Linear Wave Shaping:

RC HIGH-PASS SUMMARY

- Linear systems are those that obey homogeneity and additivity.
- Homogeneity and additivity taken together comprise the principle of superposition.
- The process by which the shape of a non-sinusoidal signal, when passed through a linear network, is altered is called linear waveshaping.
- Only a sinusoidal signal can preserve its form when transmitted through a linear high-pass network. All other signals undergo distortion in shape.
- A high-pass circuit attenuates all low-frequency signals and transmits only signals of high frequency.
- If $\boldsymbol{C}$ of the high-pass $\boldsymbol{RC}$ circuit is replaced by $\boldsymbol{L}$ and if $\boldsymbol{L}/\boldsymbol{R} = \boldsymbol{RC}$, then all the results of the $\boldsymbol{RC}$ circuits are valid for the $\boldsymbol{RL}$ circuits as well.
- A step voltage is one, which has a value zero for all times $\boldsymbol{t} < 0$, and remains at $\boldsymbol{V}$ for all times $\boldsymbol{t} \geq 0$.
- Fall time is defined as the time taken by the output waveform to fall from 90 per cent to 10 per cent of its initial value.
- The fall time is directly proportional to the time constant and inversely proportional to the lower cut-off frequency.
- The voltage across a capacitor cannot change instantaneously if the current through it remains finite.
- A waveform which is zero for $\boldsymbol{t} < 0$, and which increases linearly with time for $\boldsymbol{t} > 0$ is called a ramp.
- The transmission error, $\boldsymbol{e_t}$ for a ramp input is defined as the difference between the input and the output divided by the input.
- For most applications, the steady-state condition is reached approximately at $\boldsymbol{t} = 5\boldsymbol{\tau}$.
- A pulse may be treated as the sum of a positive step followed by a delayed negative step of the same amplitude.
- A periodic waveform which maintains itself at one constant level $\boldsymbol{V}'$ for a time $\boldsymbol{T}_1$, and then changes abruptly to another level $\boldsymbol{V}''$ and remains constant at that level for a time $\boldsymbol{T}_2$, and repeats itself with a period $\boldsymbol{T} = \boldsymbol{T}_1 + \boldsymbol{T}_2$ is called a square wave. If $\boldsymbol{T}_1 = \boldsymbol{T}_2 = \boldsymbol{T}/2$, it is called a symmetric square wave.
- The lower cut-off frequency of a high-pass circuit is given by $\boldsymbol{f}_1 = 1/2\boldsymbol{\pi RC}$.
- The capacitor in the high-pass circuit blocks the dc component of the input. Hence, no dc component is present in the output.
- The process of converting pulses into spikes by means of a circuit of a very short time constant is called peaking.
- The output of a high-pass circuit excited by a square-wave input exhibits a tilt when the time constant of the circuit is not very high.
- Ahigh-pass circuit acts as a differentiator if the time constant of the circuit is very small as compared to the time period of the input signal.
- When a high-pass circuit is used as an ideal differentiator, the phase shift between the input and the output is 90°.

- It is more convenient to introduce initial conditions in an integrator than in a differentiator.
- For double differentiation, two high-pass networks with small time constants are connected in a cascade.
- The current through an inductor cannot change instantaneously when a finite voltage is applied across it.

MULTIPLE CHOICE QUESTIONS

1. The process by which the shape of a non-sinusoidal signal when transmitted through a linear network is altered is called:
    1. Non-linear waveshaping
    2. Peaking
    3. Linear waveshaping
    4. Clamping

1. The lower half-power frequency of a high-pass circuit is:
    1. $\frac{1}{2\pi LC}$
    2. $\frac{1}{2\pi RC}$
    3. $\frac{1}{2\pi\sqrt{LC}}$
    4. $\frac{1}{2\pi\sqrt{RC}}$

1. The fall time of a high-pass circuit is:
    1. $\frac{0.35}{f_1}$
    2. $\frac{1}{2\pi LC}$
    3. $22\boldsymbol{\tau}$
    4. $\frac{35}{f_1}$

