6} \right) = Z + \frac{1}{6} YZ^2$$

$$= 131.2 \angle 72.3^\circ + \frac{1}{6} \times 10^{-3} \angle 90^\circ \times (131.2)^2 \angle 144.6^\circ$$

$$= 131.2 \angle 72.3^\circ + 2.87 \angle -125.4^\circ$$

$$= 128.5 \angle 72.7^\circ$$

$$C = Y \left( 1 + \frac{YZ}{6} \right) = 0.001 \angle 90^\circ + \frac{1}{6} \times 10^{-6} \angle 180^\circ \times 131.2 \angle 72.3^\circ$$

$$= 0.001 \angle 90^\circ$$

$$V_S = 0.938 \angle 1.2^\circ \times 127 \angle 0^\circ + 128.5 \angle 72.7^\circ \times 0.164 \angle -36.9^\circ$$

$$= 119.13 \angle 1.2^\circ + 21.07 \angle 35.8^\circ = 136.2 + j14.82$$

$$= 137 \angle 6.2^\circ \text{ kV}$$

$$|V_S|_{\text{line}} = 237.3 \text{ kV}$$

$$I_S = 0.13 \angle 16.5^\circ \text{ (same as calculated in part (b))}$$

$$\text{Sending-end pf} = \cos (16.5^\circ - 6.2^\circ = 10.3^\circ) = 0.984 \text{ leading}$$

$$\text{Sending-end power} = \sqrt{3} \times 237.3 \times 0.13 \times 0.984$$

$$= 52.58 \text{ MW}$$

The results are tabulated as:

	<i>Short line approximation</i>	<i>Nominal-<math>\pi</math></i>	<i>Exact</i>	<i>Approximation (5.28b)</i>
$ V_S _{\text{line}}$	251.2 kV	238 kV	237.23 kV	237.3 kV
$I_S$	$0.164 \angle -36.9^\circ \text{ kA}$	$0.13 \angle 16.5^\circ \text{ kA}$	$0.1286 \angle 15.3^\circ \text{ kA}$	$0.13 \angle 16.5^\circ \text{ kA}$
$\text{pf}_S$	0.745 lagging	0.984 leading	0.987 leading	0.984 leading
$P_S$	53.2 MW	52.7 MW	52.15 MW	52.58 MW

*Comments:* We find from the above example that the results obtained by the nominal- $\pi$  method and the approximation (5.28b) are practically the same and are very close to those obtained by exact calculations (part (c)). On the other hand the results obtained by the short line approximation are in considerable error. Therefore, for a line of this length (about 300 km), it is sufficiently accurate

to use the nominal- $\pi$  (or approximation (5.28b)) which results in considerable saving in computational effort.

## 5.6 INTERPRETATION OF THE LONG LINE EQUATIONS

As already said in Eq. (5.26),  $\gamma$  is a complex number which can be expressed as

$$\gamma = \alpha + j\beta$$

The real part  $\alpha$  is called the *attenuation constant* and the imaginary part  $\beta$  is called the *phase constant*. Now  $V_x$  of Eq. (5.21) can be written as

$$V_x = \left| \frac{V_R + Z_c I_R}{2} \right| e^{\alpha x} e^{j(\beta x + \phi_1)} + \left| \frac{V_R - Z_c I_R}{2} \right| e^{-\alpha x} e^{-j(\beta x - \phi_2)}$$

where

$$\phi_1 = \angle(V_R + I_R Z_c) \quad (5.36)$$

$$\phi_2 = \angle(V_R - I_R Z_c)$$

The instantaneous voltage  $v_x(t)$  can be written from Eq. (5.36) as

$$v_x(t) = \operatorname{Re} \left[ \sqrt{2} \left| \frac{V_R + Z_c I_R}{2} \right| e^{\alpha x} e^{j(\omega t + \beta x + \phi_1)} + \sqrt{2} \left| \frac{V_R - Z_c I_R}{2} \right| e^{-\alpha x} e^{j(\omega t - \beta x + \phi_2)} \right] \quad (5.37)$$

The instantaneous voltage consists of two terms each of which is a function of two variables—time and distance. Thus they represent two travelling waves, i.e.

$$v_x = v_{x_1} + v_{x_2} \quad (5.38)$$

Now

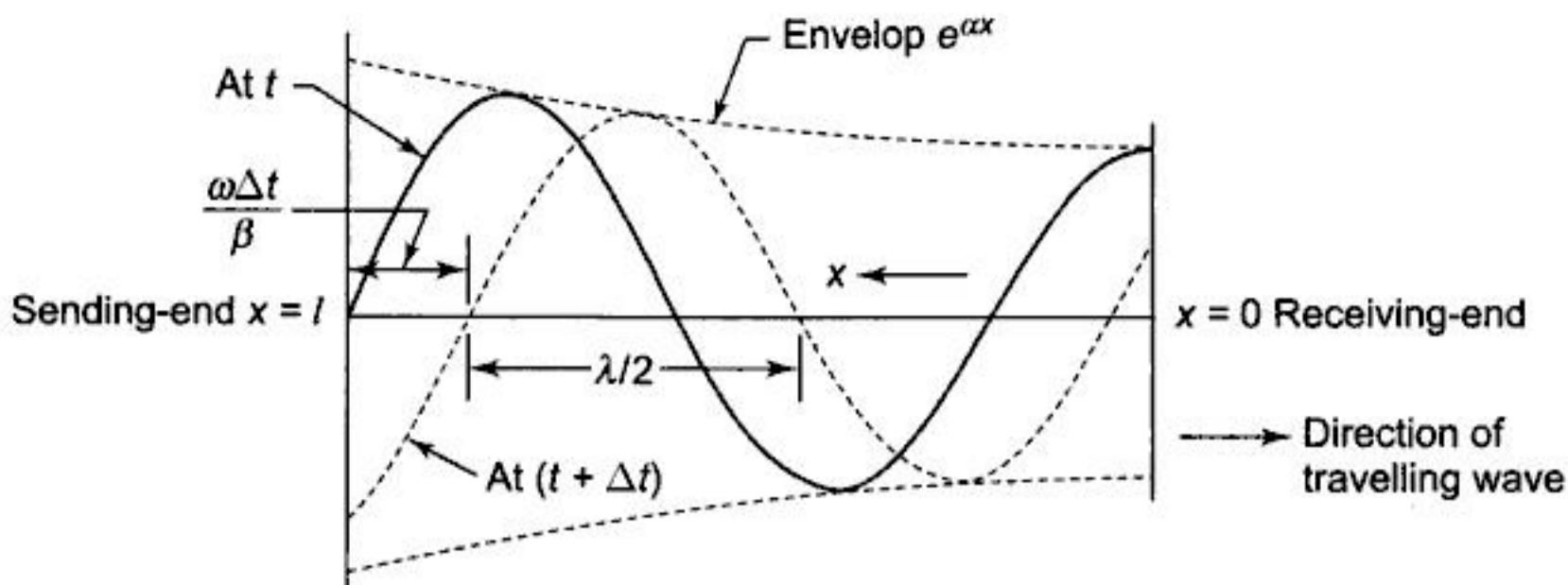
$$v_{x_1} = \sqrt{2} \left| \frac{V_R + Z_c I_R}{2} \right| e^{\alpha x} \cos(\omega t + \beta x + \phi_1) \quad (5.39)$$

At any instant of time  $t$ ,  $v_{x_1}$  is sinusoidally distributed along the distance from the receiving-end with amplitude increasing exponentially with distance, as shown in Fig. 5.13 ( $\alpha > 0$  for a line having resistance).

After time  $\Delta t$ , the distribution advances in distance phase by  $(\omega \Delta t / \beta)$ . Thus this wave is travelling towards the receiving-end and is the *incident wave*. Line losses cause its amplitude to decrease exponentially in going from the sending- to the receiving-end.

Now

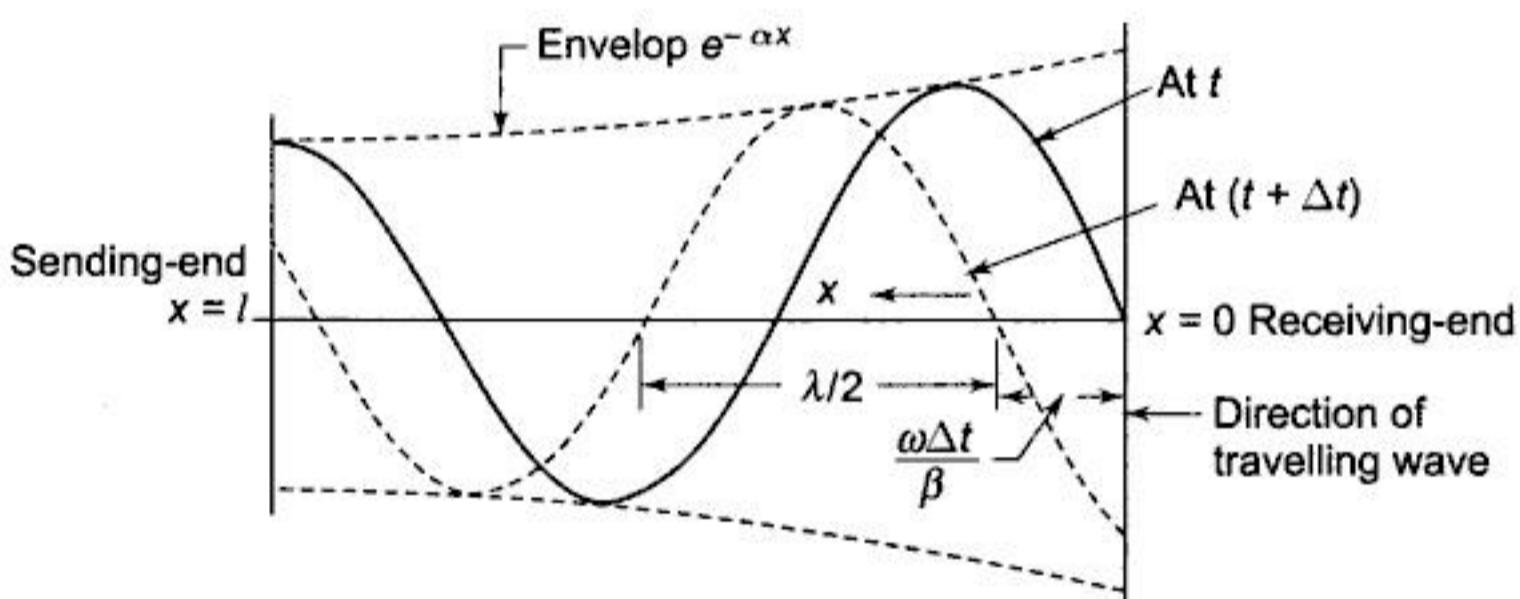
$$v_{x_2} = \sqrt{2} \left| \frac{V_R - Z_c I_R}{2} \right| e^{-\alpha x} \cos(\omega t - \beta x + \phi_2) \quad (5.40)$$



**Fig. 5.13** Incident wave

After time  $\Delta t$  the voltage distribution retards in distance phase by  $(\omega \Delta t / \beta)$ . This is the *reflected wave* travelling from the receiving-end to the sending-end with amplitude decreasing exponentially in going from the receiving-end to the sending-end, as shown in Fig. 5.14.

At any point along the line, the voltage is the sum of incident and reflected voltage waves present at the point [Eq. (5.38)]. The same is true of current waves. Expressions for incident and reflected current waves can be similarly written down by proceeding from Eq. (5.21). If  $Z_c$  is pure resistance, current waves can be simply obtained from voltage waves by dividing by  $Z_c$ .



**Fig. 5.14** Reflected wave

If the load impedance  $Z_L = \frac{V_R}{I_R} = Z_c$ , i.e. the line is terminated in its characteristic impedance, the reflected voltage wave is zero ( $V_R - Z_c I_R = 0$ ).

A line terminated in its characteristic impedance is called the *infinite line*. The incident wave under this condition cannot distinguish between a termination and an infinite continuation of the line.

Power system engineers normally call  $Z_c$  the *surge impedance*. It has a value of about 400 ohms for an overhead line and its phase angle normally varies from  $0^\circ$  to  $-15^\circ$ . For underground cables  $Z_c$  is roughly one-tenth of the value for overhead lines. The term surge impedance is, however, used in connection with surges (due to lightning or switching) or transmission lines, where the line loss can be neglected such that

$$Z_c = Z_s = \left( \frac{j\omega L}{-j\omega C} \right)^{1/2} = \left( \frac{L}{C} \right)^{1/2}, \text{ a pure resistance}$$

*Surge Impedance Loading (SIL)* of a transmission line is defined as the power delivered by a line to purely resistive load equal in value to the surge impedance of the line. Thus for a line having 400 ohms surge impedance,

$$\begin{aligned} \text{SIL} &= \sqrt{3} \frac{|V_R|}{\sqrt{3} \times 400} |V_R| \times 1000 \text{ kW} \\ &= 2.5 |V_R|^2 \text{ kW} \end{aligned} \quad (5.41)$$

where  $|V_R|$  is the line-to-line receiving-end voltage in kV. Sometimes, it is found convenient to express line loading in per unit of SIL, i.e. as the ratio of the power transmitted to surge impedance loading.

At any time the voltage and current vary harmonically along the line with respect to  $x$ , the space coordinate. A complete voltage or current cycle along the line corresponds to a change of  $2\pi$  rad in the angular argument  $\beta x$ . The corresponding line length is defined as the *wavelength*.

If  $\beta$  is expressed in rad/m,

$$\lambda = 2\pi/\beta \text{ m} \quad (5.42)$$

Now for a typical power transmission line

$$g \text{ (shunt conductance/unit length)} \approx 0$$

$$r \ll \omega L$$

$$\begin{aligned} \therefore \gamma &= (yz)^{1/2} = (j\omega C(r + j\omega L))^{1/2} \\ &= j\omega(LC)^{1/2} \left( 1 - j \frac{r}{\omega L} \right)^{1/2} \end{aligned}$$

or

$$\gamma = \alpha + j\beta \approx j\omega(LC)^{1/2} \left( 1 - j \frac{r}{2\omega L} \right)$$

$$\therefore \alpha \approx \frac{r}{2} \left( \frac{C}{L} \right)^{1/2} \quad (5.43)$$

$$\beta \approx \omega(LC)^{1/2} \quad (5.44)$$

Now time for a phase change of  $2\pi$  is  $(1/f)$  sec., where  $f = \omega/2\pi$  is the frequency in cycles/s. During this time the wave travels a distance equal to  $\lambda$ , i.e. one wavelength.

$$\therefore \text{Velocity of propagation of wave, } v = \frac{\lambda}{1/f} = f\lambda \text{ m/s} \quad (5.45)$$

which is a well known result.

For a lossless transmission line ( $R = 0, G = 0$ ),

$$\gamma = (yz)^{1/2} = j\omega(LC)^{1/2}$$

such that  $\alpha = 0, \beta = \omega(LC)^{1/2}$

$$\therefore \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega(LC)^{1/2}} = \frac{1}{f(LC)^{1/2}} \text{ m} \quad (5.46)$$

and

$$v = f\lambda = 1/(LC)^{1/2} \text{ m/s} \quad (5.47)$$

For a single-phase transmission line

$$L = \frac{\mu_0}{2\pi} \ln \frac{D}{r'}$$

$$C = \frac{2\pi k_0}{\ln D/r}$$

$$\therefore v = \frac{1}{\left( \frac{\mu_0}{2\pi} \ln \frac{D}{r'} \frac{2\pi k_0}{\ln D/r} \right)^{1/2}}$$

Since  $r$  and  $r'$  are quite close to each other, when log is taken, it is sufficiently accurate to assume that  $\ln \frac{D}{r'} \approx \ln D/r$ .

$$\therefore v \approx \frac{1}{(\mu_0 k_0)^{1/2}} = \text{velocity of light} \quad (5.48)$$

The actual velocity of the propagation of wave along the line would be somewhat less than the velocity of light.

Wavelength of a 50 Hz power transmission is approximately given by

$$\lambda \approx \frac{3 \times 10^8}{50} = 6,000 \text{ km}$$

Practical transmission lines are much shorter than this (usually several hundred kilometres). It needs to be pointed out here that the waves drawn in Figs 5.13 and 5.14 are for illustration only and do not pertain to a real power transmission line.

**Example 5.6** A three-phase 50 Hz transmission line is 400 km long. The voltage at the sending-end is 220 kV. The line parameters are  $r = 0.125 \text{ ohm/km}$ ,  $x = 0.4 \text{ ohm/km}$  and  $y = 2.8 \times 10^{-6} \text{ mho/km}$ .

Find the following:

- The sending-end current and receiving-end voltage when there is no-load on the line.

- (b) The maximum permissible line length if the receiving-end no-load voltage is not to exceed 235 kV.  
 (c) For part (a), the maximum permissible line frequency, if the no-load voltage is not to exceed 250 kV.

**Solution**

The total line parameters are:

$$R = 0.125 \times 400 = 50.0 \Omega$$

$$X = 0.4 \times 400 = 160.0 \Omega$$

$$Y = 2.8 \times 10^{-6} \times 400 \angle 90^\circ = 1.12 \times 10^{-3} \angle 90^\circ \Omega$$

$$Z = R + jX = (50.0 + j160.0) = 168.0 \angle 72.6^\circ \Omega$$

$$\begin{aligned} YZ &= 1.12 \times 10^{-3} \angle 90^\circ \times 168 \angle 72.6^\circ \\ &= 0.188 \angle 162.6^\circ \end{aligned}$$

(a) At no-load

$$V_S = AV_R; I_S = CV_R$$

$A$  and  $C$  are computed as follows:

$$\begin{aligned} A &\approx 1 + \frac{1}{2}YZ = 1 + \frac{1}{2} \times 0.188 \angle 162.6^\circ \\ &= 0.91 + j0.028 \end{aligned}$$

$$|A| = 0.91$$

$$\begin{aligned} C &= Y(1 + YZ/6) = 1.12 \times 10^{-3} \angle 90^\circ \left(1 + \frac{0.188}{6} \angle 162.6^\circ\right) \\ &= 1.09 \times 10^{-3} \angle 90.55^\circ \end{aligned}$$

Now

$$|V_R|_{\text{line}} = \frac{220}{|A|} = \frac{220}{0.91} = 242 \text{ kV}$$

$$|I_S| = |C| |V_R| = 1.09 \times 10^{-3} \times \frac{242}{\sqrt{3}} \times 10^3 = 152 \text{ A}$$

It is to be noted that under no-load conditions, the receiving-end voltage (242 kV) is more than the sending-end voltage. This phenomenon is known as the Ferranti effect and is discussed at length in Sec. 5.7.

(b) Maximum permissible no-load receiving-end voltage = 235 kV.

$$|A| = \left| \frac{V_S}{V_R} \right| = \frac{220}{235} = 0.936$$

Now

$$\begin{aligned} A &\approx 1 + \frac{1}{2}YZ \\ &= 1 + \frac{1}{2}l^2 \times j2.8 \times 10^{-6} \times (0.125 + j0.4) \\ &= (1 - 0.56 \times 10^{-6}l^2) + j0.175 \times 10^{-6}l^2 \end{aligned}$$

Since the imaginary part will be less than  $\frac{1}{10}$  th of the real part,  $|A|$  can be approximated as

$$|A| = 1 - 0.56 \times 10^{-6}l^2 = 0.936$$

$$\therefore l^2 = \frac{1 - 0.936}{0.56 \times 10^{-6}}$$

or  $l = 338 \text{ km}$

(c)  $|A| = \frac{220}{250} = 0.88$

$$A \approx 1 + \frac{1}{2} \times j1.12 \times 10^{-3} \times \frac{f}{50} \left( 50 + j160 \times \frac{f}{50} \right)$$

Neglecting the imaginary part, we can write

$$|A| = 1 - \frac{1}{2} \times 1.12 \times 10^{-3} \times 160 \times \frac{f^2}{(50)^2} = 0.88$$

Simplifying, we obtain the maximum permissible frequency as

$$f = 57.9 \text{ Hz}$$

**Example 5.7** If in Example 5.6 the line is open-circuited with a receiving-end voltage of 220 kV, find the rms value and phase angle of the following:

- (a) The incident and reflected voltages to neutral at the receiving-end.
- (b) The incident and reflected voltages to neutral at 200 km from the receiving-end.
- (c) The resultant voltage at 200 km from the receiving-end.

*Note:* Use the receiving-end line to neutral voltage as reference.

**Solution**

From Example 5.6, we have following line parameters:

$$r = 0.125 \Omega/\text{km}; x = 0.4 \Omega/\text{km}; y = j2.8 \times 10^{-6} \Omega/\text{km}$$

$$\therefore z = (0.125 + j0.4) \Omega/\text{km} = 0.42 \angle 72.6^\circ \Omega/\text{km}$$

$$\begin{aligned} \gamma &= (yz)^{1/2} = (2.8 \times 10^{-6} \times 0.42 \angle (90^\circ + 72.6^\circ))^{1/2} \\ &= 1.08 \times 10^{-3} \angle 81.3^\circ \end{aligned}$$

$$= (0.163 + j1.068) \times 10^{-3} = \alpha + j\beta$$

$$\therefore \alpha = 0.163 \times 10^{-3}; \beta = 1.068 \times 10^{-3}$$

- (a) At the receiving-end;

For open circuit  $I_R = 0$

$$\begin{aligned} \text{Incident voltage} &= \frac{V_R + Z_c I_R}{2} = \frac{V_R}{2} \\ &= \frac{220 / \sqrt{3}}{2} = 63.51 \angle 0^\circ \text{ kV (to neutral)} \end{aligned}$$

$$\text{Reflected voltage} = \frac{V_R - Z_c I_R}{2} = \frac{V_R}{2}$$

$$= 63.51 \angle 0^\circ \text{ kV (to neutral)}$$

(b) At 200 km from the receiving-end:

$$\text{Incident voltage} = \left. \frac{V_R}{2} e^{\alpha x} e^{j\beta x} \right|_{x=200 \text{ km}}$$

$$= 63.51 \exp(0.163 \times 10^{-3} \times 200)$$

$$\times \exp(j1.068 \times 10^{-3} \times 200)$$

$$= 65.62 \angle 12.2^\circ \text{ kV (to neutral)}$$

$$\text{Reflected voltage} = \left. \frac{V_R}{2} e^{-\alpha x} e^{-j\beta x} \right|_{x=200 \text{ km}}$$

$$= 63.51 e^{-0.0326} e^{-j0.2135}$$

$$= 61.47 \angle -12.2^\circ \text{ kV (to neutral)}$$

(c) Resultant voltage at 200 km from the receiving-end

$$= 65.62 \angle 12.2^\circ + 61.47 \angle -12.2^\circ$$

$$= 124.2 + j0.877 = 124.2 \angle 0.4^\circ$$

Resultant line-to-line voltage at 200 km

$$= 124.2 \times \sqrt{3} = 215.1 \text{ kV}$$

## 5.7 FERRANTI EFFECT

As has been illustrated in Example 5.6, the effect of the line capacitance is to cause the no-load receiving-end voltage to be more than the sending-end voltage. The effect becomes more pronounced as the line length increases. This phenomenon is known as the *Ferranti effect*. A general explanation of this effect is advanced below:

Substituting  $x = l$  and  $I_R = 0$  (no-load) in Eq. (5.21), we have

$$V_S = \frac{V_R}{2} e^{\alpha l} e^{j\beta l} + \frac{V_R}{2} e^{-\alpha l} e^{-j\beta l} \quad (5.49)$$

The above equation shows that at  $l = 0$ , the incident ( $E_{i0}$ ) and reflected ( $E_{r0}$ ) voltage waves are both equal to  $V_R/2$ . With reference to Fig. 5.15, as  $l$  increases, the incident voltage wave increases exponentially in magnitude  $\left(\frac{V_R}{2} e^{\alpha l}\right)$  and

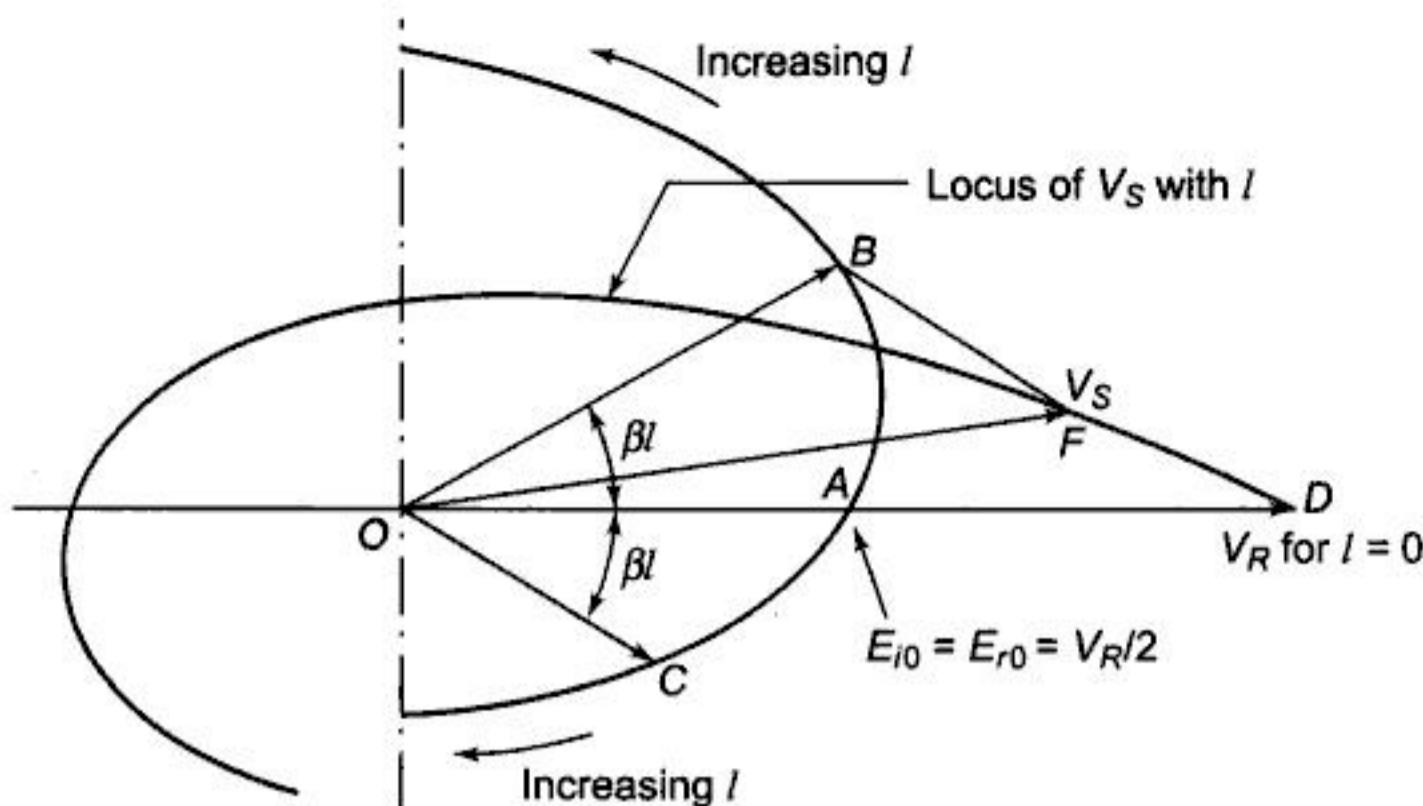


Fig. 5.15

turns through a positive angle  $\beta l$  (represented by phasor  $OB$ ); while the reflected voltage wave decreases in magnitude exponentially  $\left(\frac{V_R}{2} e^{-\alpha l}\right)$  and

turns through a negative angle  $\beta l$  (represented by phasor  $OC$ ). It is apparent from the geometry of this figure that the resultant phasor voltage  $V_S$  ( $OF$ ) is such that  $|V_R| > |V_S|$ .

A simple explanation of the Ferranti effect on an approximate basis can be advanced by lumping the inductance and capacitance parameters of the line. As shown in Fig. 5.16 the capacitance is lumped at the receiving-end of the line.

