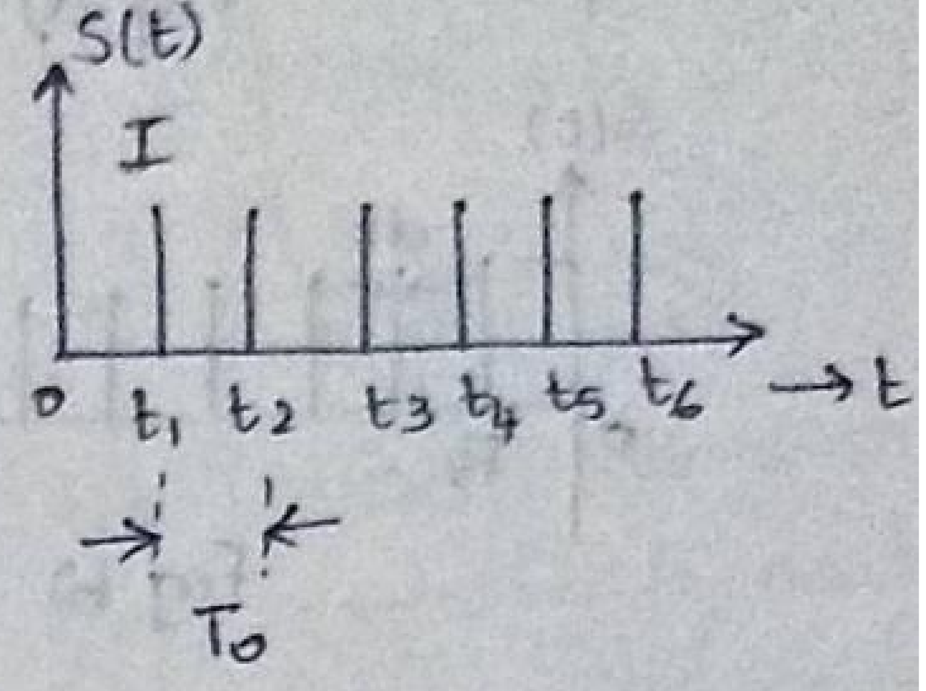
1. Analog to Digital Conversion

→ **Sampling:** Sampling of signals is the fundamental operation in signal processing by which a continuous - time s/g is first converted to discrete time s/g.

→ If the samples are in the form of impulses, the sampling is called instantaneous sampling.



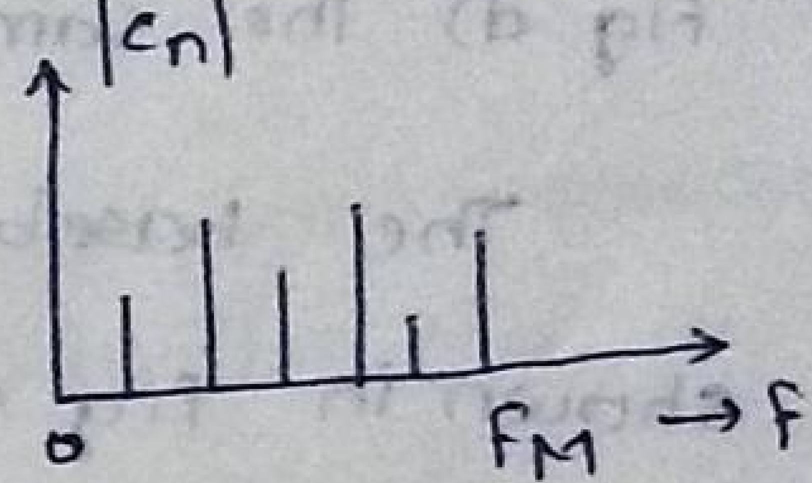
→ For an impulse train with strength I & repetition time T_0 is

$$S(t) = I \sum_{K=-\infty}^{\infty} \delta(t - KT_0)$$

$$= \frac{I}{T_0} + \sum_{n=1}^{\infty} \frac{2I}{T_0} \cos \frac{2\pi nt}{T_0}$$

→ But if the samples are in the form of pulses, then these systems are called pulse modulation systems.

→ A s/g is said to be bandlimited to a freq f_M, if that s/g has spectral components which extend upto a highest freq f_M in the upper freq direction.



→ Sampling Theorem (or) Nyquist Sampling Theorem :-

If a base band s/g m(t) is band limited to freq 'f_M' then '$2f_M$' samples/sec will completely characterize that s/g. The samples are periodic in the interval 'T_s' sec, where the sampling time T_s is $\frac{1}{2f_M}$. The s/g can be reconstructed if the sampling freq is $\boxed{f_s \geq 2f_M}$

$$\Rightarrow T_s \leq \frac{1}{2f_M}$$

Proof :- {For showing how the s/g can be reconstructed from its samples}

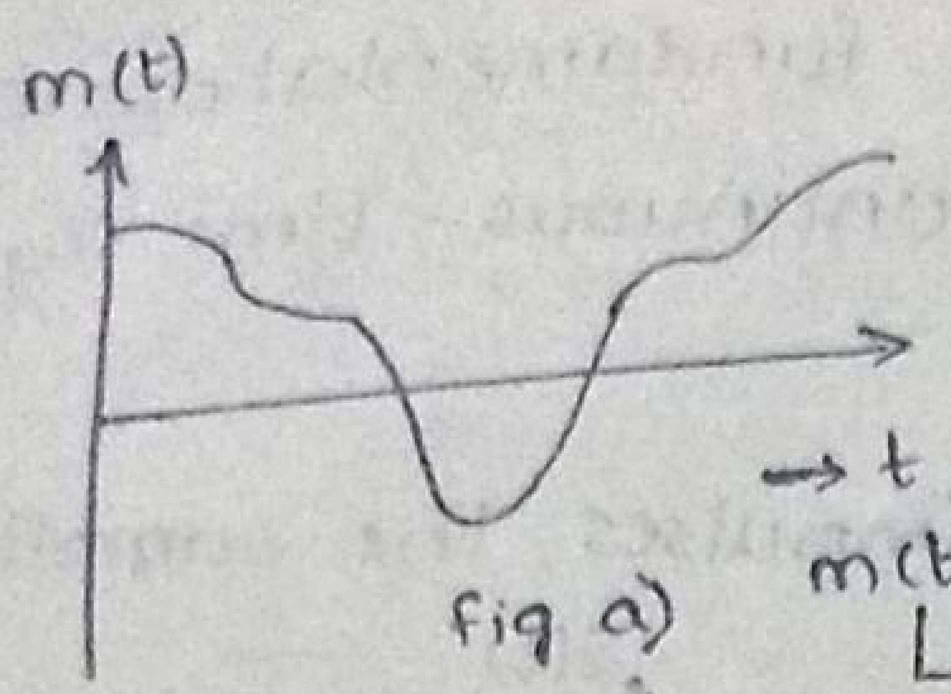
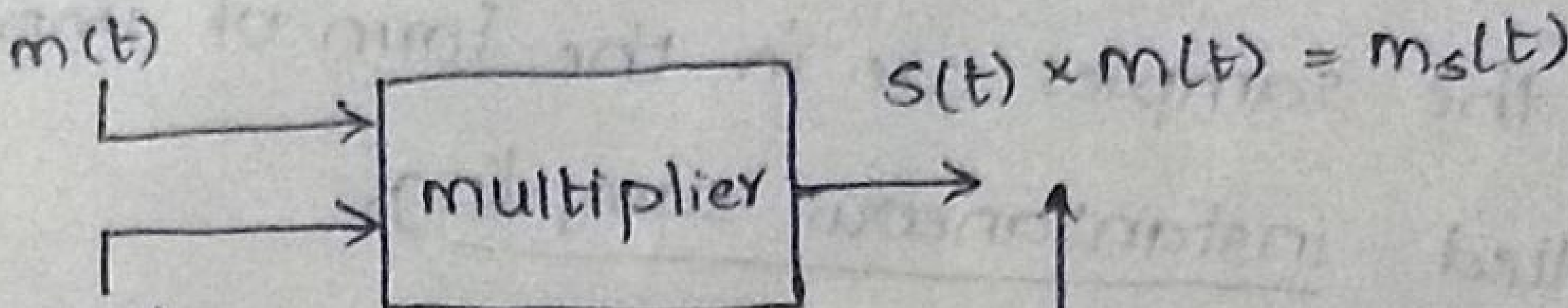


fig a)



