→ MSK (Minimum Shift Keying) :

$$v_{MSK}(t) = \sqrt{2P_s}\left[b_e(t)\sin\frac{2\pi t}{4T_b}\right]\cos\omega_0 t + \sqrt{2P_s}\left[b_o(t)\cos\frac{2\pi t}{4T_b}\right]\sin\omega_0 t$$

$$= \sqrt{2P_s}\left[\frac{b_o(t)+b_e(t)}{2}\right]\sin(\omega_0+\Omega)t + \sqrt{2P_s}\left[\frac{b_o(t)-b_e(t)}{2}\right]\sin(\omega_0-\Omega)t$$

$$= \sqrt{2P_s}\,C_H(t)\sin\omega_H t + \sqrt{2P_s}\,C_L(t)\sin\omega_L t$$

where we consider carrier at two different frequencies,

$\omega_H = \omega_0+\Omega$; $\Omega = \frac{2\pi}{4T_b}$; $C_H(t) = \frac{b_o(t)+b_e(t)}{2}$; $C_L(t) = \frac{b_o(t)-b_e(t)}{2}$

$\omega_L = \omega_0-\Omega$

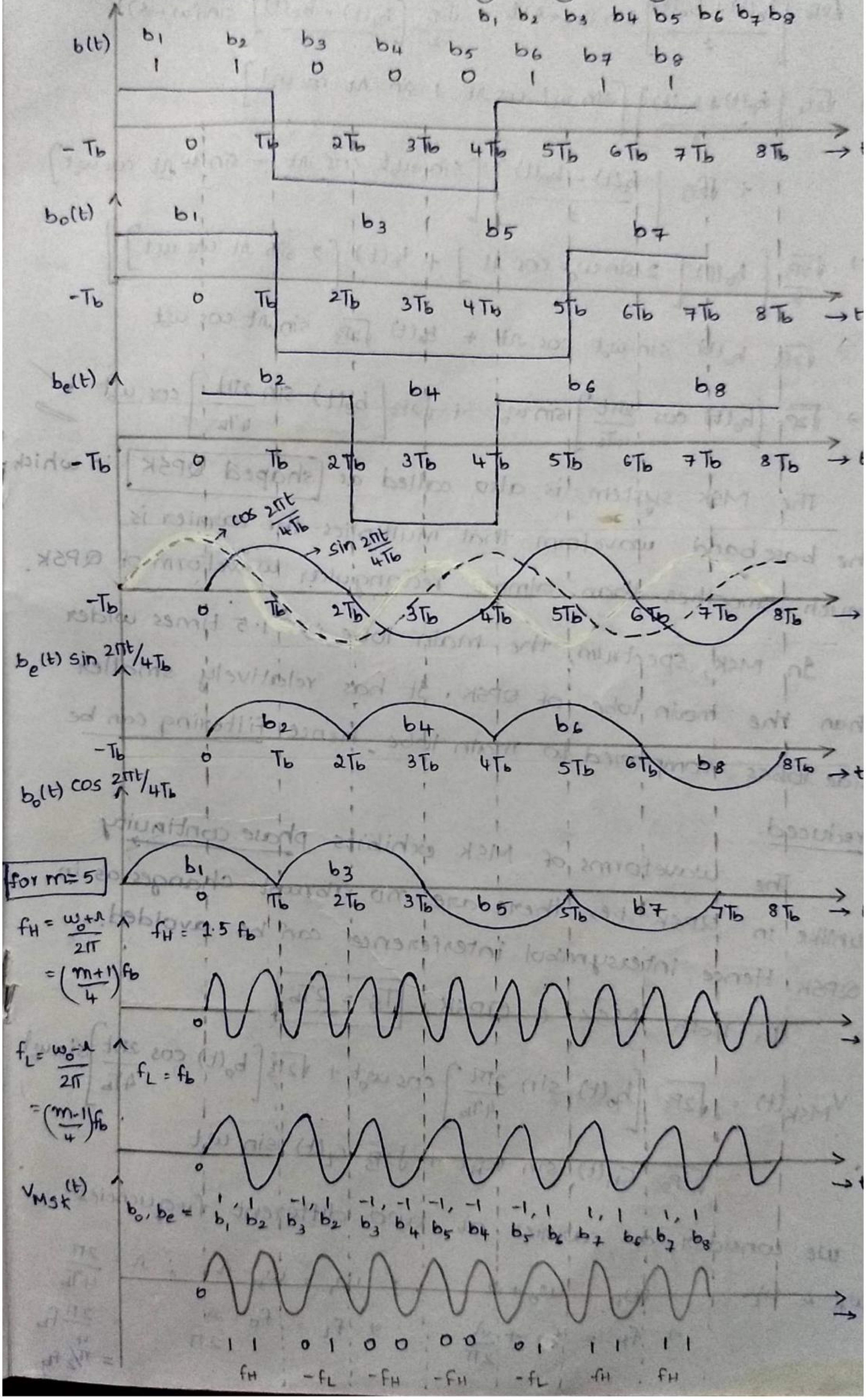
Text Book →

b_e	b_o	$v_{MSK}(t)/\sqrt{2P_s}$
-1	-1	$-\sin(\omega_0+\Omega)t$
-1	1	$\sin(\omega_0-\Omega)t$
1	-1	$-\sin(\omega_0-\Omega)t$
1	1	$\sin(\omega_0+\Omega)t$

own →

b_o	b_e	$v_{MSK}(t)/\sqrt{2P_s}$
0	0	$-\sin\omega_H t$
0	1	$-\sin\omega_L t$
1	0	$\sin\omega_L t$
1	1	$\sin\omega_H t$

