

# → Delta Modulation Systems

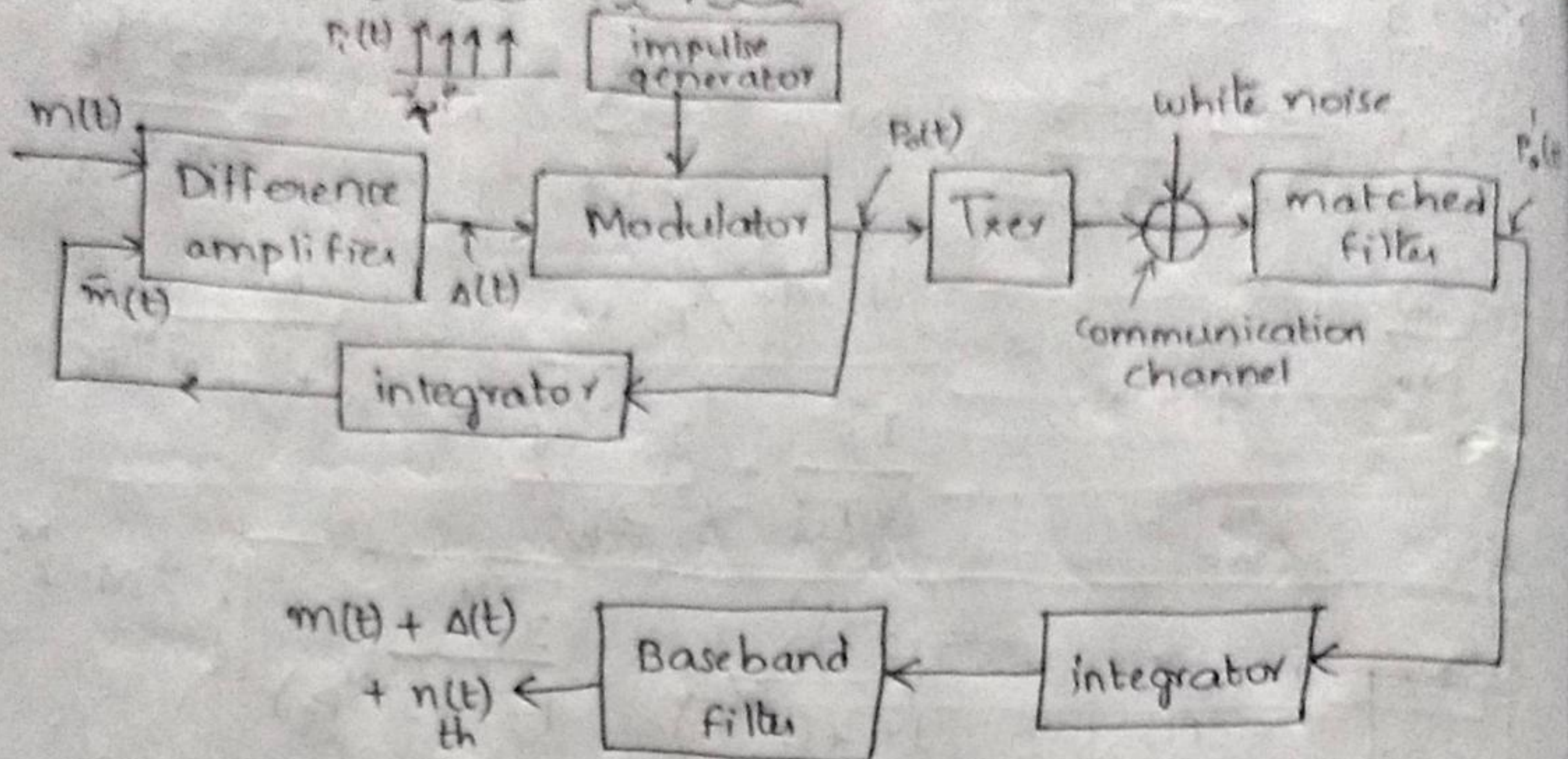
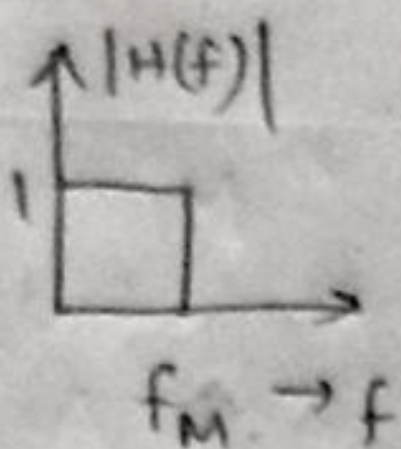