1. A dc component is associated with a periodic input waveform applied to a high-pass circuit. The dc component in the output is:
    1. ∞
    2. 0
    3. $2\boldsymbol{v_i}$
    4. Finite value

1. If the shape of the square wave is to be preserved in the output of a high-pass circuit, then the percentage tilt should be:
    1. ∞
    2. 0
    3. 50%
    4. 100%

1. When a ramp is applied to a high-pass circuit, the transmission error is given as:
    1. $\frac{T}{2\tau}$
    2. $\boldsymbol{T}$
    3. $2\boldsymbol{\tau}$
    4. $\boldsymbol{\tau}^2$

1. When a high-pass circuit is used as differentiator, it means that the output is:
    1. Same as the input
    2. Integral of the input
    3. Differential of the input
    4. None of the above

1. A double differentiator is called:
    1. An $\boldsymbol{RC}$-coupled amplifier
    2. A feedback amplifier
    3. A rate-of-rise amplifier
    4. A tuned amplifier

SHORT ANSWER QUESTIONS

1. What is a linear network? Explain the working of a high-pass $\boldsymbol{RC}$ circuit.
2. What is meant by linear waveshaping?
3. Obtain the expression for the lower cut-off frequency of a high-pass $\boldsymbol{RC}$ circuit.
4. Define fall time of a high-pass circuit for a step input.
5. Specify the condition for which a high-pass circuit behaves as a differentiator and draw the input and output waveforms.
6. Draw the circuit of a double differentiator.
7. Define per cent tilt.

8. Draw the output of a high-pass circuit for a ramp input when the time constant τ is: (i) very small, and (ii) very large when compared to the time period of the input signal.
9. Plot the response of a high-pass circuit for $\tau = 0.1T$, $\tau = T$ and $\tau = 10T$ when the input is a square wave.
10. Derive the condition under which high-pass RC circuit behaves as a peaking circuit.
11. Explain why a double differentiator is called a rate-of-rise amplifier.

LONG ANSWER QUESTIONS

1. A positive step input of magnitude V is applied to high-pass RC circuit at $t = 0$. Derive the expression for the output voltage and also calculate the fall time and show that the fall time is $0.35/f_1$ where f_1 is the lower cut-off frequency.
2. A pulse is applied as an input to a high-pass RC circuit. Derive the expression for the output voltage.
3. Derive the expression for per cent tilt, 'P' when a square wave is the input to a high-pass RC circuit.
4. Prove that, for any periodic input waveform, the average level of the steady-state output signal from the high-pass RC circuit is always zero.
5. The input to a high-pass circuit is an exponential $v_i = V(1 - e^{-t/\tau_1})$. Derive the expression for its output voltage:
   (i) When $\tau = \tau_1$ and (ii) when $\tau \neq \tau_1$ where, τ is the time constant of high-pass RC circuit.
6. The input to a high-pass circuit is a ramp $v_i = \alpha\ t$. Obtain the expression for the output voltage and also calculate the transmission error, e_t and show that e_t is propotional to the lower cut-off frequency.
7. Show how a high-pass circuit having a time constant smaller than the time period of an input signal behaves as a differentiator.
8. What is a double differentiator? Show that a double differentiator is a rate-of-rise amplifier.

SOLVED PROBLEMS

Example 1: A pulse of amplitude 10 V and duration 10 μs is applied to a high-pass RC circuit. Sketch the output waveform indicating the voltage levels for (i) $RC = t_p$, (ii) $RC = 0.5t_p$ and (iii) $RC = 2t_p$.