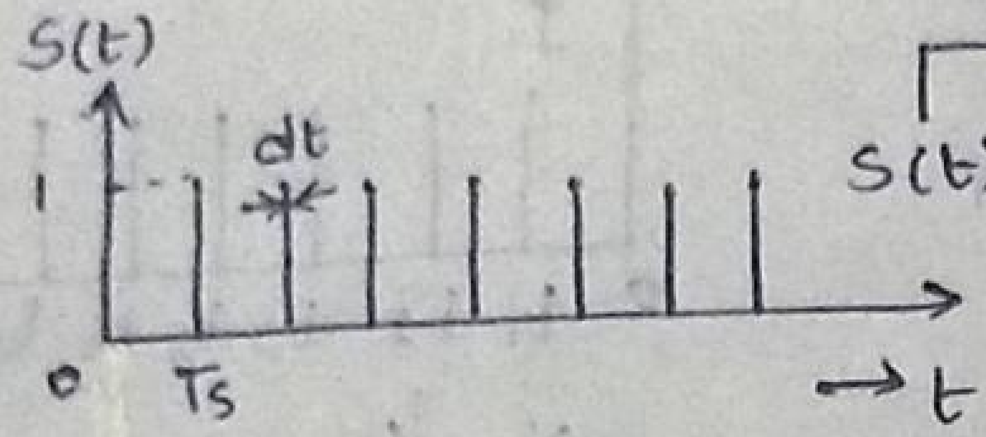


fig b)

fig c)

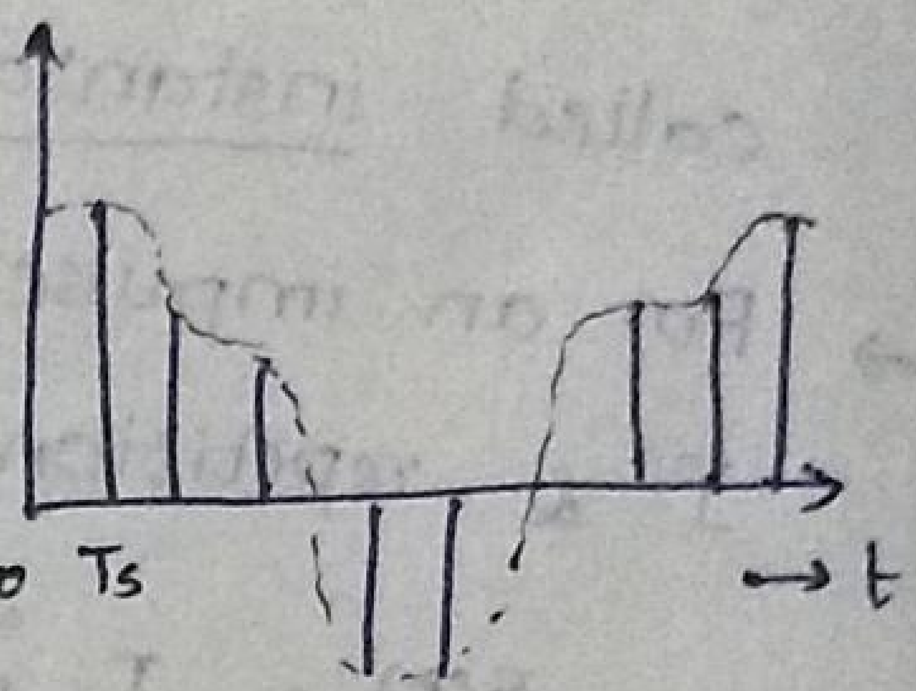


fig d)

fig a) A s/g m(t) which is to be sampled

fig b) The sampling fn s(t) contains train of very narrow unit amplitude pulses

fig c) The sampling operation is performed in multiplier

fig d) The samples of s/g m(t)

The baseband s/g m(t) which is to be sampled is shown in fig a). Let us consider a periodic train of pulses s(t) of unit amplitude & of period T_s as shown in fig b) The pulses are arbitrarily narrow having width dt.

The two s/gs m(t) & s(t) are applied to multiplier as shown in fig (c) which provides an o/p s(t) m(t).

The s/g s(t) is periodic with period T_s & has the fourier expansion $\{$replace $I = dt$ & $T_0 = T_s\}$

$$s(t) = \frac{dt}{T_s} + \frac{2\,dt}{T_s}\left[\cos\frac{2\pi t}{T_s} + \cos\frac{4\pi t}{T_s} + \cdots\right]$$

Then the sampled o/p, $m_s(t) = m(t)\, s(t)$

$$\therefore\ m(t)\,s(t) = \underbrace{\frac{dt}{T_s}\, m(t)}_{\text{Base band s/g}} + \frac{dt}{T_s}\left[\underbrace{2\, m(t)\cos\frac{2\pi t}{T_s}}_{\text{DSB-SC s/g with carrier freq } (f_s = 2f_M)} + \underbrace{2\, m(t)\cos\frac{4\pi t}{T_s}}_{\text{DSB-SC s/g with carrier freq } \{4f_M = 2f_s\}} + \cdots\right]$$