Here

$$I_S = \frac{V_S}{\left( \frac{1}{j\omega Cl} + j\omega Ll \right)}$$

Since  $C$  is small compared to  $L$ ,  $\omega Ll$  can be neglected in comparison to  $1/\omega Cl$ . Thus

$$I_S \approx jV_S \omega Cl$$

Now

$$\begin{aligned} V_R &= V_S - I_S(j\omega Ll) = V_S + V_S \omega^2 CLl^2 \\ &= V_S(1 + \omega^2 CLl^2) \end{aligned} \quad (5.50)$$

Magnitude of voltage rise =  $|V_S| \omega^2 CLl^2$

$$= |V_S| \frac{\omega^2 l^2}{v^2} \quad (5.51)$$

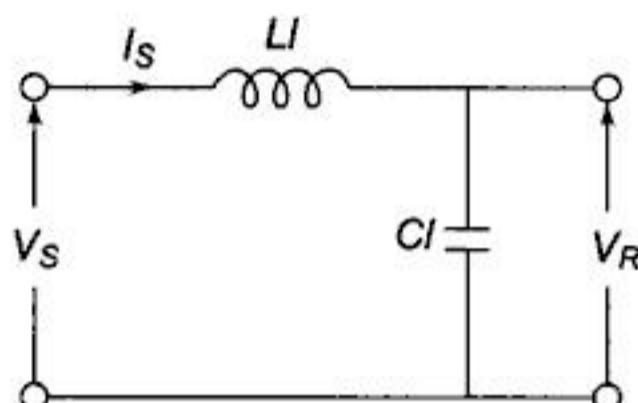


Fig. 5.16

where  $v = 1/\sqrt{LC}$  is the velocity of propagation of the electromagnetic wave along the line, which is nearly equal to the velocity of light.

### 5.8 TUNED POWER LINES

Equation (5.23) characterizes the performance of a long line. For an overhead line shunt conductance  $G$  is always negligible and it is sufficiently accurate to neglect line resistance  $R$  as well. With this approximation

$$\gamma = \sqrt{yz} = j\omega\sqrt{LC}$$

$$\cosh \gamma l = \cosh j\omega l\sqrt{LC} = \cos \omega l\sqrt{LC}$$

$$\sinh \gamma l = \sinh j\omega l\sqrt{LC} = j \sin \omega l\sqrt{LC}$$

Hence Eq. (5.23) simplifies to

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \cos \omega l\sqrt{LC} & jZ_c \sin \omega l\sqrt{LC} \\ \frac{j}{Z_c} \sin \omega l\sqrt{LC} & \cos \omega l\sqrt{LC} \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \quad (5.52)$$

Now if  $\omega l\sqrt{LC} = n\pi$ ,  $n = 1, 2, 3, \dots$

$$|V_S| = |V_R|$$

$$|I_S| = |I_R|$$

i.e. the receiving-end voltage and current are numerically equal to the corresponding sending-end values, so that there is no voltage drop on load. Such a line is called a *tuned line*.

For 50 Hz, the length of line for tuning is

$$l = \frac{n\pi}{2\pi f\sqrt{LC}}$$

Since  $1/\sqrt{LC} \approx v$ , the velocity of light

$$l = \frac{1}{2}(n\lambda) = \frac{1}{2}\lambda, \lambda, \frac{3}{2}\lambda, \dots \quad (5.53)$$

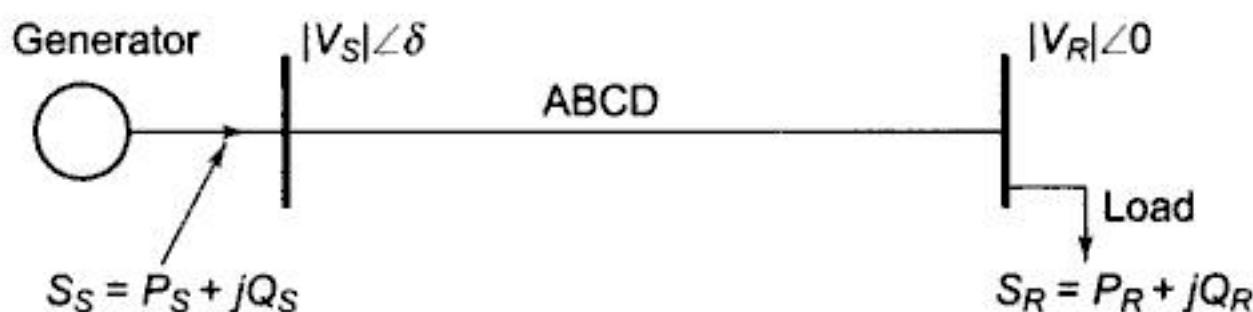
$$= 3,000 \text{ km}, 6,000 \text{ km}, \dots$$

It is too long a distance of transmission from the point of view of cost and efficiency (note that line resistance was neglected in the above analysis). For a given line, length and frequency tuning can be achieved by increasing  $L$  or  $C$ , i.e. by adding series inductances or shunt capacitances at several places along the line length. The method is impractical and uneconomical for power frequency lines and is adopted for telephony where higher frequencies are employed.

A method of tuning power lines which is being presently experimented with, uses series capacitors to cancel the effect of the line inductance and shunt inductors to neutralize line capacitance. A long line is divided into several sections which are individually tuned. However, so far the practical method of improving line regulation and power transfer capacity is to add series capacitors to reduce line inductance; shunt capacitors under heavy load conditions; and shunt inductors under light or no-load conditions.

## 5.9 POWER FLOW THROUGH A TRANSMISSION LINE

So far the transmission line performance equation was presented in the form of voltage and current relationships between sending- and receiving-ends. Since loads are more often expressed in terms of real (watts/kW) and reactive (VARs/kVAR) power, it is convenient to deal with transmission line equations in the form of sending- and receiving-end complex power and voltages. While the problem of flow of power in a general network will be treated in the next chapter, the principles involved are illustrated here through a single transmission line (2-node/2-bus system) as shown in Fig. 5.17.



**Fig. 5.17** A two-bus system

Let us take receiving-end voltage as a reference phasor ( $V_R = |V_R| \angle 0^\circ$ ) and let the sending-end voltage lead it by an angle  $\delta$  ( $V_S = |V_S| \angle \delta$ ). The angle  $\delta$  is known as the torque angle whose significance has been explained in Ch. 4 and will further be taken up in Ch. 12 while dealing with the problem of stability.

The complex power leaving the receiving-end and entering the sending-end of the transmission line can be expressed as (on per phase basis)

$$S_R = P_R + jQ_R = V_R I_R^* \quad (5.54)$$

$$S_S = P_S + jQ_S = V_S I_S^* \quad (5.55)$$

Receiving- and sending-end currents can, however, be expressed in terms of receiving- and sending-end voltages [by rearranging Eq. (5.1)] as

$$I_R = \frac{1}{B} V_S - \frac{A}{B} V_R \quad (5.56)$$

$$I_S = \frac{D}{B} V_S - \frac{1}{B} V_R \quad (5.57)$$

Let  $A, B, D$ , the transmission line constants, be written as

$$A = |A| \angle \alpha, B = |B| \angle \beta, D = |D| \angle \alpha \text{ (since } A = D)$$

Therefore, we can write

$$I_R = \left| \frac{1}{B} \right| |V_S| \angle (\delta - \beta) - \left| \frac{A}{B} \right| |V_R| \angle (\alpha - \beta)$$

$$I_S = \left| \frac{D}{B} \right| |V_S| \angle (\alpha + \delta - \beta) - \left| \frac{1}{B} \right| |V_R| \angle -\beta$$

Substituting for  $I_R$  in Eq. (5.54), we get

$$\begin{aligned} S_R &= |V_R| \angle 0 \left[ \left| \frac{1}{B} \right| |V_S| \angle (\beta - \delta) - \left| \frac{A}{B} \right| |V_R| \angle (\beta - \alpha) \right] \\ &= \frac{|V_S| |V_R|}{|B|} \angle (\beta - \delta) - \left| \frac{A}{B} \right| |V_R|^2 \angle (\beta - \alpha) \end{aligned} \quad (5.58)$$

Similarly,

$$S_S = \left| \frac{D}{B} \right| |V_S|^2 \angle (\beta - \alpha) - \frac{|V_S| |V_R|}{|B|} \angle (\beta + \delta) \quad (5.59)$$

In the above equations  $S_R$  and  $S_S$  are per phase complex voltamperes, while  $V_R$  and  $V_S$  are expressed in per phase volts. If  $V_R$  and  $V_S$  are expressed in kV line, then the three-phase receiving-end complex power is given by

$$S_R \text{ (three-phase VA)} = 3 \left\{ \frac{|V_S| |V_R| \times 10^6}{\sqrt{3} \times \sqrt{3} |B|} \angle (\beta - \delta) - \left| \frac{A}{B} \right| \frac{|V_R|^2 \times 10^6}{3} \angle (\beta - \alpha) \right\}$$

$$S_R \text{ (three-phase MVA)} = \frac{|V_S| |V_R|}{|B|} \angle (\beta - \delta) - 3 \left| \frac{A}{B} \right| |V_R|^2 \angle (\beta - \alpha) \quad (5.60)$$

This indeed is the same as Eq. (5.58). The same result holds for  $S_S$ . Thus we see that Eqs (5.58) and (5.59) give the three-phase MVA if  $V_S$  and  $V_R$  are expressed in kV line.

If Eq. (5.58) is expressed in real and imaginary parts, we can write the real and reactive powers at the receiving-end as

$$P_R = \frac{|V_S| |V_R|}{|B|} \cos (\beta - \delta) - \left| \frac{A}{B} \right| |V_R|^2 \cos (\beta - \alpha) \quad (5.61)$$

$$Q_R = \frac{|V_S| |V_R|}{|B|} \sin (\beta - \delta) - \left| \frac{A}{B} \right| |V_R|^2 \sin (\beta - \alpha) \quad (5.62)$$

Similarly, the real and reactive powers at sending-end are

$$P_S = \left| \frac{D}{B} \right| |V_S|^2 \cos(\beta - \alpha) - \frac{|V_S| |V_R|}{|B|} \cos(\beta + \delta) \quad (5.63)$$

$$Q_S = \left| \frac{D}{B} \right| |V_S|^2 \sin(\beta - \alpha) - \frac{|V_S| |V_R|}{|B|} \sin(\beta + \delta) \quad (5.64)$$

It is easy to see from Eq. (5.61) that the received power  $P_R$  will be maximum at

$$\delta = \beta$$

such that

$$P_R (\text{max}) = \frac{|V_S| |V_R|}{|B|} - \frac{|A| |V_R|^2}{|B|} \cos(\beta - \alpha) \quad (5.65)$$

The corresponding  $Q_R$  (at max  $P_R$ ) is

$$Q_R = -\frac{|A| |V_R|^2}{|B|} \sin(\beta - \alpha)$$

Thus the load must draw this much leading MVAR in order to receive the maximum real power.

Consider now the special case of a short line with a series impedance  $Z$ . Now

$$A = D = 1 \angle 0; B = Z = |Z| \angle \theta$$

Substituting these in Eqs (5.61) to (5.64), we get the simplified results for the short line as

$$P_R = \frac{|V_S| |V_R|}{|Z|} \cos(\theta - \delta) - \frac{|V_R|^2}{|Z|} \cos \theta \quad (5.66)$$

$$Q_R = \frac{|V_S| |V_R|}{|Z|} \sin(\theta - \delta) - \frac{|V_R|^2}{|Z|} \sin \theta \quad (5.67)$$

for the receiving-end and for the sending-end

$$P_S = \frac{|V_S|^2}{|Z|} \cos \theta - \frac{|V_S| |V_R|}{|Z|} \cos(\theta + \delta) \quad (5.68)$$

$$Q_S = \frac{|V_S|^2}{|Z|} \sin \theta - \frac{|V_S| |V_R|}{|Z|} \sin(\theta + \delta) \quad (5.69)$$

The above short line equation will also apply for a long line when the line is replaced by its equivalent- $\pi$  (or nominal- $\pi$ ) and the shunt admittances are lumped with the receiving-end load and sending-end generation. In fact, this technique is always used in the load flow problem to be treated in the next chapter.

From Eq.(5.66), the maximum receiving-end power is received, when  $\delta = \theta$ , so that

$$P_R(\max) = \frac{|V_S||V_R|}{|Z|} - \frac{|V_R|^2}{|Z|} \cos \theta$$

Now

$$\cos \theta = R/|Z|,$$

$$\therefore P_R(\max) = \frac{|V_S||V_R|}{|Z|} - \frac{|V_R|^2}{|Z|^2} R \quad (5.70)$$

Normally the resistance of a transmission line is small compared to its reactance (since it is necessary to maintain a high efficiency of transmission), so that  $\theta = \tan^{-1} X'R \approx 90^\circ$ ; where  $Z = R + jX$ . The receiving-end Eqs (5.66) and (5.67) can then be approximated as

$$P_R = \frac{|V_S||V_R|}{X} \sin \delta \quad (5.71)$$

$$Q_R = \frac{|V_S||V_R|}{X} \cos \delta - \frac{|V_R|^2}{X} \quad (5.72)$$

Equation (5.72) can be further simplified by assuming  $\cos \delta \approx 1$ , since  $\delta$  is normally small\*. Thus

$$Q_R = \frac{|V_R|}{X} (|V_S| - |V_R|) \quad (5.73)$$

Let  $|V_S| - |V_R| = |\Delta V|$ , the magnitude of voltage drop across the transmission line.

$$\therefore Q_R = \frac{|V_R|}{X} |\Delta V| \quad (5.74)$$

Several important conclusions that easily follow from Eqs (5.71) to (5.74) are enumerated below:

1. For  $R \approx 0$  (which is a valid approximation for a transmission line) the real power transferred to the receiving-end is proportional to  $\sin \delta$  ( $\approx \delta$  for small values of  $\delta$ ), while the reactive power is proportional to the magnitude of the voltage drop across the line.
2. The real power received is maximum for  $\delta = 90^\circ$  and has a value  $|V_S||V_R|/X$ . Of course,  $\delta$  is restricted to values well below  $90^\circ$  from considerations of stability to be discussed in Ch. 12.

---

\* Small  $\delta$  is necessary from considerations of system stability which will be discussed at length in Ch. 12.

3. Maximum real power transferred for a given line (fixed  $X$ ) can be increased by raising its voltage level. It is from this consideration that voltage levels are being progressively pushed up to transmit larger chunks of power over longer distances warranted by large size generating stations.

For very long lines, voltage level cannot be raised beyond the limits placed by present-day high voltage technology. To increase power transmitted in such cases, the only choice is to reduce the line reactance. This is accomplished by adding series capacitors in the line. This idea will be pursued further in Ch. 12. Series capacitors would of course increase the severity of line over voltages under switching conditions.

4. The VARs (lagging reactive power) delivered by a line is proportional to the line voltage drop and is independent of  $\delta$ . Therefore, in a transmission system if the VARs demand of the load is large, the voltage profile at that point tends to sag rather sharply. To maintain a desired voltage profile, the VARs demand of the load must be met locally by employing positive VAR generators (condensers). This will be discussed at length in Sec. 5.10.

A somewhat more accurate yet approximate result expressing line voltage drop in terms of active and reactive powers can be written directly from Eq. (5.5), i.e.

$$\begin{aligned} |\Delta V| &= |I_R| R \cos \phi + |I_R| X \sin \phi \\ &= \frac{|V_R| |I_R| R \cos \phi + |V_R| |I_R| X \sin \phi}{|V_R|} \\ &= \frac{RP_R + XQ_R}{|V_R|} \end{aligned} \quad (5.75)$$

This result reduces to that of Eq. (5.74) if  $R = 0$ .

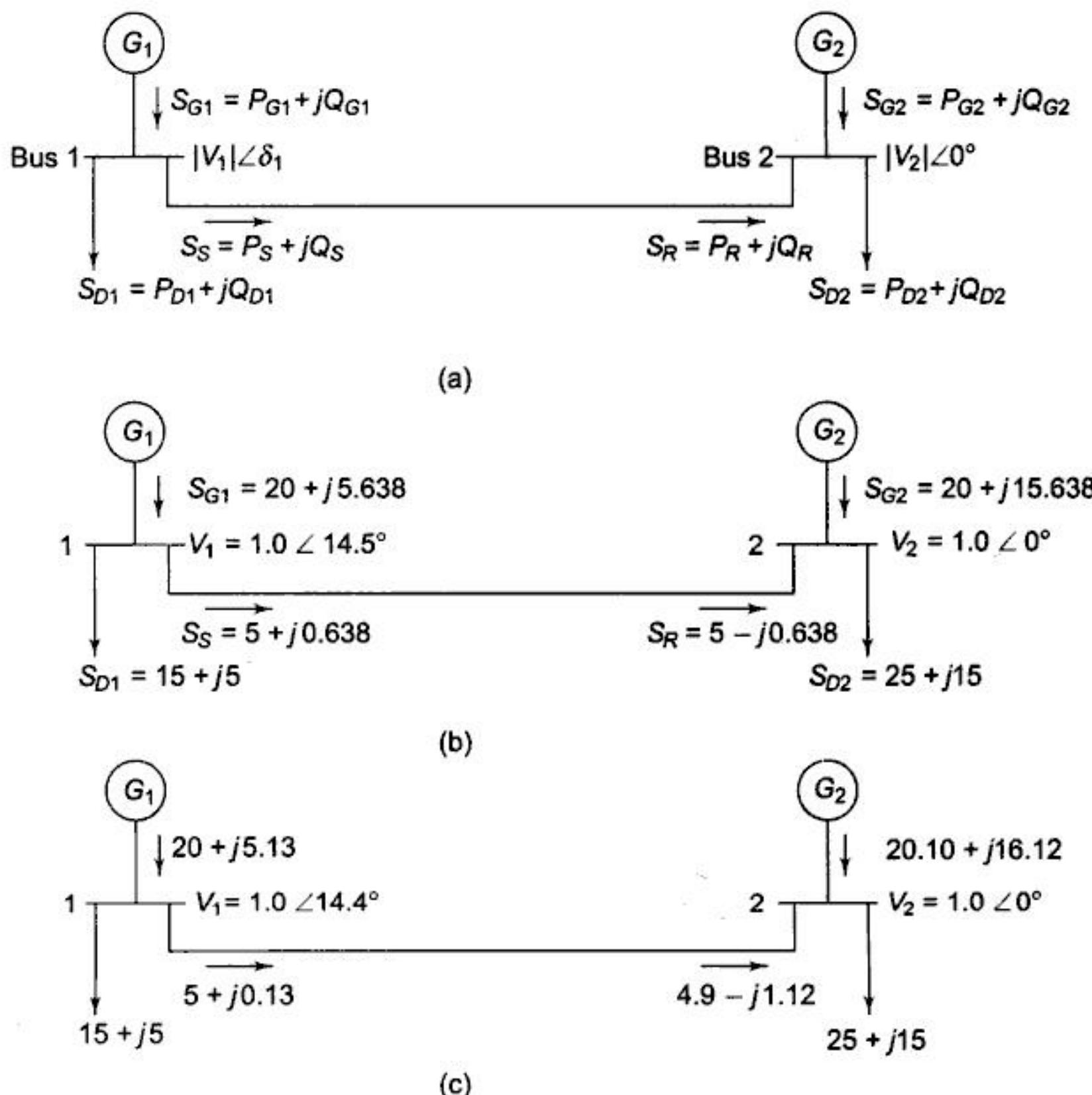
**Example 5.8** An interconnector cable links generating stations 1 and 2 as shown in Fig. 5.18. The desired voltage profile is flat, i.e.  $|V_1| = |V_2| = 1$  pu. The total demands at the two buses are

$$\begin{aligned} S_{D1} &= 15 + j5 \text{ pu} \\ S_{D2} &= 25 + j15 \text{ pu} \end{aligned}$$

The station loads are equalized by the flow of power in the cable. Estimate the torque angle and the station power factors: (a) for cable  $Z = 0 + j0.05$  pu, and (b) for cable  $Z = 0.005 + j0.05$  pu. It is given that generator  $G_1$  can generate a maximum of 20.0 pu real power.

*Solution*

The powers at the various points in the fundamental (two-bus) system are defined in Fig. 5.18(a).



**Fig. 5.18** Two-bus system

*Case (a):* Cable impedance =  $j0.05$  pu.

Since cable resistance is zero, there is no real power loss in the cable. Hence

$$P_{G1} + P_{G2} = P_{D1} + P_{D2} = 40 \text{ pu}$$

For equalization of station loads,

$$P_{G1} = P_{G2} = 20 \text{ pu}$$

Equalization means that  $P_S = P_R = 5 \text{ MW}$

The voltage of bus 2 is taken as reference, i.e.  $V_2 \angle 0^\circ$  and voltage of bus 1 is  $V_1 \angle \delta_1$ . Further, for flat voltage profile  $|V_1| = |V_2| = 1$ .

Real power flow from bus 1 to bus 2 is obtained from Eq. (5.58) by recognizing that since  $R = 0$ ,  $\theta = 90^\circ$ .

Hence

$$P_S = P_R = \frac{|V_1||V_2|}{X} \sin \delta_1$$

$$S = \frac{1 \times 1}{0.05} \sin \delta_1$$

or

$$\delta_1 = 14.5^\circ$$

$$\therefore V_1 = 1 \angle 14.5^\circ$$

From Eq. (5.69)

$$Q_S = \frac{|V_1|^2}{X} - \frac{|V_1||V_2|}{X} \cos \delta_1$$

$$= \frac{1}{0.05} - \frac{1}{0.05} \times 0.968 = 0.638 \text{ pu}$$

From Eq. (5.67)

$$Q_R = \frac{|V_1||V_2|}{X} \cos \delta_1 - \frac{|V_1|^2}{X} = -Q_S = -0.638 \text{ pu}$$

Reactive power loss\* in the cable is

$$Q_L = Q_S - Q_R = 2Q_S = 1.276 \text{ pu}$$

$$\begin{aligned} \text{Total load on station 1} &= (15 + j5) + (5 + j0.638) \\ &= 20 + j5.638 \end{aligned}$$

$$\text{Power factor at station 1} = \cos \left( \tan^{-1} \frac{5.638}{20} \right) = 0.963 \text{ lagging}$$

$$\begin{aligned} \text{Total load on station 2} &= (25 + j15) - (5 - j0.638) \\ &= 20 + j15.638 \end{aligned}$$

$$\text{Power factor at station 2} = \cos \left( \tan^{-1} \frac{15.638}{20} \right) = 0.788 \text{ lagging}$$

The station loads, load demands, and line flows are shown in Fig. 5.18(b). It may be noted that to maintain a flat voltage profile, the generators are required to supply reactive powers  $Q_{G1} = 5.638$  and  $Q_{G2} = 15.638$ , respectively.

*Case (b):* Cable impedance =  $0.005 + j0.05 = 0.0502 \angle 84.3^\circ$  pu. In this case the cable resistance causes real power loss which is not known a priori. The real load flow is thus not obvious as was in the case of  $R = 0$ . We specify the generation at station 1 as

$$P_{G1} = 20 \text{ pu}$$

The consideration for fixing this generation is economic as we shall see in Ch. 7.

---

\* Reactive power loss can also be computed as  $|I|^2 X = \frac{5^2 + (0.638)^2}{1} \times 0.05 = 1.27 \text{ pu}$ .

The generation at station 2 will be 20 pu plus the cable loss. The unknown variables in the problem are

$$P_{G2}, \delta_1, Q_{G1}, Q_{G2}$$

Let us now examine as to how many system equations can be formed.

From Eqs (5.68) and (5.69),

$$\begin{aligned} P_{G1} - P_{D1} = P_S &= \frac{|V_1|^2}{|Z|} \cos \theta - \frac{|V_1||V_2|}{|Z|} \cos(\theta + \delta_1) \\ 5 &= \frac{1}{0.0502} \cos 84.3^\circ - \frac{1}{0.0502} \cos(84.3^\circ + \delta_1) \end{aligned} \quad (\text{i})$$

$$\begin{aligned} Q_{G1} - Q_{D1} = Q_S &= \frac{|V_1|^2}{|Z|} \sin \theta - \frac{|V_1||V_2|}{|Z|} \sin(\theta + \delta_1) \\ Q_{G1} - 5 &= \frac{1}{0.0502} \sin 84.3^\circ - \frac{1}{0.0502} \sin(84.3^\circ + \delta_1) \end{aligned} \quad (\text{ii})$$

From Eqs (5.66) and (5.67)

$$\begin{aligned} P_{D2} - P_{G2} = P_R &= \frac{|V_1||V_2|}{|Z|} \cos(\theta - \delta_1) - \frac{|V_1|^2}{|Z|} \cos \theta \\ 25 - P_{G2} &= \frac{1}{0.0502} \cos(84.3^\circ - \delta_1) - \frac{1}{0.0502} \cos 84.3^\circ \end{aligned} \quad (\text{iii})$$

$$\begin{aligned} Q_{D2} - Q_{G2} = Q_R &= \frac{|V_1||V_2|}{|Z|} \sin(\theta - \delta_1) - \frac{|V_1|^2}{|Z|} \sin \theta \\ 15 - Q_{G2} &= \frac{1}{0.0502} \sin(84.3^\circ - \delta_1) - \frac{1}{0.0502} \sin 84.3^\circ \end{aligned} \quad (\text{iv})$$

Thus we have four equations, Eqs (i) to (iv), in four unknowns  $P_{G2}, \delta_1, Q_{G1}, Q_{G2}$ . Even though these are non-linear algebraic equations, solution is possible in this case. Solving Eq. (i) for  $\delta_1$ , we have

$$\delta_1 = 14.4^\circ$$

Substituting  $\delta_1$  in Eqs (ii), (iii) and (iv), we get

$$Q_{G1} = 5.13, Q_{G2} = 16.12, P_{G2} = 20.10$$

The flow of real and reactive powers for this case is shown in Fig. 5.18 (c).

It may be noted that the real power loss of 0.1 pu is supplied by  $G_2 (P_{G2} = 20.10)$ .

The problem presented above is a two-bus load flow problem. Explicit solution is always possible in a two-bus case. The reader should try the case when

$$Q_{G2} = j10 \text{ and } |V_2| = ?$$

The general load flow problem will be taken up in Ch. 6. It will be seen that explicit solution is not possible in the general case and iterative techniques have to be resorted to.

**Example 5.9** A 275 kV transmission line has the following line constants:

$$A = 0.85 \angle 5^\circ; B = 200 \angle 75^\circ$$

- Determine the power at unity power factor that can be received if the voltage profile at each end is to be maintained at 275 kV.
- What type and rating of compensation equipment would be required if the load is 150 MW at unity power factor with the same voltage profile as in part (a)?
- With the load as in part (b), what would be the receiving-end voltage if the compensation equipment is not installed?