Solution:

1. When $RC = t_p = \tau$

   At $t = t_p$

$$V_1 = 10\, e^{-(10\times10-6)/(10\times10-6)} = 10^{-1} = 3.678 \text{ V}$$

$$v_{o1} = 10\, e^{-t/(10\times10-6)} \text{ for } t < t_p$$

$$v_o(t > t_p) = (V_1 - 10)e^{-(t-10\times10-6)/(10\times10-6)} = -6.322\, e^{-(t-10\times10-6)/(10\times10-6)}$$

2. When $RC = \tau = 0.5t_p$

At $\boldsymbol{t} = \boldsymbol{t_p}$

$$V_1 = 10\, e^{-(10 \times 10-6)/(0.5 \times 10 \times 10-6)} = 10e^{-2} = 1.35 \text{ V}$$

$$v_{o1} = 10\, e^{-t/(0.5 \times 10 \times 10-6)} \text{ for } t < t_p$$

$$\boldsymbol{v_o}(\boldsymbol{t} > \boldsymbol{t_p}) = -8.65\, \boldsymbol{e}^{-(t-10 \times 10-6)/(0.5 \times 10 \times 10-6)}$$

3. When $\boldsymbol{RC} = \boldsymbol{\tau} = 2\boldsymbol{t_p}$

At $\boldsymbol{t} = \boldsymbol{t_p}$

$$\boldsymbol{V}_1 = 10\, \boldsymbol{e}(^{-10 \times 10-6)/(2 \times 10 \times 10-6)} = 10\boldsymbol{e}^{-0.5} = 6.05 \text{ V}$$

$$\boldsymbol{v_o}_1 = 10\, \boldsymbol{e}^{-t/(2 \times 10 \times 10-6)} \text{ for } \boldsymbol{t} < \boldsymbol{t_p}$$

$$\boldsymbol{v_o}(\boldsymbol{t} > \boldsymbol{t_p}) = -3.935\, \boldsymbol{e}^{-(t-10 \times 10-6)/(2 \times 10 \times 10-6)}$$

Based on these results, the output waveforms are sketched as in Fig. 1.1(a), (b) and (c) corresponding to cases (i), (ii) and (iii), respectively.

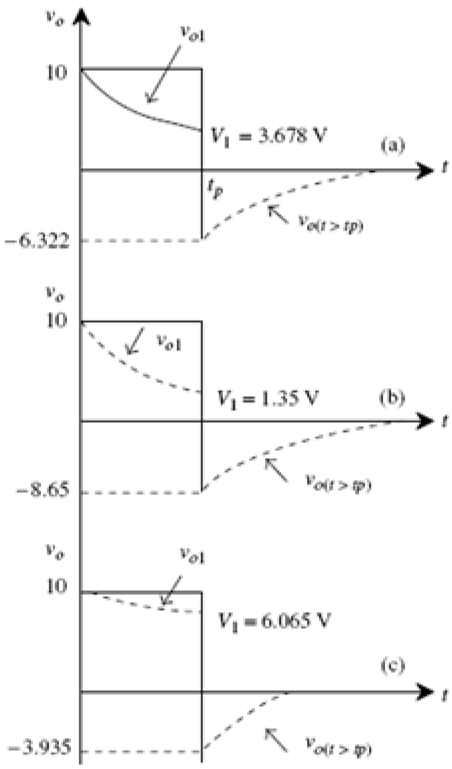

FIGURE 1.1 The response of a high-pass circuit for different values of $\boldsymbol{\tau}$

Example 2: A 10 Hz square wave whose peak-to-peak amplitude is 2 V is fed to an amplifier. Calculate and plot the output waveform if the lower 3-dB frequency is 0.3 Hz.

Solution: Let $\boldsymbol{C}$ be the condenser through which the signal is connected to the amplifier, having an input resistance $\boldsymbol{R}$, as shown in Fig. 1.2(a). This is essentially the high-pass circuit in Fig. 1.2(a).