Let the s/g $m(t)$ has spectral density $F[m(t)] = M(j\omega)$

llarly s/g $m_s(t)$ has spectral density $F[m_s(t)] = F[m(t)\,s(t)]$

$= M(j\omega)\,S(j\omega)$

<u>case</u> ① : $T_s = \frac{1}{2f_M}$

$|F(s(t)m(t))| = |M(j\omega)S(j\omega)|$

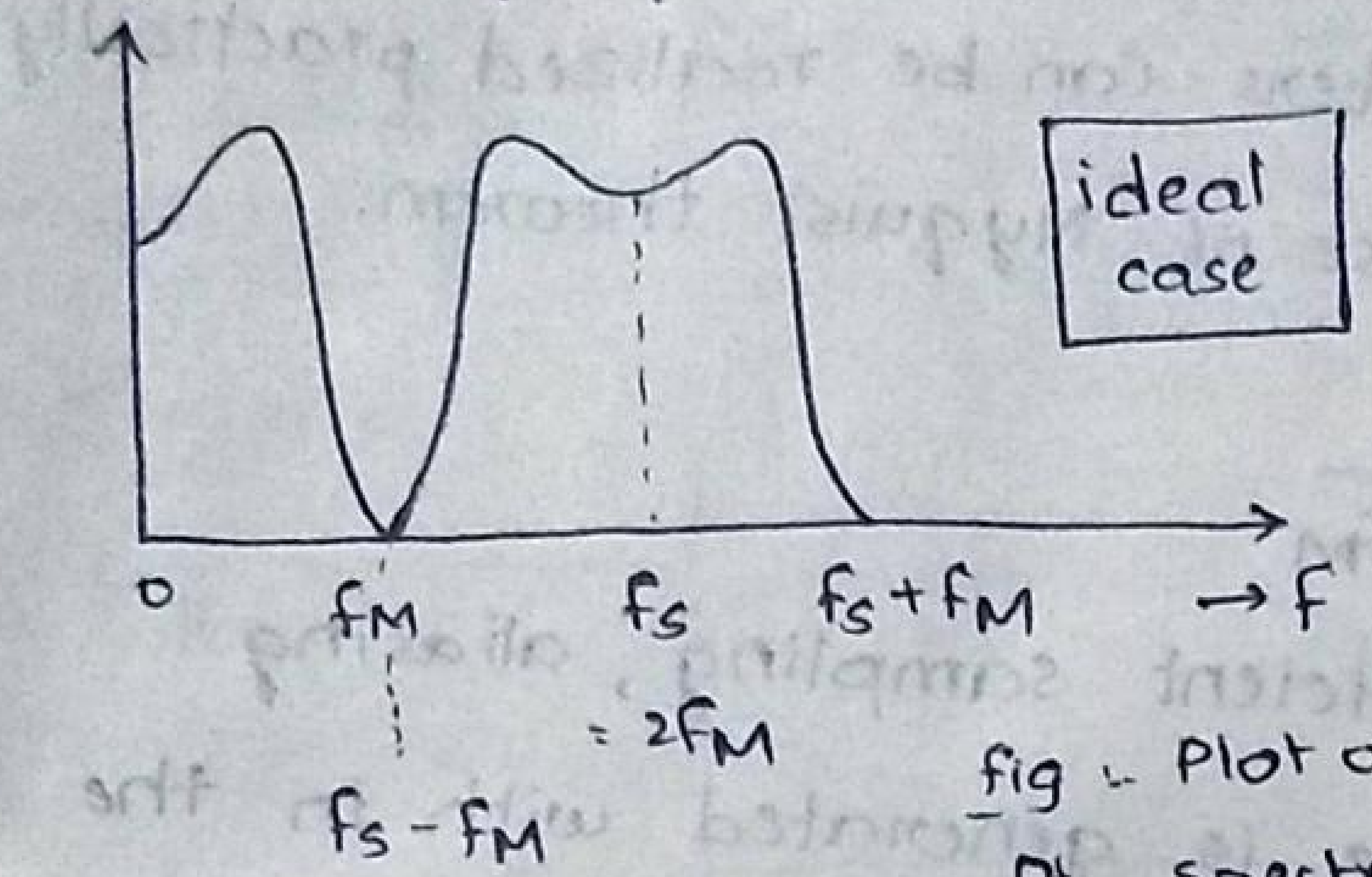


fig :- Plot of amplitude of spectrum of sampled s/g.

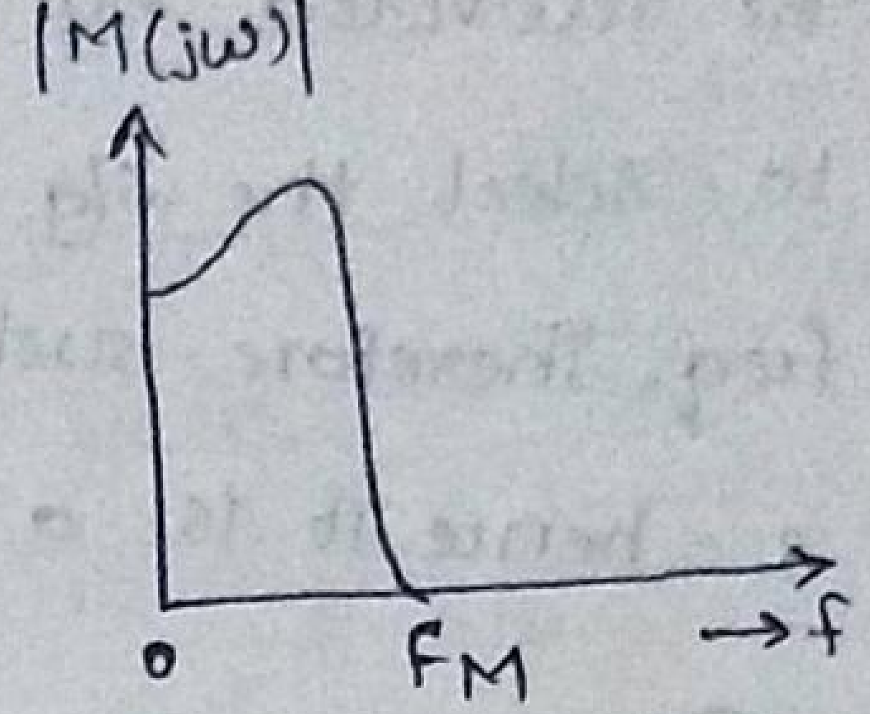
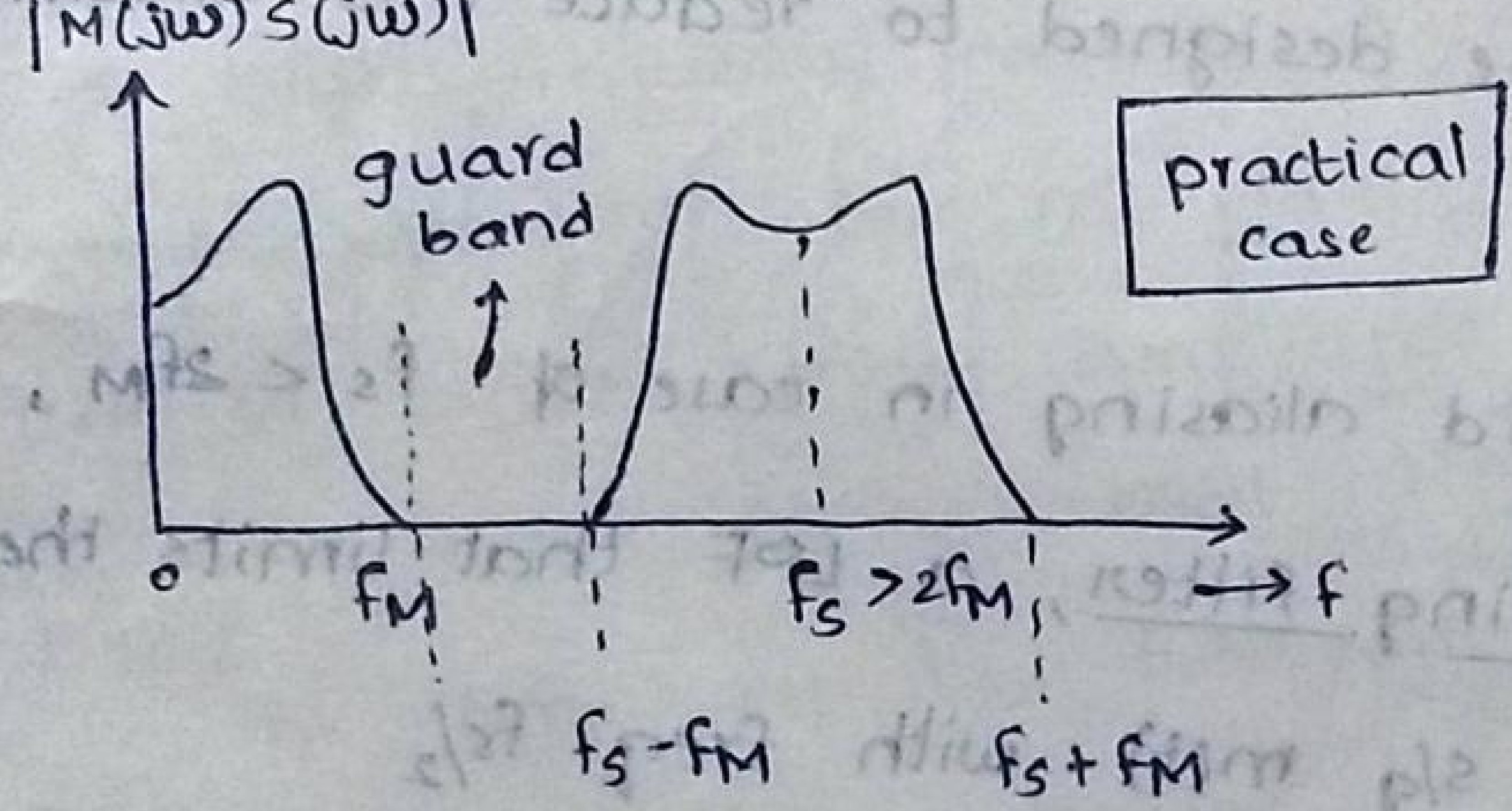


fig :- magnitude plot of spectral density of s/g bandlimited to f_M.

