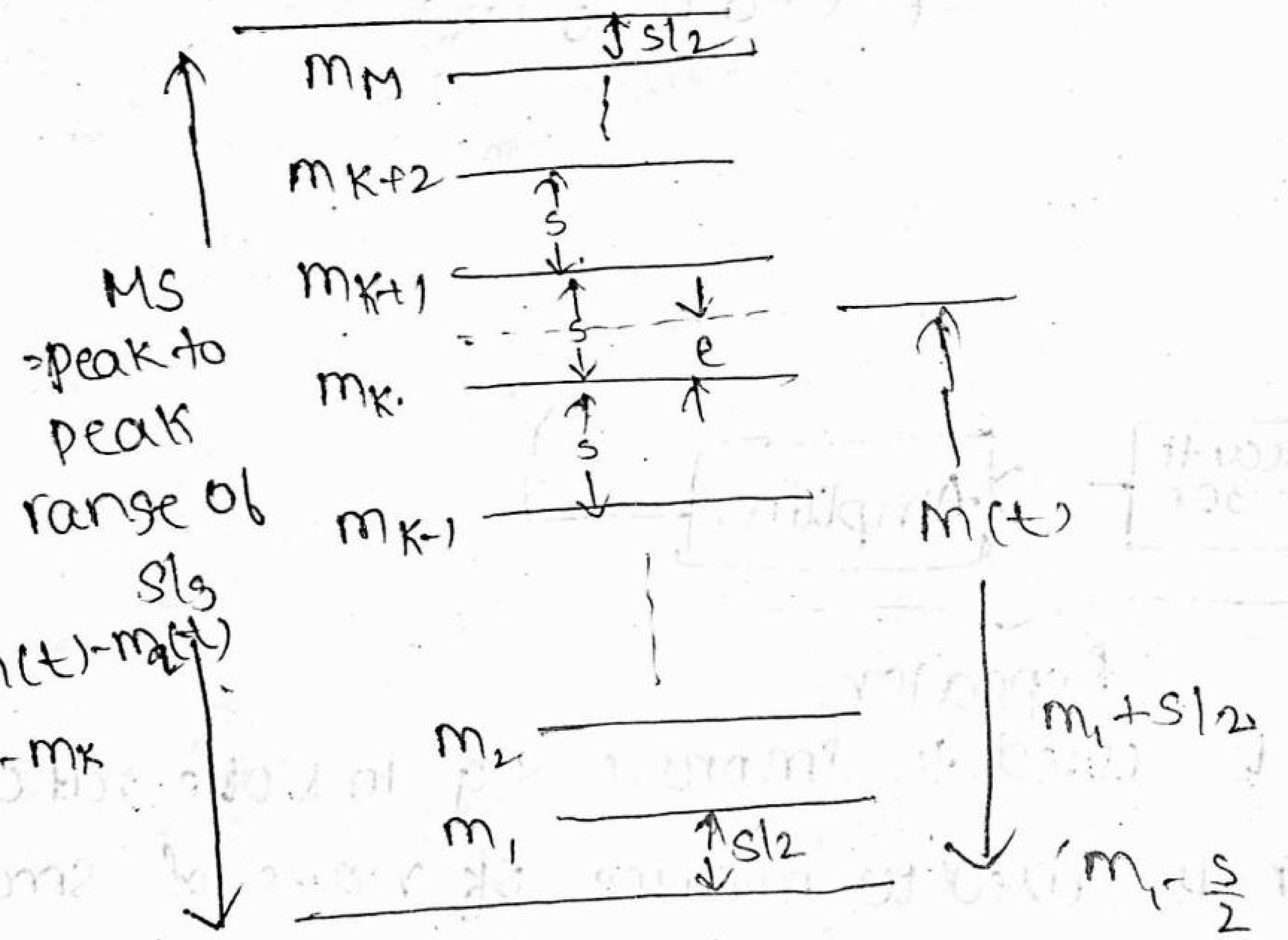
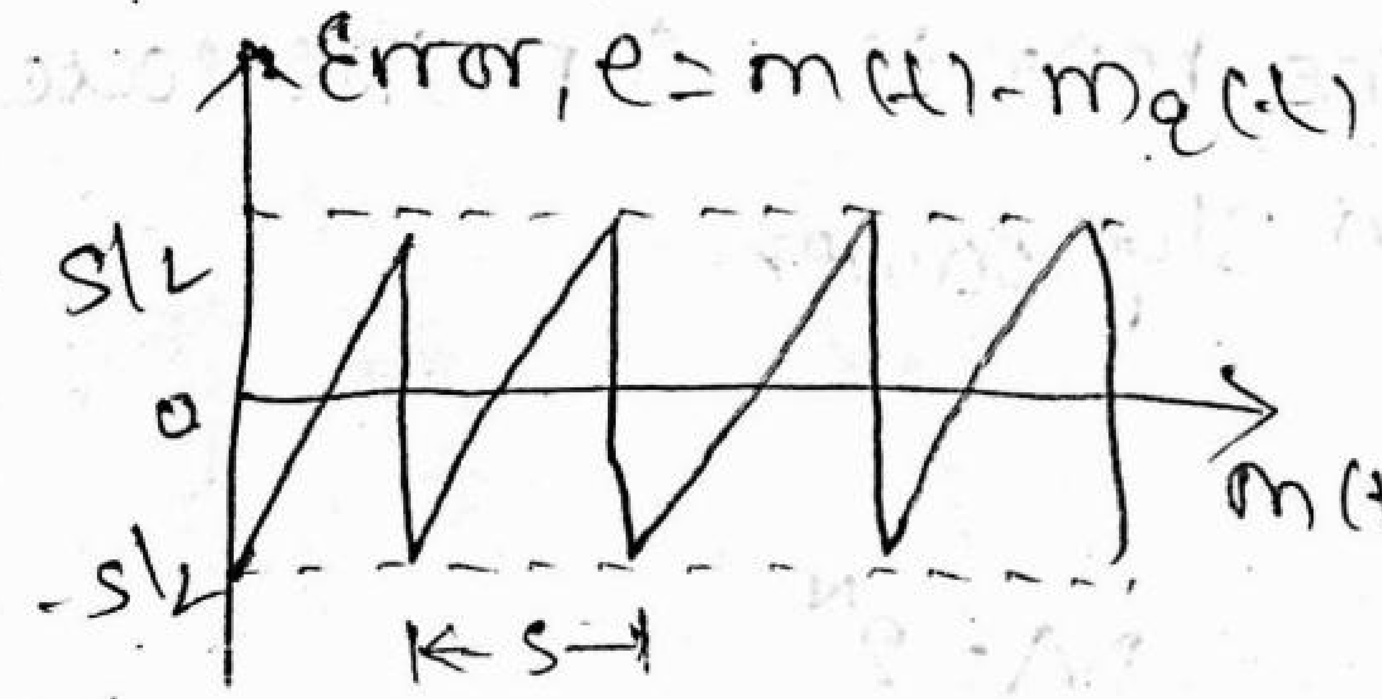
Note:
As long as the noise has an instaneous amplitude less than $S/2$ the noise will not appear at the output but if this noise exceeds $S/2$, an Error in level will occur