The lower 3-dB frequency f_1 = 0.3 Hz

Input frequency is f = 10 Hz

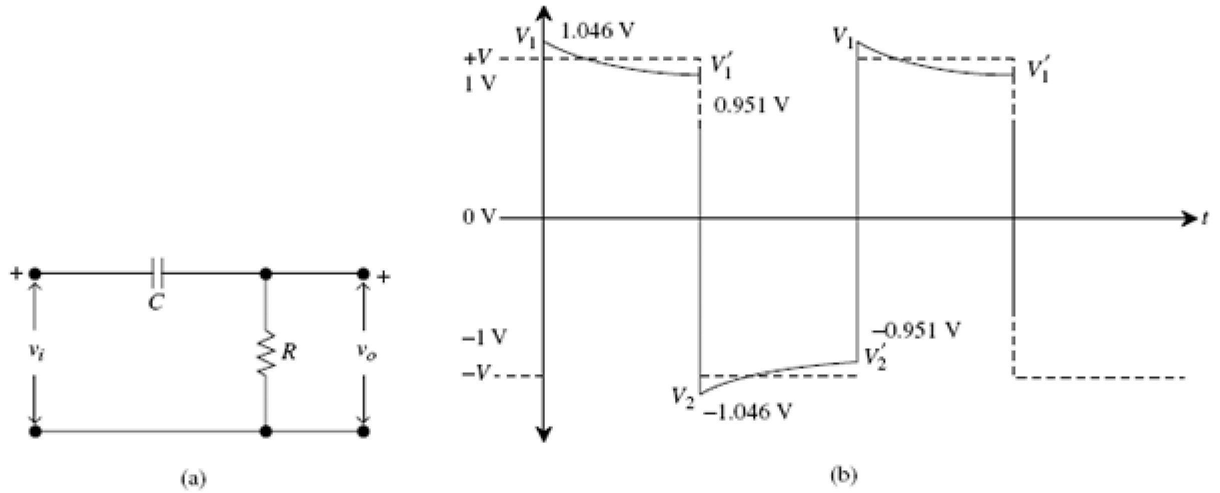

FIGURE 1.2 (a) The coupling network and (b) The response of the circuit

$$\tau = RC = \frac{1}{2\pi f_1} = \frac{1}{2\pi(0.3)} = 0.53\ \mathrm{s}$$

$$T = \frac{1}{f} = \frac{1}{10} = 0.1\ \mathrm{s}$$

Therefore,

$$\frac{T}{2} = 0.05\ \mathrm{s}$$

$$V_1 = \frac{V}{1 + e^{-T/2\tau}} = \frac{2}{1 + e^{-0.05/0.53}} = 1.046\ \mathrm{V} \qquad V_1' = V_1 e^{-T/2\tau} = 1.046 e^{-0.05/0.53} = 0.951\ \mathrm{V}$$

$$V_1 = -V_2 \quad \text{and} \quad V_1' = -V_2'$$

$$V_1 = \mid V_2 \mid = 1.046\ \mathrm{V} \qquad V_1' = \mid V_2' \mid = 0.951\ \mathrm{V}$$

The response of the circuit is shown in Fig. 1.2(b).

Example 3: A 20-Hz symmetrical square wave, shown in Fig. 1.3(a), with peak-to-peak amplitude of 2 V is impressed on a high-pass circuit shown in Fig. 1.3(a) whose lower 3-dB frequency is 10 Hz. Calculate and sketch the output waveform. What is the peak-to-peak output amplitude of the above waveform?

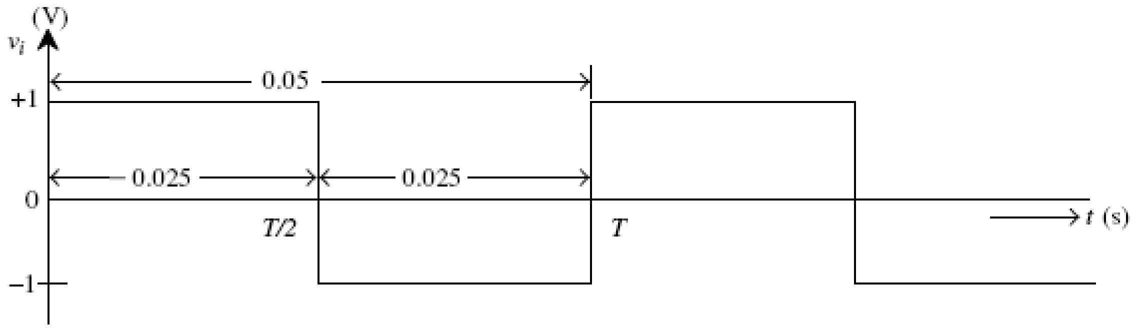

FIGURE 1.3(a) Input to the high-pass circuit

Solution: The lower 3-dB frequency:

$$f_1 = \frac{1}{2\pi RC} = 10\,\text{Hz} \qquad RC = \tau = \frac{1}{2\pi f_1} = \frac{1}{2\pi \times 10} = 0.0159\,\text{s}$$