<u>case</u> ② :- $T_s < \frac{1}{2f_M} \Rightarrow f_s > 2f_M$



Guard band $= (f_s - f_M) - f_M$

$= f_s - 2f_M$

fig :- Guard band appears when $f_s > 2f_M$

<u>case</u> ③ : $T_s > \frac{1}{2f_M} \Rightarrow f_s < 2f_M$

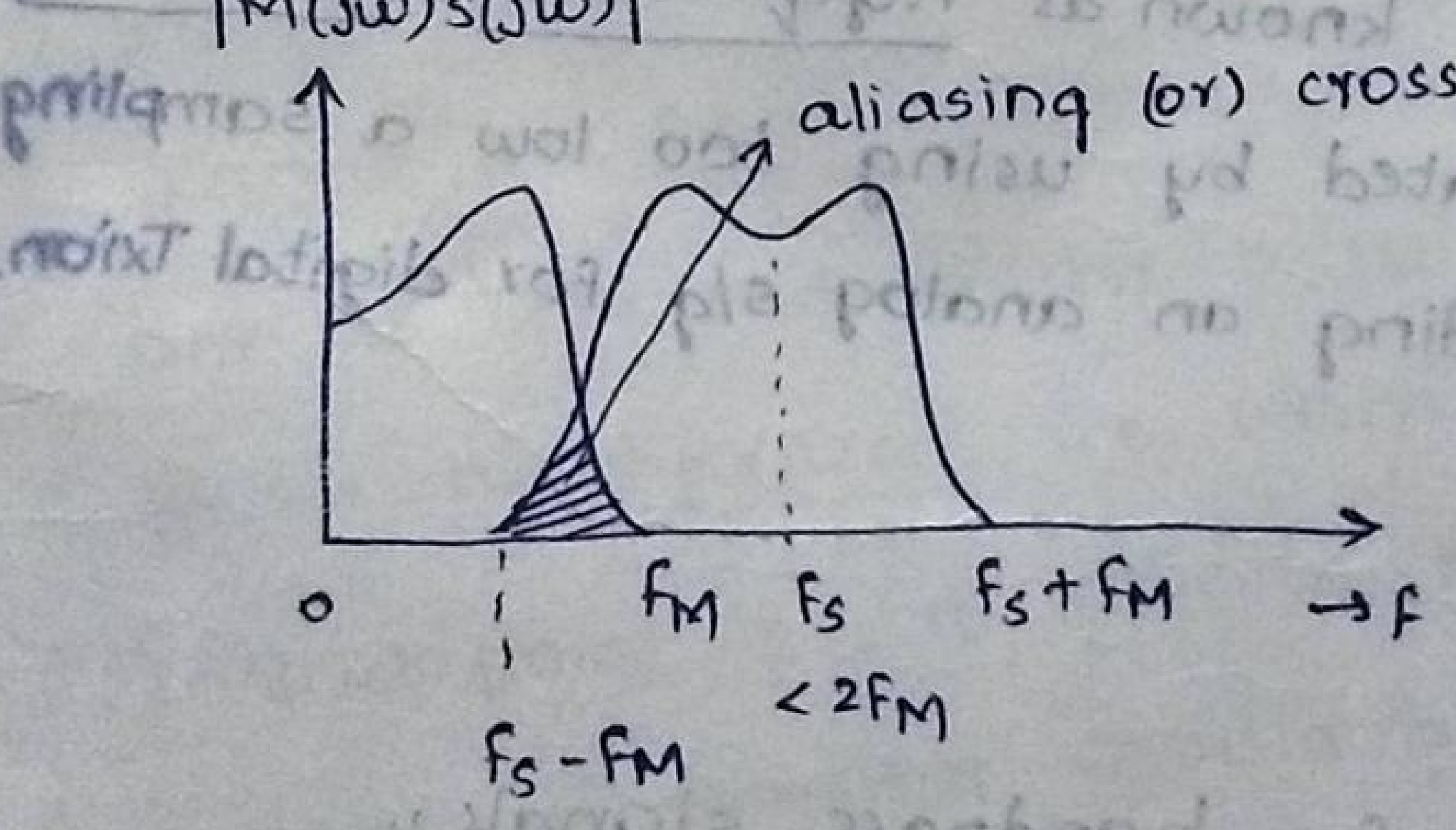


fig :- Over-lapping of spectr[a] when $f_s < 2f_M$

<u>case</u> ① :- $f_s = 2f_M$

From the above spectrums when $f_s = 2f_M$, the s/g can be recovered by passing the Rxed s/g through a LPF having a sharp cut off freq at f_M. But practically such type of filters cannot be realized. Hence it is an ideal case for Nyquist theorem.

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② case ② :- when $f_s > 2f_M$, a guardband of $f_s - 2f_M$ $\{(f_s - f_M) - f_M = f_s - 2f_M\}$ results as shown in fig. Hence the s/g can be recovered by using a LPF. In this case, the LPF used to select the s/g need not have infinitely sharp cut-off freq. Therefore such type of filters can be realized practically & hence it is a practical case of Nyquist theorem.

case ③ :- $f_s < 2f_M \Rightarrow T_s > \frac{1}{2f_M}$

when $f_s < 2f_M$, due to insufficient sampling, aliasing (distortion (or) crosstalk) noise is generated with in the freq band of 0 to f_M as shown in fig.

No filter can be designed to reduce this aliasing effect.

→ When necessary, to avoid aliasing in case of $f_s < 2f_M$,

① we use an antialiasing filter, a LPF that limits the freq band of msg s/g m(t) with freq $f_s/2$

② $f_s > 2f_M$

→ The min sampling rate is known as Nyquist rate, $\boxed{f_s = 2f_M}$

→ Aliasing :- Distortion created by using too low a sampling rate ($f_s < 2f_M$) when coding an analog s/g for digital Txion

→ Sampling rate :- f_s

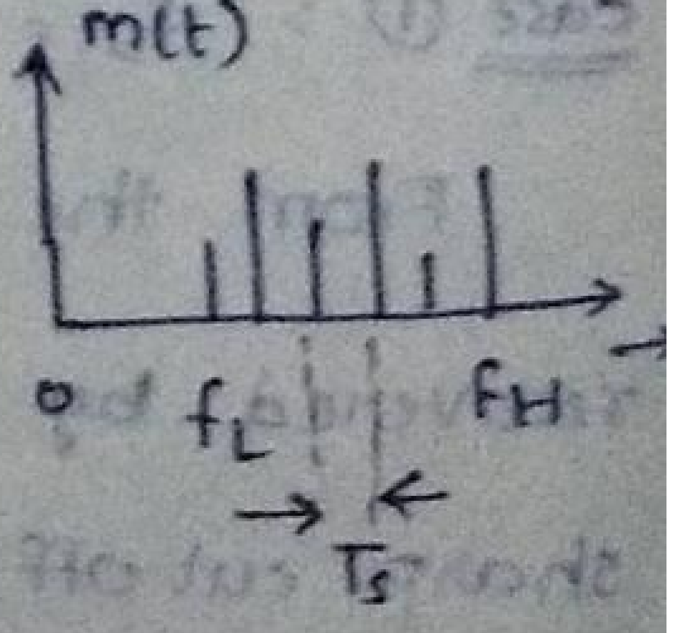
→ Sampling time : T_s

→ Nyquist sampling theorem for bandpass signals :-

$\boxed{f_s \geq 2(f_H - f_L)}$ $\Rightarrow T_s \leq \frac{1}{2(f_H - f_L)}$

f_H → Highest spectral frequency component

f_L → Lowest spectral frequency component.