*Solution*

- Given  $|V_S| = |V_R| = 275 \text{ kV}$ ;  $\alpha = 5^\circ$ ,  $\beta = 75^\circ$ . Since the power is received at unity power factor,

$$Q_R = 0$$

Substituting these values in Eq. (5.62), we can write

$$\begin{aligned} 0 &= \frac{275 \times 275}{200} \sin(75^\circ - \delta) - \frac{0.85}{200} \times (275)^2 \sin(75^\circ - 5^\circ) \\ 0 &= 378 \sin(75^\circ - \delta) - 302 \end{aligned}$$

which gives

$$\delta = 22^\circ$$

From Eq. (5.61)

$$\begin{aligned} P_R &= \frac{275 \times 275}{200} \cos(75^\circ - 22^\circ) - \frac{0.85}{200} \times (275)^2 \cos 70^\circ \\ &= 227.6 - 109.9 = 117.7 \text{ MW} \end{aligned}$$

- Now  $|V_S| = |V_R| = 275 \text{ kV}$

Power demanded by load = 150 MW at UPF

$$\therefore P_D = P_R = 150 \text{ MW}; Q_D = 0$$

From Eq. (5.61)

$$150 = \frac{275 \times 275}{200} \cos(75^\circ - \delta) - \frac{0.85}{200} \times (275)^2 \cos 70^\circ$$

$$150 = 378 \cos(75^\circ - \delta) - 110$$

or  $\delta = 28.46^\circ$

From Eq. (5.62)

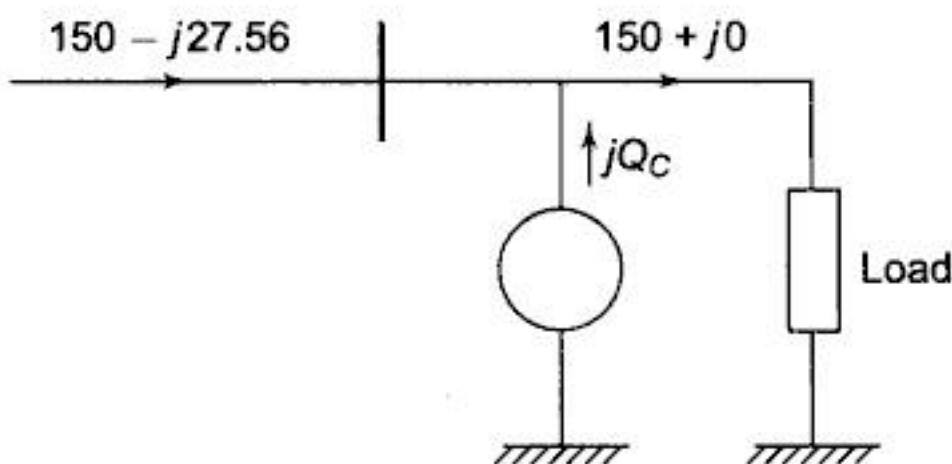
$$\begin{aligned} Q_R &= \frac{275 \times 275}{200} \sin(75^\circ - 28.46^\circ) - \frac{0.85}{200} \times (275)^2 \sin 70^\circ \\ &= 274.46 - 302 = -27.56 \text{ MVAR} \end{aligned}$$

Thus in order to maintain 275 kV at a receiving-end,  $Q_R = -27.56$  MVAR must be drawn along with the real power of  $P_R = 150$  MW. The load being 150 MW at unity power factor, i.e.  $Q_D = 0$ , compensation equipment must be installed at the receiving-end. With reference to Fig. 5.19, we have

$$-27.56 + Q_C = 0$$

or  $Q_C = +27.56$  MVAR

i.e. the compensation equipment must feed positive VARs into the line. See subsection 5.10 for a more detailed explanation.



**Fig. 5.19**

(c) Since no compensation equipment is provided

$$P_R = 150 \text{ MW}, Q_R = 0$$

Now,

$$|V_S| = 275 \text{ kV}, |V_R| = ?$$

Substituting this data in Eqs (5.61) and (5.62), we have

$$150 = \frac{275 |V_R|}{200} \cos(75^\circ - \delta) - \frac{0.85}{200} |V_R|^2 \cos 70^\circ \quad (\text{i})$$

$$0 = \frac{275 |V_R|}{200} \sin(75^\circ - \delta) - \frac{0.85}{200} |V_R|^2 \sin 70^\circ \quad (\text{ii})$$

From Eq. (ii), we get

$$\sin(75^\circ - \delta) = 0.0029 |V_R|$$

$$\therefore \cos(75^\circ - \delta) = (1 - (0.0029)^2 |V_R|^2)^{1/2}$$

Substituting in Eq. (i), we obtain

$$150 = 1.375 |V_R| (1 - (0.0029)^2 |V_R|^2)^{1/2} - 0.00145 |V_R|^2$$

Solving the quadratic and retaining the higher value of  $|V_R|$ , we obtain

$$|V_R| = 244.9 \text{ kV}$$

*Note:* The second and lower value solution of  $|V_R|$ , though feasible, is impractical as it corresponds to abnormally low voltage and efficiency.

It is to be observed from the results of this problem that larger power can be transmitted over a line with a fixed voltage profile by installing compensation equipment at the receiving-end capable of feeding positive VARs into the line.

### Circle Diagrams

It has been shown above that the flow of active and reactive power over a transmission line can be handled computationally. It will now be shown that the locus of complex sending- and receiving-end power is a circle. Since circles are convenient to draw, the circle diagrams are a useful aid to visualize the load flow problem over a single transmission line.

The expressions for complex number receiving- and sending-end powers are reproduced below from Eqs (5.58) and (5.59),

$$S_R = - \left| \frac{A}{B} \right| |V_R|^2 \angle(\beta - \alpha) + \frac{|V_S| |V_R|}{|B|} \angle(\beta - \delta) \quad (5.58)$$

$$S_S = \left| \frac{D}{B} \right| |V_R|^2 \angle(\beta - \alpha) - \frac{|V_S| |V_R|}{|B|} \angle(\beta + \delta) \quad (5.59)$$

**The units for  $S_R$  and  $S_S$  are MVA (three-phase) with voltages in kV line.** As per the above equations,  $S_R$  and  $S_S$  are each composed of two phasor components—one a constant phasor and the other a phasor of fixed magnitude but variable angle. The loci for  $S_R$  and  $S_S$  would, therefore, be circles drawn from the tip of constant phasors as centres.

It follows from Eq. (5.58) that the centre of receiving-end circle is located at the tip of the phasor

$$-\left| \frac{A}{B} \right| |V_R|^2 \angle(\beta - \alpha) \quad (5.76)$$

in polar coordinates or in terms of rectangular coordinates.

Horizontal coordinate of the centre

$$= - \left| \frac{A}{B} \right| |V_R|^2 \cos(\beta - \alpha) \text{ MW} \quad (5.77)$$

Vertical coordinate of the centre

$$= - \left| \frac{A}{B} \right| |V_R|^2 \sin(\beta - \alpha) \text{ MVAR}$$

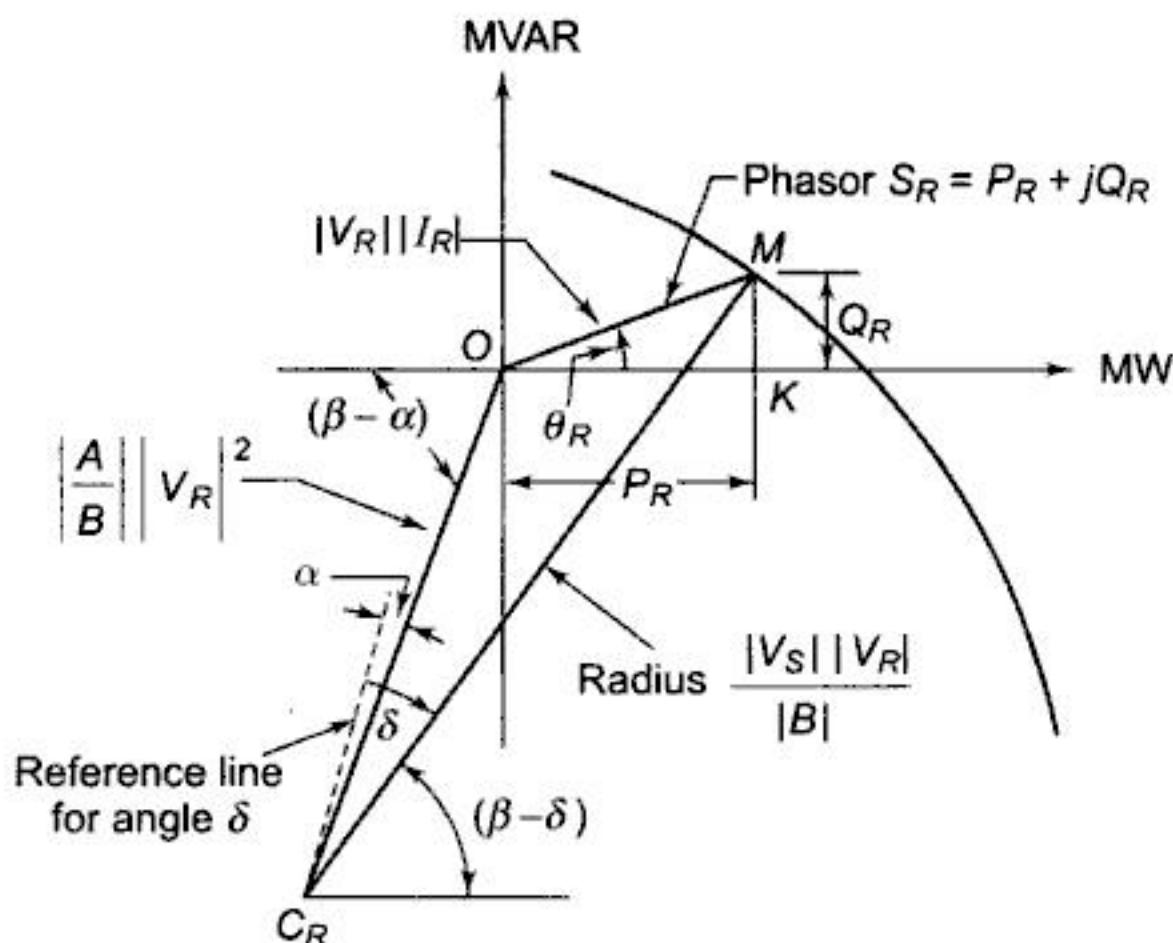
The radius of the receiving-end circle is

$$\frac{|V_S| |V_R|}{|B|} \text{ MVA} \quad (5.78)$$

The receiving-end circle diagram is drawn in Fig. 5.20. The centre is located by drawing  $OC_R$  at an angle  $(\beta - \alpha)$  in the positive direction from the negative MW-axis. From the centre  $C_R$  the receiving-end circle is drawn with the radius  $|V_S| |V_R|/|B|$ . The operating point  $M$  is located on the circle by means of the received real power  $P_R$ . The corresponding  $Q_R$  (or  $\theta_R$ ) can be immediately read from the circle diagram. The torque angle  $\delta$  can be read in accordance with the positive direction indicated from the reference line.

For constant  $|V_R|$ , the centre  $C_R$  remains fixed and concentric circles result for varying  $|V_S|$ . However, for the case of constant  $|V_S|$  and varying  $|V_R|$  the centres of circles move along the line  $OC_R$  and have radii in accordance to  $|V_S| |V_R|/|B|$ .

Similarly, it follows from Eq. (5.59) that the centre of the sending-end circle is located at the tip of the phasor



**Fig. 5.20** Receiving-end circle diagram

$$\left| \frac{D}{B} \right| |V_S|^2 \angle(\beta - \alpha) \quad (5.79)$$

in the polar coordinates or in terms of rectangular coordinates.

Horizontal coordinate of the centre

$$= \left| \frac{D}{B} \right| |V_S|^2 \cos (\beta - \alpha) \text{ MW} \quad (5.80)$$

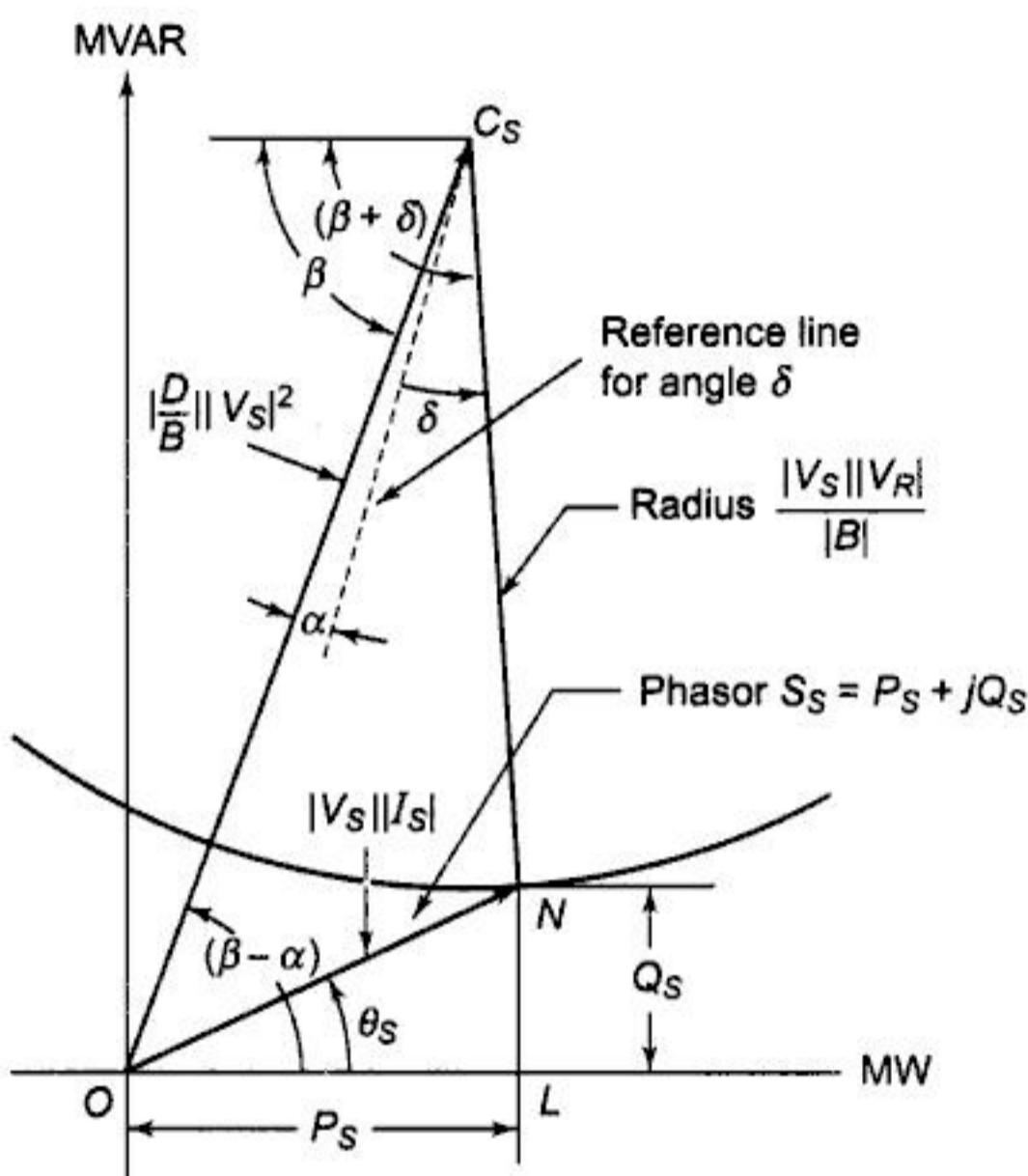
Vertical coordinate of the centre

$$= \left| \frac{D}{B} \right| |V_S|^2 \sin (\beta - \alpha) \text{ MVAR}$$

The radius of the sending-end circle is

$$\frac{|V_S| |V_R|}{|B|} \quad (5.81)$$

The sending-end circle diagram is shown in Fig. 5.21. The centre is located by drawing  $OC_S$  at angle  $(\beta - \alpha)$  from the positive MW-axis. From the centre the sending-end circle is drawn with a radius  $\frac{|V_S| |V_R|}{|B|}$  (same as in the case of receiving-end). The operating point  $N$  is located by measuring the torque angle  $\delta$  (as read from the receiving-end circle diagram) in the direction indicated from the reference line.



**Fig. 5.21** Sending-end circle diagram

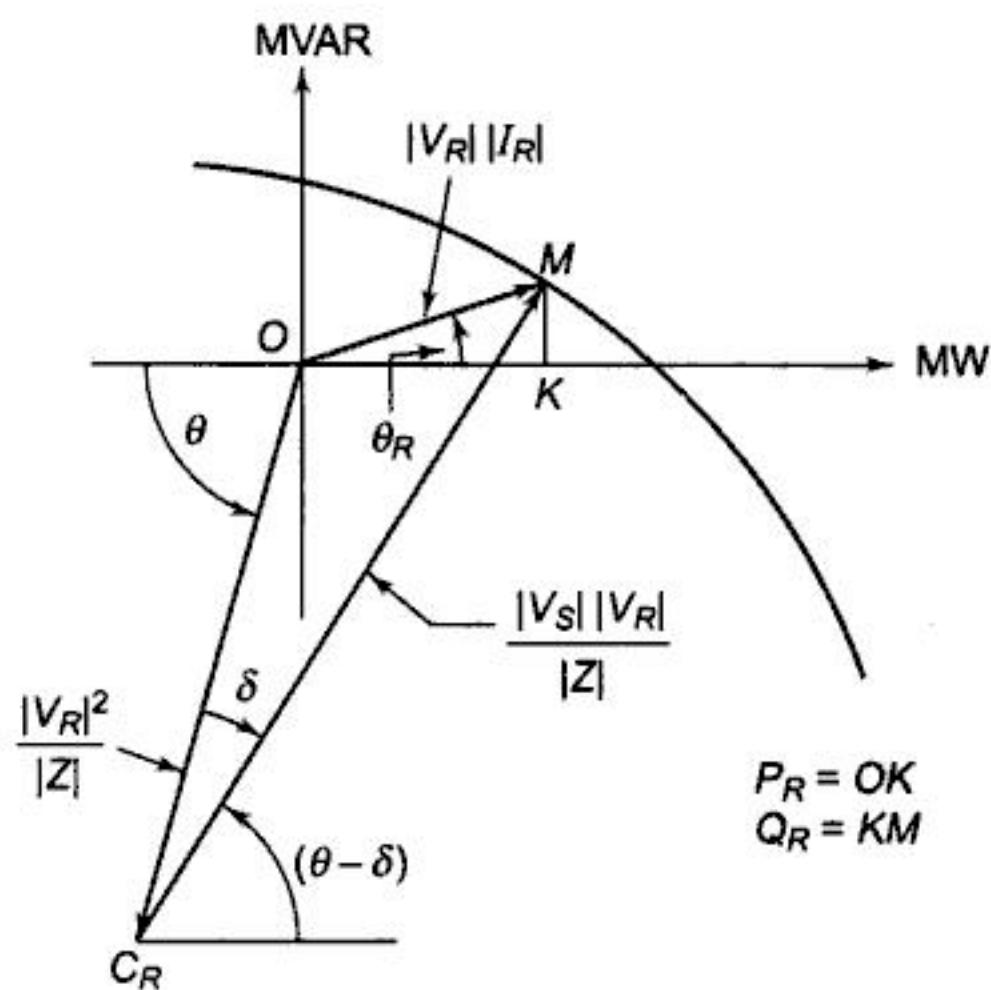
For constant  $|V_S|$  the centre  $C_S$  remains fixed and concentric circles result for varying  $|V_R|$ . However, if  $|V_R|$  is fixed and  $|V_S|$  varies, the centres of the circles move along the line  $OC_S$  and have radii in accordance to  $|V_S| |V_R| / |B|$ .

For the case of a short line with a series impedance  $|Z| \angle \theta$ , the simplified circle diagrams can be easily drawn by recognizing

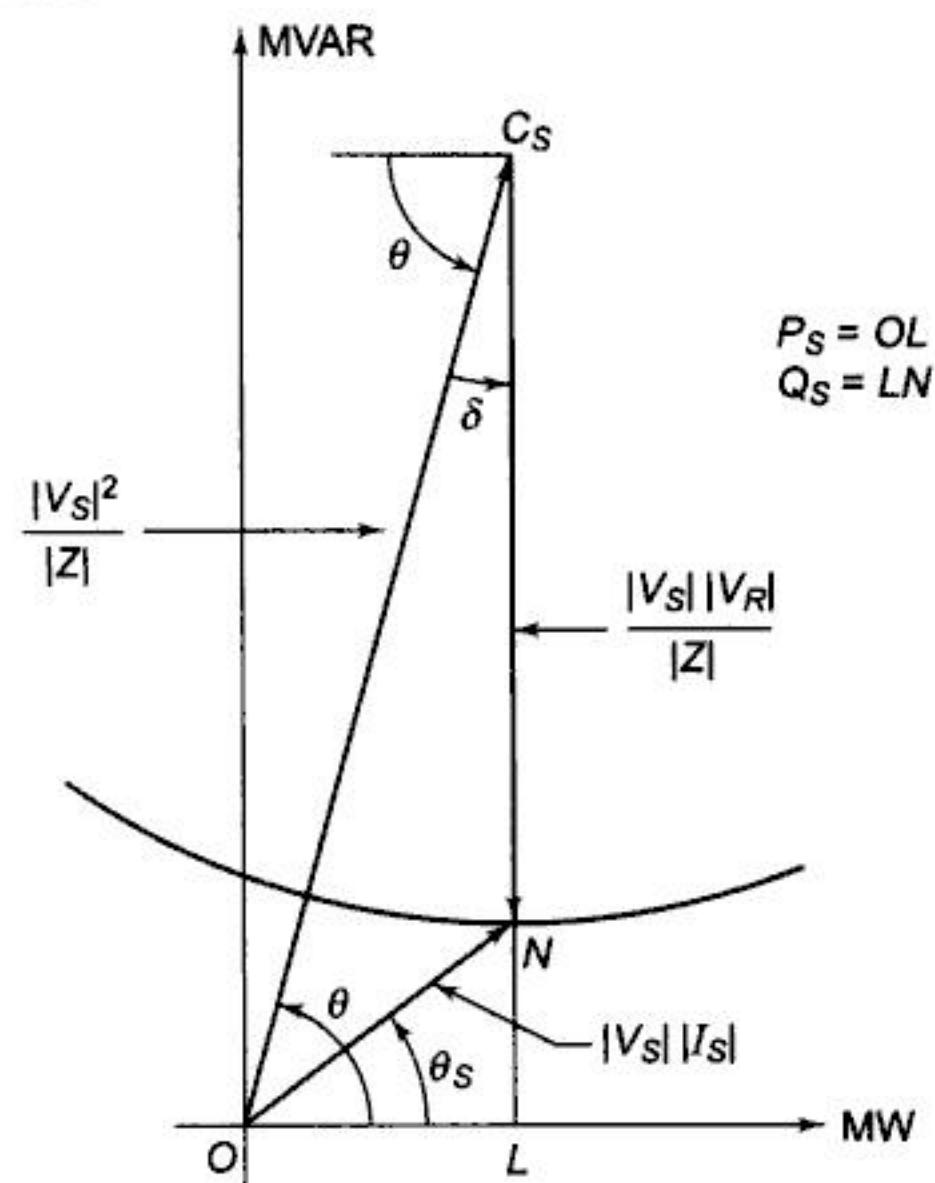
$$|A| = |D| = 1, \alpha = 0$$

$$|B| = |Z|, \beta = \theta$$

The corresponding receiving- and sending-end circle diagrams have been drawn in Figs 5.22 and 5.23.



**Fig. 5.22** Receiving-end circle diagram for a short line



**Fig. 5.23** Sending-end circle diagram for a short line

The use of circle diagrams is illustrated by means of the two examples given below:

**Example 5.10** A 50 Hz, three-phase, 275 kV, 400 km transmission line has the following parameters:

$$\text{Resistance} = 0.035 \Omega/\text{km per phase}$$

$$\text{Inductance} = 1.1 \text{ mH/km per phase}$$

$$\text{Capacitance} = 0.012 \mu\text{F/km per phase}$$

If the line is supplied at 275 kV, determine the MVA rating of a shunt reactor having negligible losses that would be required to maintain 275 kV at the receiving-end when the line is delivering no load. Use nominal- $\pi$  method.

**Solution**

$$R = 0.035 \times 400 = 14 \Omega$$

$$X = 314 \times 1.1 \times 10^{-3} \times 400 = 138.2 \Omega$$

$$Z = 14 + j138 = 138.7 \angle 84.2^\circ \Omega$$

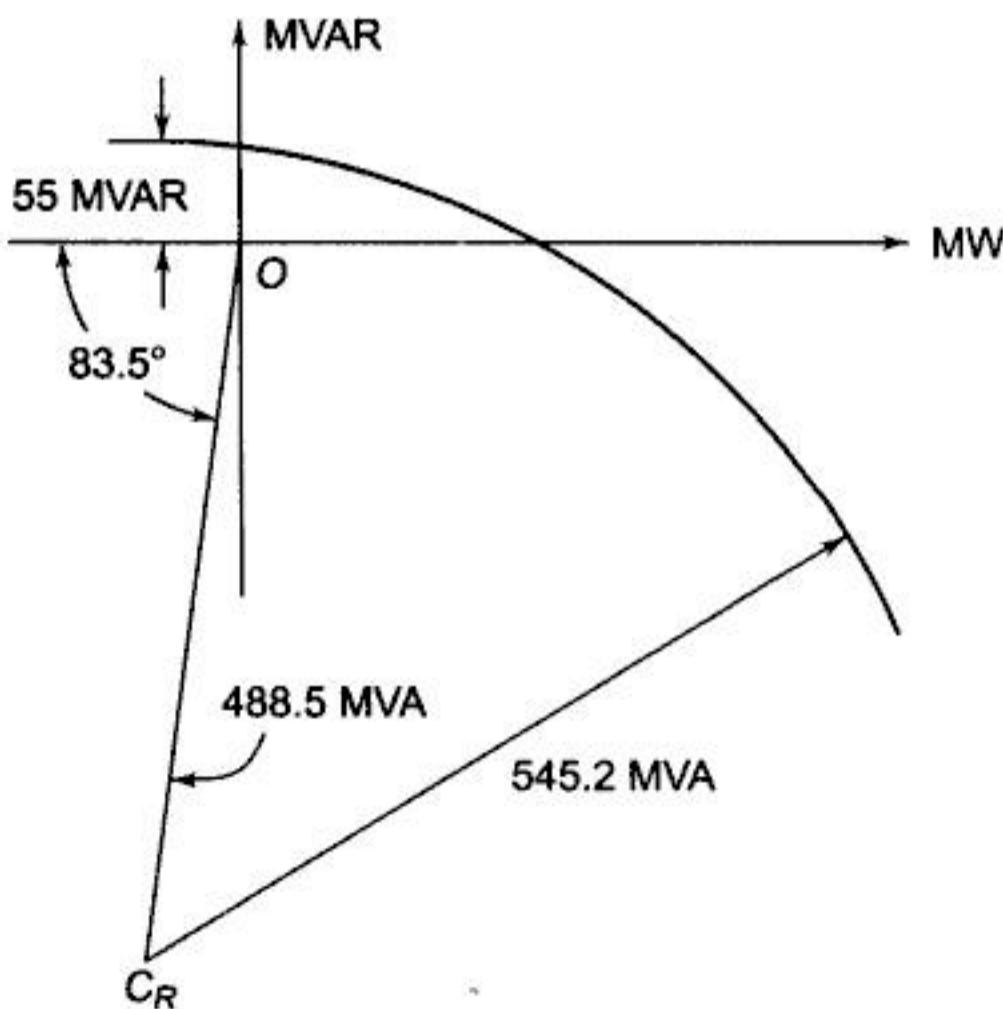
$$Y = 314 \times 0.012 \times 10^{-6} \times 400 \angle 90^\circ = 1.507 \times 10^{-3} \angle 90^\circ \Omega$$

$$A = \left( 1 + \frac{1}{2} YZ \right) = 1 + \frac{1}{2} \times 1.507 \times 10^{-3} \times 138.7 \angle 174.2^\circ$$

$$= (0.896 + j0.0106) = 0.896 \angle 0.7^\circ$$

$$B = Z = 138.7 \angle 84.2^\circ$$

$$|V_S| = 275 \text{ kV}, |V_R| = 275 \text{ kV}$$



**Fig. 5.24** Circle diagram for Example 5.10

$$\text{Radius of receiving-end circle} = \frac{|V_S| |V_R|}{|B|} = \frac{275 \times 275}{138.7} = 545.2 \text{ MVA}$$

Location of the centre of receiving-end circle,

$$\left| \frac{A}{B} \right| |V_R|^2 = \frac{275 \times 275 \times 0.896}{138.7} = 488.5 \text{ MVA}$$

$$\angle(\beta - \alpha) = 84.2^\circ - 0.7^\circ = 83.5^\circ$$

From the circle diagram of Fig. 5.24, +55 MVAR must be drawn from the receiving-end of the line in order to maintain a voltage of 275 kV. Thus rating of shunt reactor needed = 55 MVA.