Let the bit sequence, b(t) is 1 1 0 0 0 1 1 1
b_1 b_2 b_3 b_4 b_5 b_6 b_7 b_8



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$$\sqrt{2P_s}\left[\frac{b_e(t)+b_o(t)}{2}\right]\sin(\omega_o+\Omega)t + \frac{\sqrt{2P_s}}{2}\left[b_o(t)-b_e(t)\right]\sin(\omega_o-\Omega)t$$

$$\Rightarrow \sqrt{2P_s}\left[\frac{b_e(t)+b_o(t)}{2}\right]\left[\sin\omega_o t\cos\Omega t + \sin\Omega t\cos\omega_o t\right]$$

$$+ \sqrt{2P_s}\left[\frac{b_o(t)-b_e(t)}{2}\right]\left[\sin\omega_o t\cos\Omega t - \sin\Omega t\cos\omega_o t\right]$$

$$\Rightarrow \frac{\sqrt{2P_s}}{2}\left[b_o(t)\left[2\sin\omega_o t\cos\Omega t\right] + b_e(t)\left[2\sin\Omega t\cos\omega_o t\right]\right]$$

$$\Rightarrow \sqrt{2P_s}\, b_o(t)\sin\omega_o t\cos\Omega t + b_e(t)\sqrt{2P_s}\sin\Omega t\cos\omega_o t$$

$$\Rightarrow \sqrt{2P_s}\left[b_o(t)\cos\frac{2\pi t}{4T_b}\right]\sin\omega_o t + \sqrt{2P_s}\left[b_e(t)\sin\frac{2\pi t}{4T_b}\right]\cos\omega_o t$$

The MSK system is also called as $\boxed{\text{shaped QPSK}}$ in which the base band waveform that multiplies the carrier is much smoother than almost rectangular waveform of QPSK.

In MSK spectrum, the main lobe is 1.5 times wider than the main lobe of QPSK. It has relatively smaller side lobes compared to main lobe, hence filtering can be reduced.

The waveforms of MSK exhibits phase continuity unlike in QPSK i.e, there are no abrupt changes as in QPSK. Hence intersymbol interference can be avoided.

In both MSK & QPSK, $\boxed{T_s = 2T_b}$

$$V_{MSK}(t) = \sqrt{2P_s}\left[b_e(t)\sin\frac{2\pi t}{4T_b}\right]\cos\omega_o t + \sqrt{2P_s}\left[b_o(t)\cos\frac{2\pi t}{4T_b}\right]\sin\omega_o t$$

$$= \sqrt{2P_s}\, C_H(t)\sin\omega_H t + \sqrt{2P_s}\, C_L(t)\sin\omega_L t$$

we consider a carrier at two different frequencies ω_H & ω_L.

$$\omega_H = \omega_o + \Omega \quad ; \quad \omega_L = \omega_o - \Omega \quad ; \quad \Omega = \frac{2\pi}{4T_b}$$

$$\Rightarrow f_H = f_o + \frac{\Omega}{2\pi} \qquad \Rightarrow f_L = f_o - \frac{\Omega}{2\pi} \qquad = \frac{2\pi}{4}f_b$$

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$\Rightarrow f_H = f_0 + \frac{\pi/2\, f_b}{2\pi}$; $\boxed{f_L = f_0 - \frac{f_b}{4}} \rightarrow (3)$ $\Rightarrow \boxed{\Omega = \frac{\pi}{2} f_b} \longrightarrow (1)$

$\Rightarrow \boxed{f_H = f_0 + \frac{f_b}{4}} \rightarrow (2)$

In MSK, two frequencies f_H & f_L are chosen to insure that the two possible s/gs are orthogonal over the bit interval T_b.

$$\therefore \int_0^{T_b} \sin \omega_H t \sin \omega_L t \; dt = 0.$$

$$\Rightarrow \frac{1}{2}\int_0^{T_b} \left[\cos(\omega_H - \omega_L)t - \cos(\omega_H + \omega_L)t \right] dt = 0$$

$$\Rightarrow \left[\frac{\sin(\omega_H - \omega_L) T_b}{\omega_H - \omega_L} - \frac{\sin(\omega_H + \omega_L) T_b}{\omega_H + \omega_L} \right] = 0.$$

$(\omega_H - \omega_L) T_b = n\pi$; $(\omega_H + \omega_L) T_b = m\pi$

$\Rightarrow (f_H - f_L) = \frac{n\pi}{2\pi T_b}$ $\Rightarrow f_H + f_L = \frac{m\pi}{2\pi T_b}$