→ Problems (2) {0, 1,}

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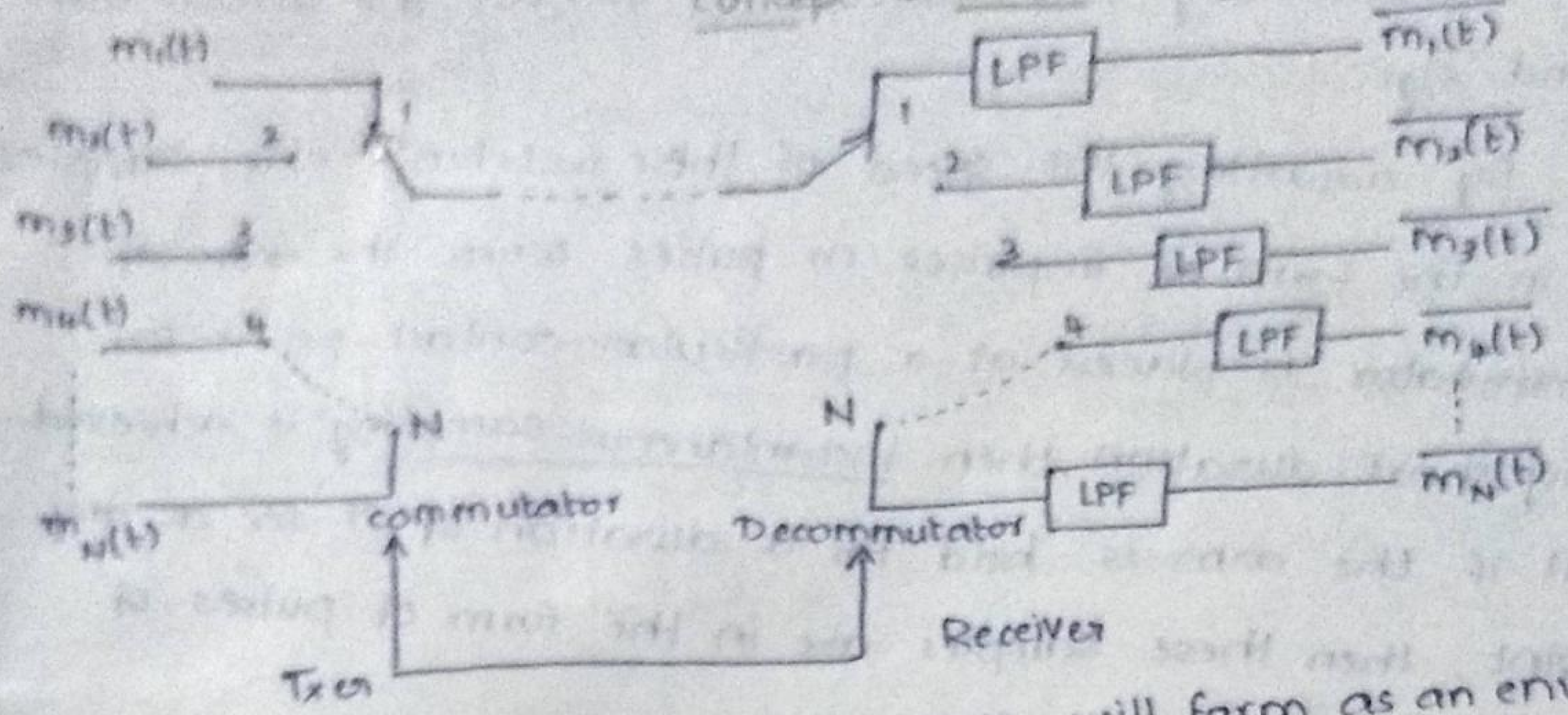
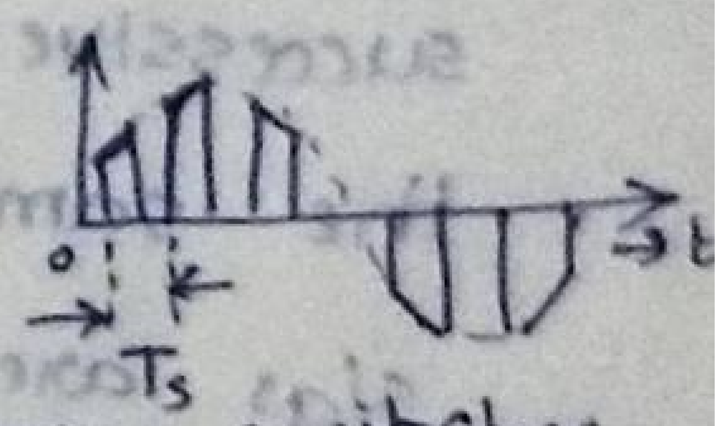
→ Time Division Multiplexing. Pulse Amplitude Modulation system (TDM-PAM system) Pulse Amplitude Modulation and concept of Time Division Multiplexing



In AM system, a base band s/g $m(t)$ will form as an envelope if a high frequency carrier $A \cos \omega_c t$ to form a modulated carrier waveform. But in PAM, the samples are in the form of pulses whose amplitude is equal to the amplitude of base band s/g at that time instant.

The time gap b/w two successive leading edges of this PAM samples is defined as sampling time T_s.



In order to multiplex 'N' number of signals in PAM systems, consider two rotatory switches, one at Txer end & the other at Rxer end, which needs to be synchronized. i.e, when the arm of commutator is at contact point '1', it collects the samples of $m_1(t)$, then the arm of decommutator should be at contact point '1' so that samples of $m_1(t)$ are passed through LPF designed to recover $m_1(t)$ only.

The switch at Txer is called commutator which (or) collects the samples of all 'N' number of baseband s/gs. At the receiving end, a similar type of switching mechanism called decommutator is present, which Rxes these samples as i/p & separates these samples & applies them to the corresponding filter circuits to recover the actual base band s/gs.

By adjusting the speed of these switching ckts, samples are in the form of impulses or pulses. When the arm of commutator is placed at a particular contact point for very short duration, then instantaneous sampling is achieved. But if the arm is held for a duration of 'τ' sec at any point, then these samples are in the form of pulses of duration τ sec.

→ Channel BW for PAM :-

Interlacing of samples :-

If N no. of base band s/gs i.e., $m_1(t), m_2(t), \ldots m_N(t)$ are to be multiplexed then b/w two successive samples of $m_1(t)$ the samples of remaining $(N-1)$ s/gs are to be Txed.

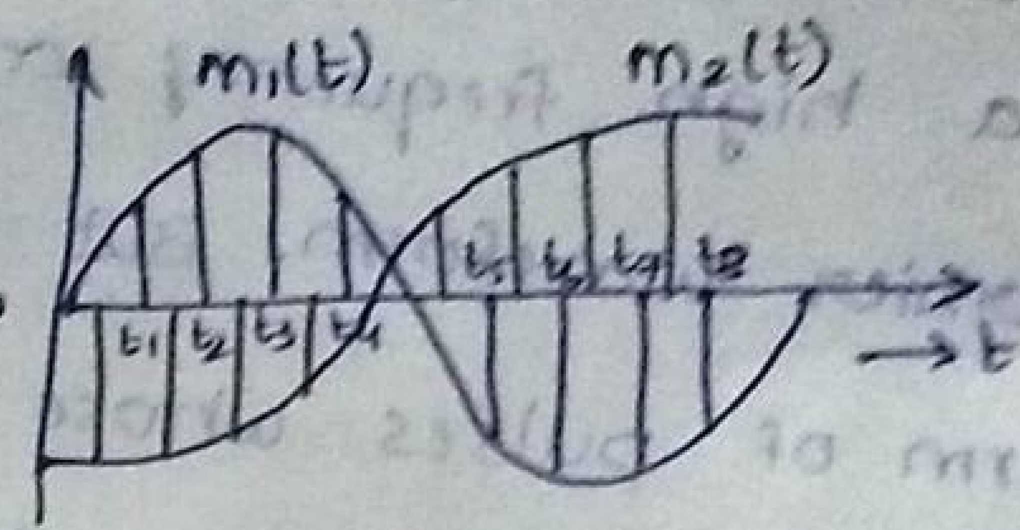


fig :- Interlacing of samples for 2 base band s/gs.

If f_M is the bandlimited freq, then $T_s = \frac{1}{2f_M}$. Hence the interval of separation b/w two successive samples is

$$\boxed{\frac{T_s}{N} = \frac{1}{2f_M N}}$$

If the BW of the channel is sufficient, then all the 'N' number of s/gs can be recovered without any distortion, but insufficient BW results crosstalk (overlapping of response).