Fig. A Delta  
① modulation  
system



$$\Delta(t) = m(t) - \bar{m}(t) ; \Delta(t) \text{ is +ve if } m(t) > \bar{m}(t)$$

$$\Delta(t) \text{ is -ve if } m(t) < \bar{m}(t)$$

where  $\bar{m}(t) = \frac{s}{I} \int P_o(t) dt$  ie, o/p of integrator, it has been adjusted so that its response to i/p impulse of strength  $I$  is a step of size 's'.  
 $P_o(t)$  is o/p of modulator whose polarity depends on  $\Delta(t)$ .

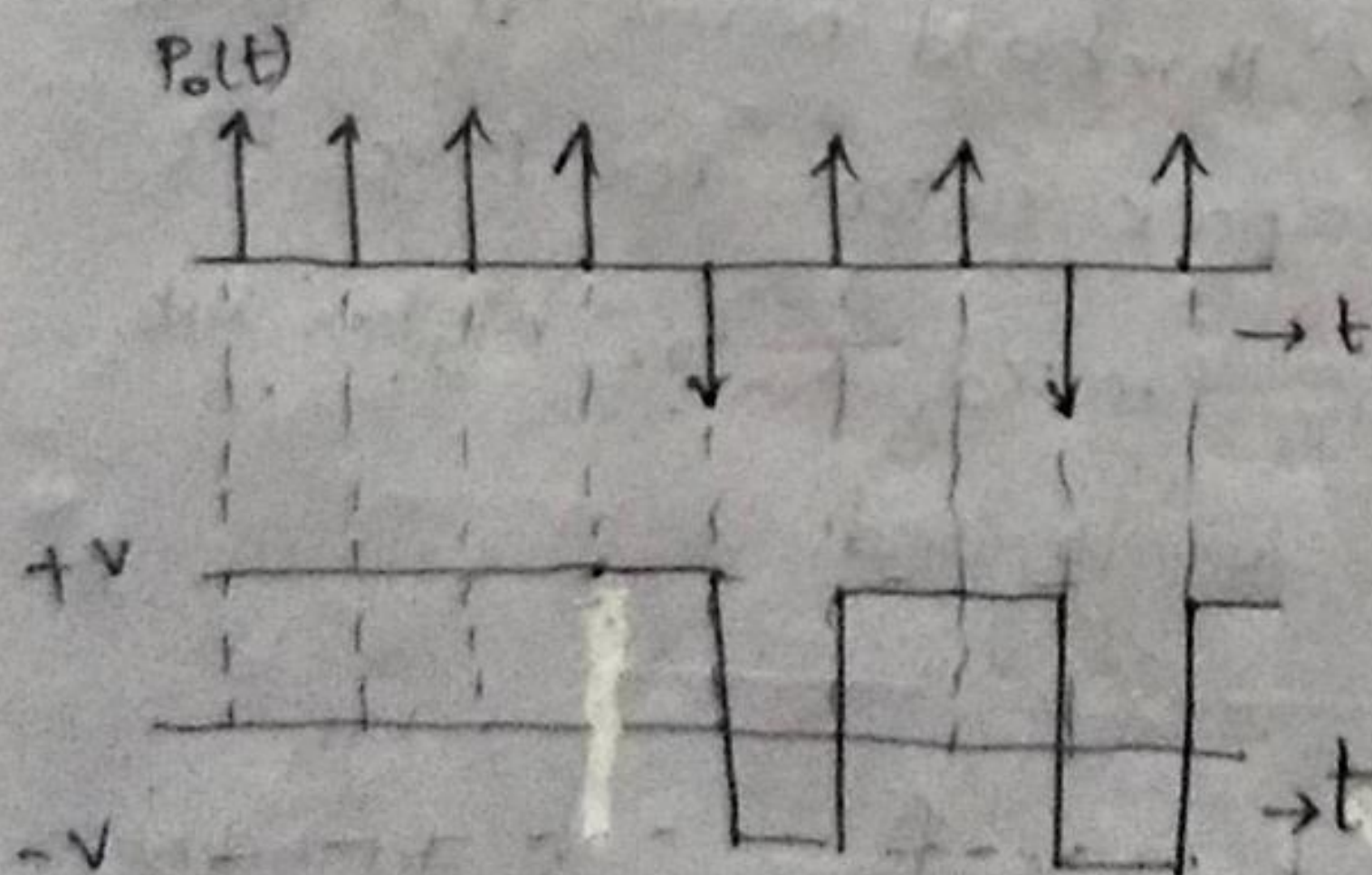


fig a) impulse train  $P_o(t)$  appearing at the o/p of modulator.

fig b) 2-level s(t) Txed over the communication channel

$P_o(t)$  is impulse train, but before Txion, it is converted to 2-level waveform, since the later waveform has greater power than a train of narrow pulses. This conversion

is accomplished by a block in  $T_{xer}$ . After detection by the matched filter, this binary waveform will be converted to a sequence of impulses  $P_o'(t)$ .

In the absence of thermal noise  $P_o'(t) = P_o(t)$  & if the integrators used at both  $T_{xer}$  &  $R_{xer}$  are identical,  $\bar{m}(t)$  is Recovered at the  $R_{xer}$ .

Quantization noise in Delta Modulation ( $N_q$ ) :-

$\Delta(t)$  is source of quantization noise in DM. Minimize the slope overloading such that error  $\Delta(t) < 's'$

