

## Delta Modulation Systems

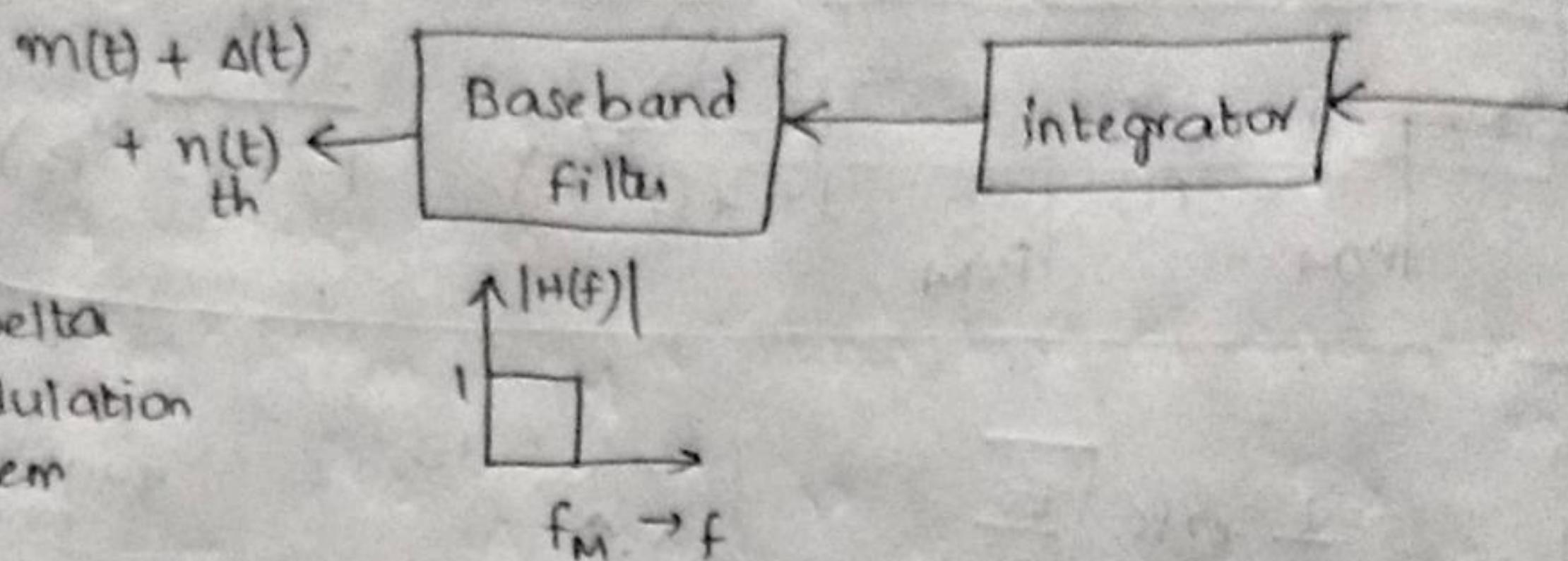
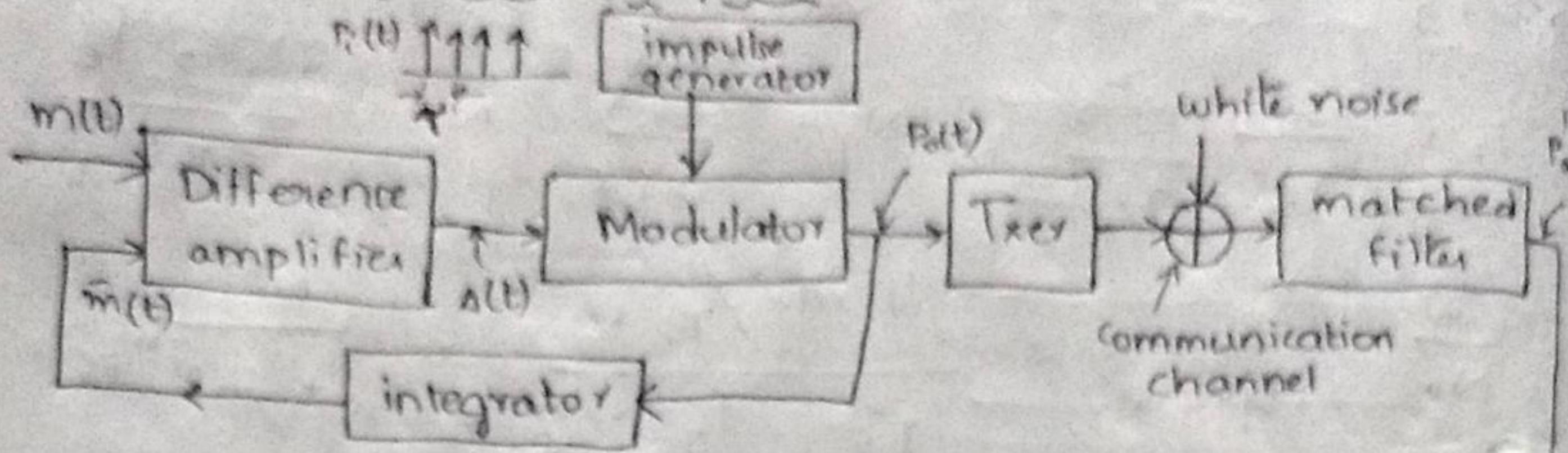
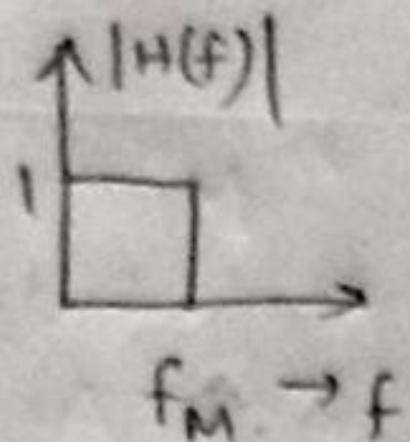