Input signal frequency $f = 20$ Hz

Time period of the input

$$T = \frac{1}{f} = \frac{1}{20} = 0.05\,\text{s}$$

$$\frac{T}{2} = \frac{0.05}{2} = 0.025\,\text{s}$$

Therefore, τ is small compared to $T/2$; So the capacitor charges and discharges appreciably in each half-cycle. Since the input is a symmetrical square wave, $V_1 = -V_2$, i.e., $|V_1| = |V_2|$, $V_1' = -V_2'$ i.e., $|V_1'| = |V_2'|$.

The peak-to-peak input = 2 V. Hence,

$$V_1 = \frac{V}{1 + e^{(-T/2)/\tau}} = \frac{2}{1 + e^{-0.025/0.0159}} = 1.656\,\text{V} \qquad V_2 = -V_1 = -1.656\,\text{V}$$

Peak-to-peak value of output = $V_1 - V_2 = 3.312$ V.

$$V_1' = V_1 e^{(-T/2)/\tau} = 1.656\, e^{-(0.025/0.0159)} = 0.344\,\text{V}$$

$$V_1' = -V_2' = 0.344\,\text{V}$$

The output is plotted in Fig. 1.4(b).

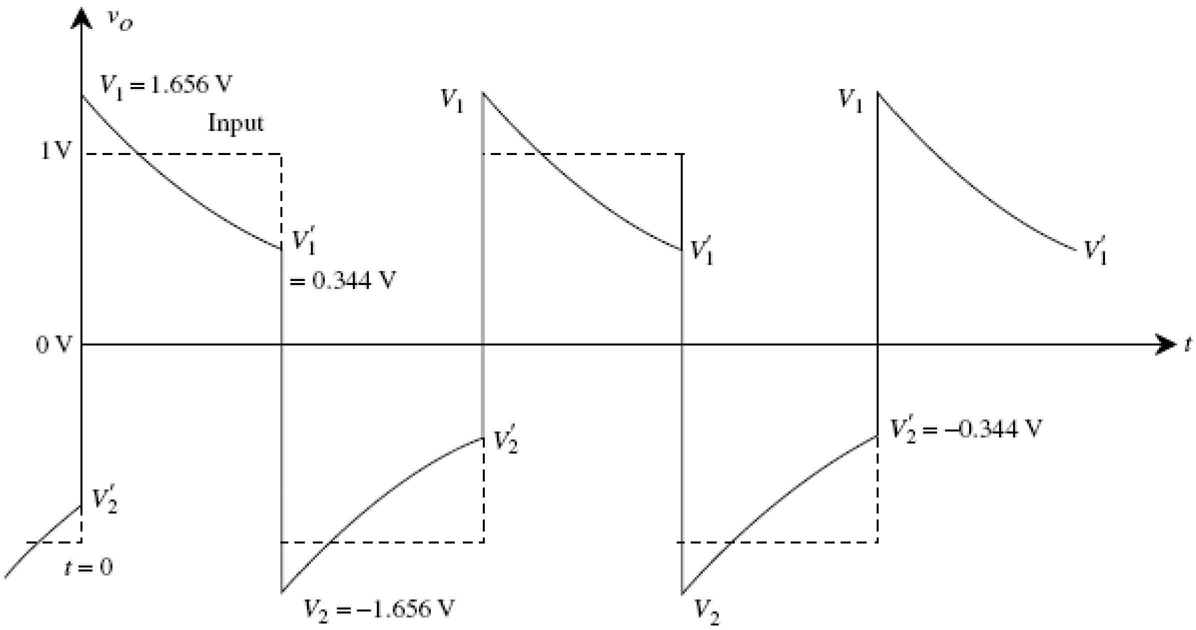

FIGURE 1.4(b) Output of the high-pass circuit for the given input

Example 4: An unsymmetric square wave has a peak-to-peak amplitude of 2 V and is referenced to the zero level. The duration of the positive section is 0.2 s, and of the negative section is 0.4 s. If this waveform is applied as input to the high-pass circuit shown in Fig. 1.2(a) whose time constant is 0.4 s, what are the steady-state maximum and minimum values of the output waveform? Prove that the area under the positive section is equal to the area under the negative section of the output waveform.