In this TDM, if the sample is a pulse of width τ sec, then max no. of s/gs that can be multiplexed is

$$\boxed{N_{max} = \frac{T_s}{\tau}}$$

where T_s is sampling time, if $N_{max} > \frac{T_s}{\tau}$ crosstalk will result.

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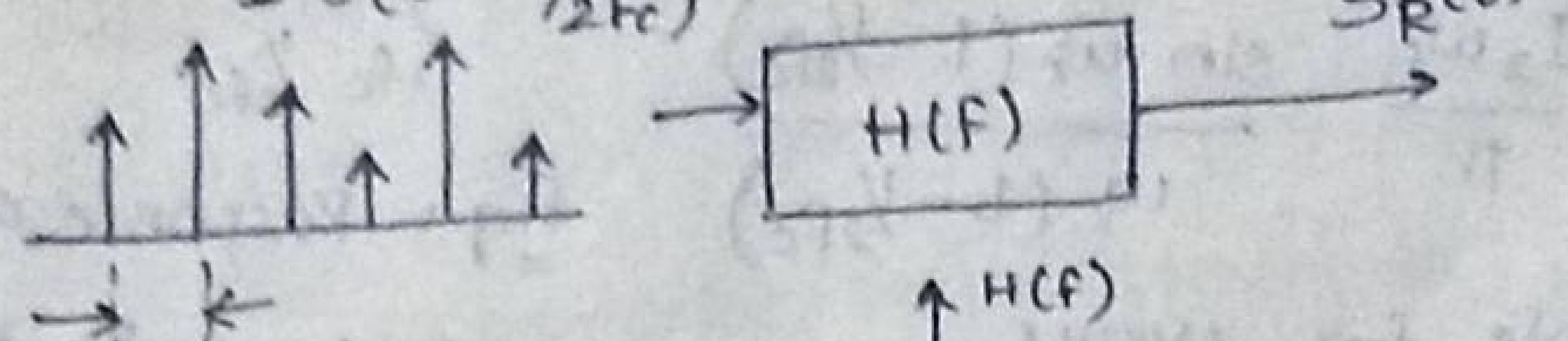
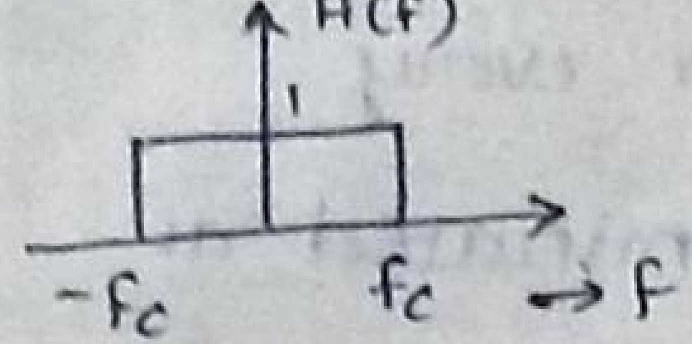
→ Channel BW of PAM s/g :-

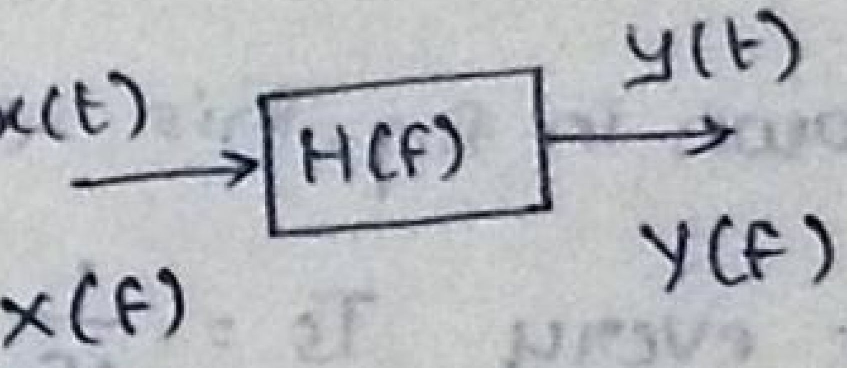
Consider the sequence of samples in the form of impulses are applied as i/p to comm. channel which has a characteristics of an ideal LPF with angular cut-off freq $\omega_c = 2\pi f_c$ having unity gain & no delay.



where $S_R(t)$ is the response fn developed by the channel at the Rxing end.



$$H(f) = 1 \quad -f_c \le f \le f_c$$
$$= 0 \quad \text{elsewhere}$$



$$H(f) = \frac{Y(f)}{X(f)}$$
$$\Rightarrow Y(f) = H(f)X(f)$$
$$\Rightarrow y(t) = F^{-1}\left[H(f)X(f)\right]$$

$$S_R(t) = F^{-1}\left[H(f)\, F\left[I\,\delta\left(t-\frac{K}{2f_c}\right)\right]\right]$$

$$\left\{ \begin{aligned} F[\delta(t-t_1)] &= e^{-j2\pi f t_1} \\ F[v(t)] &= V(f) = \int_{-\infty}^{\infty} v(t)\, e^{-j2\pi f t}\, dt \\ F^{-1}[V(f)] &= v(t) = \int_{-\infty}^{\infty} V(f)\, e^{j2\pi f t}\, df \end{aligned} \right\}$$

$$\Rightarrow S_R(t) = F^{-1}\left[H(f)\, I\, e^{-j2\pi f K/2f_c}\right]$$

$$= I\int_{-f_c}^{f_c} 1 \cdot e^{-j2\pi f K/2f_c}\, e^{j2\pi f t}\, df$$

$$= I\int_{-f_c}^{f_c} e^{j2\pi f(t-K/2f_c)}\, df = I\left[\frac{e^{j2\pi f(t-K/2f_c)}}{j2\pi\left(t-\frac{K}{2f_c}\right)}\right]_{-f_c}^{f_c}$$

$$\left\{ \frac{e^{jx}-e^{-jx}}{2j} = \sin x \right.$$

$$\Rightarrow S_R(t) = \frac{I\,\cancel{2j}\,\sin 2\pi f_c(t-K/2f_c)}{\cancel{j}2\pi(t-K/2f_c)} = \frac{I\omega_c}{\pi}\,\frac{\sin \omega_c(t-K/2f_c)}{\omega_c(t-K/2f_c)}$$

$$\therefore s_R(t) = \frac{I\omega_c}{\pi} \frac{\sin \omega_c(t-K/2f_c)}{\omega_c(t-K/2f_c)}$$

For $t=0$, $m_1(t)$ is sampled

$t=\frac{1}{2f_c}$, $m_2(t)$ is sampled

For $K=0$, $s_{R_1}(t) = \frac{I_1\omega_c}{\pi} \frac{\sin \omega_c t}{\omega_c t}$

$K=1$, $s_{R_2}(t) = \frac{I_2\omega_c}{\pi} \frac{\sin \omega_c(t-1/2f_c)}{\omega_c(t-1/2f_c)}$