**Example 5.11** A 275 kV, three-phase line has the following line parameters:

$$A = 0.93 \angle 1.5^\circ, B = 115 \angle 77^\circ$$

If the receiving-end voltage is 275 kV, determine:

- The sending-end voltage required if a load of 250 MW at 0.85 lagging pf is being delivered at the receiving-end.
- The maximum power that can be delivered if the sending-end voltage is held at 295 kV.
- The additional MVA that has to be provided at the receiving-end when delivering 400 MVA at 0.8 lagging pf, the supply voltage being maintained at 295 kV.

*Solution*

In Fig. 5.25 the centre of the receiving-end circle is located at

$$\left| \frac{A}{B} \right| |V_R|^2 = \frac{275 \times 275 \times 0.93}{115} = 611.6 \text{ MVA}$$

$$\cos^{-1} 0.85 = 31.8^\circ$$

$$\angle(\beta - \alpha) = 77^\circ - 1.5^\circ = 75.5^\circ$$

- Locate  $OP$  corresponding to the receiving-end load of 250 MW at 0.85 lagging pf ( $+31.8^\circ$ ). Then

$$C_R P = 850 = \frac{|V_S| |V_R|}{|B|} = \frac{275 |V_S|}{115}$$

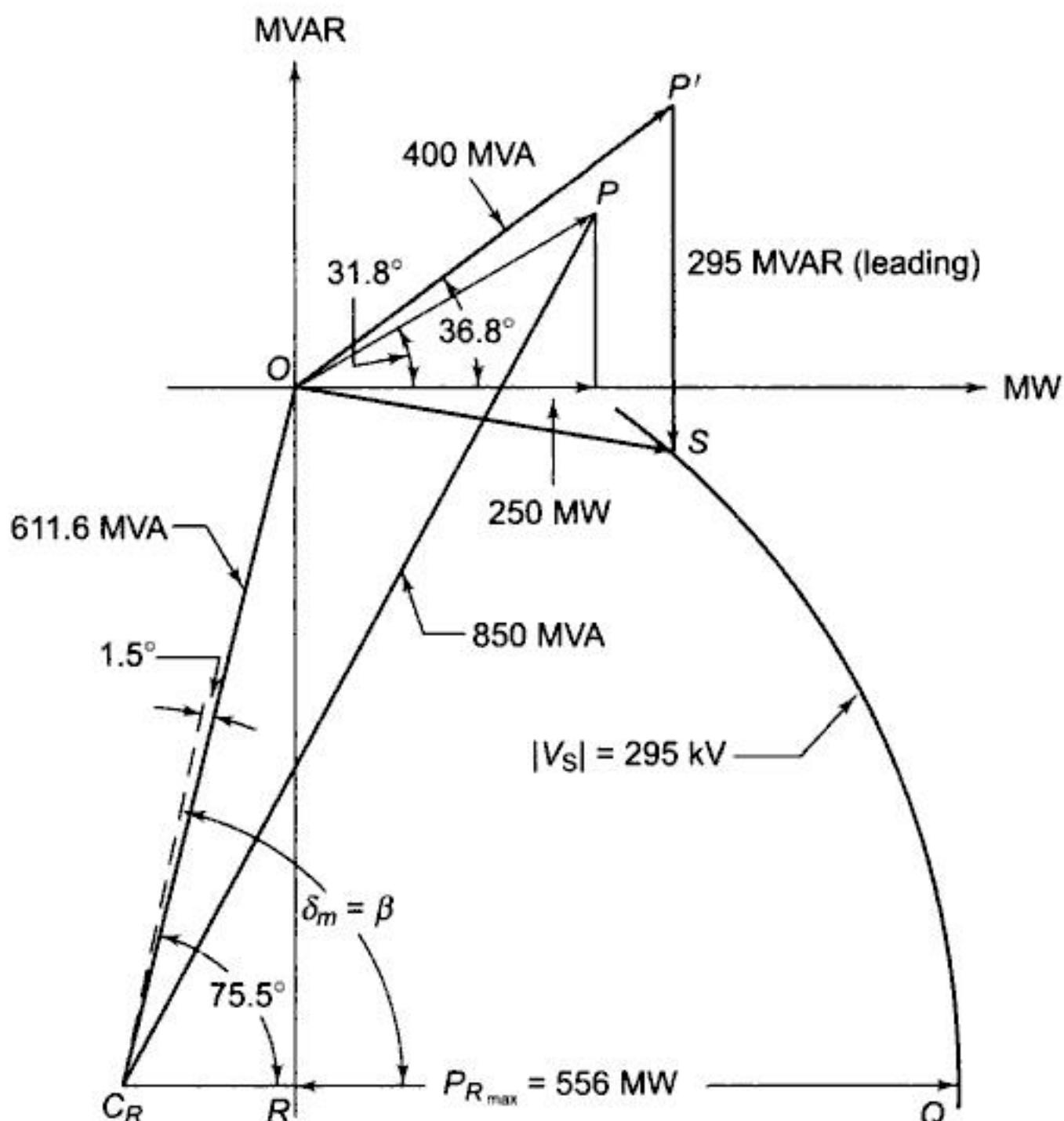
$$\therefore |V_S| = 355.5 \text{ kV}$$

- Given  $|V_S| = 295 \text{ kV}$

$$\text{Radius of circle diagram} = \frac{295 \times 275}{115} = 705.4 \text{ MVA}$$

Drawing the receiving-end circle (see Fig. 5.25) and the line  $C_R Q$  parallel to the MW-axis, we read

$$P_{R \max} = RQ = 556 \text{ MW}$$



**Fig. 5.25** Circle diagram for Example 5.11

- (c) Locate  $OP'$  corresponding to 400 MVA at 0.8 lagging pf (+ 36.8°). Draw  $P'S$  parallel to MVAR-axis to cut the circle drawn in part (b) at  $S$ . For the specified voltage profile, the line load should be  $OS$ . Therefore, additional MVA to be drawn from the line is

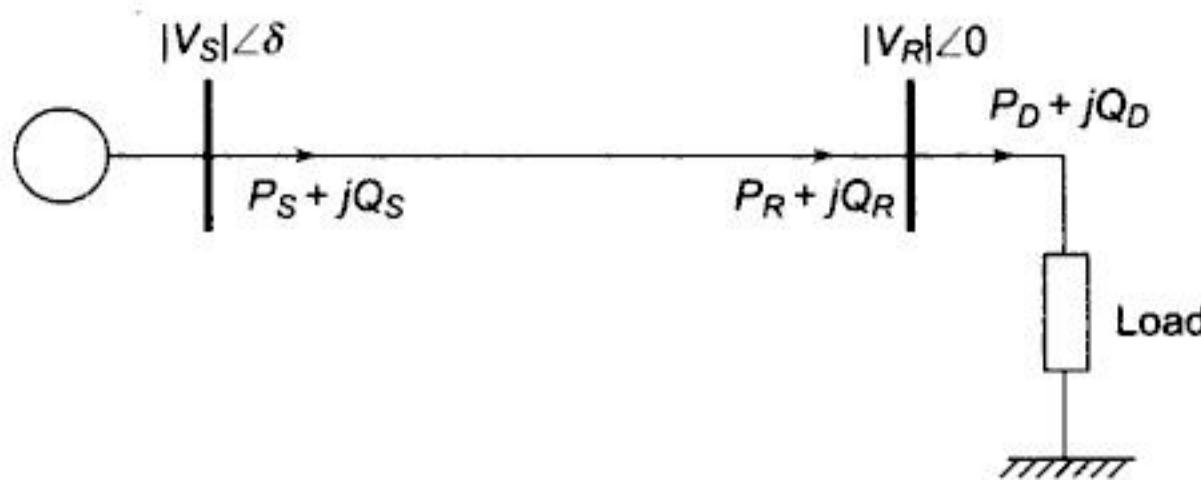
$$P'S = 295 \text{ MVAR or } 295 \text{ MVA leading}$$

## 5.10 METHODS OF VOLTAGE CONTROL

Practically all equipments used in power system is rated for a certain voltage with a permissible band of voltage variations. Voltage at various buses must, therefore, be controlled within a specified regulation figure. This article will discuss the two methods by means of which voltage at a bus can be controlled.

Consider the two-bus system shown in Fig. 5.26 (already exemplified in Sec. 5.9). For the sake of simplicity let the line be characterized by a series reactance (i.e. it has negligible resistance). Further, since the torque angle  $\delta$  is small under practical conditions, real and reactive powers delivered by the line for fixed sending-end voltage  $|V_s|$  and a specified receiving-end voltage  $|V_R^s|$  can be written as below from Eqs (5.71) and (5.73).

$$P_R = \frac{|V_s| |V_R^s|}{X} \sin \delta \quad (5.82)$$



**Fig. 5.26** A two-bus system

$$Q_R^s = \frac{|V_R^s|}{X} (|V_S| - |V_R^s|) \quad (5.83)$$

Equation (5.83) upon quadratic solution \* can also be written as

$$|V_R^s| = \frac{1}{2}|V_S| + \frac{1}{2}|V_S|(1 - 4XQ_R^s/|V_S|^2)^{1/2} \quad (5.84)$$

Since the real power demanded by the load must be delivered by the line,

$$P_R = P_D$$

Varying real power demand  $P_D$  is met by consequent changes in the torque angle  $\delta$ .

It is, however, to be noted that the received reactive power of the line must remain fixed at  $Q_R^s$  as given by Eq. (5.83) for fixed  $|V_S|$  and specified  $|V_R^s|$ . The line would, therefore, operate with specified receiving-end voltage for only one value of  $Q_D$  given by

$$Q_D = Q_R^s$$

Practical loads are generally lagging in nature and are such that the VAR demand  $Q_D$  may exceed  $Q_R^s$ . It easily follows from Eq. (5.83) that for  $Q_D > Q_R^s$  the receiving-end voltage must change from the specified value  $|V_R^s|$  to some value  $|V_R|$  to meet the demanded VARs. Thus

$$Q_D = Q_R = \frac{|V_R|}{X} (|V_S| - |V_R|) \text{ for } (Q_D > Q_R^s)$$

The modified  $|V_R|$  is then given by

$$|V_R| = \frac{1}{2}|V_S| + \frac{1}{2}|V_S|(1 - 4XQ_R/|V_S|^2)^{1/2} \quad (5.85)$$

Comparison of Eqs (5.84) and (5.85) reveals that for  $Q_D = Q_R = Q_R^s$ , the receiving-end voltage is  $|V_R^s|$ , but for  $Q_D = Q_R > Q_R^s$ ,

$$|V_R| < |V_R^s|$$

\* Negative sign in the quadratic solution is rejected because otherwise the solution would not match the specified receiving-end voltage which is only slightly less than the sending-end voltage (the difference is less than 12%).

Thus a VAR demand larger than  $Q_R^s$  is met by a consequent fall in receiving-end voltage from the specified value. Similarly, if the VAR demand is less than  $Q_R^s$ , it follows that

$$|V_R| > |V_R^s|$$

Indeed, under light load conditions, the charging capacitance of the line may cause the VAR demand to become negative resulting in the receiving-end voltage exceeding the sending-end voltage (this is the Ferranti effect already illustrated in Subsection 5.7).

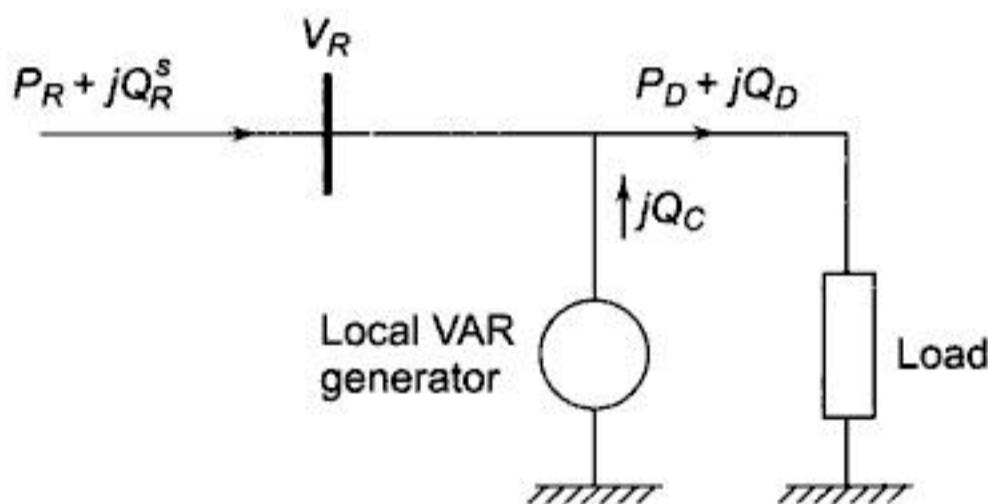
In order to regulate the line voltage under varying demands of VARs, the two methods discussed below are employed.

### Reactive Power Injection

It follows from the above discussion that in order to keep the receiving-end voltage at a specified value  $|V_R^s|$ , a fixed amount of VARs ( $Q_R^s$ ) must be drawn from the line.\* To accomplish this under conditions of a varying VAR demand  $Q_D$ , a local VAR generator (controlled reactive source/compensating equipment) must be used as shown in Fig. 5.27. The VAR balance equation at the receiving-end is now

$$Q_R^s + Q_C = Q_D$$

Fluctuations in  $Q_D$  are absorbed by the *local VAR generator*  $Q_C$  such that the VARs drawn from the line remain fixed at  $Q_R^s$ . The receiving-end voltage would thus remain fixed at  $|V_R^s|$  (this of course assumed a fixed sending-end voltage  $|V_S|$ ). Local VAR compensation can, in fact, be made automatic by using the signal from the VAR meter installed at the receiving-end of the line.



**Fig. 5.27** Use of local VAR generator at the load bus

Two types of VAR generators are employed in practice—*static type* and *rotating type*. These are discussed below:

#### Static VAR generator

It is nothing but a bank of three-phase static capacitors and/or inductors. With reference to Fig. 5.28, if  $|V_R|$  is in line kV, and  $X_C$  is the per phase capacitive

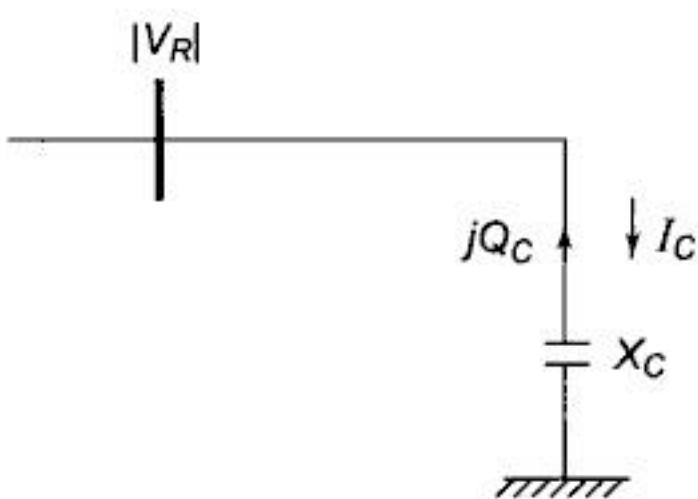
\* Of course, since  $|V_R^s|$  is specified within a band,  $Q_R^s$  may vary within a corresponding band.

reactance of the capacitor bank on an equivalent star basis, the expression for the VARs fed into the line can be derived as under.

$$I_C = j \frac{|V_R|}{\sqrt{3} X_C} \text{ kA}$$

$$jQ_C (\text{three-phase}) = 3 \frac{|V_R|}{\sqrt{3}} (-I^* C)$$

$$= j3 \times \frac{|V_R|}{\sqrt{3}} \times \frac{|V_R|}{\sqrt{3} X_C} \text{ MVA}$$



**Fig. 5.28** Static capacitor bank

$$\therefore Q_C (\text{three-phase}) = \frac{|V_R|^2}{X_C} \text{ MVAR} \quad (5.86)$$

If inductors are employed instead, VARs fed into the line are

$$Q_L (\text{three-phase}) = - \frac{|V_R|^2}{X_L} \text{ MVAR} \quad (5.87)$$

Under heavy load conditions, when positive VARs are needed, capacitor banks are employed; while under light load conditions, when negative VARs are needed, inductor banks are switched on.

The following observations can be made for static VAR generators:

- (i) Capacitor and inductor banks can be switched on in steps. However, stepless (smooth) VAR control can now be achieved using SCR (Silicon Controlled Rectifier) circuitry.
- (ii) Since  $Q_C$  is proportional to the square of terminal voltage, for a given capacitor bank, their effectiveness tends to decrease as the voltage sags under full load conditions.
- (iii) If the system voltage contains appreciable harmonics, the fifth being the most troublesome, the capacitors may be overloaded considerably.
- (iv) Capacitors act as short circuit when switched 'on'.
- (v) There is a possibility of series resonance with the line inductance particularly at harmonic frequencies.

### **Rotating VAR generator**

It is nothing but a synchronous motor running at no-load and having excitation adjustable over a wide range. It feeds positive VARs into the line under overexcited conditions and feeds negative VARs when underexcited. A machine thus running is called a *synchronous condenser*.

Figure 5.29 shows a synchronous motor connected to the receiving-end bus bars and running at no load. Since the motor draws negligible real power from the bus bars,  $E_G$  and  $V_R$  are nearly in phase.  $X_S$  is the synchronous reactance of the motor which is assumed to have negligible resistance. If  $|E_G|$  and  $|V_R|$  are in line kV, we have

$$I_C = \frac{(|V_R| - |E_G|) \angle 0^\circ}{\sqrt{3} \times jX_S} \text{ kA}$$

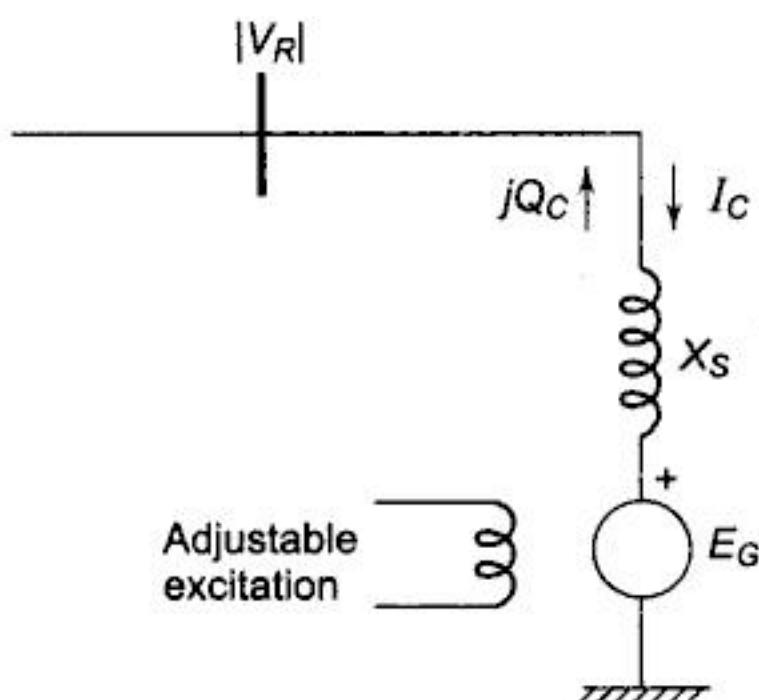
$$jQ_C = 3 \frac{|V_R| \angle 0^\circ}{\sqrt{3}} (-I_C^*)$$

$$= 3 \frac{|V_R|}{\sqrt{3}} \left( -\frac{|V_R| - |E_G|}{-jX_S \sqrt{3}} \right)$$

$$= j|V_R|(|E_G| - |V_R|)/X_S \text{ MVA}$$

$$\therefore Q_C = |V_R| (|E_G| - |V_R|)/X_S \text{ MVAR}$$

(5.88)



**Fig. 5.29** Rotating VAR generation

It immediately follows from the above relationship that the machine feeds positive VARs into the line when  $|E_G| > |V_R|$  (overexcited case) and injects negative VARs if  $|E_G| < |V_R|$  (underexcited case). VARs are easily and continuously adjustable by adjusting machine excitation which controls  $|E_G|$ .

In contrast to static VAR generators, the following observations are made in respect of rotating VAR generators.

- (i) These can provide both positive and negative VARs which are continuously adjustable.
- (ii) VAR injection at a given excitation is less sensitive to changes in bus voltage. As  $|V_R|$  decreases and  $(|E_G| - |V_R|)$  increases with consequent smaller reduction in  $Q_C$  compared to the case of static capacitors.

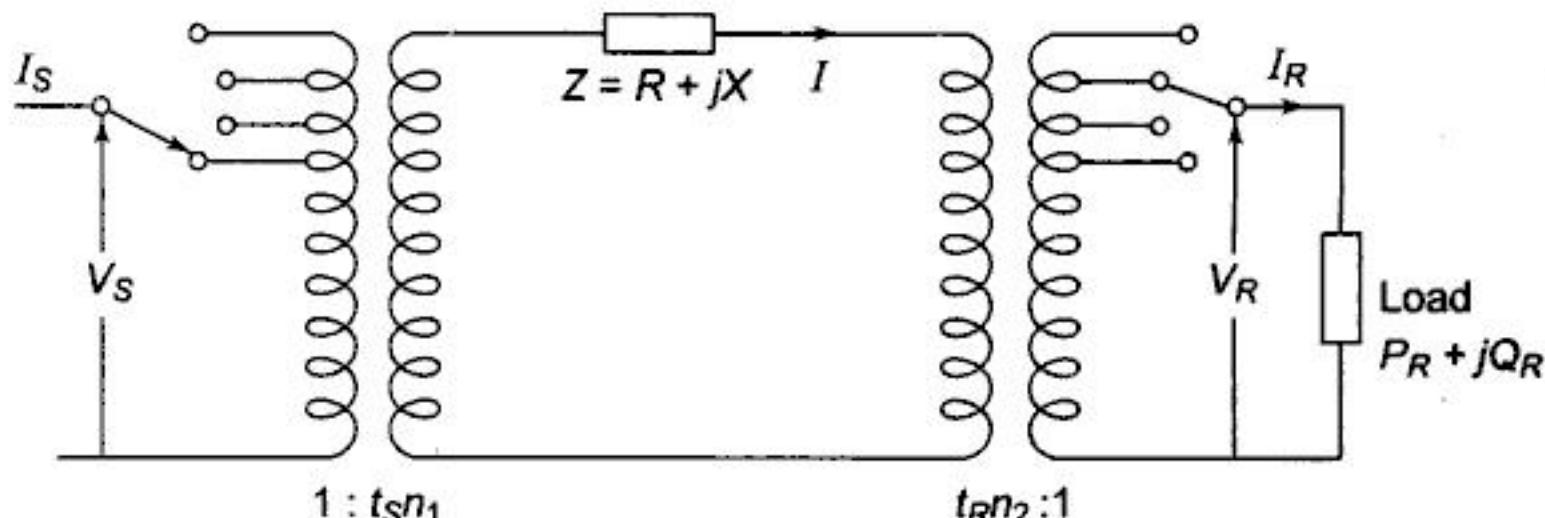
From the observations made above in respect of static and rotating VAR generators, it seems that rotating VAR generators would be preferred. However, economic considerations, installation and maintenance problems limit their practical use to such buses in the system where a large amount of VAR injection is needed.

### Control by Transformers

The VAR injection method discussed above lacks the flexibility and economy of voltage control by transformer tap changing. The transformer tap changing is obviously limited to a narrow range of voltage control. If the voltage correction needed exceeds this range, tap changing is used in conjunction with the VAR injection method.

Receiving-end voltage which tends to sag owing to VARs demanded by the load, can be raised by simultaneously changing the taps of sending- and receiving-end transformers. Such tap changes must be made 'on-load' and can be done either manually or automatically, the transformer being called a Tap Changing Under Load (TCUL) transformer.

Consider the operation of a transmission line with a tap changing transformer at each end as shown in Fig. 5.30. Let  $t_S$  and  $t_R$  be the fractions of the nominal transformation ratios, i.e. the tap ratio/nominal ratio. For example, a transformer with nominal ratio 3.3 kV/11 kV when tapped to give 12 kV with 3.3 kV input has  $t_S = 12/11 = 1.09$ .



**Fig. 5.30** Transmission line with tap changing transformer at each end

With reference to Fig. 5.30 let the impedances of the transformer be lumped in  $Z$  along with the line impedance. To compensate for voltage in the line and transformers, let the transformer taps be set at off nominal values,  $t_S$  and  $t_R$ . With reference to the circuit shown, we have

$$t_S n_1 V_S = t_R n_2 V_R + IZ \quad (5.89)$$

From Eq. (5.75) the voltage drop referred to the high voltage side is given by

$$|\Delta V| = \frac{RP_R + XQ_R}{t_R n_2 |V_R|} \quad (5.90)$$

Now

$$|\Delta V| = t_S n_1 |V_S| - t_R n_2 |V_R|$$

$$\therefore t_S n_1 |V_S| = t_R n_2 |V_R| + \frac{RP_R + XQ_R}{t_R n_2 |V_R|} \quad (5.91)$$

In order that the voltage on the HV side of the two transformers be of the same order and the tap setting of each transformer be the minimum, we choose

$$t_S t_R = 1 \quad (5.92)$$

Substituting  $t_R = 1/t_S$  in Eq. (5.91) and reorganizing, we obtain

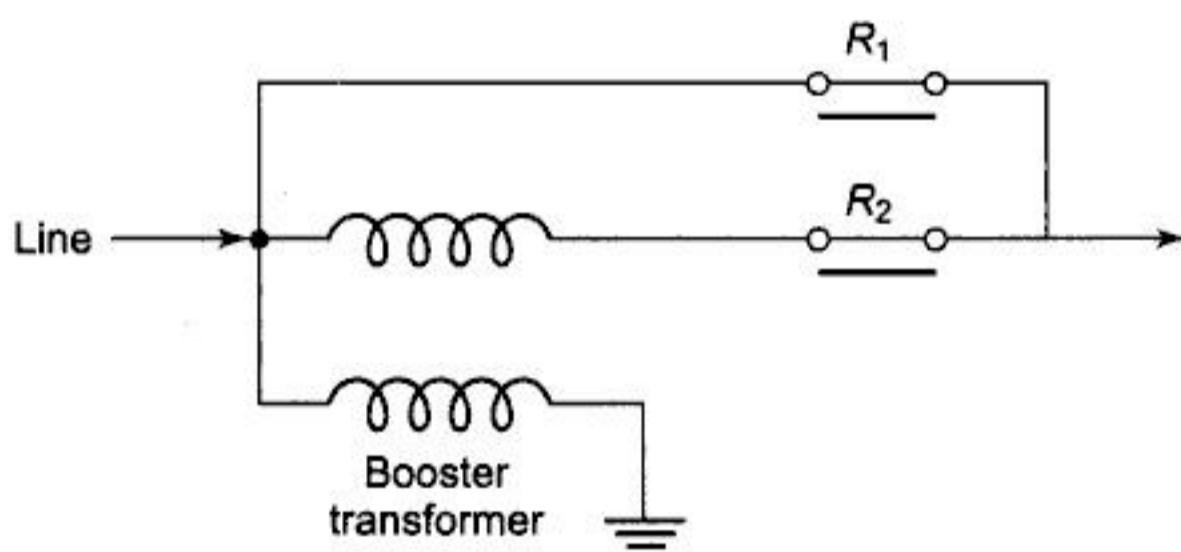
$$t_S^2 \left( 1 - \frac{RP_R + XQ_R}{n_1 n_2 |V_S| |V_R|} \right) = \frac{n_2 |V_R|}{n_1 |V_S|} \quad (5.93)$$

For complete voltage drop compensation, the right hand side of Eq. (5.93) should be unity.

It is obvious from Fig. 5.30 that  $t_S > 1$  and  $t_R < 1$  for voltage drop compensation. Equation (5.90) indicates that  $t_R$  tends to increase\* the voltage  $|\Delta V|$  which is to be compensated. Thus merely tap setting as a method of voltage drop compensation would give rise to excessively large tap setting if compensation exceeds certain limits. Thus, if the tap setting dictated by Eq. (5.93), to achieve a desired receiving-end voltage exceeds the normal tap setting range (usually not more than  $\pm 20\%$ ), it would be necessary to simultaneously inject VARs at the receiving-end in order to maintain the desired voltage level.