fig: A Delta  
① modulation  
system



$$\Delta(t) = m(t) - \bar{m}(t); \Delta(t) \text{ is } +\text{ve if } m(t) > \bar{m}(t)$$

$$\Delta(t) \text{ is } -\text{ve if } m(t) < \bar{m}(t)$$

where  $\bar{m}(t) = \frac{s}{I} \int P_o(t) dt$  ie, o/p of integrator, it has been adjusted so that its response to i/p impulse of strength I is a step of size 's'.

$P_o(t)$  is o/p of modulator whose polarity depends on  $\Delta(t)$ .

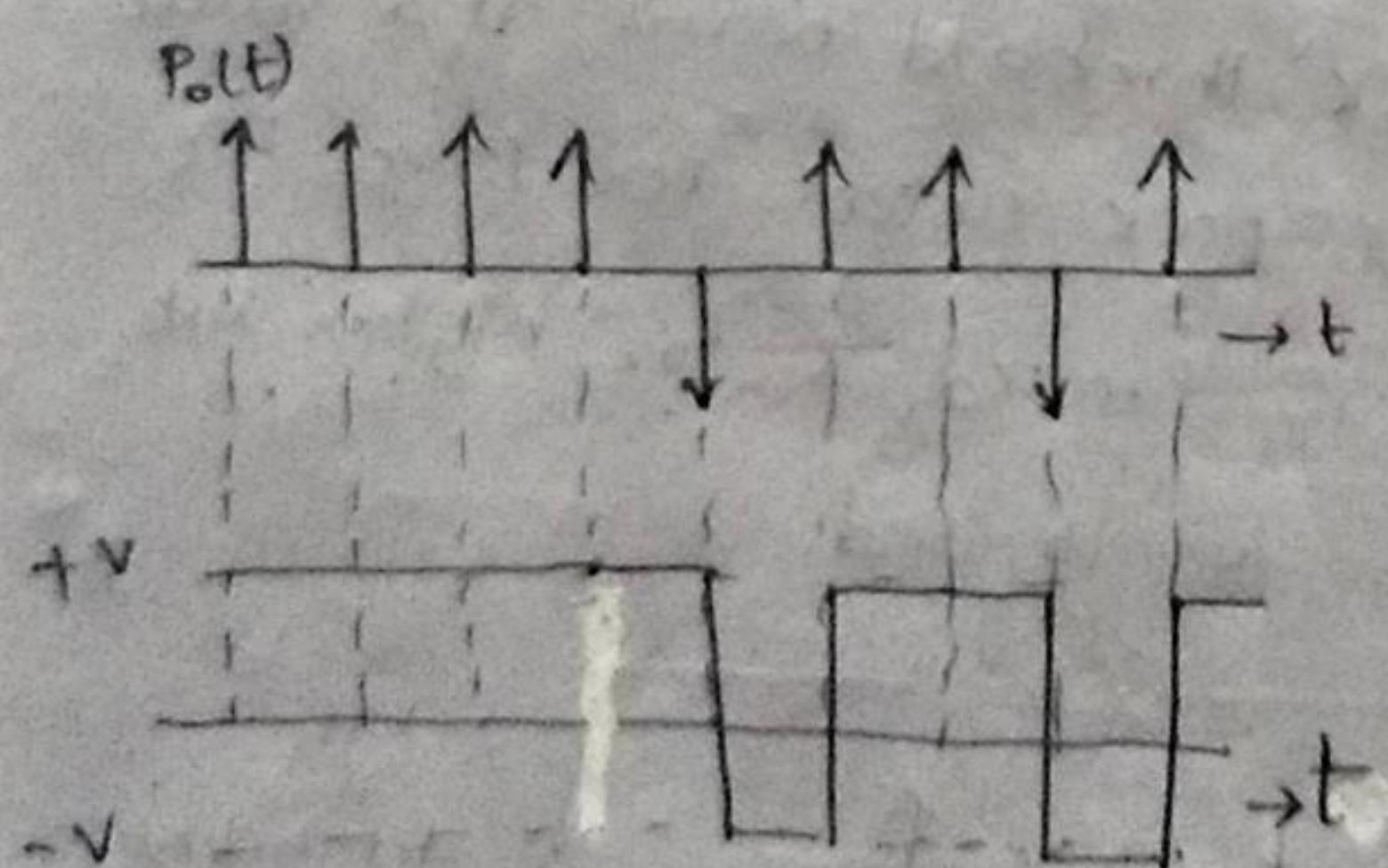


fig a) impulse train  $P_o(t)$   
appearing at the o/p  
of modulator.