If  $\Delta(t)$  is a s/g making excursions between  $+s$  &  $-s$

where ' $s$ ' is a uniform stepsize, Then pdf of  $\Delta(t)$  is

$$f(\Delta) = \frac{1}{2s} \quad -s \leq \Delta \leq s$$

The normalized power of waveform of  $\Delta(t)$  is

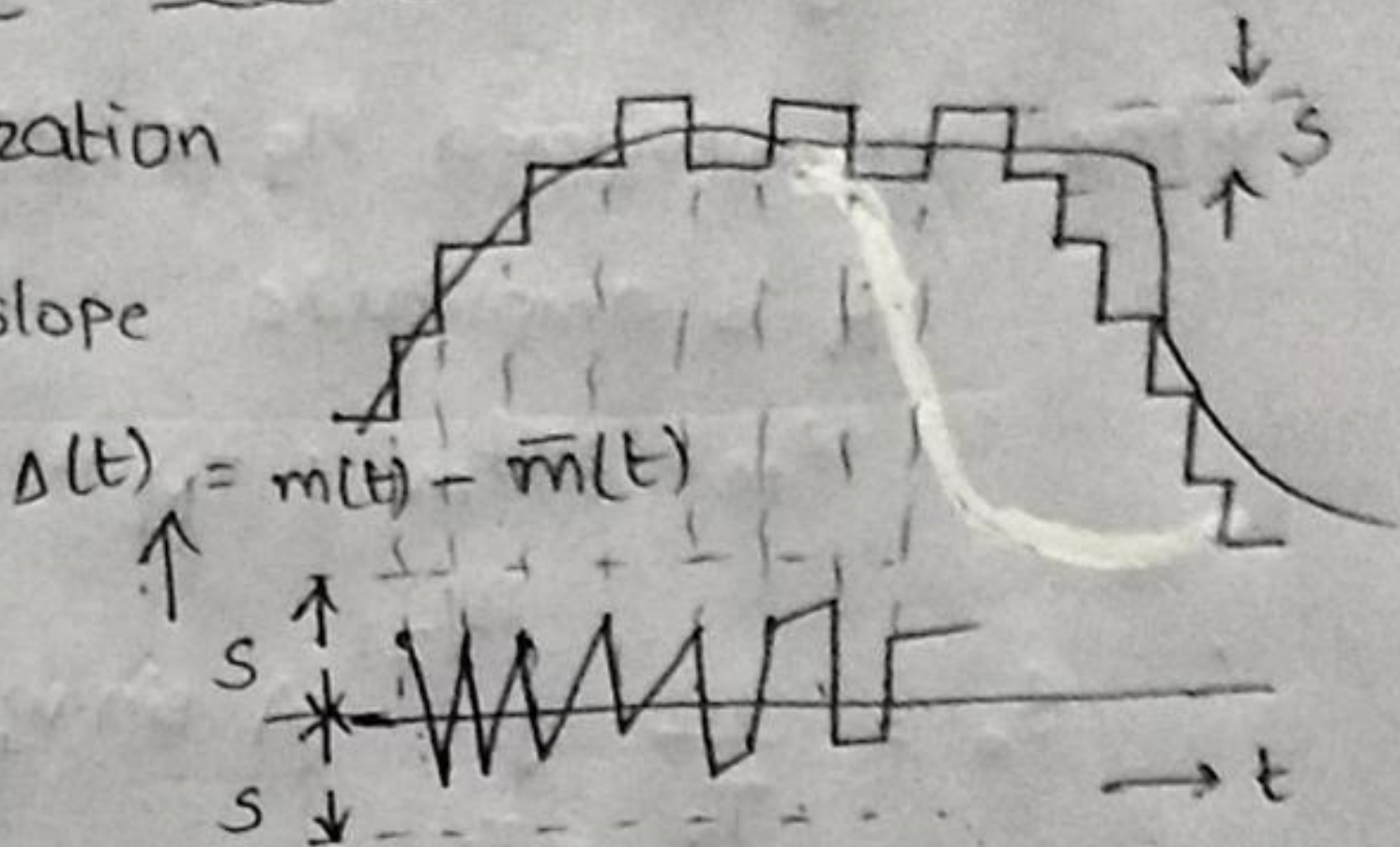
$$\begin{aligned} \overline{[\Delta(t)]^2} &= \int_{-s}^s \Delta^2 f(\Delta) d\Delta = \left[ \frac{\Delta^3}{3} \right]_{-s}^s \Rightarrow \frac{1}{6s} [2s^3] \\ &= \int_{-s}^s \Delta^2 \frac{1}{2s} d\Delta \Rightarrow \frac{2s^3}{6s} \end{aligned}$$

$$\Rightarrow \overline{[\Delta(t)]^2} = \frac{s^2}{3}$$

The spectrum of  $\Delta(t)$  extends continuously over a frequency range which begins near zero. Consider a LPF with adjustable cut-off freq

$$f_c = \frac{1}{T} = f_b \quad \left\{ \begin{array}{l} f_b \text{ is bit rate} \\ T \text{ is step duration} \end{array} \right\}, \text{ so that } \Delta(t)$$

passes through this filter with min amount of distortion. So that spectrum of  $\Delta(t)$  extends from 0 to  $f_b$ , but the cut off freq of final base band filter is  $f_m$



