→ Noise Bandwidth :

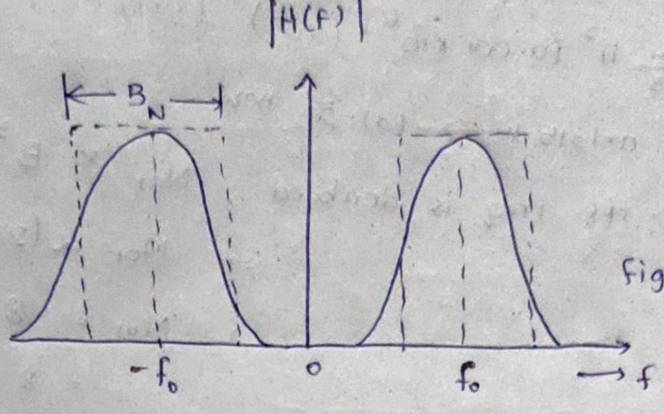

fig ①

Let the white noise is present at the i/p to a Rxer and a filter with transfer fn H(f) centered at f_0 as shown in fig① by a solid curve is being used to restrict the noise power actually passed on to the Rxer.

Consider a contemplated rectangular filter whose bandwidth is B_N is also centered at f_0 indicated by dotted plot in fig①. Let the rectangular filter bandwidth B_N be adjusted so that the real filter & rectangular filter transmit same noise power. Then the bandwidth B_N is called noise bandwidth of the real filter

When white noise with power spectral density $\eta/2$ is applied as i/p, then o/p noise power

$$N_o(\text{RC low pass filter}) = \frac{\pi}{2}\eta f_c$$

$$N_o(\text{Rectangular contemplated filter}) = \eta B_N$$

setting N_o (RC low pass filter) = N_o (rectangular contemplated filter)

we find Noise Bandwidth to be $\frac{\pi}{2}\eta f_c = \eta B_N$

$$\Rightarrow \boxed{B_N = \frac{\pi}{2} f_c}$$

$\Rightarrow \pi/_2\, \eta f_c = \eta B_N$

$\Rightarrow \boxed{\eta f_c = B_N}$

$\therefore$ Noise Bandwidth $B_N = \eta f_c$

Problem :-

→ Calculate the noise bandwidth for a Gaussian filter whose transfer function is

$$|H(\omega)|^2 = e^{-\omega^2}$$

Ans) $N_o = \int_{-\infty}^{\infty} G_{no}(f)\, df = \int_{-\infty}^{\infty} G_{ni}(f)\, |H(f)|^2\, df$

$$= \int_{-\infty}^{\infty} e^{-\omega^2}\, \eta/_2\, df. \qquad \{\because \omega = 2\pi f\}$$

$$= \int_{-\infty}^{\infty} e^{-4\pi^2 f^2}\, \eta/_2\, df$$

let $x = 2\pi f$

$dx = 2\pi\, df$

$$= \frac{\eta}{2} \int_{-\infty}^{\infty} e^{-x^2} \frac{dx}{2\pi}$$

$$\boxed{\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}}$$

$$= \frac{\eta}{2} \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-x^2} dx$$

$$\Rightarrow N_o = \frac{\eta}{2} \frac{1}{2\pi} \sqrt{\pi} = \frac{\eta}{4\pi} \sqrt{\pi}$$

$$= \frac{\eta}{4\sqrt{\pi}}$$

$\therefore$ Noise Bandwidth, $B_N = ?$

N_o (contemplated rectangular filter) = N_o (Gaussian filter with transfer function $|H(\omega)|^2 = e^{-\omega^2}$)

$$\Rightarrow \eta B_N = \frac{\eta}{4\sqrt{\pi}}$$

$$\Rightarrow \boxed{B_N = \frac{1}{4\sqrt{\pi}}}$$

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Problem :-

A low pass s/g within <u>4 KHz</u> is of strength 0.001 watts is passed through a distorting channel with transfer fn $H(f) = \frac{4000}{f+4000j}$ & is corrected with additive white noise whose magnitude is $\eta/2 = 10^{-8}$ watts/Hz. At receiver end there is an equalizer which exactly matches the channel in the frequency of interest & zero elsewhere. Find the S/N ratio at <u>o/p equalizer.</u>

Ans) <u>Given</u> :- Distorting channel Transfer fn, $H(f) = \frac{4000}{f+4000j}$