fig b) 2-level sig Txed over  
the communication channel

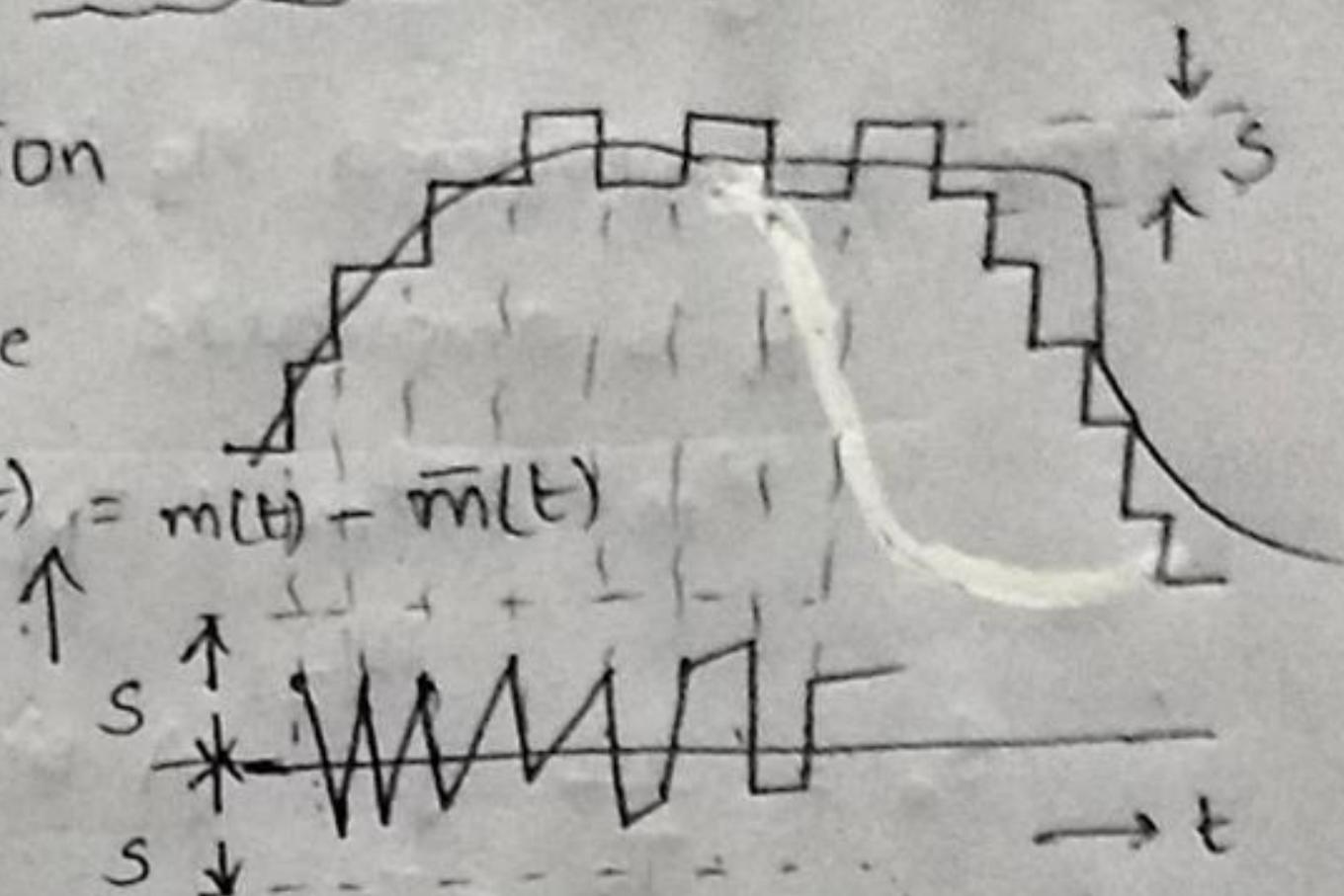
$P_o(t)$  is impulse train, but before transmission, it is converted to 2-level waveform, since the latter waveform has greater power than a train of narrow pulses. This conversion

is accomplished by a block in Txer. After detection by the matched filter, this binary waveform will be converted to a sequence of impulses  $P_0(t)$ .

In the absence of thermal noise  $P_o'(t) = P_o(t)$  & if the integrators used at both Txer & Rxer are identical,  $m(t)$  is recovered at the Rxer.

Quantization noise in Delta Modulation (N<sub>q</sub>) :-

$\Delta(t)$  is source of quantization noise in DM. Minimize the slope overloading such that error  $\Delta(t) < s'$



If  $\Delta(t)$  is a sig making excursions between  $+s$  &  $-s$  where ' $s$ ' is a uniform stepsize, Then pdf of  $\Delta(t)$  is

$$f(\Delta) = \frac{1}{2s} \quad -s \leq \Delta \leq s$$