& the amount of noise power passing through the filter is quantization noise

$$N_q = \frac{f_M S^2}{3f_b}$$

o/p s/q power ( $S_o$ ) :

let the s/q be  $m(t) = A \sin \omega_M t$

where we considered s/q is sinusoidal

A is amplitude

$\omega_M = 2\pi f_M$ ,  $f_M$  is upper limit of base band freq range

$$\begin{aligned} S_o &= \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} (A \sin \omega_M t)^2 dt = \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} m^2(t) dt \\ &= \frac{A^2}{T_s} \int_{-T_s/2}^{T_s/2} \left[ \frac{1 - \cos 2\omega_M t}{2} \right] dt \\ &= \frac{A^2}{2T_s} \left[ t - \frac{\sin 2\omega_M t}{2\omega_M} \right]_{-T_s/2}^{T_s/2} \end{aligned}$$

$$\Rightarrow S_o = \frac{A^2}{2T_s} [T_s - 0] = \frac{A^2}{2}$$

$$\therefore S_o = \frac{A^2}{2}$$

To avoid slope overloading,

$$\begin{aligned} A &= \frac{S}{\gamma \omega_M} = \frac{S f_b}{\omega_M} \\ \therefore S_o &= \frac{A^2}{2} = \frac{S^2 f_b^2}{\omega_M^2 2} \end{aligned}$$

$$\begin{aligned} \begin{cases} \text{max slope of } m(t) &= 2\pi f_b A \\ \Rightarrow \frac{S}{\gamma} &= 2\pi f_b A \\ \Rightarrow \frac{S}{\gamma} &= \omega_M A \\ \& \gamma &= \frac{1}{f_b} \end{cases} \end{aligned}$$

$$\therefore \frac{S_o}{N_q} = \frac{S^2 f_b^2 / 2\omega_M^2}{S^2 f_M / 3f_b} = \frac{3f_b^3}{f_M (2\omega_M^2)} = \frac{3f_b^3}{f_M (2 \cdot 4\pi^2 f_M^2)}$$

$$\Rightarrow \frac{S_0}{N_q} = \frac{3f_b^3}{8\pi^2 f_m^3} = \frac{3}{8\gamma^3 \pi^2 f_m^3}$$

$$\boxed{\therefore \frac{S_0}{N_q} = \frac{3}{80} \frac{1}{(\gamma f_m)^3} = \frac{3}{80} \left(\frac{f_b}{f_m}\right)^3}$$

$$= \frac{3}{80 (\gamma f_m)^3} \quad \left\{ \begin{array}{l} \pi^2 = 9.86 \\ \approx 10 \end{array} \right.$$

let the sampling time is  $T_s = \frac{1}{2f_m}$

word length is  $N$

$$\gamma_{\max} \left( \begin{array}{l} \text{max bit duration} \\ \text{without guard time} \end{array} \right) = \frac{T_s}{N}$$

$$\Rightarrow \gamma_{\max} = \frac{T_s}{N} = \frac{1}{2f_m N} \quad \left\{ \because T_s = \frac{1}{2f_m} \right\}$$

$$\therefore \frac{S_0}{N_q} = \frac{3}{80} \left(\frac{f_b}{f_m}\right)^3 = \frac{3}{80} \frac{1}{(\gamma f_m)^3} = \frac{3 (2f_m N)^3}{80 f_m^3}$$