### Control by Mid-Line Boosters

It may be desirable on technical or economic grounds, to increase the voltage at an intermediate point in a line rather than at the ends as with tap-changing transformers. Boosters are generally used in distribution feeders where the cost of tap-changing transformers is not warranted. Fig. 5.31 shows the connection of in-phase booster transformer.



**Fig. 5.31** Booster transformer

The booster can be pressed into the circuit by closure of relay  $R_2$  and the opening of relay  $R_1$  and vice versa.

Owing to increasing voltages and line lengths, and also the greater use of cables, the light-load reactive problem for an interconnected system becomes significant, specially with latest generators of limited VAR absorption capability. At peak load, transmission systems are required to increase their VAR generation, and as the load reduces during light load period, they need to reduce the generated VARs by the following methods listed in order of economic viability:

- (i) switch out shunt capacitors,
- (ii) switch in shunt reactors,
- (iii) run hydro plant on maximum VAR absorption,
- (iv) switch out one cable in a double-circuit link,
- (v) tap-stagger transformers,
- (vi) run base load generators at maximum VAR absorption.

\* This is so because  $t_R < 1$  increases the line current  $I$  and hence voltage drop.

## Compensation of Transmission Lines

The performance of long EHV AC transmission systems can be improved by reactive compensation of series or shunt (parallel) type. Series capacitors and shunt reactors are used to reduce artificially the series reactance and shunt susceptance of lines and thus they act as the line compensators. Compensation of lines results in improving the system stability (Ch. 12) and voltage control, in increasing the efficiency of power transmission, facilitating line energization and reducing temporary and transient overvoltages.

Series capacitor compensation reduces the series impedance of the line which causes voltage drop and is the most important factor in finding the maximum power transmission capability of a line (Eq. 5.70).  $A$ ,  $C$  and  $D$  constants are functions of  $Z$  and therefore are also affected by change in the value of  $Z$ , but these changes are small in comparison to the change in  $B$  as  $B = Z$  for the nominal- $\pi$  and equals  $Z(\sinh \gamma l / \gamma l)$  for the equivalent- $\pi$ .

The voltage drop  $\Delta V$  due to series compensation is given by

$$\Delta V \approx IR \cos \phi_r + I(X_L - X_C) \sin \phi_r \quad (5.94)$$

Here  $X_C$  is the capacitive reactance of the series capacitor bank per phase and  $X_L$  is the total inductive reactance of the line/phase. In practice,  $X_C$  may be so selected that the factor  $(X_L - X_C) \sin \phi_r$  becomes negative and equals (in magnitude)  $R \cos \phi_r$  so that  $\Delta V$  becomes zero. The ratio  $X_C/X_L$  is called "compensation factor" and when expressed as a percentage is known as the "percentage compensation".

The extent of effect of compensation depends on the number, location and circuit arrangements of series capacitor and shunt reactor stations. While planning long-distance lines, besides the average degree of compensation required, it is required to find out the most appropriate location of the reactors and capacitor banks, the optimum connection scheme and the number of intermediate stations. For finding the operating conditions along the line, the  $ABCD$  constants of the portions of line on each side of the capacitor bank, and  $ABCD$  constants of the bank may be first found out and then equivalent constants of the series combination of line-capacitor-line can then be arrived at by using the formulae given in Appendix B.

In India, in states like UP, series compensation is quite important since super thermal plants are located (east) several hundred kilometers from load centres (west) and large chunks of power must be transmitted over long distances. Series capacitors also help in balancing the voltage drop of two parallel lines.

When series compensation is used, there are chances of sustained overvoltage to the ground at the series capacitor terminals. This overvoltage can be the power limiting criterion at high degree of compensation. A spark gap with a high speed contactor is used to protect the capacitors under overvoltage conditions.

Under light load or no-load conditions, charging current should be kept less than the rated full-load current of the line. The charging current is approximately given by  $B_C|V|$  where  $B_C$  is the total capacitive susceptance of the line

and  $|V|$  is the rated voltage to neutral. If the total inductive susceptance is  $B_L$  due to several inductors connected (shunt compensation) from line to neutral at appropriate places along the line, then the charging current would be

$$I_{\text{chg}} = (B_C - B_L) |V| = B_C |V| \left( 1 - \frac{B_L}{B_C} \right) \quad (5.95)$$

Reduction of the charging current is by the factor of  $(1 - B_L/B_C)$  and  $B_L/B_C$  is the shunt compensation factor. Shunt compensation at no-load also keeps the receiving-end voltage within limits which would otherwise be quite high because of the Ferranti effect. Thus reactors should be introduced as load is removed for proper voltage control.

As mentioned earlier, the shunt capacitors are used across an inductive load so as to provide part of the reactive VARs required by the load to keep the voltage within desirable limits. Similarly, the shunt reactors are kept across capacitive loads or in light load conditions, as discussed above, to absorb some of the leading VARs for achieving voltage control. Capacitors are connected either directly to a bus or through tertiary winding of the main transformer and are placed along the line to minimize losses and the voltage drop.

It may be noted that for the same voltage boost, the reactive power capacity of a shunt capacitor is greater than that of a series capacitor. The shunt capacitor improves the pf of the load while the series capacitor has hardly any impact on the pf. Series capacitors are more effective for long lines for improvement of system stability.

Thus, we see that in both series and shunt compensation of long transmission lines it is possible to transmit large amounts of power efficiently with a flat voltage profile. Proper type of compensation should be provided in proper quantity at appropriate places to achieve the desired voltage control. The reader is encouraged to read the details about the Static Var Systems (SVS) in Refs [7, 8, 16].

### **5.11 SUMMARY**

In this chapter modelling, characteristics and performance of power transmission lines have been dealt with in considerable details. This is a prerequisite for future chapters and further studies in power systems.

## **Problems**

- 5.1 A three-phase voltage of 11 kV is applied to a line having  $R = 10 \Omega$  and  $X = 12 \Omega$  per conductor. At the end of the line is a balanced load of  $P$  kW at a leading power factor. At what value of  $P$  is the voltage regulation zero when the power factor of the load is (a) 0.707, (b) 0.85?

- 5.2 A long line with  $A = D = 0.9 \angle 1.5^\circ$  and  $B = 150 \angle 65^\circ \Omega$  has at the load end a transformer having a series impedance  $Z_T = 100 \angle 67^\circ \Omega$ . The load voltage and current are  $V_L$  and  $I_L$ . Obtain expressions for  $V_S$  and  $I_S$  in form of

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_L \\ I_L \end{bmatrix}$$

and evaluate these constants.

- 5.3 A three-phase overhead line 200 km long has resistance =  $0.16 \Omega/\text{km}$  and conductor diameter of 2 cm with spacing 4 m, 5 m and 6 m transposed. *Find:* (a) the  $ABCD$  constants using Eq. (5.28b), (b) the  $V_S$ ,  $I_S$ ,  $\text{pf}_S$ ,  $P_S$  when the line is delivering full load of 50 MW at 132 kV and 0.8 lagging pf, (c) efficiency of transmission, and (d) the receiving-end voltage regulation.
- 5.4 A short 230 kV transmission line with a reactance of  $18 \Omega/\text{phase}$  supplies a load at 0.85 lagging power factor. For a line current of 1,000 A the receiving- and sending-end voltages are to be maintained at 230 kV. Calculate (a) rating of synchronous capacitor required, (b) the load current, (c) the load MVA. Power drawn by the synchronous capacitor may be neglected.
- 5.5 A 40 MVA generating station is connected to a three-phase line having

$$Z = 300 \angle 75^\circ \Omega \quad Y = 0.0025 \angle 90^\circ \Omega^{-1}$$

The power at the generating station is 40 MVA at unity power factor at a voltage of 120 kV. There is a load of 10 MW at unity power factor at the mid point of the line. Calculate the voltage and load at the distant end of the line. Use nominal- $T$  circuit for the line.

- 5.6 The generalized circuit constants of a transmission line are

$$A = 0.93 + j0.016$$

$$B = 20 + j140$$

The load at the receiving-end is 60 MVA, 50 Hz, 0.8 power factor lagging. The voltage at the supply end is 220 kV. Calculate the load voltage.

- 5.7 Find the incident and reflected currents for the line of Problem 5.3 at the receiving-end and 200 km from the receiving-end.
- 5.8 If the line of Problem 5.6 is 200 km long and delivers 50 MW at 220 kV and 0.8 power factor lagging, determine the sending-end voltage, current, power factor and power. Compute the efficiency of transmission, characteristic impedance, wavelength and velocity of propagation.
- 5.9 For Example 5.7, find the parameters of the equivalent- $\pi$  circuit for the line.
- 5.10 An interconnector cable having a reactance of  $6 \Omega$  links generating stations 1 and 2 as shown in Fig. 5.18(a). The desired voltage profile is  $|V_1| = |V_2| = 22 \text{ kV}$ . The loads at the two-bus bars are 40 MW at 0.8 lagging

power factor and 20 MW at 0.6 lagging power factor, respectively. The station loads are equalized by the flow of power in the cable. Estimate the torque angle and the station power factors.

- 5.11 A 50 Hz, three-phase, 275 kV, 400 km transmission line has the following parameters (per phase):

$$\text{Resistance} = 0.035 \Omega/\text{km}$$

$$\text{Inductance} = 1 \text{ mH/km}$$

$$\text{Capacitance} = 0.01 \mu\text{F/km}$$

If the line is supplied at 275 kV, determine the MVA rating of a shunt reactor having negligible losses that would be required to maintain 275 kV at the receiving-end, when the line is delivering no-load. Use nominal- $\pi$  method.

- 5.12 A three-phase feeder having a resistance of  $3 \Omega$  and a reactance of  $10 \Omega$  supplies a load of 2.0 MW at 0.85 lagging power factor. The receiving-end voltage is maintained at 11 kV by means of a static condenser drawing 2.1 MVAR from the line. Calculate the sending-end voltage and power factor. What is the voltage regulation and efficiency of the feeder?
- 5.13 A three-phase overhead line has resistance and reactance of 5 and  $20 \Omega$ , respectively. The load at the receiving-end is 30 MW, 0.85 power factor lagging at 33 kV. Find the voltage at the sending-end. What will be the kVAR rating of the compensating equipment inserted at the receiving-end so as to maintain a voltage of 33 kV at each end? Find also the maximum load that can be transmitted.
- 5.14 Construct a receiving-end power circle diagram for the line of Example 5.7. Locate the point corresponding to the load of 50 MW at 220 kV with 0.8 lagging power factor. Draw the circle passing through the load point. Measure the radius and determine therefrom  $|V_S|$ . Also draw the sending-end circle and determine therefrom the sending-end power and power factor.
- 5.15 A three-phase overhead line has resistance and reactance per phase of 5 and  $25 \Omega$ , respectively. The load at the receiving-end is 15 MW, 33 kV, 0.8 power factor lagging. Find the capacity of the compensation equipment needed to deliver this load with a sending-end voltage of 33 kV. Calculate the extra load of 0.8 lagging power factor which can be delivered with the compensating equipment (of capacity as calculated above) installed, if the receiving-end voltage is permitted to drop to 28 kV.

## References

### Books

1. *Transmission Line Reference Book—345 kV and Above*, 2nd edn, Electric Power Research Institute, Palo Alto CA., 1982.

2. McCombe, J. and F.J. Haigh, *Overhead-line Practice*, Macdonald, London, 1966.
3. Stevenson, W.D., *Elements of Power System Analysis*, 4th edn, McGraw-Hill, New York, 1982.
4. Arrillaga, J., High Voltage Direct Current Transmission, *IEE Power Engineering Series 6*, Peter Peregrinus Ltd., London, 1983.
5. Kimbark, E.W., *Direct Current Transmission*, vol. 1, Wiley, New York, 1971.
6. Uhlmann, E., *Power Transmission by Direct Current*, Springer-Verlag, Berlin, 1975.
7. Miller, T.J.E., *Reactive Power Control in Electric Systems*, Wiley, New York 1982.
8. Mathur, R.M. (Ed.), *Static Compensators for Reactive Power Control*, Context Pub., Winnipeg, 1984.
9. Desphande, M.V., *Electrical Power System Design*, Tata McGraw-Hill, New Delhi, 1984.

## Papers

10. Dunlop, R.D., R. Gautam and D.P. Marchenko, "Analytical Development of Loadability Characteristics for EHV and UHV Transmission Lines", *IEEE Trans.*, PAS, 1979, 98: 606.
11. "EHV Transmission", (Special Issue), *IEEE Trans.*, June 1966, No. 6, PAS-85.
12. Goodrich, R.D., "A Universal Power Circle Diagram", *AIEE Trans.*, 1951, 70: 2042.
13. Indulkar, C.S., Parmod Kumar and D.P. Kothari, "Sensitivity Analysis of a Multiconductor Transmission Line", *Proc. IEEE*, March 1982, 70: 299.
14. Indulkar, C.S., Parmod Kumar and D.P. Kothari, "Some Studies on Carrier Propagation in Overhead Transmission Lines", *IEEE Trans.*, on PAS, no. 4, 1983, 102: 942.
15. Bijwe, P.R., D.P. Kothari, J. Nanda and K.S. Lingamurthy, "Optimal Voltage Control Using Constant Sensitivity Matrix", *Electric Power System Research*, Oct. 1986, 3: 195.
16. Kothari, D.P., et al., "Microprocessors Controlled Static VAR Systems", *Proc. Int. Conf. Modelling & Simulation*, Gorakhpur, Dec. 1985, 2: 139.

# **Chapter 6**

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## **Load Flow Studies**

### **6.1 INTRODUCTION**

With the background of the previous chapters, we are now ready to study the operational features and electrical performance of a composite power system. The symmetrical steady state is, in fact, the most important mode of operation of a power system. Three major problems encountered in this mode of operation are listed below in their hierarchical order.

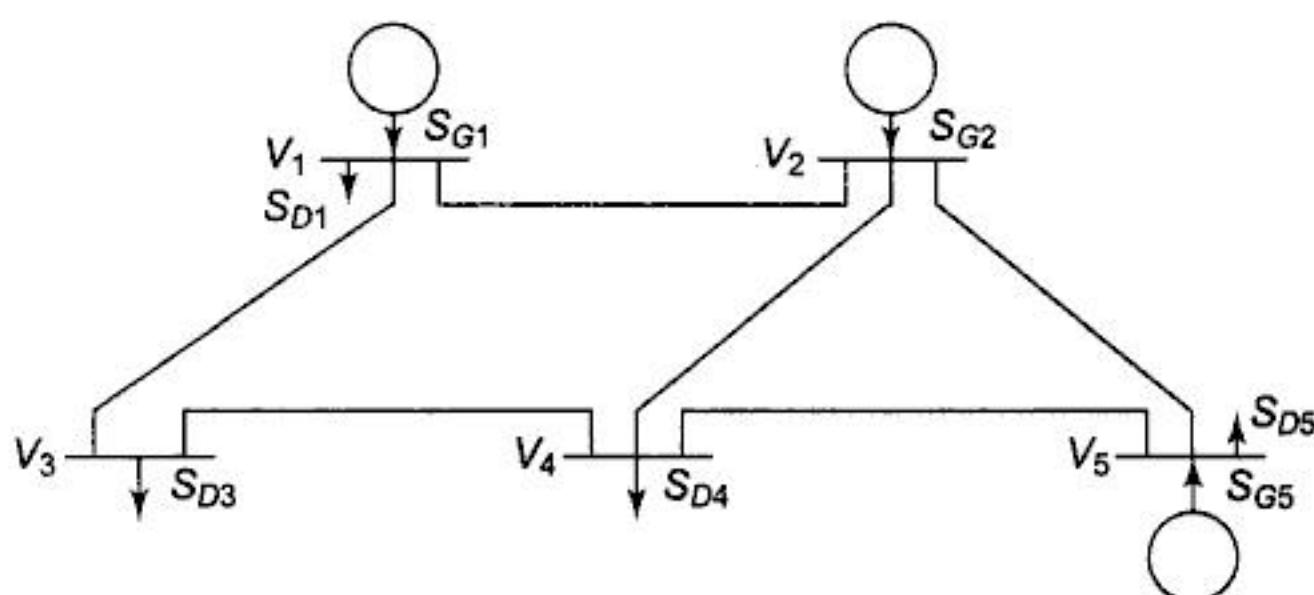
1. Load flow problem
2. Optimal load scheduling problem
3. Systems control problem

This chapter is devoted to the load flow problem, while the other two problems will be treated in later chapters.

Load flow study in power system parlance is the steady state solution of the power system network. The power system is modelled by an electric network and solved for the steady-state powers and voltages at various buses. The direct analysis of the circuit is not possible, as the loads are given in terms of complex powers rather than impedances, and the generators behave more like power sources than voltage sources. The main information obtained from the load flow study comprises of magnitudes and phase angles of load bus voltages, reactive powers and voltage phase angles at generator buses, real and reactive power flow on transmission lines together with power at the reference bus, other variables being specified. This information is essential for the continuous monitoring of the current state of the system and for analyzing the effectiveness of the alternative plans for the future, such as adding new generator sites, meeting increased load demand and locating new transmission sites.

In load flow analysis, we are mainly interested in voltages at various buses and power injection into the transmission system. Figure 6.1 shows the one-line diagram of a power system having five buses.

Here  $S_Gs$  and  $S_Ds$  represent the complex powers injected by generators and complex powers drawn by the loads and  $Vs$  represent the complex voltages at the various buses. Thus, there results a net injection of power into the transmission system. In a practical system, there may be thousands of buses



**Fig. 6.1** One-line diagram of a five bus system

and transmission links. We shall concentrate mainly on the transmission system with the generators and loads modelled by the complex powers. The transmission system may be a primary transmission system, which transmits bulk power from the generators to the bulk power substation, or a subtransmission system which transmits power from substations or some old generators to the distribution substations. The transmission system is to be designed in such a manner that the power system operation is reliable and economic, and no difficulties are encountered in its operation. The likely difficulties are, one or more transmission lines becoming overloaded, generator(s) becoming overloaded, or the stability margin for a transmission link being too small, etc. Also, there may be emergencies, such as the loss of one or more transmission links, shut-down of generators, etc. which gives rise to overloading of some generators and transmission links. In system operation and planning, the voltages and powers are kept within certain limits and alternative plans are developed for easy and reliable operation. At the same time, it is also necessary to consider the economy of operation with respect to fuel costs to generate all the power needed. It may happen that each of the objectives mentioned above gives conflicting results. Usually a compromise has to be made with such results so that the system is reliable to operate in an emergency, and at the same time, is economical in operation.

The power system network of today is highly complicated consisting of hundreds of buses and transmission links. Thus, the load flow study involves extensive calculations. Before the advent of digital computers, the AC calculating board was the only means of carrying out load flow studies. These studies were, therefore, tedious and time consuming. With the availability of fast and large sized digital computers, all kinds of power system studies, including load flow study can now be carried out conveniently. In fact, some of the advanced level sophisticated studies which were almost impossible to carry out on the AC calculating board, have now become possible. The AC calculating board has been rendered obsolete.



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Thus, in general,

$$\begin{aligned} I_i &= Y_{i1} V_1 + Y_{i2} V_2 + \dots + Y_{ii} V_i + \dots + Y_{in} V_n \\ &= \sum_{k=1}^n (Y_{ik} V_k) \end{aligned} \quad (6.2b)$$

where  $i = 1, \dots, n$   
where

$$Y_{ik} (i \neq k) = \frac{I_i}{V_k} \text{ (all } V = 0 \text{ except } V_k\text{)} \quad (6.3)$$

= Short circuit transfer admittance between  $i$ th and  $k$ th bus

$$\text{and } Y_{ii} = \frac{I_i}{V_i} \text{ (all } V = 0 \text{ except } V_i\text{)} \quad (6.4)$$

= Short circuit driving point admittance  
or self-admittance at the  $i$ th bus

From Eqs (6.2a) and (6.3)

$$Y_{ik} (i \neq k) = -y_{ik} = \text{Negative of the total admittance connected}\text{ between } i\text{th and } k\text{th bus} \quad (6.5a)$$

( $Y_{ik} (i \neq k) = 0$  if there is no transmission line between  $i$ th and  $k$ th bus)

From Eqs (6.2a) and (6.4)

$$Y_{ii} = y_{i0} + y_{i1} + y_{i2} + \dots + y_{i, i-1} + y_{i, i+1} + \dots + y_{in} \quad (6.5b)$$

= Sum of the admittances directly connected to  $i$ th bus

Writing Eq. (6.2b) for all the  $n$  buses we can write its matrix form as

$$\mathbf{I}_{\text{BUS}} = \mathbf{Y}_{\text{BUS}} \mathbf{V}_{\text{BUS}} \quad (6.6)$$

where  $\mathbf{I}_{\text{BUS}}$  is  $n \times 1$  column vector of bus currents

$\mathbf{V}_{\text{BUS}}$  is  $n \times 1$  column vector of bus voltages

$\mathbf{Y}_{\text{BUS}}$  is  $n \times n$  matrix of admittances given as

$$\mathbf{Y}_{\text{BUS}} = \begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1n} \\ Y_{21} & Y_{22} & \dots & Y_{2n} \\ \vdots & \vdots & & \vdots \\ Y_{n1} & Y_{n2} & \dots & Y_{nn} \end{bmatrix}_{n \times n}$$

It follows that,

- (1) The diagonal element of  $\mathbf{Y}_{\text{BUS}}$  is given by Eq. (6.5b) and is the self-admittance. The off-diagonal element of  $\mathbf{Y}_{\text{BUS}}$  is given by Eq. (6.5a) and is the transfer admittance.

- (2)  $\mathbf{Y}_{\text{BUS}}$  is  $n \times n$  matrix where  $n$  is the number of buses.
- (3)  $\mathbf{Y}_{\text{BUS}}$  is a symmetric matrix ( $Y_{ik} = Y_{ki}$  ( $k \neq i$ )) if the regulating transformers are not involved. So only  $\frac{n \times n - n}{2} + n = \frac{n(n+1)}{2}$  terms are to be stored for an  $n$ -bus system.
- (4)  $Y_{ik}$  ( $i \neq k$ ) = 0 if  $i$ th and  $k$ th buses are not connected.

Since in a power network each bus is connected only to a few other buses (two or three), the  $\mathbf{Y}_{\text{BUS}}$  of a large network is very sparse, i.e. it has a large number of zero elements.

Equation (6.6) can also be written in the form

$$\mathbf{V}_{\text{BUS}} = \mathbf{Z}_{\text{BUS}} \mathbf{I}_{\text{BUS}} \quad (6.7)$$

where  $\mathbf{Z}_{\text{BUS}}$  (Bus Impedance Matrix) =  $\mathbf{Y}_{\text{BUS}}^{-1}$  (6.8)

$$\mathbf{Z}_{\text{BUS}} = \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1n} \\ Z_{21} & Z_{22} & \dots & Z_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{n1} & Z_{n2} & \dots & Z_{nn} \end{bmatrix}_{n \times n \text{ for } n \text{ bus system}}$$

1. The diagonal elements are short circuit driving point impedances, and the off-diagonal elements are short circuit transfer impedances.
2. Symmetric  $\mathbf{Y}_{\text{BUS}}$  yields symmetric  $\mathbf{Z}_{\text{BUS}}$ .
3.  $\mathbf{Z}_{\text{BUS}}$  is a full-matrix, i.e. zero elements in  $\mathbf{Y}_{\text{BUS}}$  become non-zero elements in the corresponding  $\mathbf{Z}_{\text{BUS}}$ .

$\mathbf{Y}_{\text{BUS}}$  is often used in solving load flow problems. It has gained widespread application owing to its simplicity in data preparation, and the ease with which it can be formed and modified for network changes. One of its greatest advantages is its sparsity, as it heavily reduces computer memory and time requirements.

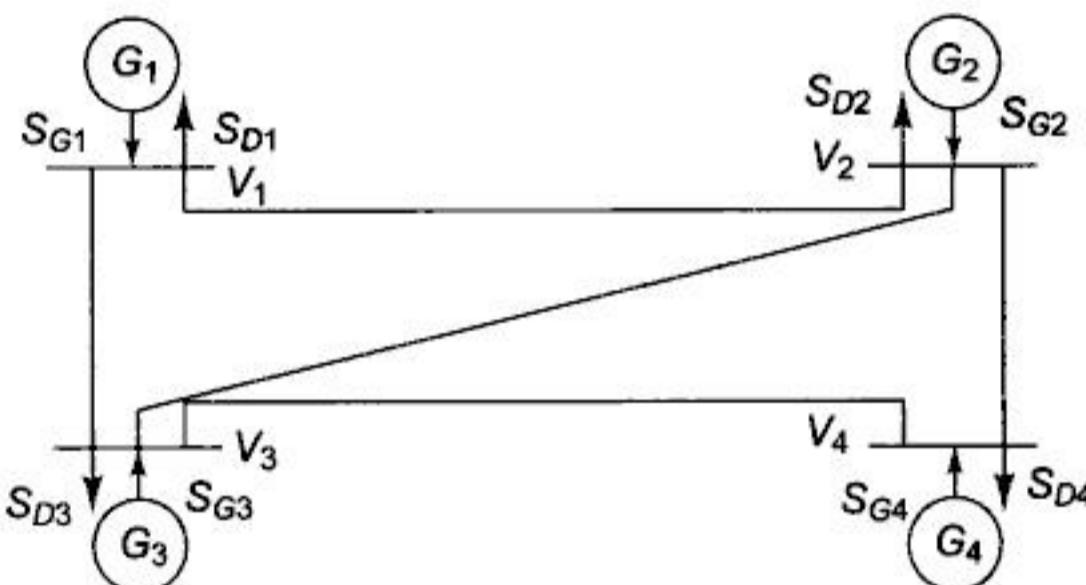
The formation of a bus impedance matrix requires either matrix inversion or use of involved algorithms.  $\mathbf{Z}_{\text{BUS}}$  is, however, most useful for short circuit studies and will be elaborated in the relevant chapter.

*Note:* 1. Tinney and associates at Bonneville Power Authority were the first to exploit the sparsity feature of  $\mathbf{Y}_{\text{BUS}}$  in greatly reducing numerical computations and in minimizing memory requirements.

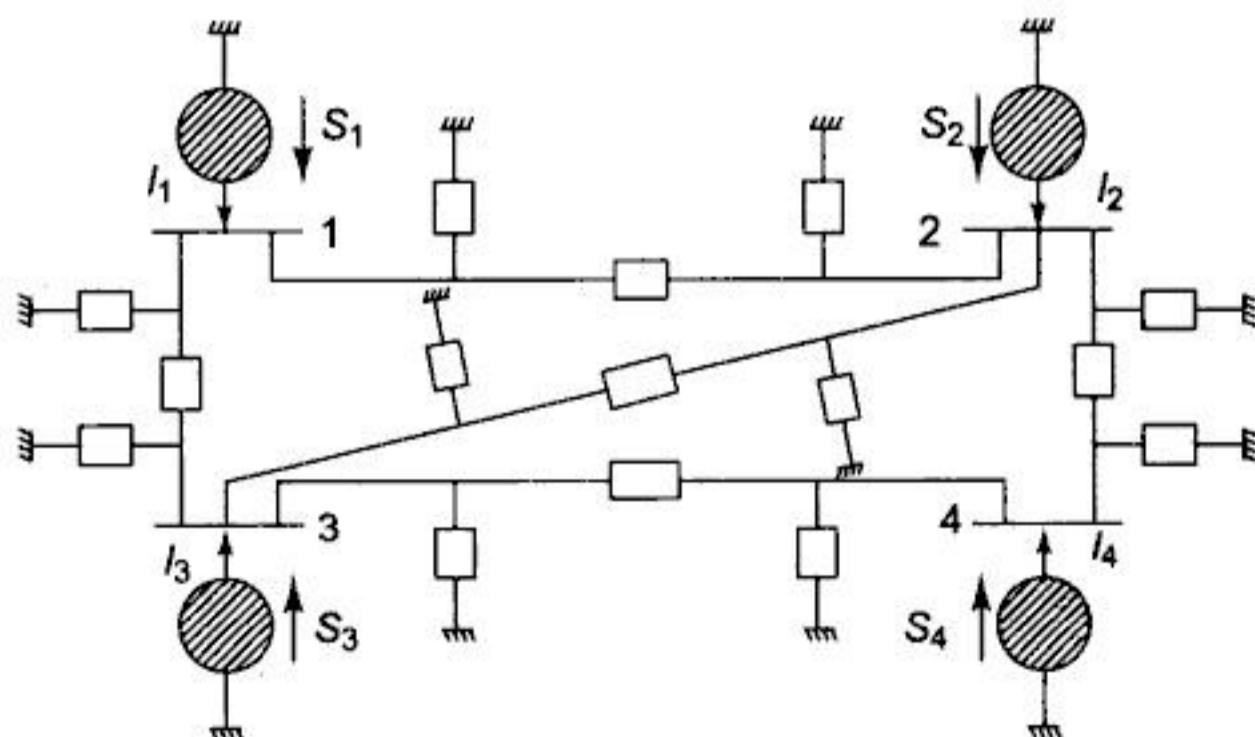
2. In more sophisticated load flow studies of large power systems, it has been shown that a certain ordering of buses (nodes) produces faster convergence and solution. This ordering is known as optimal ordering.