$\Rightarrow f_H - f_L = \frac{n}{2} f_b$ $\Rightarrow f_H + f_L = \frac{m}{2} f_b$

From eqs (2) & (3) From eqs (2) & (3)

$f_H - f_L = \frac{\cancel{2} f_b}{\cancel{4}_2} = \frac{n}{2} \cancel{f_b}$ $2f_0 = \frac{m}{2} f_b$

$\Rightarrow \boxed{n = 1}$ $\Rightarrow \boxed{f_0 = \frac{m}{4} f_b}$

Substituting these values of n & f_0 in eqs (2) & (3) we get

$f_H = f_0 + \frac{f_b}{4}$ lly $f_L = f_0 - \frac{f_b}{4}$

$= \frac{m}{4} f_b + \frac{f_b}{4}$ $= \frac{m}{4} f_b - \frac{f_b}{4}$

$\Rightarrow \boxed{f_H = \left(\frac{m+1}{4}\right) f_b}$ $\Rightarrow \boxed{f_L = \left(\frac{m-1}{4}\right) f_b}$

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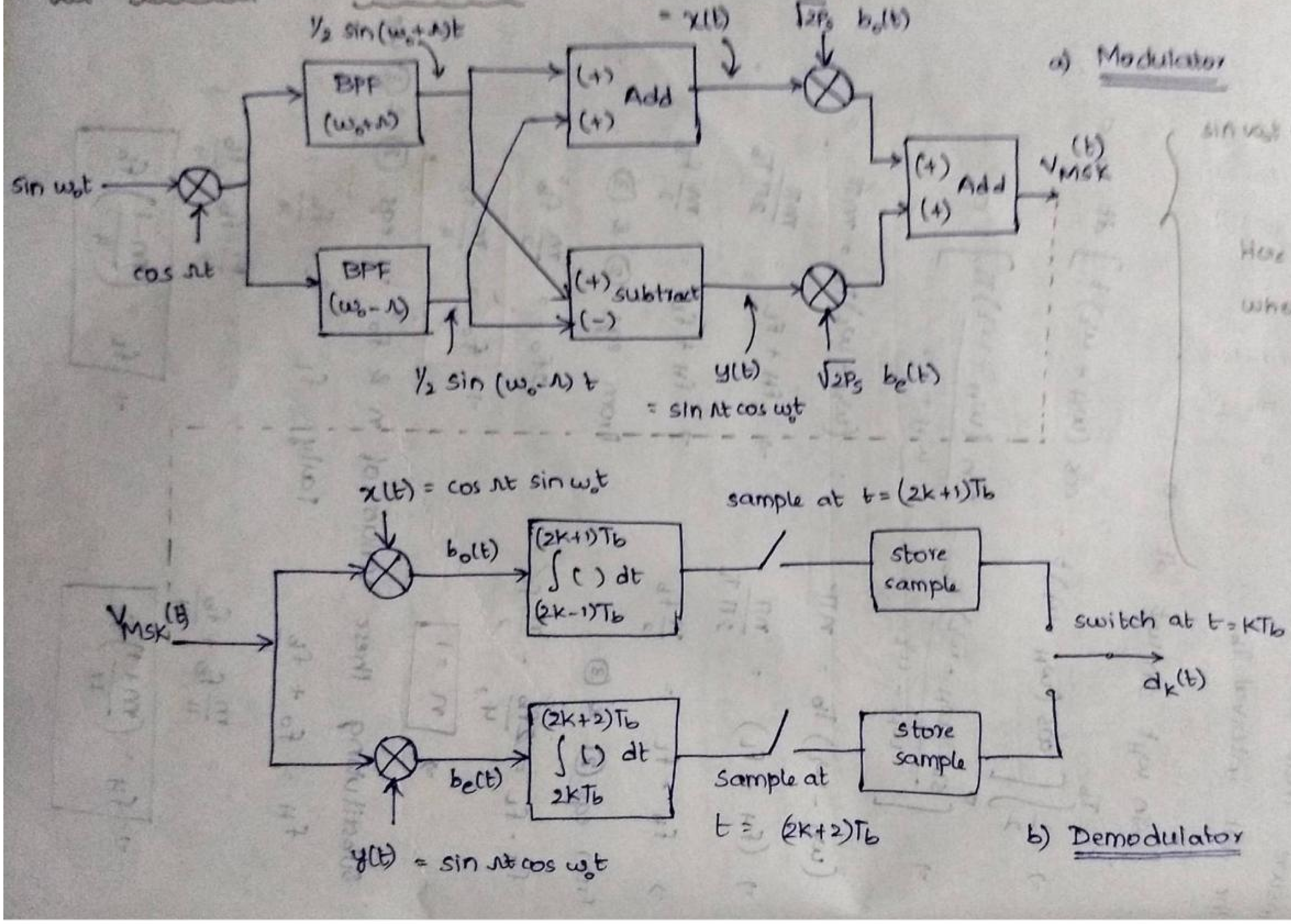
→ MSK modulator & demodulator:



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