$\therefore$ Transfer fn of equalizer is $H(f) = \frac{f+4000j}{4000}$

$\eta/2 = 10^{-8}$ watts/Hz ; $S_0 = 0.001$ watts

$$\therefore G_{no}(f) = |H(f)|^2 \left(\eta/2\right) \rightarrow G_{ni}(f) \text{ for white noise}$$

$$G_{no}(f) = |H(f)|^2 G_{ni}(f) \quad \text{where } G_{ni}(f) = \eta/2 \text{ for white noise}$$

$$= \left[\frac{f+4000j}{4000}\right]\left[\frac{f-4000j}{4000}\right] \eta/2$$

$$= \frac{f^2+4000^2}{16\times10^6} \times 10^{-8} \qquad \left\{\because \text{Given } \eta/2 = 10^{-8}\right\}$$

$$\therefore N_o = \int_{-\infty}^{\infty} G_{no}(f)\, df = \int_{-4000}^{4000} \frac{f^2+4000^2}{2\times16\times10^6} \times 10^{-8}\, df$$

$$= \frac{1}{16\times10^6} \times 10^{-8} \int_{-4000}^{4000} \left(f^2+4000^2\right) df$$

$$= \frac{10^{-8}}{16\times10^6} \left[\frac{f^3}{3} + 4000^2 (f)\right]_{-4000}^{4000}$$

$$= \frac{10^{-8}}{16\times10^6} \left[\frac{2(4000)^3}{3} + (4000)^2 (4000+4000)\right.$$

$\Rightarrow N_o = \frac{10^{-8}}{16 \times 10^{6}} \, 4000^{2} \left[\frac{2(4000)}{3} + 8000 \right]$

$\Rightarrow N_o = \frac{10^{-8} \times \cancel{16 \times 10^{6}}}{\cancel{16 \times 10^{6}}} \left[\frac{8000}{3} + 8000 \right]$

$\Rightarrow N_o = 10^{-8} \times 10666.67$ watts

$= 10666.67 \times 10^{-8}$ watts

$\therefore$ S/N ratio $= \frac{S_o}{N_o} = \frac{0.001}{10^{-8} \times 10666.67} = \frac{1}{10^{-8}} \times 9.375 \times 10^{-8}$

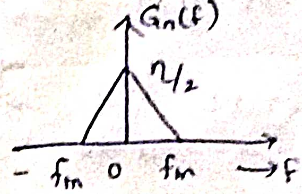
$= 9.375$

$\therefore$ S/N ratio in dB $= 10 \log \frac{S_o}{N_o}$

$= 10 \log 9.375 = 9.7197$ dB

→ Problem :-

The noise $n(t)$ has spectral density as shown in fig. write an expression for noise power spectral density $G_n(f)$



Ans] for $0 < f < f_m$:- $\underset{x_1 \; y_1}{(f_m, 0)}$ $\underset{x_2 \; y_2}{(0, \eta/2)}$

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$

$\Rightarrow G_n(f) - 0 = \frac{\eta/2 - 0}{0 - f_m} (f - f_m)$

$\Rightarrow G_n(f) = \frac{-\eta}{2f_m} (f - f_m)$ for $0 < f < f_m$

for $-f_m < f < 0$:- $\underset{x_1 \; y_1}{(-f_m, 0)}$ $\underset{x_2 \; y_2}{(0, \eta/2)}$

$\Rightarrow G_n(f) - 0 = \frac{\eta/2 - 0}{0 - (-f_m)} (f - (-f_m))$

$\Rightarrow G_n(f) = \frac{\eta}{2f_m} (f + f_m)$ for $-f_m < f < 0$

$\therefore G_n(f) = \begin{cases} \frac{-\eta}{2f_m}(f - f_m) & \text{for } 0 < f < f_m \\ \eta/2f_m \, (f + f_m) & \text{for } -f_m < f < 0 \end{cases}$

→ Quadrature Component of noise [Narrow band representation of noise

Let us consider noise $n(t)$ as the superposition of spectral components of noise which is represented as

$$n(t) = \lim_{\Delta f \to 0} \sum_{k=1}^{\infty} (a_k \cos 2\pi k\Delta f t + b_k \sin 2\pi k \Delta f t) \longrightarrow (1)$$

where a_k & b_k are gaussian random variables with zero mean, equal variance & are uncorrelated to each other.