Solution: Given $T_1 = 0.2$ s, $T_2 = \tau = 0.4$ s

The steady-state output waveform is drawn by calculating V_1, V_1', V_2 and V_2'.

At $t = 0^-$, $v_o = V_2'$ and at $t = 0^+$, $v_o = V_1$

For $0 < t < T_1$, $v_o = V_1 e^{-t/\tau}$

At $t = T_1$, $v_o = V_1' = V_1 e^{-T_1/\tau} = V_1 e^{-0.2/0.4} = 0.606\, V_1$

For $T_1 < t < (T_1 + T_2)$, $v_o = V_2 e^{-T_2/\tau}$

At $t = T_2$, $v_0 = V_2' = V_2 e^{-T_2/\tau} = V_2 e^{-0.4/0.4} = 0.367 V_2$

The peak-to-peak input is 2 V.

$V_1' - V_2 = 2 \quad 0.606 V_1 - V_2 = 2$

$V_1 - V_2' = 2 \quad V_1 - 0.367 V_2 = 2$

Solving the above equations, $V_1 = 1.628$ V and $V_2 = -1.016$ V.

$V_1' = 0.606 \times 1.628 = 0.986$ V and $V_2' = -0.367 \times 1.106 = -0.372$ V.

The area under positive swing of output:

$$A_1 = \int_0^{T_1} V_1 e^{-t/\tau}\, dt = V_1\tau(1 - e^{-T_1/\tau}) = 1.628 \times 0.4\ (1 - e^{-0.2/0.4}) = 0.256 \text{ V-s}$$

The area under the negative swing of output is:

$$A_2 = \int_{T_1}^{T_2} |V_2|\, e^{-(t-T_1)/\tau}\, dt = |V_2|\tau(1 - e^{-T_2/\tau}) = 1.016 \times 0.4\ (1 - e^{-0.4/0.4}) = 0.256 \text{ V-s}$$

Thus, $A_1 = A_2$. The output waveform is shown in Fig. 1.5.