3.  $\mathbf{Y}_{\text{BUS}}/\mathbf{Z}_{\text{BUS}}$  constitute models of the passive portions of the power network.

**Example 6.1** Figure 6.5(a) shows the one-line diagram of a four-bus system, which is replaced by its equivalent circuit in Fig. 6.5(b), where the equivalent power source at each bus is represented by a shaded circle. The equivalent power source at the  $i$ th bus injects currents  $I_i$  into the bus. The structure of the power system is such that all the sources are always connected to a common ground node. The transmission lines are replaced by their nominal- $\pi$  equivalent. Figure 6.5(b) is redrawn in Fig. 6.5(c), after lumping the shunt admittances at the buses. The line admittance between nodes  $i$  and  $k$  is depicted by  $y_{ik}$ . Also,  $y_{ik} = y_{ki}$ . Further, the mutual admittances between the lines is assumed to be zero. In Fig. 6.5(c), there are five nodes, viz. the ground node zero and the nodes corresponding to the four buses. Applying KCL at nodes 1, 2, 3 and 4 respectively, we get four equations as follows:



(a) One-line diagram of a four-bus system



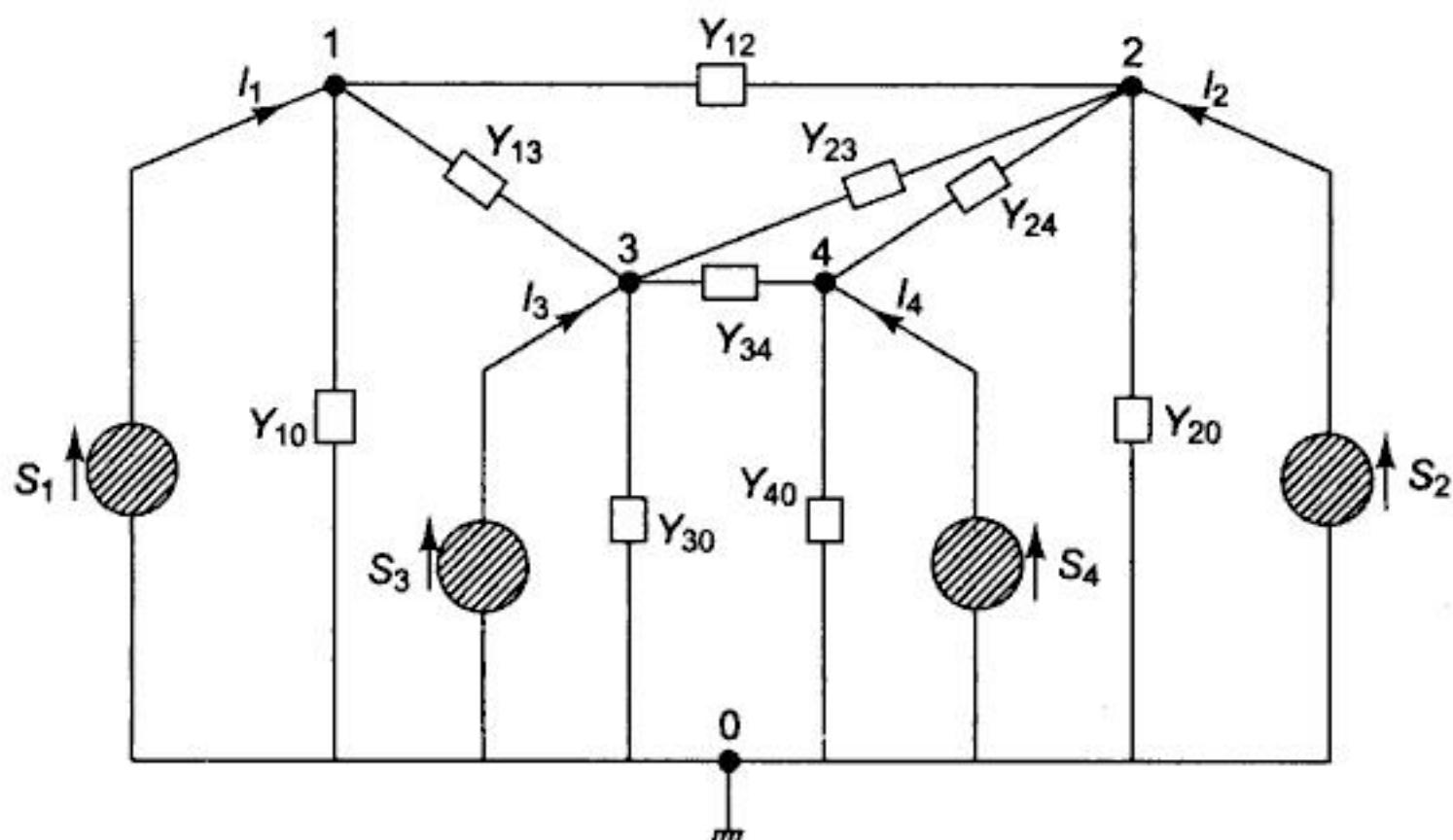
(b) Equivalent circuit of the power system of (a)

$$I_1 = y_{10} V_1 + y_{12} (V_1 - V_2) + y_{13} (V_1 - V_3)$$

$$I_2 = y_{20} V_2 + y_{12} (V_2 - V_1) + y_{23} (V_2 - V_3) + y_{24} (V_2 - V_4)$$

$$I_3 = y_{30} V_3 + y_{13} (V_3 - V_1) + y_{23} (V_3 - V_2) + y_{34} (V_3 - V_4)$$

$$I_4 = y_{40} V_4 + y_{24} (V_4 - V_2) + y_{34} (V_4 - V_3)$$



(c) Reduced circuit diagram of the power system

**Fig. 6.5 (a)-(c)** Sample four-bus system

Rearranging and writing in matrix form, we get

$$\begin{array}{|c|c|c|c|c|} \hline I_1 & (Y_{10} + Y_{12} + Y_{13}) & -Y_{12} & -Y_{13} & 0 \\ \hline I_2 & -Y_{12} & (Y_{20} + Y_{12} + Y_{23} + Y_{24}) & -Y_{23} & -Y_{24} \\ \hline I_3 & -Y_{13} & -Y_{23} & (Y_{30} + Y_{13} + Y_{23} + Y_{34}) & -Y_{34} \\ \hline I_4 & 0 & -Y_{24} & -Y_{34} & (Y_{40} + Y_{24}) \\ \hline \end{array} = \begin{array}{|c|} \hline V_1 \\ \hline V_2 \\ \hline V_3 \\ \hline V_4 \\ \hline \end{array}$$

Writing in the standard form, we get

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

or

$$I_{\text{BUS}} = Y_{\text{BUS}} V_{\text{BUS}}$$

where

$$Y_{\text{BUS}} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix}$$

As already concluded in Eqs (6.5a and b), we observe that the diagonal elements of  $Y_{\text{BUS}}$  are self-admittances, and are given by

$$\begin{aligned}Y_{11} &= y_{10} + y_{12} + y_{13} \\Y_{22} &= y_{20} + y_{12} + y_{23} + y_{24} \\Y_{33} &= y_{30} + y_{13} + y_{23} + y_{34}, \text{ and} \\Y_{44} &= y_{40} + y_{24} + y_{34}\end{aligned}$$

The off-diagonal elements of  $\mathbf{Y}_{\text{BUS}}$  are transfer admittances, and are given by

$$\begin{aligned}Y_{12} &= Y_{21} = -y_{12} \\Y_{13} &= Y_{31} = -y_{13} \\Y_{14} &= Y_{41} = 0 \\Y_{23} &= Y_{32} = -y_{23} \\Y_{24} &= Y_{42} = -y_{24} \\Y_{34} &= Y_{43} = -y_{34}\end{aligned}$$

### Algorithm for the Formation of $\mathbf{Y}_{\text{BUS}}$ Matrix

#### (a) Assuming no Mutual Coupling between Transmission Lines

Initially all the elements of  $\mathbf{Y}_{\text{BUS}}$  are set to zero. Addition of an element of admittance  $y$  between buses  $i$  and  $j$  affects four entries in  $\mathbf{Y}_{\text{BUS}}$ , viz.,  $Y_{ii}$ ,  $Y_{ij}$ ,  $Y_{ji}$ ,  $Y_{jj}$ , as follows:

$$\begin{aligned}Y_{ii \text{ new}} &= Y_{ii \text{ old}} + y \\Y_{ij \text{ new}} &= Y_{ij \text{ old}} - y \\Y_{ji \text{ new}} &= Y_{ji \text{ old}} - y \\Y_{jj \text{ new}} &= Y_{jj \text{ old}} + y\end{aligned}\tag{6.9a}$$

Addition of an element of admittance  $y$  from bus  $i$  to ground will only affect  $Y_{ii}$ , i.e.

$$Y_{ii \text{ new}} = Y_{ii \text{ old}} + y\tag{6.9b}$$

**Example 6.2** Consider the sample four-bus system in Fig. 6.5. Initially, set all the elements of  $\mathbf{Y}_{\text{BUS}}$  to zero.

1. Addition of  $y_{10}$  affects only  $Y_{11}$

$$Y_{11 \text{ new}} = Y_{11 \text{ old}} + y_{10} = 0 + y_{10} = y_{10}$$

2. Addition of  $y_{12}$  affects  $Y_{11}$ ,  $Y_{12}$ ,  $Y_{21}$ ,  $Y_{22}$

$$Y_{11 \text{ new}} = Y_{11 \text{ old}} + y_{12} = y_{10} + y_{12}\tag{i}$$

$$Y_{12 \text{ new}} = Y_{12 \text{ old}} - y_{12} = 0 - y_{12} = -y_{12}\tag{i}$$

$$Y_{21 \text{ new}} = Y_{21 \text{ old}} - y_{12} = 0 - y_{12} = -y_{12}\tag{ii}$$

$$Y_{22 \text{ new}} = Y_{22 \text{ old}} - y_{12} = 0 + y_{12}$$

3. Addition of  $y_{13}$  affects  $Y_{11}, Y_{13}, Y_{31}, Y_{33}$

$$Y_{11 \text{ new}} = Y_{11 \text{ old}} + y_{13} = y_{10} + y_{12} + y_{13} \quad (\text{iii})$$

$$Y_{13 \text{ new}} = Y_{13 \text{ old}} - y_{13} = 0 - y_{13} = -y_{13} \quad (\text{iv})$$

$$Y_{31 \text{ new}} = Y_{31 \text{ old}} - y_{13} = 0 - y_{13} = -y_{13} \quad (\text{v})$$

$$Y_{33 \text{ new}} = Y_{33 \text{ old}} + y_{13} = 0 + y_{13} = y_{13}$$

4. Addition of  $y_{20}$  affects only  $Y_{22}$

$$Y_{22 \text{ new}} = Y_{22 \text{ old}} + y_{20} = y_{12} + y_{20}$$

5. Addition of  $y_{23}$  affects  $Y_{22}, Y_{23}, Y_{32}, Y_{33}$

$$Y_{22 \text{ new}} = Y_{22 \text{ old}} + y_{23} = (y_{20} + y_{12}) + y_{23} = y_{20} + y_{12} + y_{23}$$

$$Y_{23 \text{ new}} = Y_{23 \text{ old}} - y_{23} = 0 - y_{23} = -y_{23} \quad (\text{vi})$$

$$Y_{32 \text{ new}} = Y_{32 \text{ old}} - y_{23} = 0 - y_{23} = -y_{23} \quad (\text{vii})$$

$$Y_{33 \text{ new}} = Y_{33 \text{ old}} + y_{23} = y_{13} + y_{23}$$

6. Addition of  $y_{24}$  affects  $Y_{22}, Y_{24}, Y_{42}, Y_{44}$

$$\begin{aligned} Y_{22 \text{ new}} &= Y_{22 \text{ old}} + y_{24} = (y_{20} + y_{12} + y_{23}) + y_{24} \\ &= y_{20} + y_{12} + y_{23} + y_{24} \end{aligned} \quad (\text{viii})$$

$$Y_{24 \text{ new}} = Y_{24 \text{ old}} - y_{24} = 0 - y_{24} = -y_{24} \quad (\text{ix})$$

$$Y_{42 \text{ new}} = Y_{42 \text{ old}} - y_{24} = 0 - y_{24} = -y_{24} \quad (\text{x})$$

$$Y_{44 \text{ new}} = Y_{44 \text{ old}} + y_{24} = 0 + y_{24} = y_{24}$$

7. Addition of  $y_{30}$  affects only  $Y_{33}$

$$Y_{33 \text{ new}} = Y_{33 \text{ old}} + y_{30} = (y_{13} + y_{23}) + y_{30} + y_{13} + y_{23}$$

8. Addition of  $y_{34}$  affects  $Y_{33}, Y_{34}, Y_{43}, Y_{44}$

$$\begin{aligned} Y_{33 \text{ new}} &= Y_{33 \text{ old}} + y_{34} = (y_{30} + y_{13} + y_{23}) + y_{34} \\ &= y_{30} + y_{13} + y_{23} + y_{34} \end{aligned} \quad (\text{xi})$$

$$Y_{34 \text{ new}} = Y_{34 \text{ old}} - y_{34} = 0 - y_{34} = -y_{34} \quad (\text{xii})$$

$$Y_{43 \text{ new}} = Y_{43 \text{ old}} - y_{34} = 0 - y_{34} = -y_{34} \quad (\text{xiii})$$

$$Y_{44 \text{ new}} = Y_{44 \text{ old}} + y_{34} = y_{24} + y_{34}$$

9. Addition of  $y_{40}$  affects only  $Y_{44}$

$$Y_{44 \text{ new}} = Y_{44 \text{ old}} + y_{40} = (y_{24} + y_{34}) + y_{40} = y_{40} + y_{24} + y_{34} \quad (\text{xiv})$$

The final values of the elements of the bus admittance matrix are given by appropriate equation from Eqs (i) through (xiv).

Further  $Y_{14} = Y_{41} = 0$

### (b) Assuming Mutual Coupling between Transmission Lines

The equivalent circuit of mutually coupled transmission lines is shown in Fig. 6.6. Shunt elements are omitted for simplicity; this effect can be included

in a straight forward manner as seen in Example 6.1. The mutual impedance between the transmission lines is  $z_m$ , and the series impedances are  $z_{s1}$  and  $z_{s2}$ . From Fig. 6.6, we have

$$V_i = z_{s1} I_i + z_m I_k + V_j$$

$$V_k = z_{s2} I_k + z_m I_i + V_l$$

or

$$\begin{bmatrix} V_i \\ V_k \end{bmatrix} - \begin{bmatrix} V_j \\ V_l \end{bmatrix} = \begin{bmatrix} z_{s1} & z_m \\ z_m & z_{s2} \end{bmatrix} \begin{bmatrix} I_i \\ I_k \end{bmatrix}$$

or

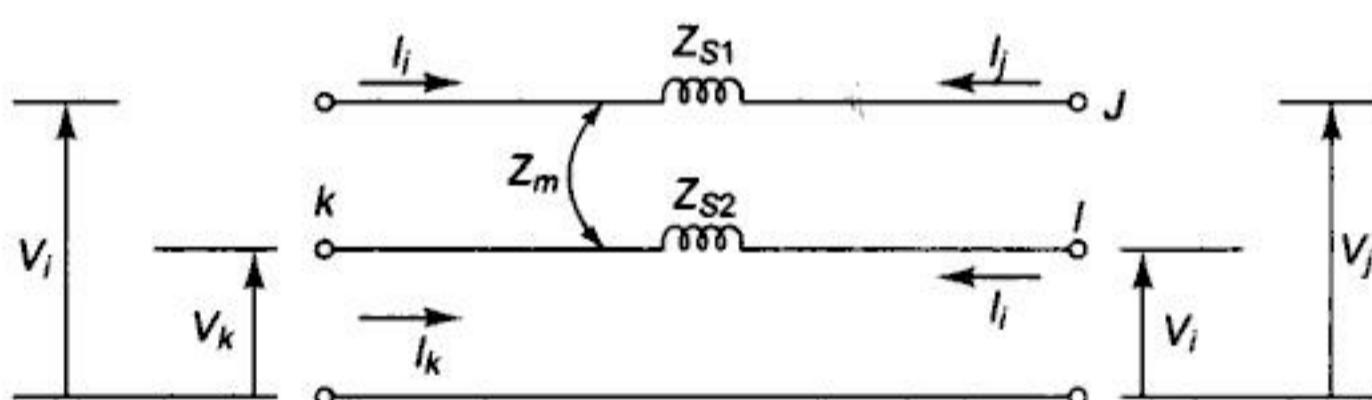
$$\begin{bmatrix} I_i \\ I_k \end{bmatrix} = \begin{bmatrix} y_{s1} & y_m \\ y_m & y_{s2} \end{bmatrix} \begin{bmatrix} V_i - V_j \\ V_k - V_l \end{bmatrix} \quad (6.10a)$$

Similarly, we have

$$\begin{bmatrix} I_j \\ I_l \end{bmatrix} = \begin{bmatrix} y_{s1} & y_m \\ y_m & y_{s2} \end{bmatrix} \begin{bmatrix} V_j - V_i \\ V_l - V_k \end{bmatrix} \quad (6.10b)$$

where

$$\begin{bmatrix} y_{s1} & y_m \\ y_m & y_{s2} \end{bmatrix} = \begin{bmatrix} z_{s1} & z_m \\ z_m & z_{s2} \end{bmatrix}^{-1}$$



**Fig. 6.6** Mutually coupled transmission lines

From Eqs 6.10(a) and (b) the elements of  $Y_{\text{BUS}}$  become

$$\left\{ \begin{array}{l} Y_{ii \text{ new}} = Y_{ii \text{ old}} + y_{s1} \\ Y_{jj \text{ new}} = Y_{jj \text{ old}} + y_{s1} \\ Y_{kk \text{ new}} = Y_{kk \text{ old}} + y_{s2} \\ Y_{ll \text{ new}} = Y_{ll \text{ old}} + y_{s2} \end{array} \right. \quad (6.11a)$$

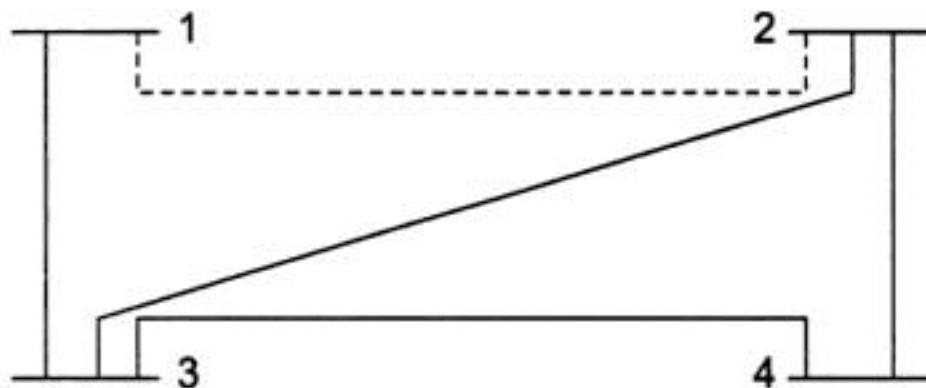
$$\left\{ \begin{array}{l} Y_{ij \text{ new}} = Y_{ji \text{ new}} = Y_{ij \text{ old}} - y_{s1} \\ Y_{kl \text{ new}} = Y_{lk \text{ new}} = Y_{kl \text{ old}} - y_{s2} \end{array} \right. \quad (6.11b)$$

$$\begin{cases} Y_{ik \text{ new}} = Y_{ki \text{ new}} = Y_{ik \text{ old}} + y_m \\ Y_{jl \text{ new}} = Y_{lj \text{ new}} = Y_{jl \text{ old}} + y_m \end{cases} \quad (6.11\text{c})$$

$$\begin{cases} Y_{il \text{ new}} = Y_{li \text{ new}} = Y_{il \text{ old}} - y_m \\ Y_{jk \text{ new}} = Y_{kj \text{ new}} = Y_{jk \text{ old}} - y_m \end{cases} \quad (6.11\text{d})$$

**Example 6.3** Figure 6.7 shows the one-line diagram of a simple four-bus system. Table 6.1 gives the line impedances identified by the buses on which these terminate. The shunt admittance at all the buses is assumed to be negligible.

- (a) Find  $Y_{\text{BUS}}$ , assuming that the line shown dotted is not connected.
- (b) What modifications need to be carried out in  $Y_{\text{BUS}}$  if the line shown dotted is connected?



**Fig. 6.7** Sample system for Example 6.3

**Solution**

- (a) From Table 6.1, Table 6.2 is obtained from which  $Y_{\text{BUS}}$  for the system can be written as

**Table 6.1**

Line, bus to bus	$R, \text{pu}$	$X, \text{pu}$
1–2	0.05	0.15
1–3	0.10	0.30
2–3	0.15	0.45
2–4	0.10	0.30
3–4	0.05	0.15

**Table 6.2**

Line	$G, \text{pu}$	$B, \text{pu}$
1–2	2.000	-6.0
1–3	1.000	-3.0
2–3	0.666	-2.0
2–4	1.000	-3.0
3–4	2.000	-6.0

$$\mathbf{Y}_{\text{BUS}} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \quad (\text{i})$$

$$\mathbf{Y}_{\text{BUS}} = \begin{bmatrix} y_{13} & 0 & -y_{13} & 0 \\ 0 & (y_{23} + y_{24}) & -y_{23} & -y_{24} \\ -y_{13} & -y_{23} & (y_{31} + y_{32} + y_{34}) & -y_{34} \\ 0 & -y_{24} & -y_{34} & (y_{43} + y_{42}) \end{bmatrix} \quad (\text{ii})$$

$$\mathbf{Y}_{\text{BUS}} = \begin{bmatrix} 1-j3 & 0 & -1+j3 & 0 \\ 0 & 1.666-j5 & -0.666+j2 & -1+j3 \\ -1+j3 & -0.666+j2 & 3.666-j11 & -2+j6 \\ 0 & -1+j3 & -2+j6 & 3-j9 \end{bmatrix} \quad (\text{iii})$$

- (b) The following elements of  $\mathbf{Y}_{\text{BUS}}$  of part (a) are modified when a line is added between buses 1 and 2.

$$Y_{11 \text{ new}} = Y_{11 \text{ old}} + (2-j6) = 3-j9$$

$$Y_{12 \text{ new}} = Y_{12 \text{ old}} - (2-j6) = -2+j6 = Y_{21 \text{ new}} \quad (\text{iv})$$

$$Y_{22 \text{ new}} = Y_{22 \text{ old}} + (2-j6) = 3.666-j11$$

Modified  $\mathbf{Y}_{\text{BUS}}$  is written as

$$\mathbf{Y}_{\text{BUS}} = \begin{bmatrix} 3-j9 & -2+j6 & -1+j3 & 0 \\ -2+j6 & 3.666-j11 & -0.666+j2 & -1+j3 \\ -1+j3 & -0.666+j2 & 3.666-j11 & -2+j6 \\ 0 & -1+j3 & -2+j6 & 3-j9 \end{bmatrix} \quad (\text{v})$$

### 6.3 FORMATION OF $\mathbf{Y}_{\text{BUS}}$ BY SINGULAR TRANSFORMATION

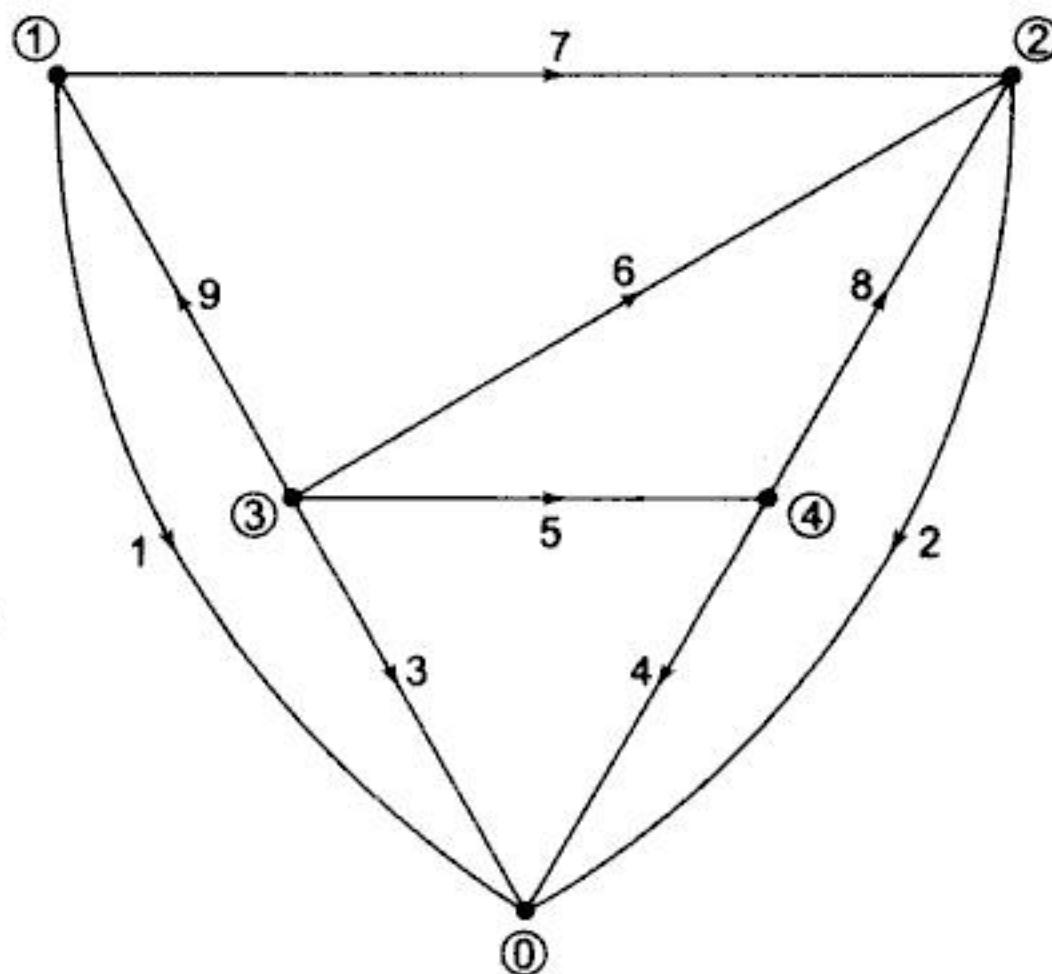
The matrix pair  $\mathbf{Y}_{\text{BUS}}$  and  $\mathbf{Z}_{\text{BUS}}$  form the network models for load flow studies. The  $\mathbf{Y}_{\text{BUS}}$  can be alternatively assembled by the use of singular transformation given by a graph theoretical approach. This approach is of great theoretical and practical significance, and is therefore discussed here. To start with, the graph theory is briefly reviewed.

#### Graph

To describe the geometrical features of a network, it is replaced by single line segments called *elements*, whose terminals are called *nodes*. The resulting figure is called the *graph* of the given network. A *linear graph* depicts the geometrical interconnection of the elements of a network. A *connected graph*

is one in which there is at least one path between every pair of nodes. If each element of a connected graph is assigned a direction\*, it is called an *oriented graph*.