Select the value of $k = K$ an arbitrary frequency $f_0 = K\Delta f$ has been introduced into the argument of eq (1)

Therefore by merging $f_0 = K\Delta f$

$$\Rightarrow 2\pi f_0 = 2\pi K \Delta f$$

$$\Rightarrow \boxed{2\pi f_0 - 2\pi K\Delta f = 0}$$ to the argument of eq. (1), we have

$$n(t) = \lim_{\Delta f \to 0} \sum_{k=1}^{\infty} \left(a_k \cos 2\pi (K + k - K) \Delta f t + b_k \sin 2\pi (K + k - K) \Delta f t \right)$$

$$= \lim_{\Delta f \to 0} \sum_{k=1}^{\infty} \left[a_k \cos 2\pi \left(f_0 + (k-K)\Delta f \right) t + b_k \sin 2\pi \left(f_0 + (k - K)\Delta f \right) t \right]$$

$$= \lim_{\Delta f \to 0} \sum_{k=1}^{\infty} \left[a_k \left(\cos 2\pi (f_0 t) \cos 2\pi (k - K)\Delta f t - \sin 2\pi f_0 t \sin 2\pi (k-K)\Delta f t \right) + b_k \left(\sin 2\pi f_0 t \cos 2\pi (k-K)\Delta f t + \cos 2\pi f_0 t \sin 2\pi (k-K)\Delta f t \right) \right]$$

$$= \lim_{\Delta f \to 0} \sum_{k=1}^{\infty} \left\{ \cos 2\pi f_0 t \left(a_k \cos 2\pi (k-K)\Delta f t + b_k \sin 2\pi (k-K \right. \right.$$
$$\left. - \sin 2\pi f_0 t \left(a_k \sin 2\pi (k-K)\Delta f t - b_k \cos 2\pi (k-K \right. \right.$$

$$\Rightarrow n(t) = \lim_{\Delta f \to 0} \sum_{k=1}^{\infty} \left\{ \begin{array}{l} \cos 2\pi f_0 t \left(a_k \cos 2\pi (k-K) \Delta f t + b_k \sin 2\pi (k-K) \Delta f t \right) \\ - \sin 2\pi f_0 t \left(a_k \sin 2\pi (k-K) \Delta f t - b_k \cos 2\pi (k-K) \Delta f t \right) \end{array} \right\}$$

$$\Rightarrow \boxed{n(t) = n_c(t) \cos 2\pi f_0 t - n_s(t) \sin 2\pi f_0 t} \rightarrow$$ Narrow band representation of noise

where

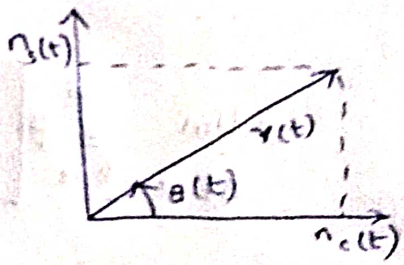
$$n_c(t) = \lim_{\Delta f \to 0} \sum_{k=1}^{\infty} \left[a_k \cos 2\pi (k-K) \Delta f t + b_k \sin 2\pi (k-K) \Delta f t \right]$$

$$\& \; n_s(t) = \lim_{\Delta f \to 0} \sum_{k=1}^{\infty} \left[a_k \sin 2\pi (k-K) \Delta f t - b_k \cos 2\pi (k-K) \Delta f t \right]$$

Here $n_c(t)$ and $n_s(t)$ are called quadrature components of noise because of appearance of sinusoidals in quadrature in the above eqs. Here $n_c(t)$ & $n_s(t)$ are also gaussian random processes where the coefficients a_k, b_k are gaussian random variables with zero mean, equal variance and uncorrelated to each other.

The noise spectral components of $n(t)$ at freq $k\Delta f$ gives rise to two quadrature components $n_c(t)$ & $n_s(t)$ of frequency $(k-K)\Delta f \Rightarrow k\Delta f - K\Delta f$

$$\Rightarrow f - f_0.$$



$r(t)$ is the resultant phasor of amplitude

$$r(t) = \sqrt{n_c^2(t) + n_s^2(t)}$$

which makes an angle

$$\theta(t) = \tan^{-1}\left(\frac{n_s(t)}{n_c(t)}\right)$$

fig :- phasor diagram of the quadrature representation of noise.

Note :- Narrow band representation of noise is frequently used with great convenience in dealing of noise confined to a relatively narrow frequency band in the neighbourhood of f_0.