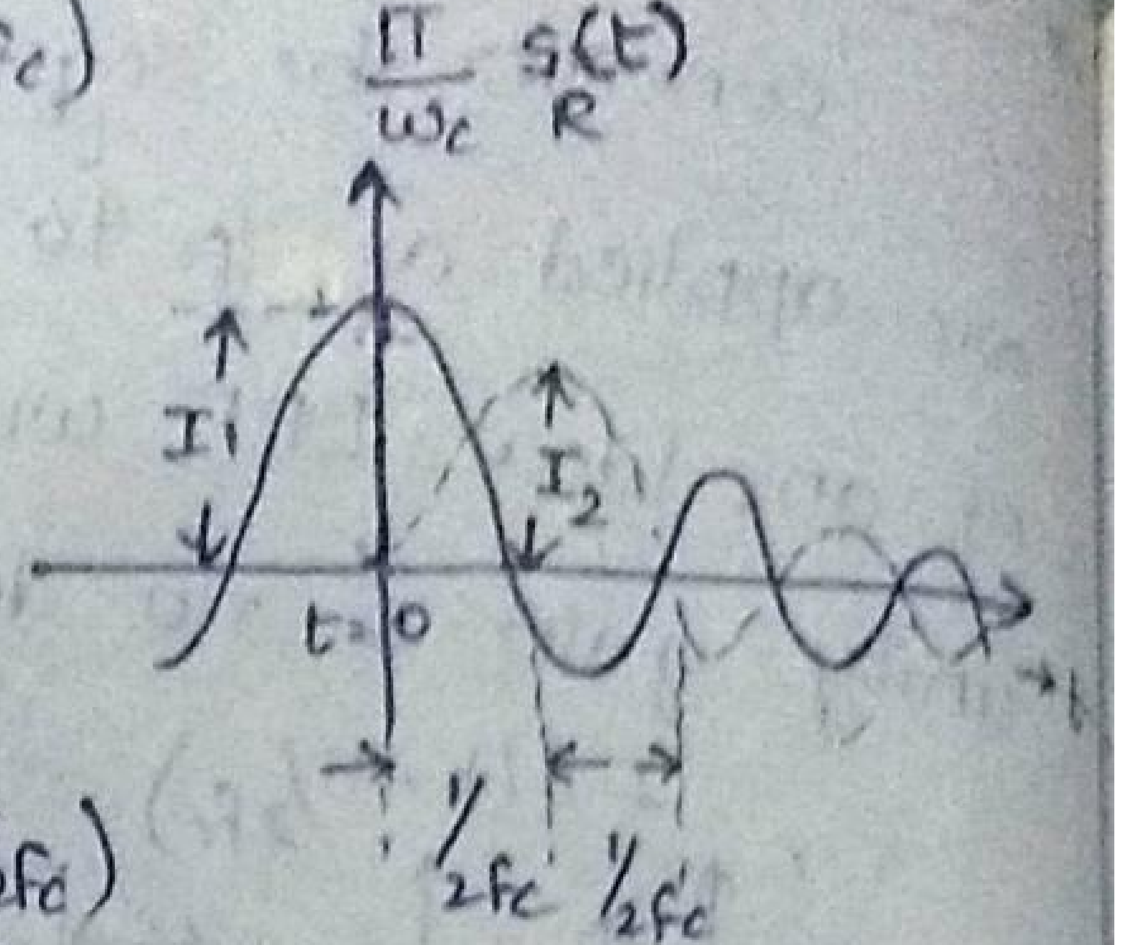


Fig: Response of ideal LPF to instantaneous samples.

By Txing, one sample for every $1/2f_c$ sec, crosstalk can be eliminated as shown in fig. This sequence should be repeated for every $T_s = \frac{1}{2f_M}$ sec. Hence total no. of s/gs that can be multiplexed is $\boxed{N = \frac{f_c}{f_M}}$

1 sample $\rightarrow$ $1/2f_c$ sec

$N(?) \rightarrow T_s = \frac{1}{2f_M}$ sec

$\Rightarrow N\frac{1}{2f_c} = \frac{1}{2f_M}$

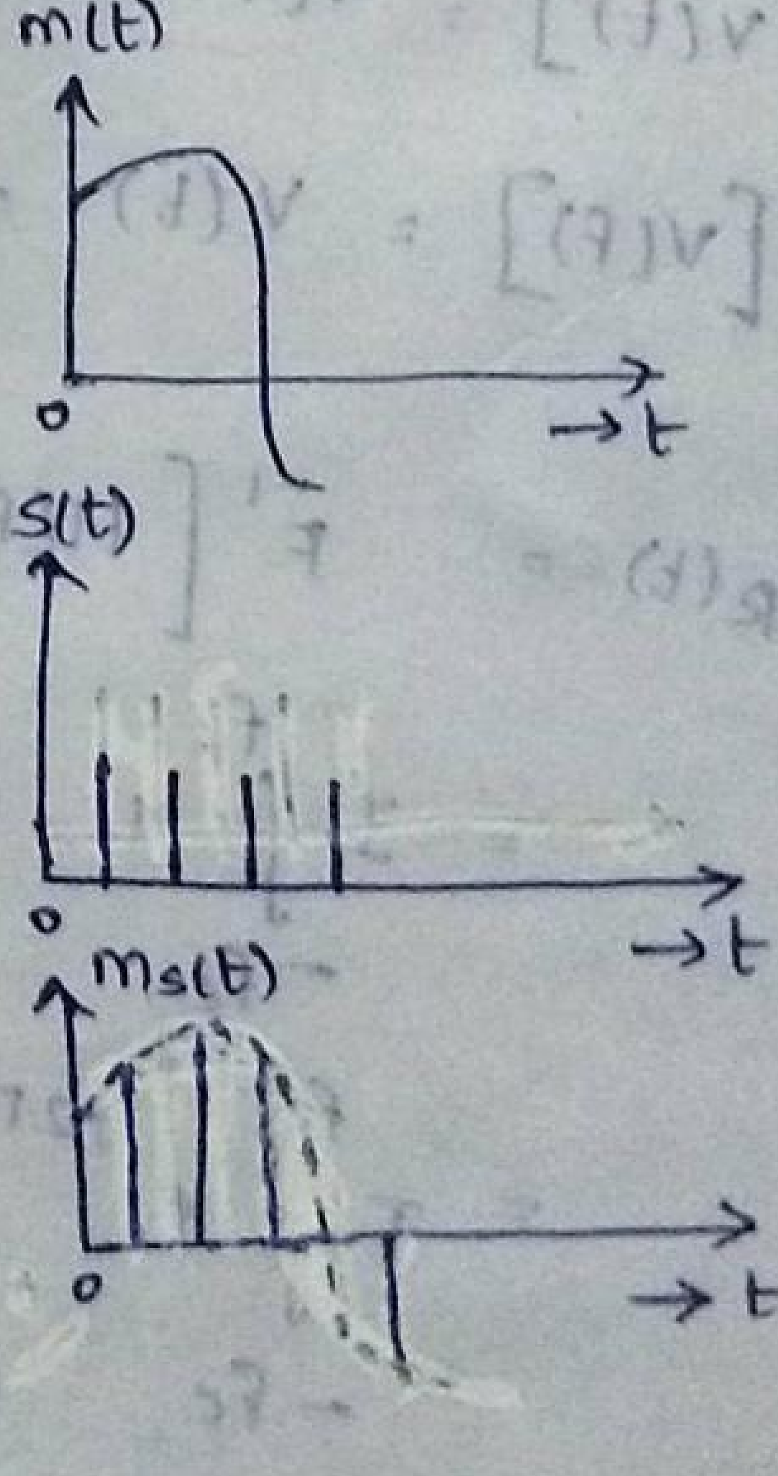
$\Rightarrow N = \frac{f_c}{f_M}$

→ Sampling Techniques :-

① Instantaneous sampling :-

Here, the sampling s/g is impulse train

$$I \sum \delta(t-KT_s)$$

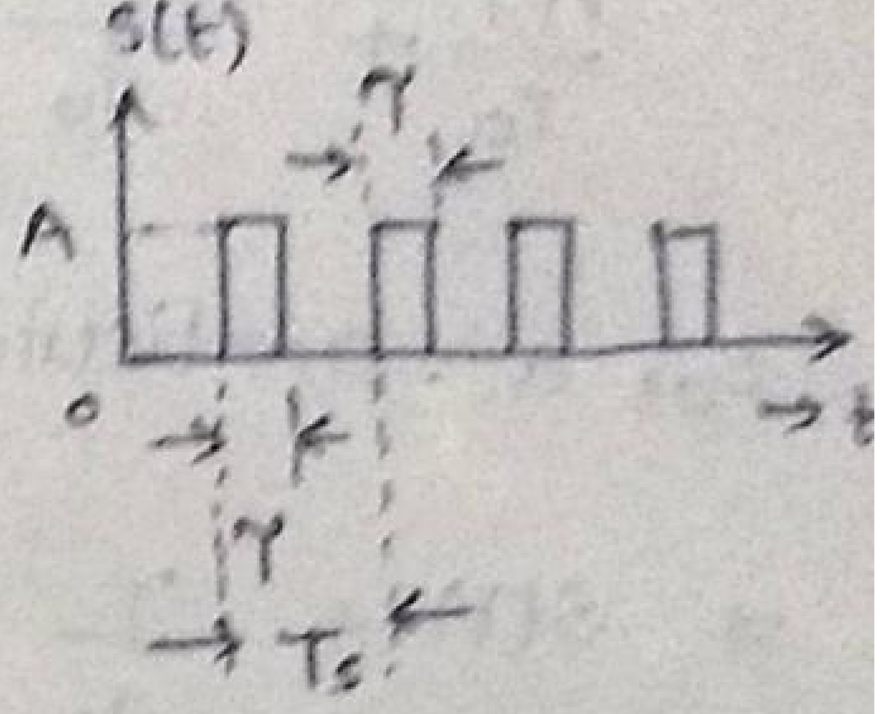


In TDM - PAM systems, instantaneous sampling is achieved when the arm of commutator is held at a particular contact point for a very short duration, so that samples in the form of impulses can be obtained.