Power networks are so structured that out of the  $m$  total number of nodes, one node (normally described by 0) is always at ground potential and the remaining  $n = m - 1$  nodes are the buses at which the source power is injected. Figure 6.8 shows the oriented linear graph of the power network of Fig. 6.5(c). Here the overall line admittance between any two buses (nodes in the corresponding graph) are represented by a single line element. Also, each source and the shunt admittance connected across it are represented by a single line element. In fact, this combination represents the most general network element, and is described under the subheading Primitive Network.



**Fig. 6.8** Oriented linear graph of the circuit in Fig. 6.5(c)

A connected subgraph containing all the nodes of a graph but having no closed paths is called a *tree*. The elements of a tree are called *branches* or *tree branches*. The number of branches  $b$  that form a tree are given by

$$b = m - 1 = n \text{ (number of buses)} \quad (6.12)$$

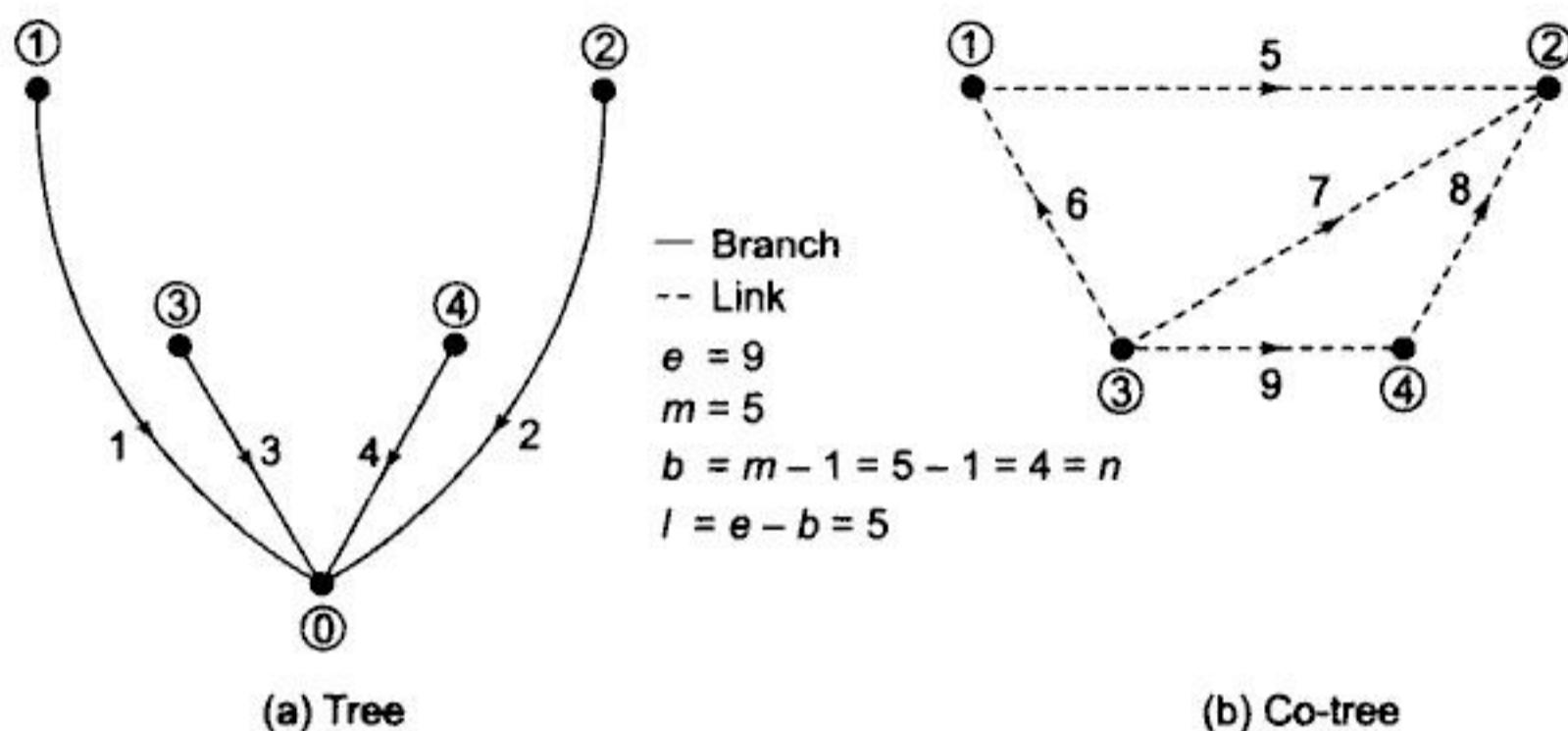
Those elements of the graph that are not included in the tree are called *links* or *link branches*, and they form a subgraph, not necessarily connected, called *co-tree*. The number of links  $l$  of a connected graph with  $e$  elements is

$$l = e - b = e - m + 1 \quad (6.13)$$

There may be more than one possible trees (and therefore, co-trees) of a graph.

A tree and the corresponding co-tree of the graph of Fig. 6.8 are shown in Fig. 6.9. The reader should try and find some other tree and co-tree pairs.

\* For convenience, direction is so assigned as to coincide with the assumed positive direction of the element current.



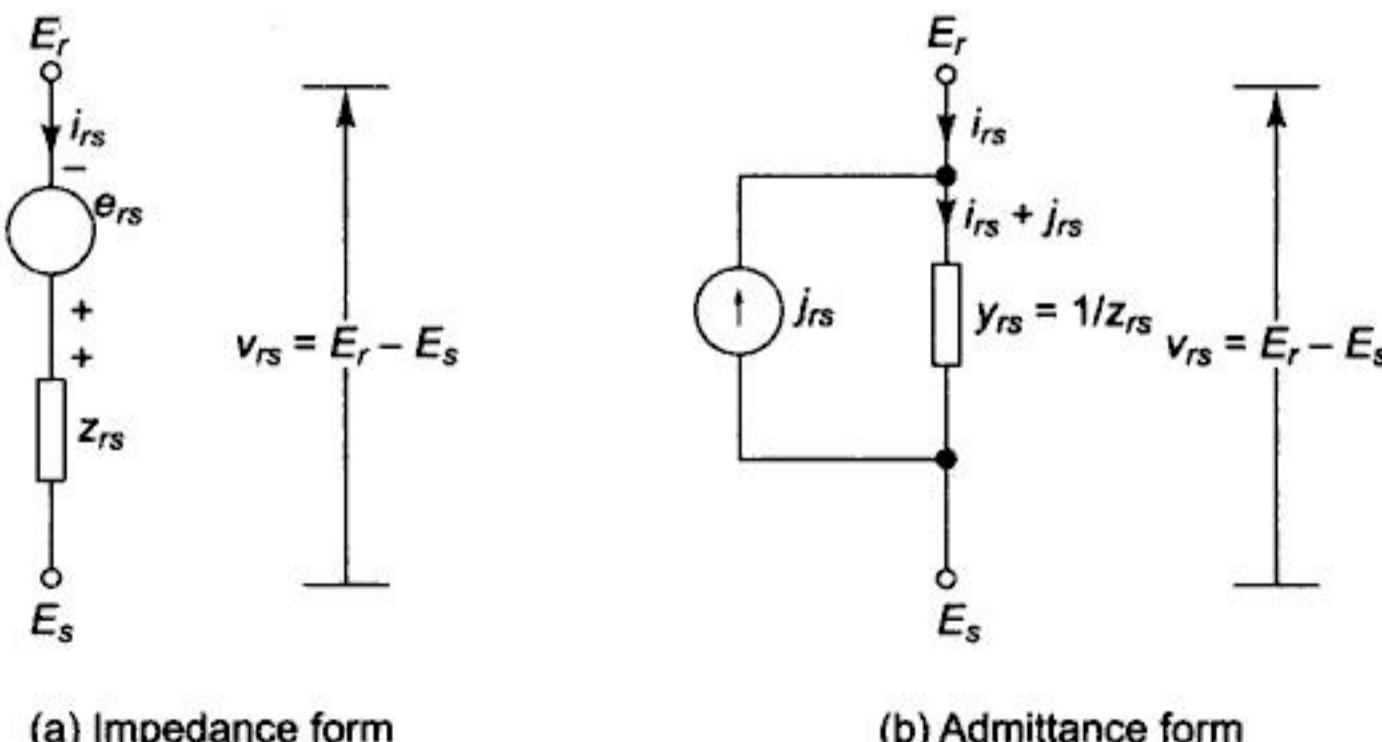
**Fig. 6.9** Tree and co-tree of the oriented connected graph of Fig. 6.6

If a link is added to the tree, the corresponding graph contains one closed path called a basic loop. Thus, a graph has as many basic loops as the number of links. A loop is distinguished from a basic loop, as it can be any loop in the original graph. Therefore, the number of loops is greater than, or at the most equal to, the number of basic loops in a graph.

### Primitive Network

A network element may in general contain active and passive components. Figure 6.10 shows a general network element, connected between nodes  $r$  and  $s$ , with its alternative impedance and admittance form. The impedance form is a voltage source  $e_{rs}$  in series with an impedance  $z_{rs}$ , while the admittance form is a current source  $j_{rs}$  in parallel with an admittance  $y_{rs}$ . The element current is  $i_{rs}$ , and the element voltage is  $v_{rs} = E_r - E_s$ , where  $E_r$  and  $E_s$  are the voltages of the element nodes  $r$  and  $s$  respectively.

It may be remembered here that for steady state AC performance, all element variables ( $v_{rs}$ ,  $E_r$ ,  $E_s$ ,  $i_{rs}$ ,  $j_{rs}$ ) are phasors, and the element parameters ( $z_{rs}$ ,  $y_{rs}$ ) are complex numbers.



**Fig. 6.10** Representation of a network element

The voltage relation for Fig. 6.10 can be written as

$$v_{rs} + e_{rs} = z_{rs} i_{rs} \quad (6.14)$$

Similarly, the current relation for Fig. 6.10 can be written as

$$i_{rs} + j_{rs} = y_{rs} v_{rs} \quad (6.15)$$

The forms of Fig. 6.10(a) and (b) are equivalent, wherein the parallel source current in admittance form is related to the series voltage in impedance form by

$$j_{rs} = y_{rs} e_{rs}$$

Also

$$y_{rs} = 1/z_{rs}$$

A set of unconnected elements is defined as a primitive network. The performance equations of a primitive network are given by:

In impedance form,

$$\mathbf{V} + \mathbf{E} = \mathbf{ZI} \quad (6.16)$$

In admittance form,

$$\mathbf{I} + \mathbf{J} = \mathbf{YV} \quad (6.17)$$

Here  $\mathbf{V}$  and  $\mathbf{I}$  are the element voltage and current vectors respectively, and  $\mathbf{J}$  and  $\mathbf{E}$  are the source vectors.  $\mathbf{Z}$  and  $\mathbf{Y}$  are referred to as the primitive impedance and admittance matrices, respectively. These are related as  $\mathbf{Z} = \mathbf{Y}^{-1}$ . If there is no mutual coupling between elements,  $\mathbf{Z}$  and  $\mathbf{Y}$  are diagonal matrices, where the diagonal entries are the impedances/admittances of the network elements and are reciprocal.

## Network Variables in Bus Frame of Reference

The linear network graph helps in the systematic assembly of a network model. The main problem in deriving mathematical models for large and complex power network is to select a minimum or zero redundancy (linearly independent) set of current or voltage variables, which is sufficient to give the information about all element voltages and currents. One set of such variables is the  $b$  tree voltages. By topological reasoning, these variables constitute a nonredundant set. The knowledge of  $b$  tree voltages allows us to compute all element voltages, and therefore, all bus currents assuming all element admittances being known.

Consider a tree graph shown in Fig. 6.9(a) where the ground node is chosen as the reference node. This is the most appropriate tree choice for the power network. With this choice, the  $b$  tree branch voltages become identical with the bus voltages as the tree branches are incidental to the ground node.

## Bus Incidence Matrix

For the specific system of Fig. 6.8, we obtain the following relations between the nine element voltages and the four bus (i.e. tree branch) voltages  $V_1$ ,  $V_2$ ,  $V_3$  and  $V_4$ .