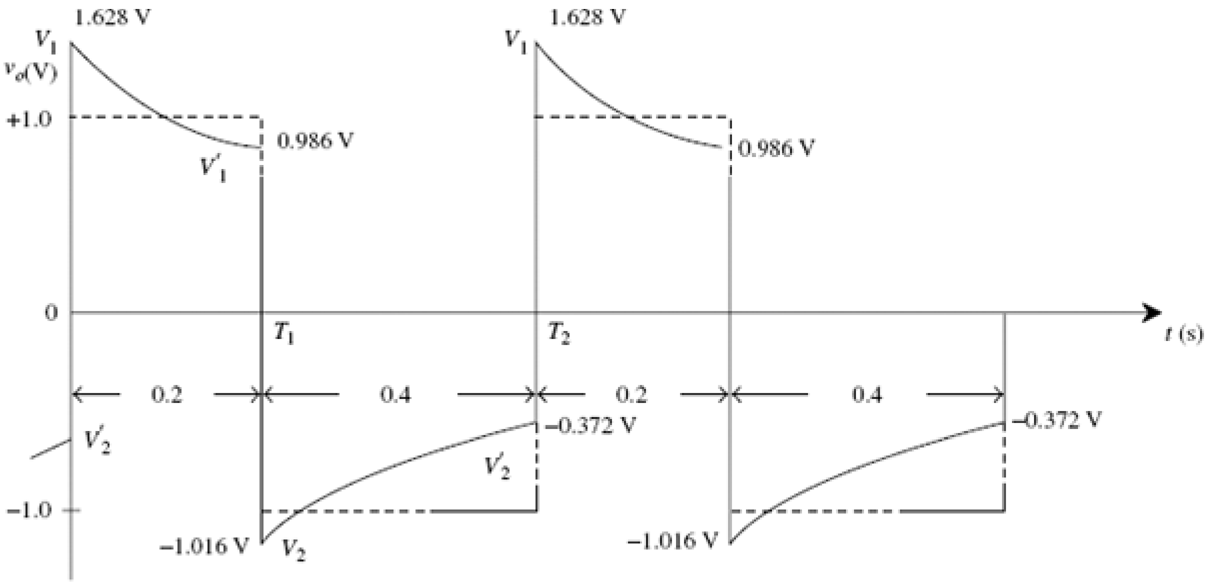

FIGURE 1.5 The output of the high-pass circuit for the specified input

Example 5: A ramp is applied to an RC differentiator, [see Fig. 1.1(a)]. Draw to scale the output waveform for the following cases: (i) $T = RC$, (ii) $T = 0.5RC$, (iii) $T = 10RC$.

Solution:

From Eq. of ramp, we can write

$$v_o = \alpha\,\tau\left(1 - e^{-t/\tau}\right) \qquad v_o = V\left(\frac{\tau}{T}\right)\left(1 - e^{-t/\tau}\right) \quad \text{as} \quad \alpha = \frac{V}{T}$$

The peak of the output will occur at $t = T$.

$$v_o(\text{peak}) = V\left(\frac{\tau}{T}\right)\left(1 - e^{-T/\tau}\right)$$

1. When $T = \tau$, $(\tau/T) = 1$ and $(T/\tau) = 1$

$$v_o(\text{peak}) = V(1)\left(1 - e^{-1}\right) = 0.632 \text{ V}$$

2. When $T = 0.5\tau$

$$\left(\frac{T}{\tau}\right) = 0.5 \quad \text{and} \quad \left(\frac{\tau}{T}\right) = 2$$

$$v_o(\text{peak}) = V(2)\left(1 - e^{-0.5}\right) = 0.788\ \text{V}$$

3. When $\boldsymbol{T} = 10\boldsymbol{\tau}$

$$\left(\frac{T}{\tau}\right) = 10 \quad \left(\frac{\tau}{T}\right) = 0.1$$

$$\boldsymbol{v_o}(\boldsymbol{peak}) = \boldsymbol{V}\,(0.1)^{\left(1 - e^{-10}\right)} = \boldsymbol{V}(0.1)(1 - 0.000045) = 0.1\ \text{V}$$

The response is plotted in Fig. 1.6.

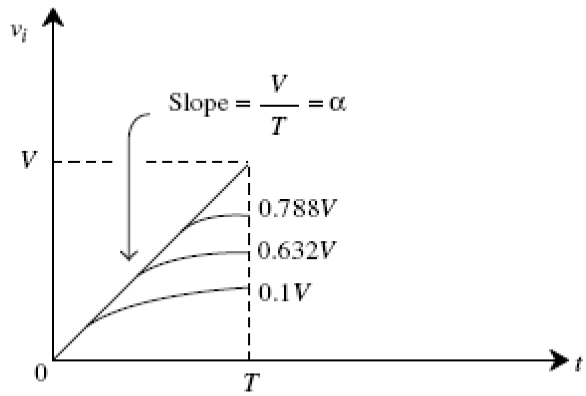

FIGURE 1.6 The response of the high-pass circuit to ramp input

UNSOLVED PROBLEMS

1. A waveform shown in Fig. 1p.1 is applied as input to a $\boldsymbol{RC}$ high pass circuit whose time constant is 250 ps. If the maximum output voltage across the resistor is 50 V, what is the peak value of the input waveform?