The normalized power of waveform of  $\Delta(t)$  is

$$\overline{[\Delta(t)]^2} = \int_{-S}^S \Delta^2 f(\Delta) d\Delta = \frac{\left[ \frac{\Delta^3}{3} \right]_S}{2S} \Rightarrow \frac{1}{6S} [2S]^3 = \frac{2S^3}{6}$$

$$\int_{-s}^s \Delta^2 \frac{1}{2s} d\Delta = \frac{\frac{1}{2}(-s)^3 - \frac{1}{2}s^3}{2s} = \frac{-\frac{1}{2}s^3 - \frac{1}{2}s^3}{2s} = \frac{-s^3}{2s} = -\frac{s^2}{2}$$

The spectrum of  $\delta(t)$  extends  $-s$

$$\Rightarrow \overline{(\Delta(t))^2} = \frac{S^2}{3}$$

continuously over a frequency range which begins near  
a suitable cut-off fre-

continuously over a range of frequencies. Consider a LPF with adjustable cut-off freq zero. consider a

$$f_C = \frac{1}{\gamma} = f_b$$

$\{ f_b \text{ is bit rate} \}$ , so that  $\Delta(t)$   
 $\{ \tau \text{ is step duration} \}$

$t_c = \frac{1}{\gamma} = t_b$  ]  $\gamma$  is step duration.  
 passed through this filter with min amount of distortion.  
 and extends from 0 to  $f_{br}$ .