But the o/p level of these samples is very small, & there is a chance of losing the data in the background of noise. (when they are Txed over the comm. channel) Hence the baseband s/g may not be reconstructed in all the cases. Hence alternate sampling methods are introduced.

② Pulse sampling : Pulse sampling is of 2 types

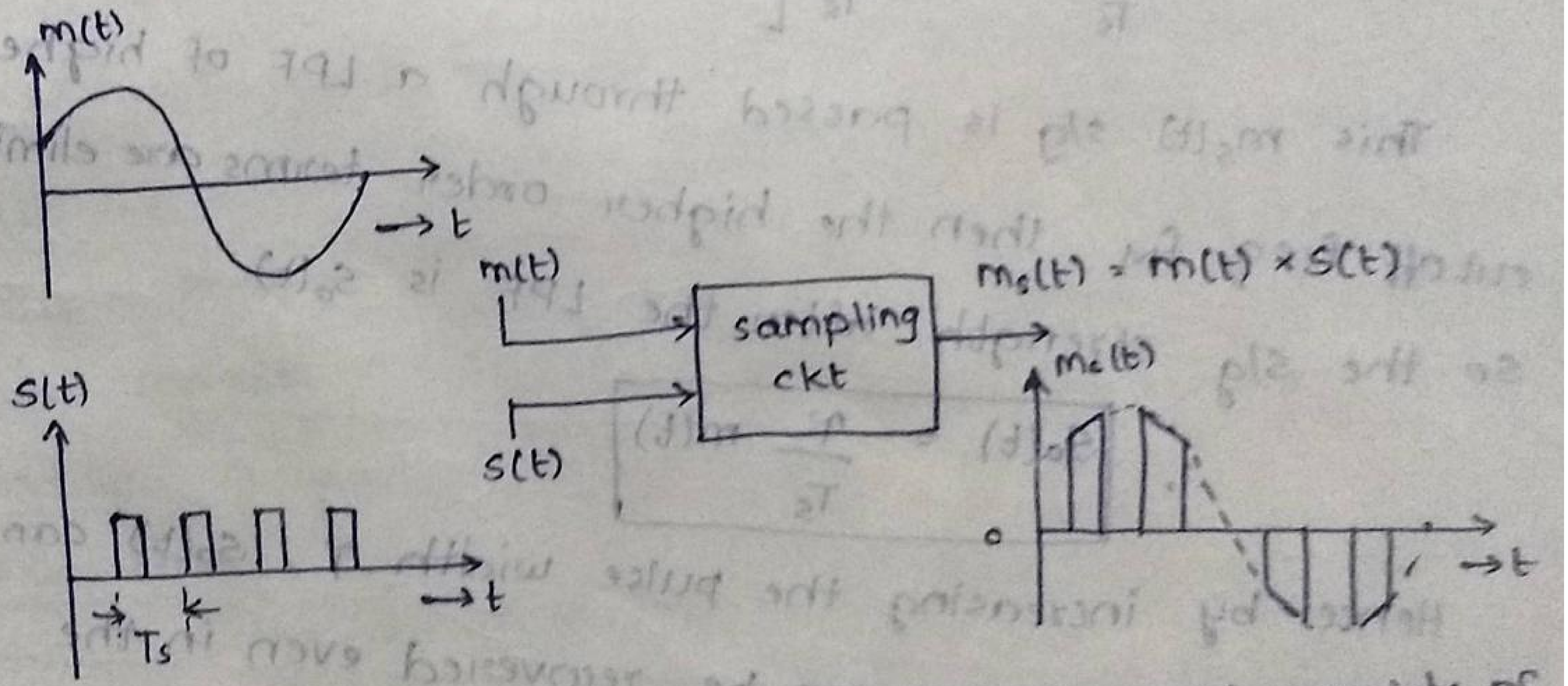
1) Natural sampling
2) Flat-Top sampling.



Pulse sampling can be used to raise the s/g level where the sampling s/g s(t) represents a periodic pulse train of amplitude A, repetition time Ts & pulse width τ sec.

$$\boxed{s(t) = A \Sigma P(t-kT_s)}$$

① Natural sampling :



In natural sampling, the sampled o/p $m_s(t)$ consists of pulses of width τ sec, repeated for every Ts sec where $T_s = \frac{1}{2f_m}$. The top portion of these sampled pulses follow the original base band s/g amplitude (i.e., retains the shape of original s/g during that pulse interval). Hence these natural samples always represent the actual base band s/g & always convey "true information" about the base band s/g. Hence the base band s/g can be reconstructed at the Rxing end with "minimum amount of distortion".

For a periodic pulse train of amplitude A & pulse width τ, repetition time T_o, the fourier series expansion is given by

$$\frac{A\tau}{T_0} + \frac{2A\tau}{T_0} \sum_{n=1}^{\infty} \frac{\sin n\pi\tau/T_0}{\frac{n\pi\tau}{T_0}} \cos \frac{2\pi n t}{T_0}$$

For a particular sampling s/g, $A = 1$, $T_0 = T_s$

$$\Rightarrow \quad S(t) = \frac{\tau}{T_s} + \frac{2\tau}{T_s} \sum_{n=1}^{\infty} C_n \cos \frac{2\pi n t}{T_s} \quad \text{where } C_n = \frac{\sin \frac{n\pi\tau}{T_0}}{\frac{n\pi\tau}{T_0}}$$

$$= \frac{\tau}{T_s} + \frac{2\tau}{T_s}\left[C_1 \cos \frac{2\pi t}{T_s} + C_2 \cos \frac{4\pi t}{T_s} + \cdots\right]$$

$$\therefore \quad m_s(t) = m(t)\, s(t)$$

$$= \frac{\tau}{T_s} m(t) + \frac{2\tau}{T_s}\left[C_1 m(t) \cos \frac{2\pi t}{T_s} + C_2 m(t) \cos \frac{4\pi t}{T_s} + \cdots\right]$$

This $m_s(t)$ s/g is passed through a LPF of higher cutoff freq f_M, then the higher order terms are eliminated. So the s/g strength after the LPF is $S_o(t)$

$$\therefore \quad \boxed{S_o(t) = \frac{\tau}{T_s} m(t)}$$

Hence by increasing the pulse width τ, $S_o(t)$ can be increased so that $m(t)$ can be recovered even in the background of noise, but in a TDM system, the no. of s/gs that can be multiplexed $\left(N = \frac{T_s}{\tau}\right)$ is effected i.e., if τ is increased, N value is reduced. But practically to reduce the effect of crosstalk, τ should be less than $\frac{T_s}{N}$ $\left\{\tau < \frac{T_s}{N}\right\}$

Hence always choose a compromise value of τ to satisfy the required conditions for $S_o(t)$ & Number of s/gs to be multiplexed. (N)

→ Note: ① Practically $\tau + \tau_g = \frac{T_s}{N}$ where τ_g is the guard time allocated b/w 2 successive ... crosstalk