Quantization Error



$e(t) = m(t) - m_q(t)$
$= m(t) - m_K$



Step $S = \frac{V_H - V_L}{M}$

fig. Error voltage $e(t)$ as a fn of instaneous value of sig $m(t)$

error $m(t) = m(t) - m_q(t)$

I. at $m(t) = m_1 - S/2$

$m_q(t) = m_1$

$\Rightarrow e(t) = m_1 - S/2 - m_1 = -S/2$

II. at $m(t) = m_1 + S/2$

$m_q(t) = m_1$

$\Rightarrow e(t) = m_1 + S/2 - m_1 = S/2$

The difference b/w original signal and quantized signal may be viewed as noise due to quantisation process & is called Quantization Error. If 'e' is the difference between original and quantized sig voltages then '$\overline{e^2}$' is the mean square quantization Error

Divide peak to peak range of msg signal m(t) x 'M' no. of quantization levels each of magnitude 's' volts. So the quantized levels are placed at the center of each voltage interval as $m_1, m_2 \ldots m_K \ldots m_M$

From fig. at time 't' m(t) is the signal which is closest to the level m_K then quantizer o/p will be m_K and the error will be equal '$m(t)-m_K$'

If f(m) is the probability density function of m(t) then f(m)dm is the probability that m(t) lies in the range '$m-\frac{dm}{2}$' to '$m+\frac{dm}{2}$' then the mean square quantization Error is.

$$\overline{e^2} = \sum_{K=1}^{M} \int_{m_K-s/2}^{m_K+\frac{s}{2}} \overline{e^2(t)}\, f(m)\, dm, = \sum_{K=1}^{M} \int_{m_K-s/2}^{m_K+s/2} (m-m_K)^2 f(m)\, dm \text{ is}$$

$$= \int_{m_1-s/2}^{m_1+s/2} (m-m_1)^2 f(m)\, dm + \int_{m_2-s/2}^{m_2+s/2} (m-m_1)^2 f(m)\, dm + \ldots$$

In general f(m) is not constant but ~~it~~ will if we increase 'M' so that 's' is reduced in b/w levels ..

Therefore f(m) is constant. So, (On the first term,

On 1st term, $f(m) = f^{(1)}$ is constant

2nd term, $f(m) = f^{(2)}$ is constant

Let $x = m - m_K$

for $K=1 \Rightarrow x = m - m_1$

Upper limit, we have $m = m_1 + S/2 \Rightarrow x = m - m_1$
$= m_1 + \frac{S}{2} - m_1 = S/2$

lower limit we have $m = m_1 - S/2$

$\Rightarrow x = m - m_1$
$= m_1 - \frac{S}{2} - m_1 = -S/2$

and $dm = dx$

$$\therefore \bar{e}^2 = \int_{-S/2}^{S/2} x^2 f^{(1)} dx + \int_{-S/2}^{S/2} x^2 f^{(2)} dx + \dots$$

$$= \left[f^{(1)} + f^{(2)} + \dots\right] \int_{-S/2}^{S/2} x^2 dx$$

$$= \left[f^{(1)} + f^{(2)} \dots\right] \left[\frac{x^3}{3}\right]_{-S/2}^{S/2}$$

$$= \left[f^{(1)} + f^{(2)} + \dots\right] \frac{1}{3}\left[\frac{S^3}{8} - \left(\frac{-S^3}{8}\right)\right]$$

$$= \left[f^{(1)} + f^{(2)} + \dots\right] \frac{1}{3} \cdot \frac{2S^3}{8_4}$$

$$\Rightarrow \bar{e}^2 = \left[f^{(1)} + f^{(2)} + \dots\right] \frac{S^3}{12} = \left[f^{(1)}S + f^{(2)}S + \dots\right] \frac{S^2}{12}$$

$f^{(1)}S$ is the probability that sig $m(t)$ lies in 1^{st} level

$$\left(f^{(1)}S + f^{(2)}S + \dots\right) = 1$$

[$\because$ Total probability = 1 of sig

$$\Rightarrow \boxed{\bar{e}^2 = \frac{S^2}{12}}$$

Mean Square Quantization Error