passes through the origin. The spectrum of  $\alpha(t)$  extends from 0 to  $f_{bg}$ .

so that spectrum of  $B_{av}$   
but the cut off freq of final base band filter is  $f_M$

& the amount of noise power passing through the filter  
is quantization noise

$$N_q = \frac{f_M s^2}{3 f_b}$$

Op s/q power ( $S_o$ ) :-

let the s/q be  $m(t) = A \sin \omega_M t$

where we considered s/q is sinusoidal

$A$  is amplitude

$\omega_M = 2\pi f_M$ ,  $f_M$  is upper limit of base band freq range

$$\begin{aligned} S_o &= \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} (A \sin \omega_M t)^2 dt = \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} m^2(t) dt \\ &= \frac{A^2}{T_s} \int_{-T_s/2}^{T_s/2} \left[ \frac{1 - \cos 2\omega_M t}{2} \right] dt \\ &= \frac{A^2}{2T_s} \left[ t - \frac{\sin 2\omega_M t}{2\omega_M} \right]_{-T_s/2}^{T_s/2} \\ \Rightarrow S_o &= \frac{A^2}{2T_s} [T_s - 0] = \frac{A^2}{2} \end{aligned}$$

$$\therefore S_o = \frac{A^2}{2}$$

To avoid slope overloading,

$$\begin{aligned} A &= \frac{s}{\gamma \omega_M} = \frac{sf_b}{\omega_M} \\ \therefore S_o &= \frac{A^2}{2} = \frac{s^2 f_b^2}{\omega_M^2 2} \end{aligned}$$

$$\begin{cases} \text{max slope} = 2\pi f_A \\ \text{of } m(t) \\ \Rightarrow \frac{s}{\gamma} = 2\pi f_A \\ \Rightarrow \frac{s}{\gamma} = \omega A \\ \& \gamma = \frac{1}{f_b} \end{cases}$$

$$\therefore \frac{S_o}{N_q} = \frac{s^2 f_b^2 / 2\omega_M^2}{s^2 f_M / 3f_b} = \frac{3f_b^3}{f_M (2\omega_M^2)} = \frac{3f_b^3}{f_M (2 \cdot 4\pi^2 f_M)}$$

$$\Rightarrow \frac{S_0}{Nq} = \frac{\frac{3f_b^3}{8\pi^2 f_m^3}}{= \frac{3}{8\pi^2 f_m^3}}$$

$$\boxed{\therefore \frac{S_0}{Nq} = \frac{3}{80} \left( \frac{f_b}{f_m} \right)^3 = \frac{3}{80} \left( \frac{f_b}{f_m} \right)^3}$$

$\left\{ \begin{array}{l} \pi^2 = 9.86 \\ \approx 10 \end{array} \right.$

let the sampling time is  $T_s = \frac{1}{2f_m}$

word length is  $N$

$$\gamma_{max} (\text{max bit duration without guard time}) = \frac{T_s}{N}$$

$$\Rightarrow \gamma_{max} = \frac{T_s}{N} = \frac{1}{2f_m N} \quad \left\{ \because T_s = \frac{1}{2f_m} \right\}$$

$$\therefore \frac{S_0}{Nq} = \frac{3}{80} \left( \frac{f_b}{f_m} \right)^3 = \frac{3}{80} \frac{1}{(\gamma f_m)^3} = \frac{3}{80} \frac{(2f_m N)^3}{f_m^3}$$