$$= \frac{3}{80} \times 8N^3$$

$$\Rightarrow \boxed{\frac{S_0}{N_q} = 0.3 N^3} \rightarrow \text{in DM}$$

for  $N=8$

$$\frac{S_0}{N_q} = 0.3 (8)^3 \Rightarrow$$

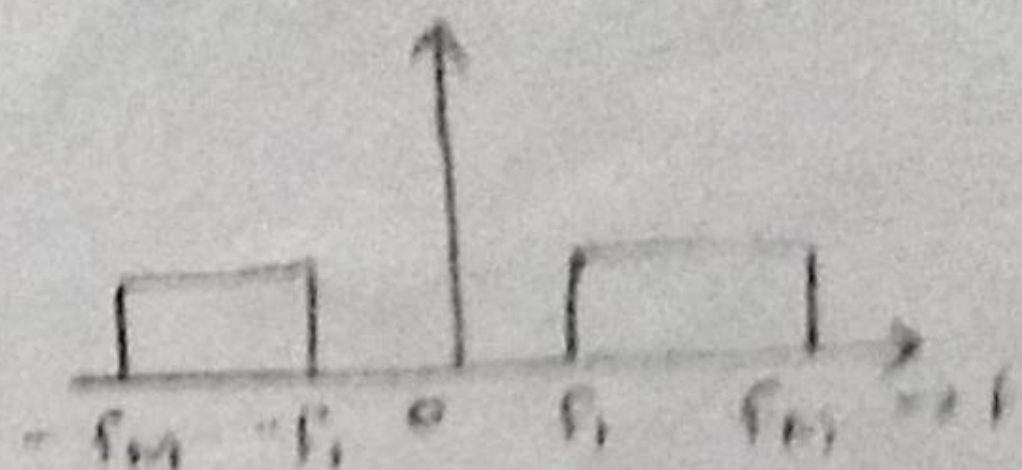
$$= 22 \text{ db}$$

$$\boxed{\begin{array}{l} \text{for } N=8 ; \frac{S_0}{N_q} = 22 \text{ db in DM} \\ ; \frac{S_0}{N_q} = 48 \text{ db in PCM} \\ \left\{ \frac{S_0}{N_q} = 2^{2N} \right\} \end{array}}$$

for  $N = 8 \Rightarrow S_0/N_q = 22 \text{ db} \rightarrow \text{in DM} \left\{ \frac{S_0}{N_q} = 0.3 \text{ dB} \right\}$   
 $S_0/N_q = 48 \text{ db} \rightarrow \text{in PCM} \left\{ \frac{S_0}{N_q} = 2^{2H} \right\}$

Thermal noise :- By considering the effect of thermal noise in a DM system

$$N_{th} = \frac{2S^2 P_e}{\pi^2 f_l \gamma} \quad \text{where } \gamma \rightarrow \text{bit duration.}$$



$$\therefore \text{SNR} = \frac{S_0}{N_0} = \frac{S_0}{N_q + N_{th}}$$

$$= \frac{S^2 / 2\gamma^2 \omega_M^2}{\frac{f_M S^2}{3f_b} + \frac{2S^2 P_e}{\pi^2 f_l \gamma}} = \frac{\frac{1}{2}\gamma^2 \omega_M^2}{\frac{f_M \gamma}{3} + \frac{2P_e}{\pi^2 f_l \gamma}} = \frac{\frac{1}{2}\gamma^2 \omega_M^2}{\frac{f_M \gamma}{3} \left( 1 + \frac{6P_e}{f_M^2 \pi^2 f_l} \right)}$$

$$= \frac{\frac{2\pi}{2\gamma^2 \omega_M^2}}{\frac{2\pi f_M \gamma}{3} \left( 1 + \frac{6P_e \times 2 \times 2}{(2\pi f_M)(2\pi f_l) \gamma^2} \right)} = \frac{3\pi / \omega_M^3 \gamma^3}{\left( 1 + \frac{24 P_e}{(\omega_M \gamma)(\omega_l \gamma)} \right)}$$

$$\therefore \text{SNR} = \frac{3\pi / \omega_M^3 \gamma^3}{1 + \frac{24 P_e}{(\omega_M \gamma)(\omega_l \gamma)}} \approx \frac{0.375 (f_b/f_M)^3}{1 + \frac{0.6 P_e f_b^2}{f_M f_l}} \quad \text{where } \gamma = 1/f_b$$

By making a plot b/w the Rxed sig energy & SNR for  $N=8$  bits, the threshold for PCM occurs at a value of  $\frac{S_i}{N_{fM}} = 22.5$  db while for DM it occurs at 20.5 db.

$\therefore$  PCM yields superior performance in terms of SNR compared to DM, but the simplicity of hardware of DM system outweighs this superior performance in several practical applications.