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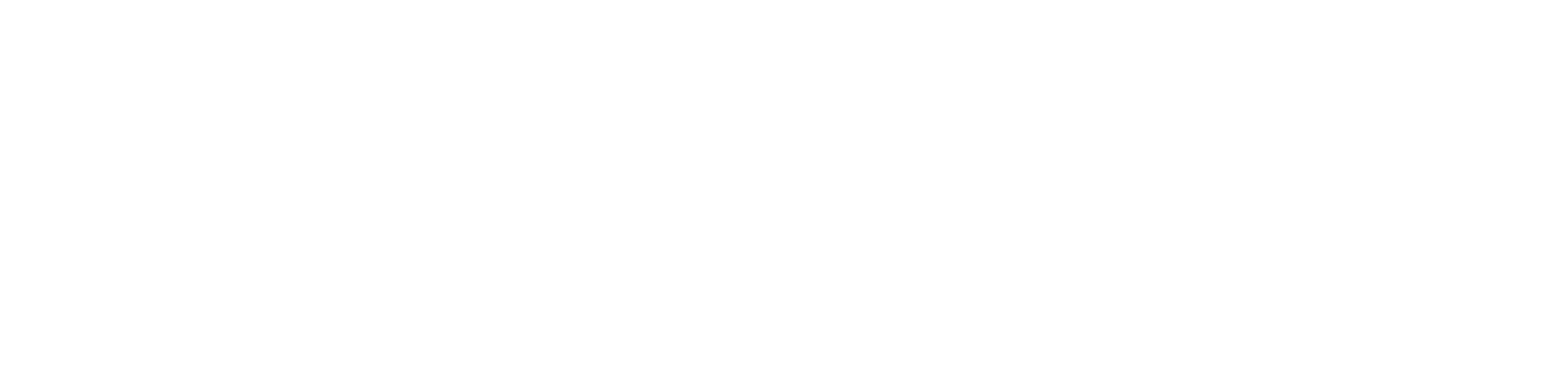
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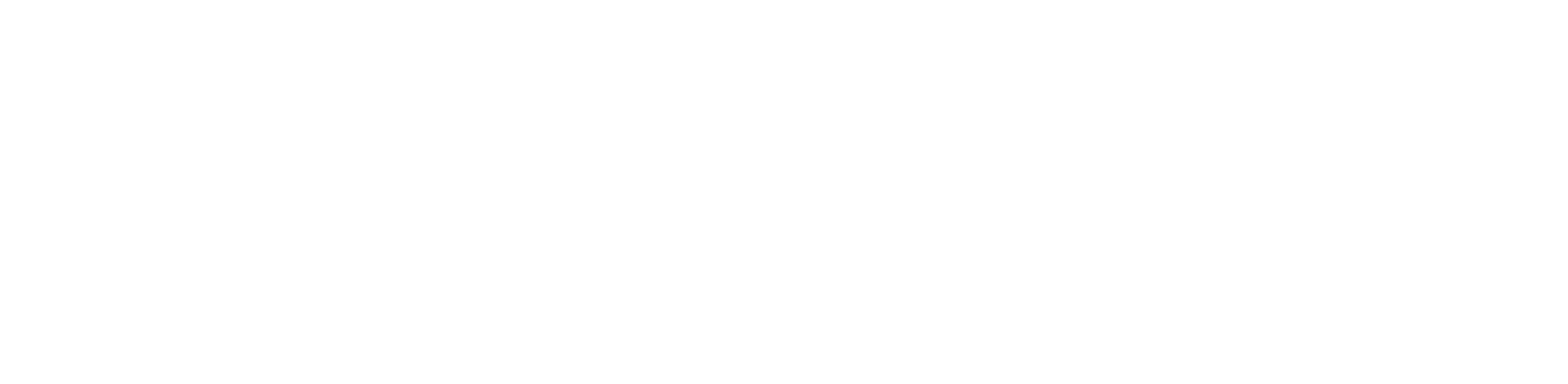
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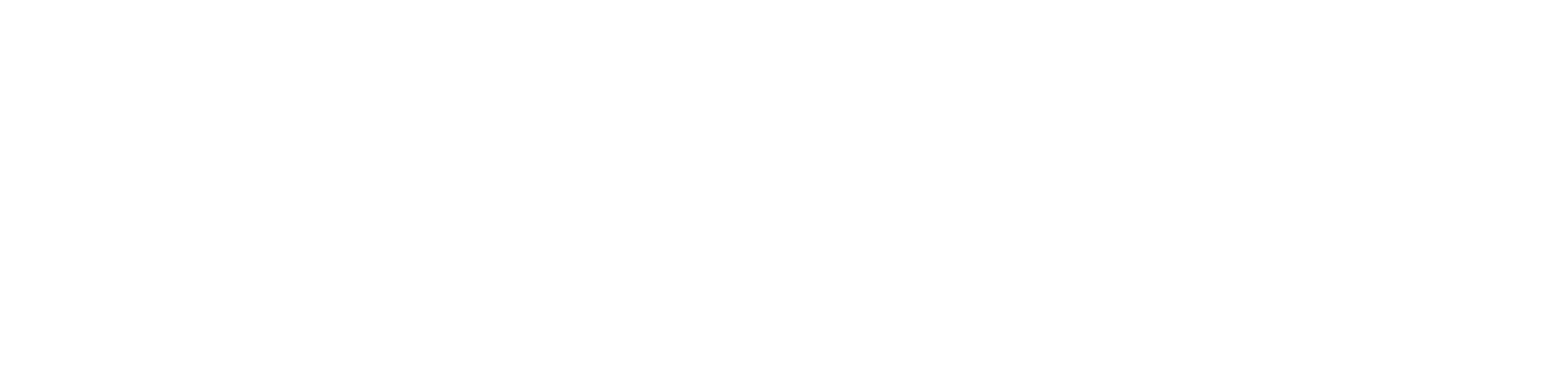
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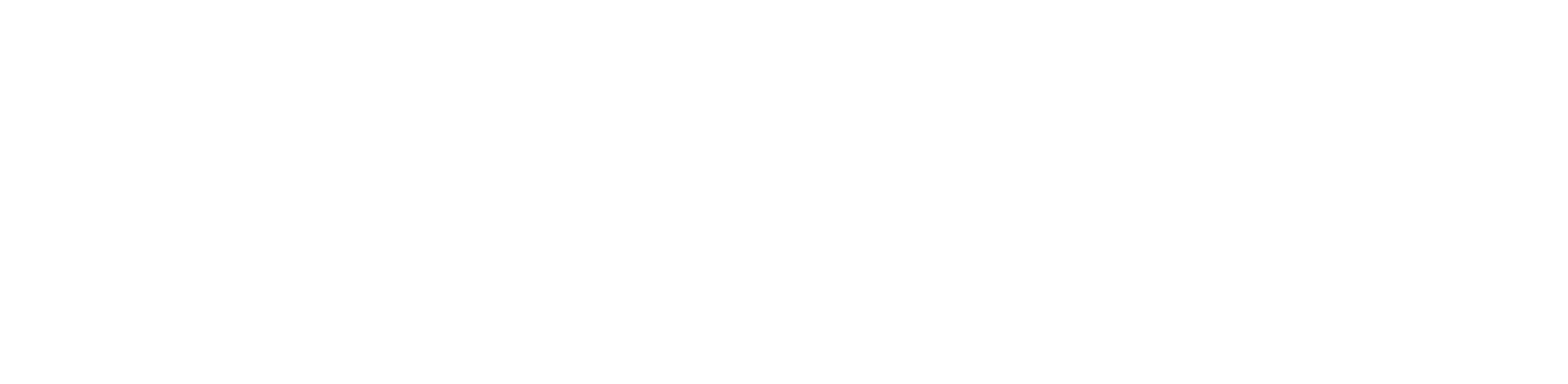
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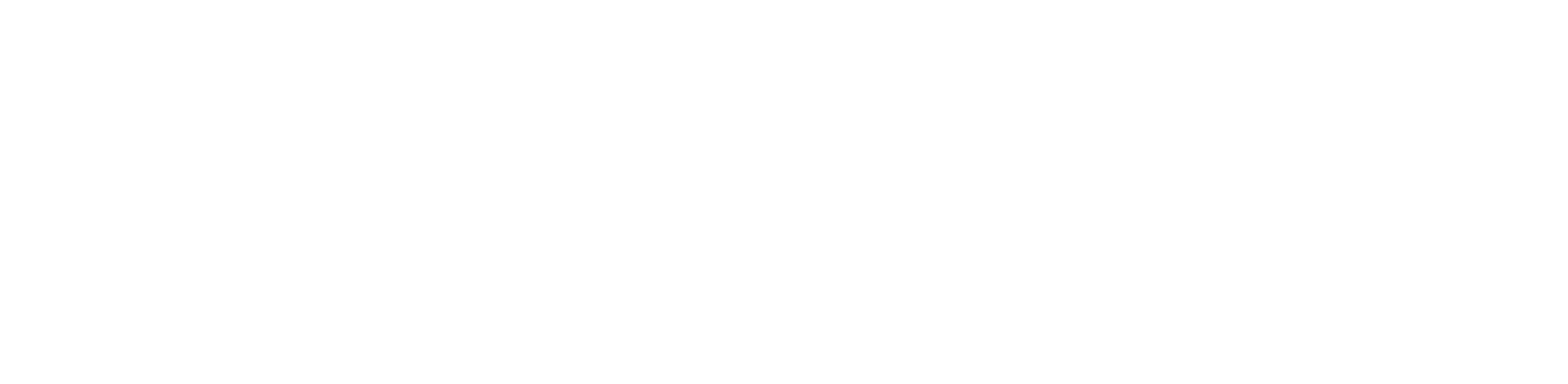
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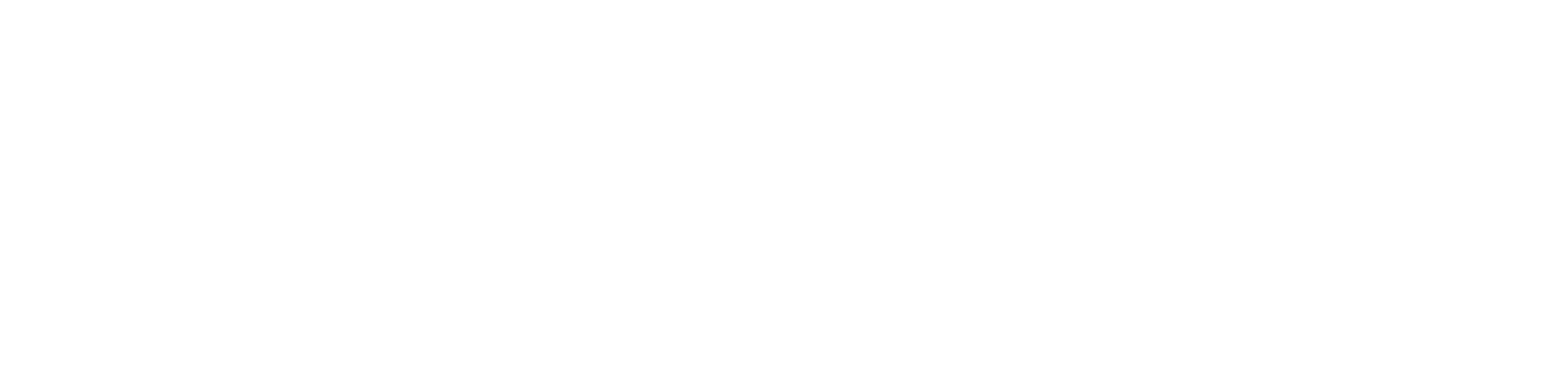
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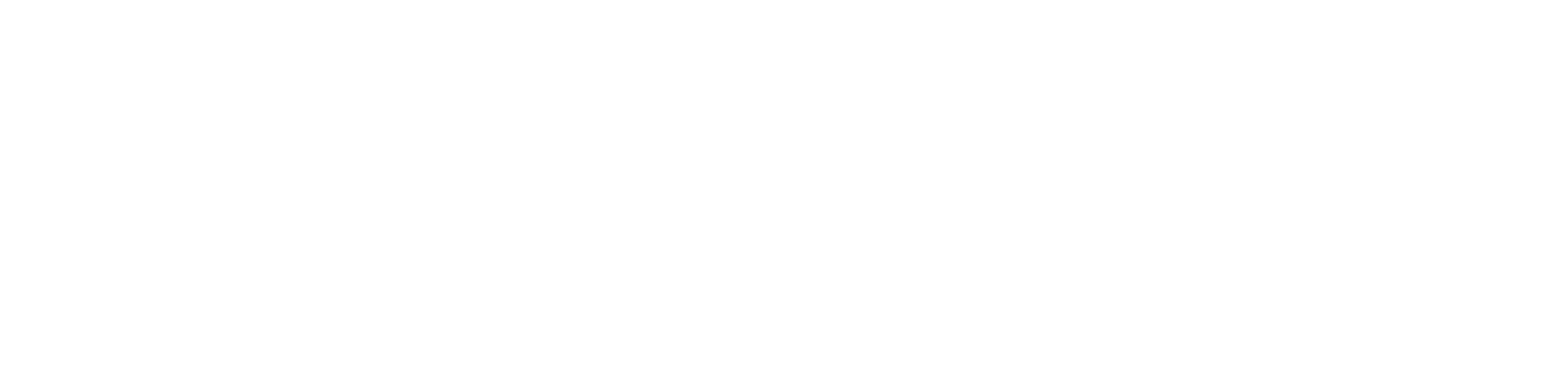
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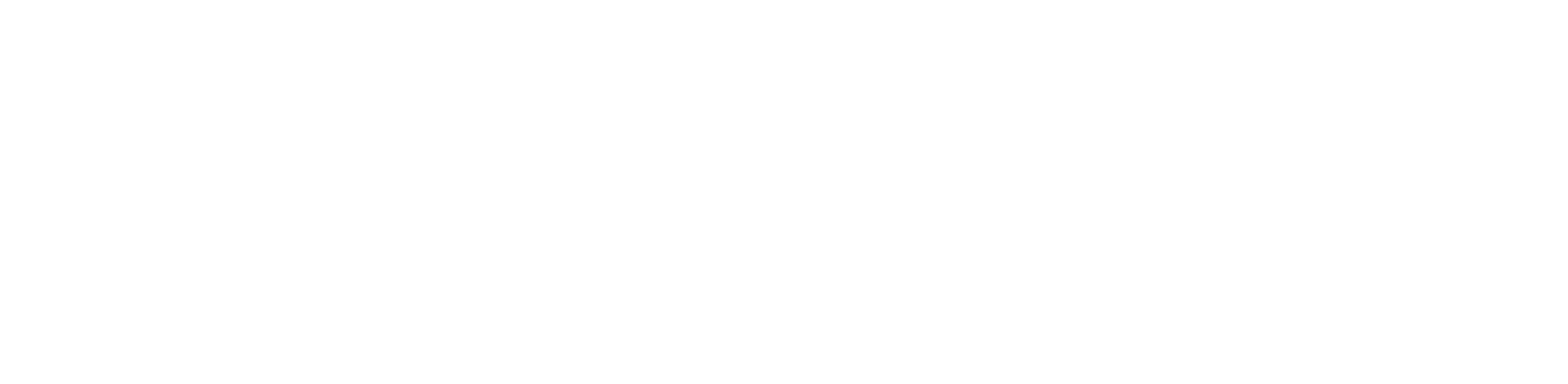
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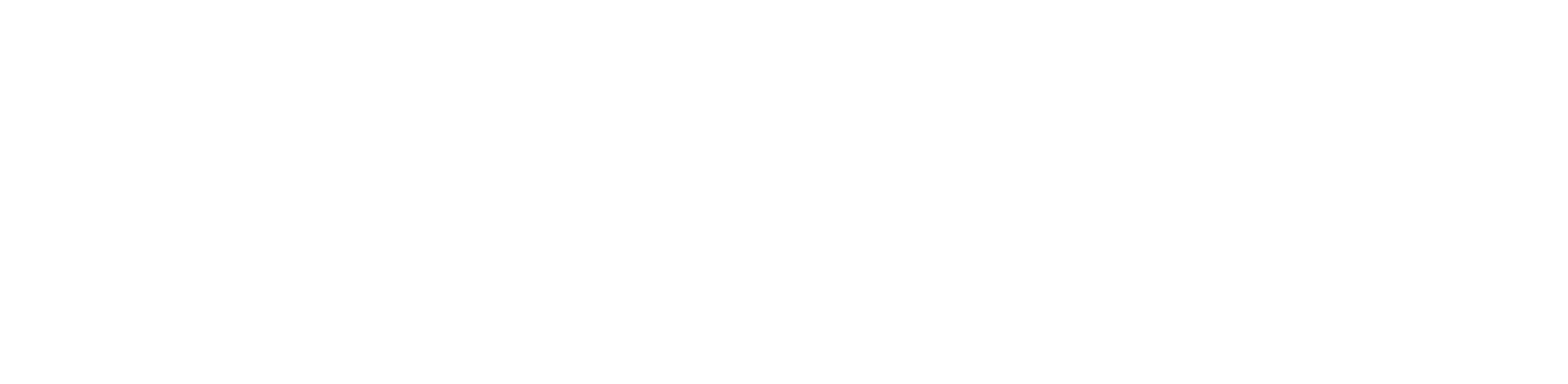
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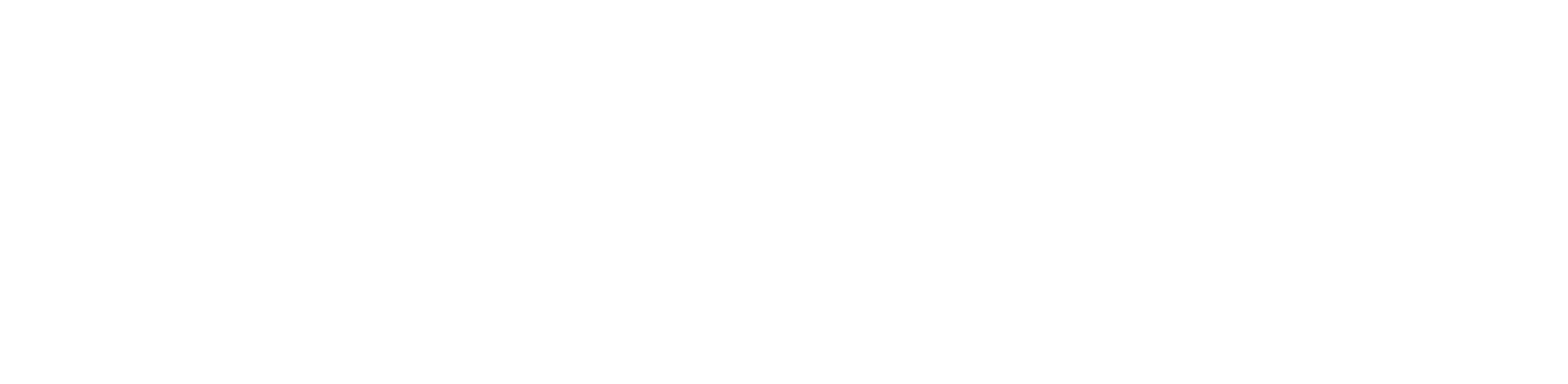
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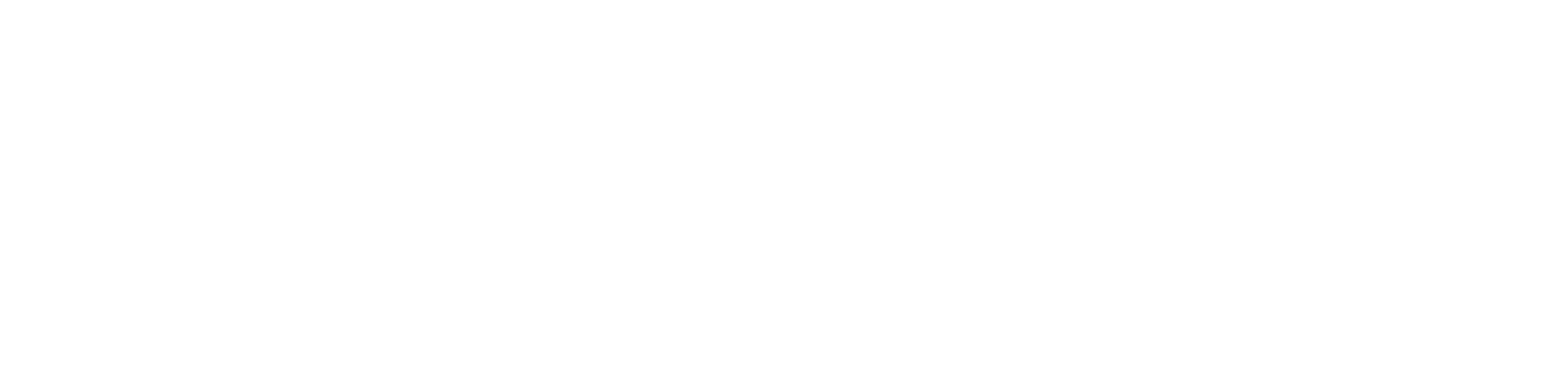
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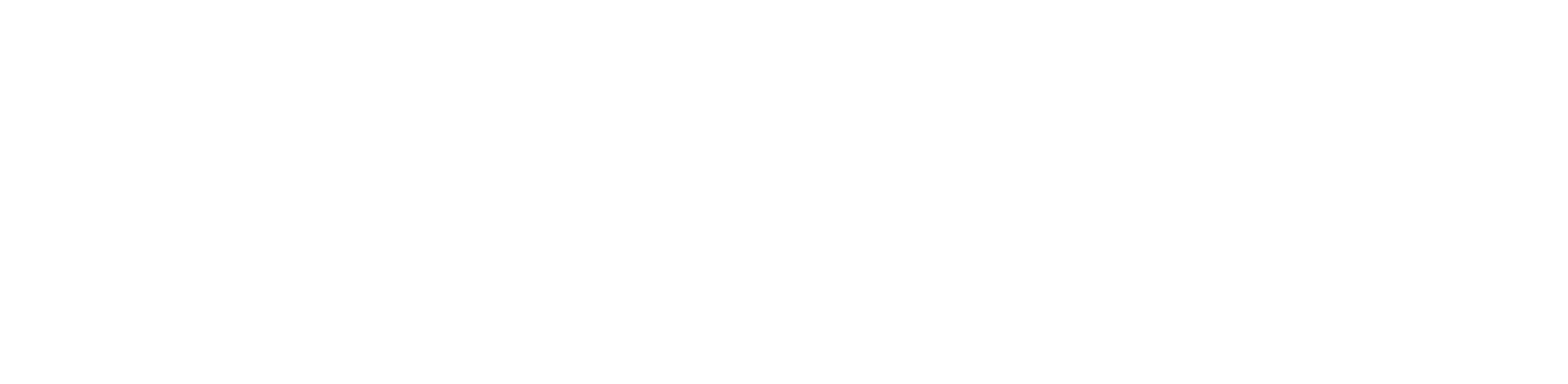
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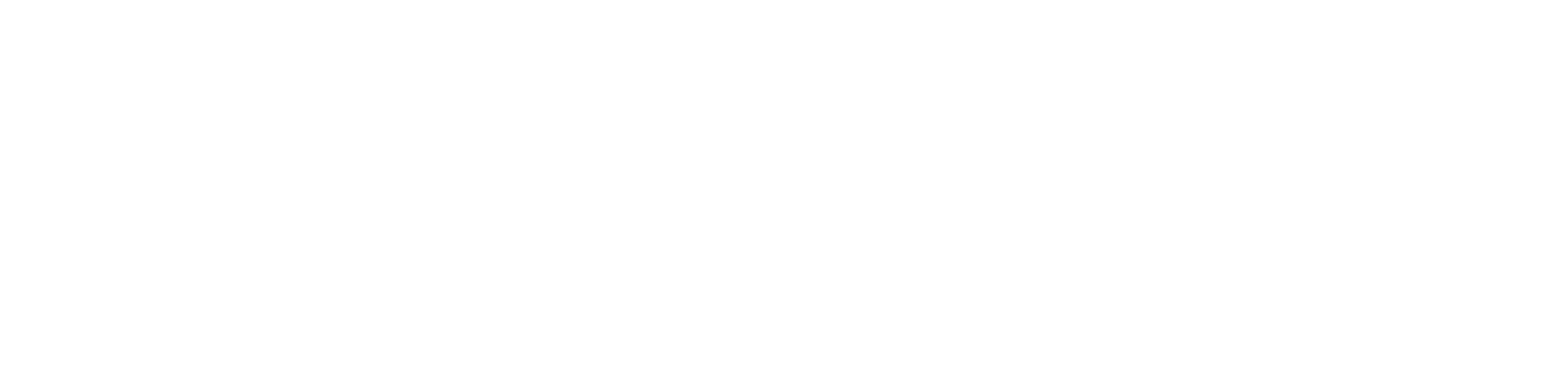
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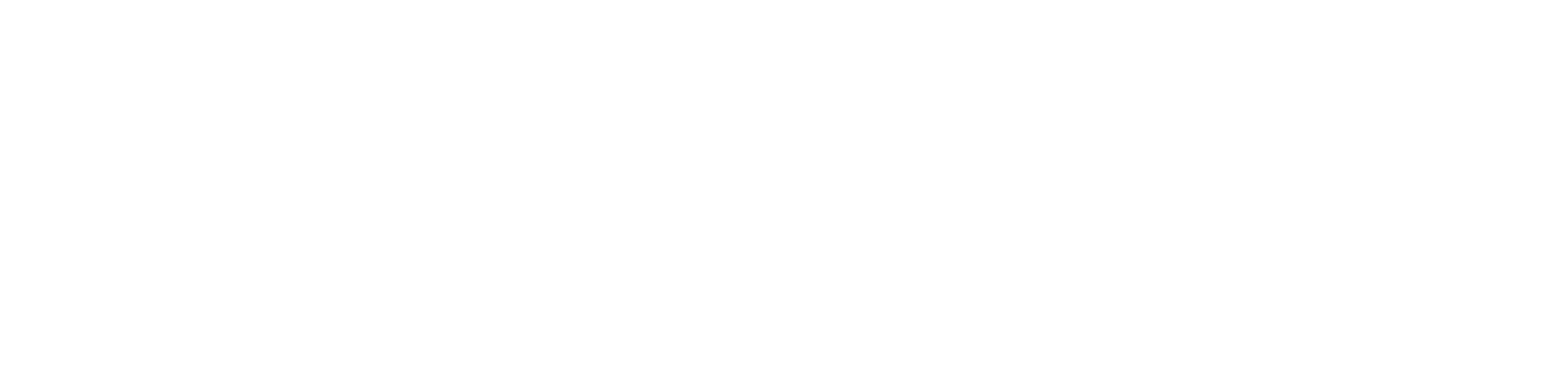
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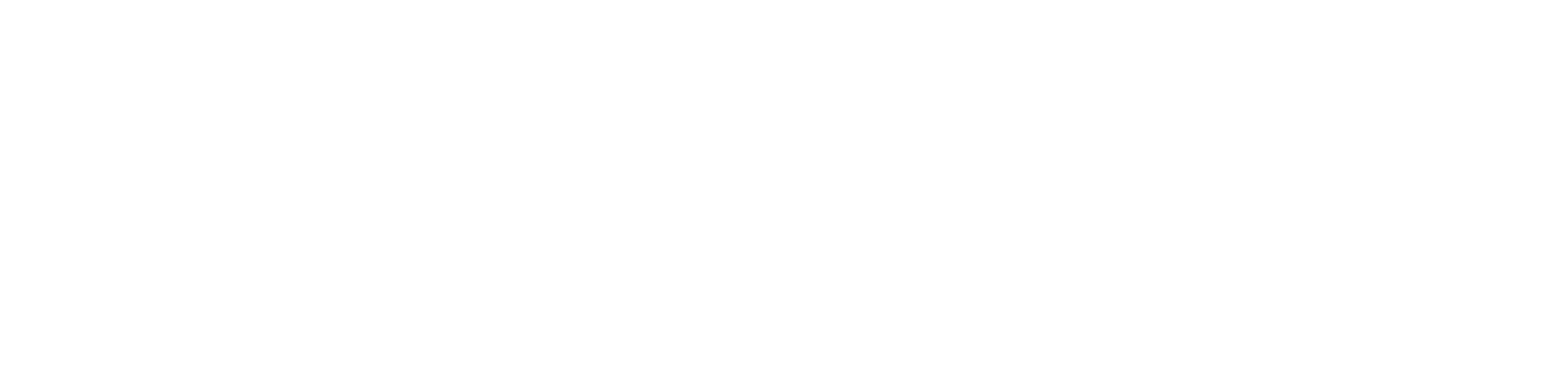
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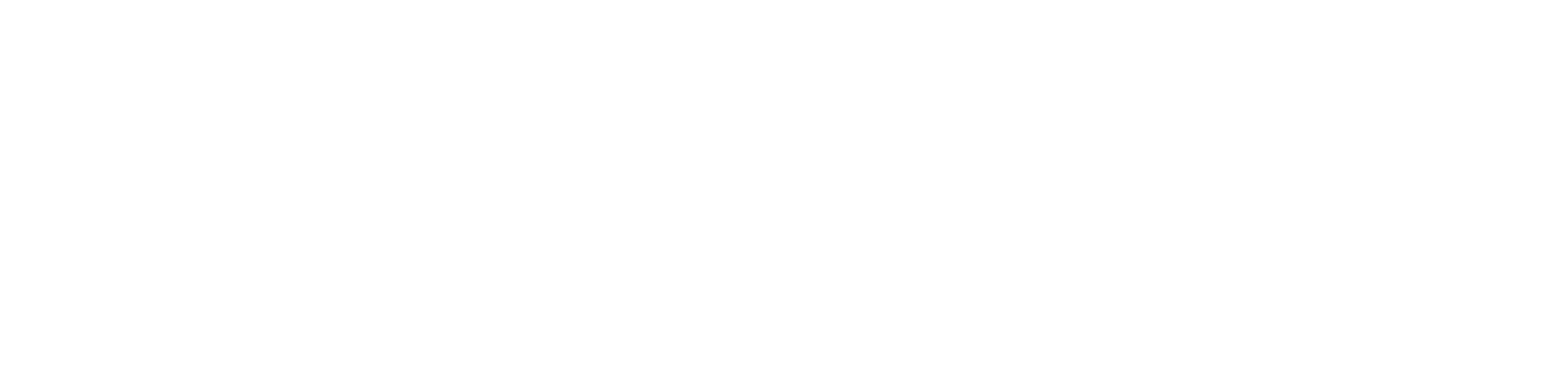
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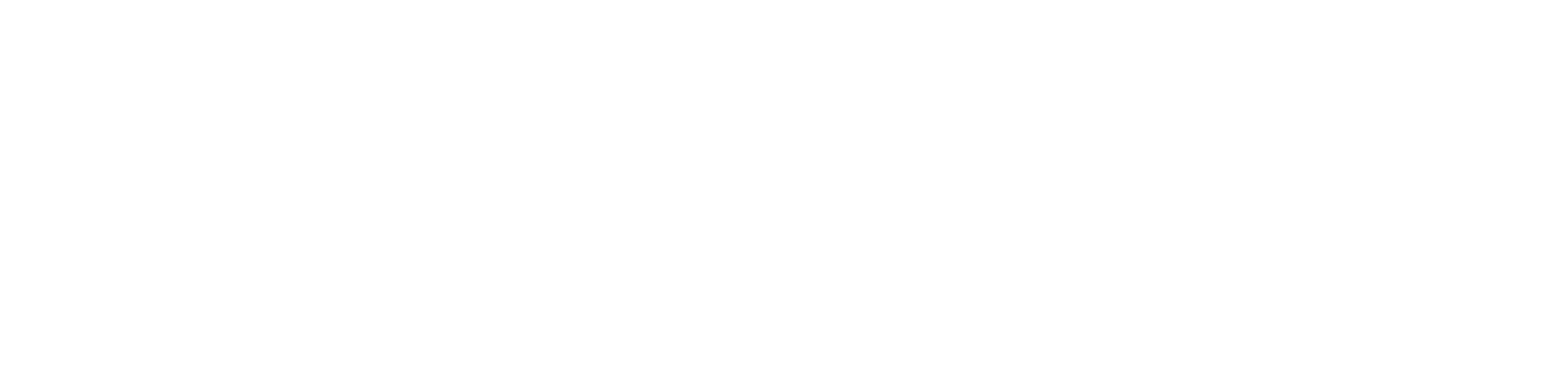
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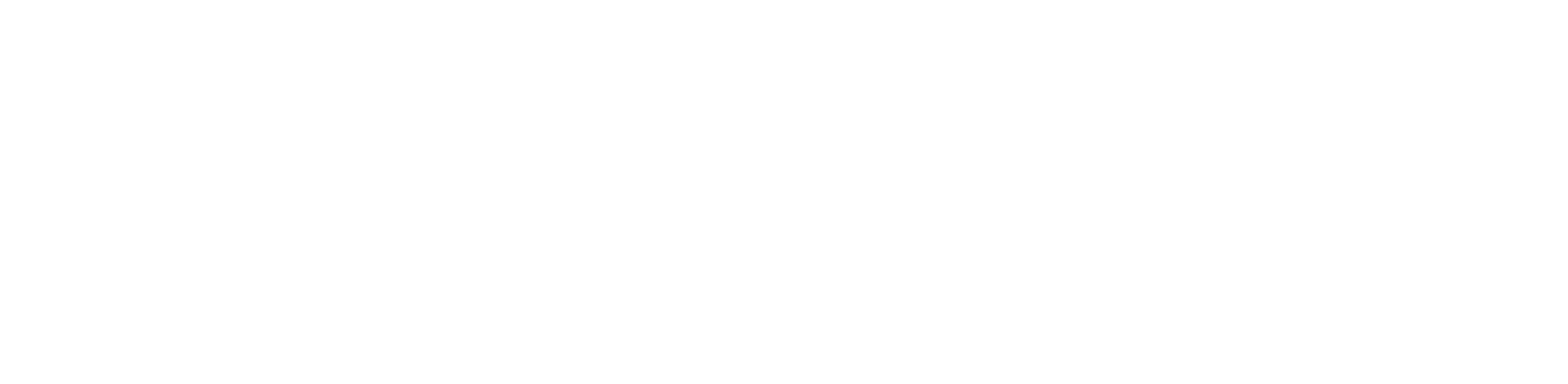
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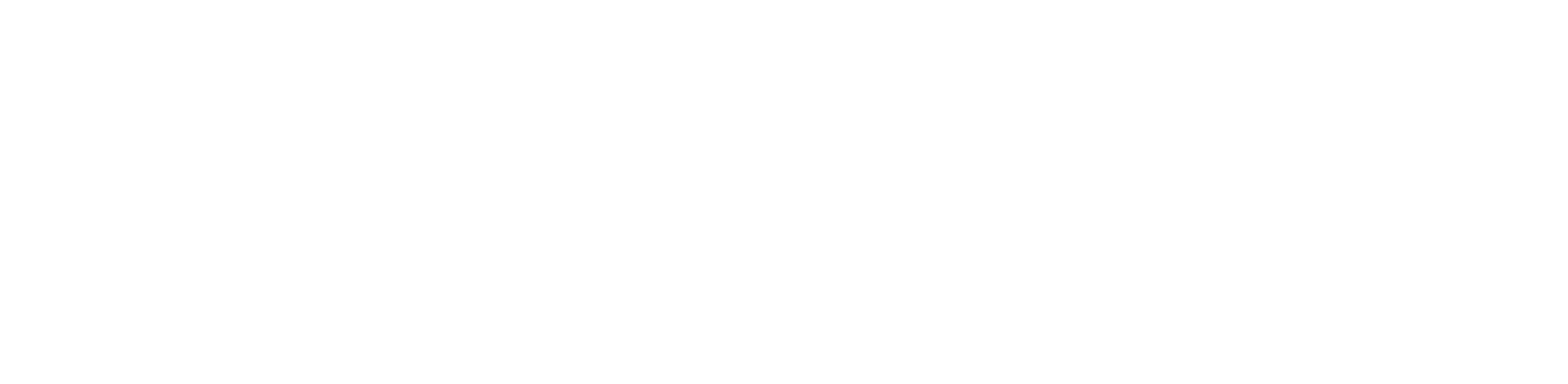
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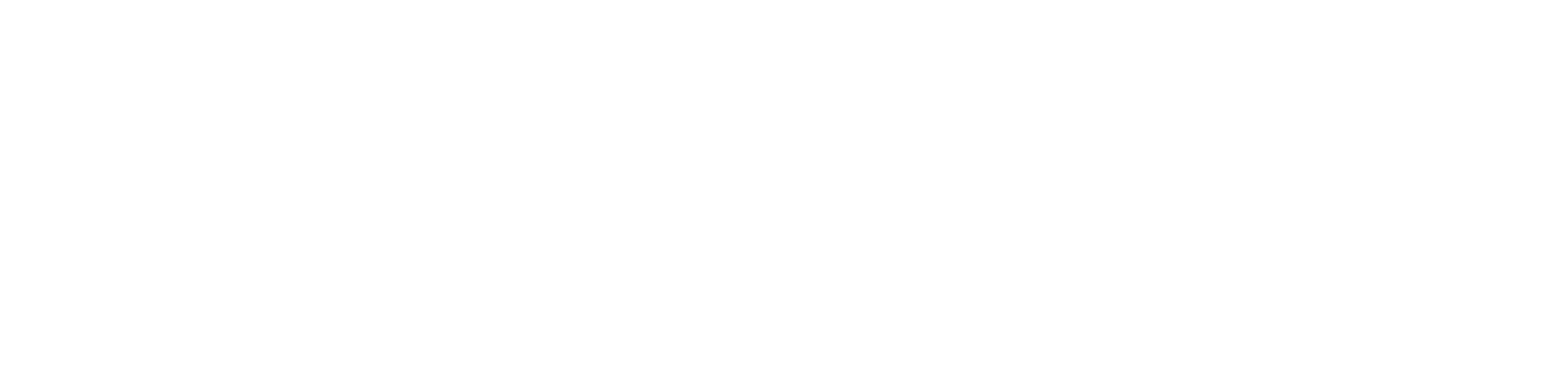
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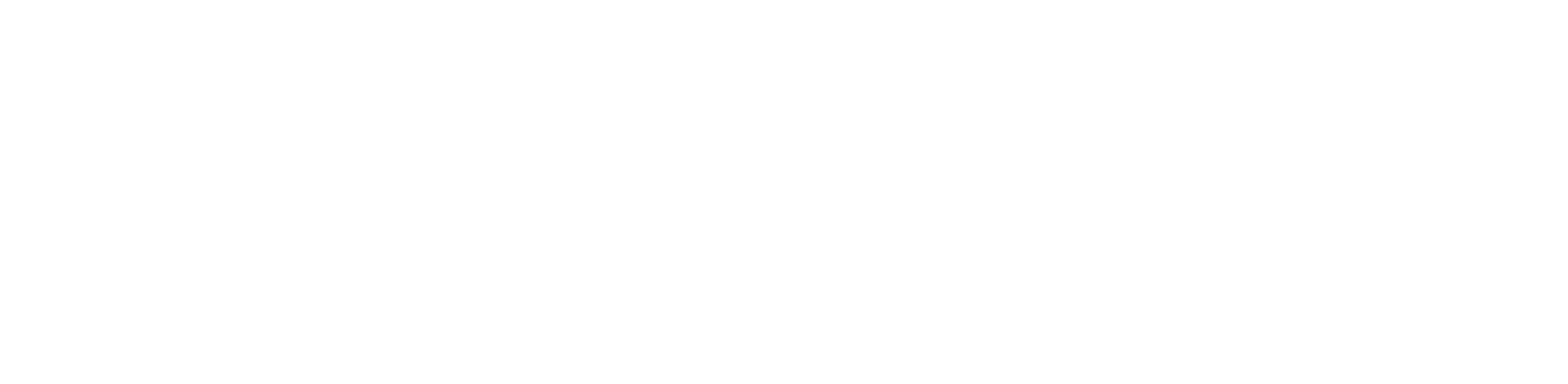
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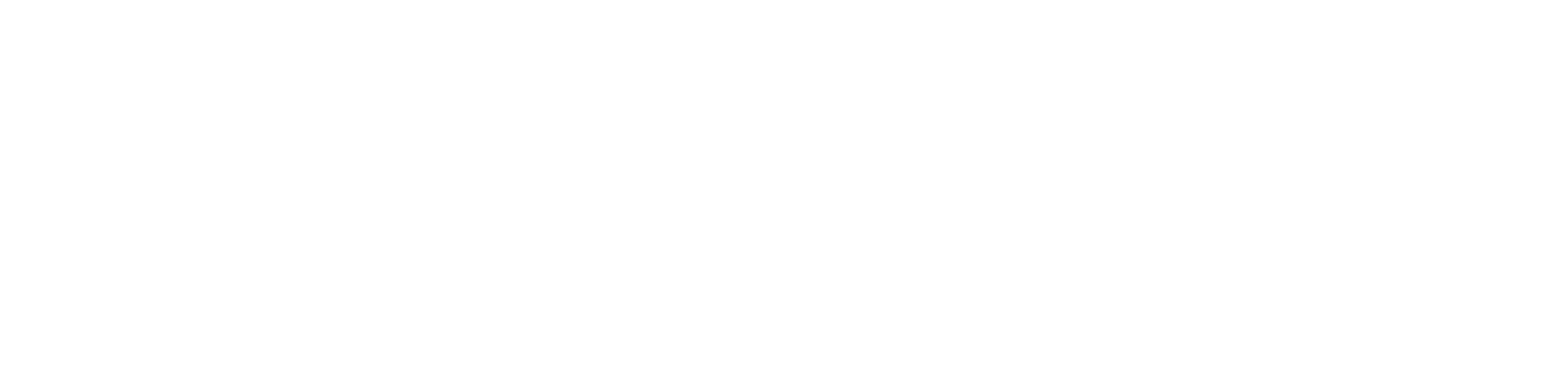
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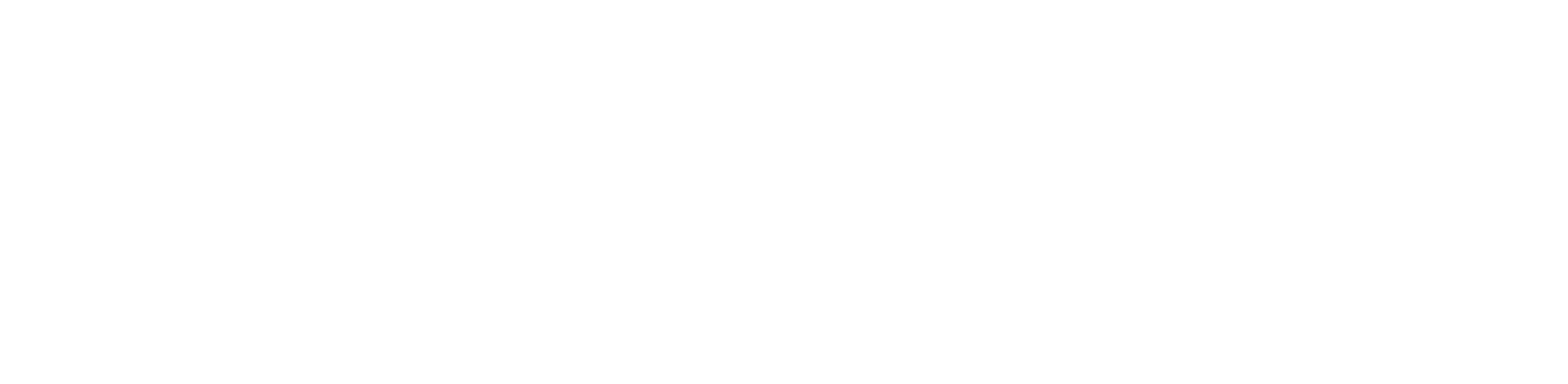
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