$$= \frac{3}{80} \times 8N^3$$

$$\Rightarrow \boxed{\frac{S_0}{Nq} = 0.3 N^3} \rightarrow \text{in DM}$$

for  $N = 8$

$$\frac{S_0}{Nq} = 0.3 (8)^3 \Rightarrow$$

$$= 22 \text{ db}$$

for  $N = 8 ; \frac{S_0}{Nq} = 22 \text{ db in DM}$

$$; \frac{S_0}{Nq} = 48 \text{ db in PCM}$$

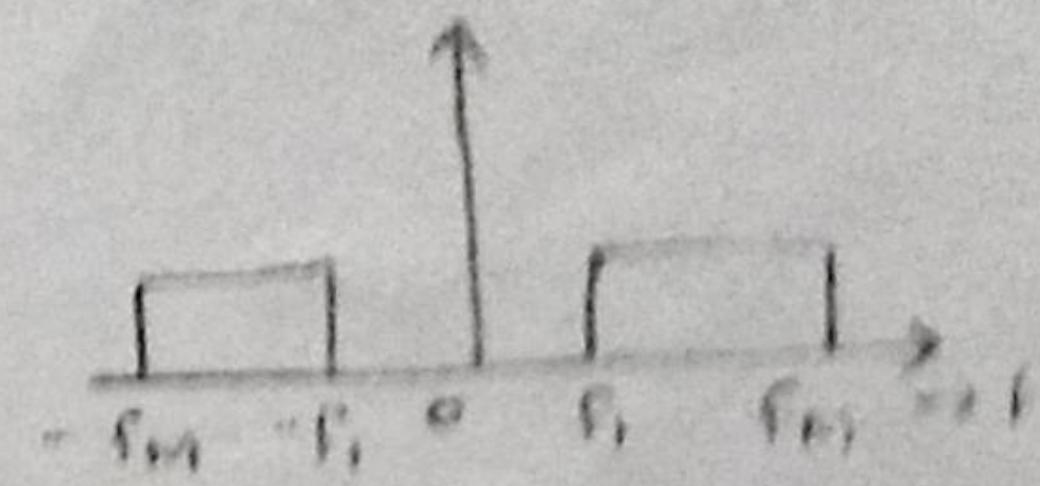
$\left\{ \frac{S_0}{Nq} = 2^{2N} \right.$

$$\boxed{Nq}$$

for  $N = 8 \Rightarrow S/N_q = 22 \text{ db} \rightarrow \text{in DM} \quad \left\{ \frac{S_0}{N_q} = 10^{11.14/20} \right.$   
 $S/N_q = 48 \text{ db} \rightarrow \text{in PCM} \quad \left\{ \frac{S_0}{N_q} = 10^{2.2/20} \right.$

Thermal noise ~ By considering the effect of thermal noise in a DM system

$$N_{th} = \frac{2S^2 P_e}{\pi^2 f_i \gamma} \quad \text{where } \gamma \rightarrow \text{bit duration.}$$



$f_l$  is the lower cut off freq of base band filer.

$$\therefore \text{SNR} = \frac{S_0}{N_0} = \frac{S_0}{N_q + N_{th}}$$

$$= \frac{\frac{S^2}{2\pi^2 w_M^2}}{\frac{f_m s^2}{3f_b} + \frac{2S^2 P_e}{\pi^2 f_i \gamma}} = \frac{\frac{1}{2\pi^2 w_M^2}}{\frac{f_m \gamma}{3} + \frac{2P_e}{\pi^2 f_i \gamma}} = \frac{\frac{1}{2\pi^2 w_M^2}}{\frac{f_m \gamma}{3} \left( 1 + \frac{6P_e}{f_m^2 \pi^2 f_i} \right)}$$

$$= \frac{\frac{2\pi}{2\pi^2 w_M^2}}{\frac{2\pi f_m \gamma}{3} \left( 1 + \frac{6P_e \times 2 \times 2}{(2\pi f_m)(2\pi f_i) \gamma^2} \right)} = \frac{\frac{3\pi}{w_M^3 \gamma^3}}{\left( 1 + \frac{24P_e}{(w_M \gamma)(w_i \gamma)} \right)}$$

$\therefore \text{SNR} = \frac{\frac{3\pi}{w_M^3 \gamma^3}}{1 + \frac{24P_e}{(w_M \gamma)(w_i \gamma)}} \approx \frac{0.375 \left( f_b/f_m \right)^3}{1 + \frac{0.6P_e f_b^2}{f_m f_i}}$	when $\gamma = \frac{f_b}{f_m}$ .
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By making a plot b/w the Rxed sig energy & SNR for N=8 bits, the threshold for PCM occurs at a value of  $\frac{S_i}{N_f M} = 22.5 \text{ db}$  while for DM it occurs at  $20.5 \text{ db}$ .

$\therefore$  PCM yields superior performance in terms of SNR compared to DM, but the simplicity of hardware of DM system outdoes this superior performance in several practical applications.