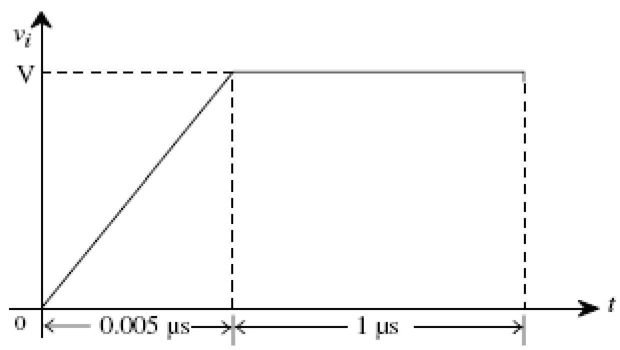

FIGURE 1p.1 Input to the high-pass $\boldsymbol{RC}$ circuit

2. A limited ramp shown in Fig. 1p.2(a) is applied to a $\boldsymbol{RC}$ high-pass circuit of Fig. 2.2(a). The time constant of the $\boldsymbol{RC}$ circuit is 2 ms. Calculate the maximum value of the output voltage and the output at the end of the input waveform.

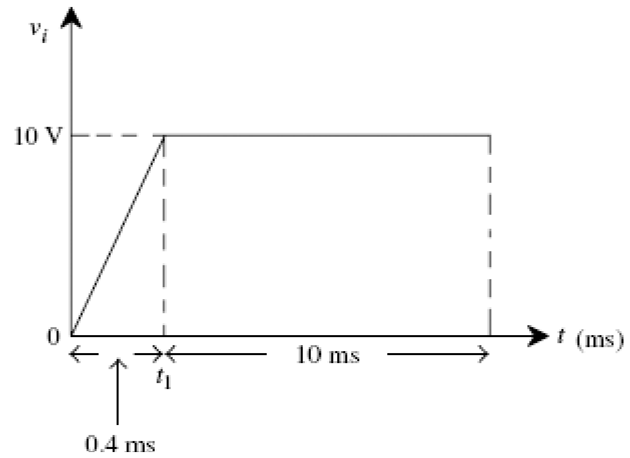

FIGURE 1p.2 Input to the high-pass circuit

3. The periodic waveform shown in Fig. 1p.3 is applied to an $\boldsymbol{RC}$ differentiating circuit whose time constant is 10 $\boldsymbol{\mu}$s. Sketch the output and calculate the maximum and minimum values of the output voltage with respect to the ground.

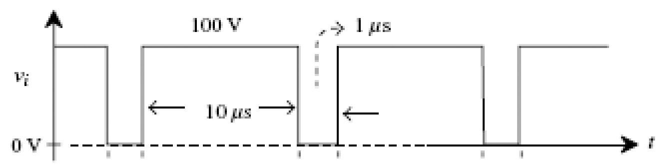

FIGURE 1p.3 Periodic square wave as an input to the high-pass circuit

4. The periodic ramp voltage as shown in Fig. 1p.4 is applied to a high-pass $\boldsymbol{RC}$ circuit. Find equations from which to determine the steady-state output waveform when $\boldsymbol{T}_1 = \boldsymbol{T}_2 = \boldsymbol{RC}$.

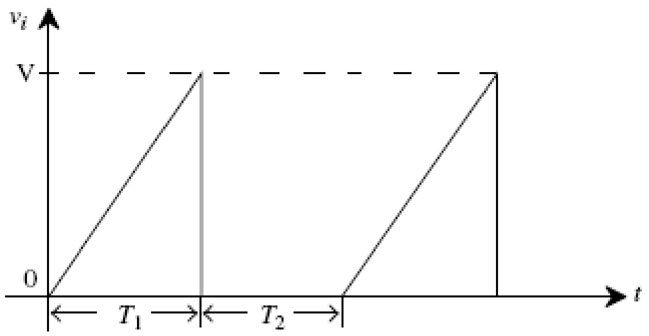

FIGURE 1p.4 A periodic ramp as input

5. A square wave of pulse width 2 ms and peak amplitude of 12 V as shown in Fig. 1p.5 is applied to high-pass $\boldsymbol{RC}$ circuit with time constant 4 ms. Plot the first four cycles of the output waveform. $\boldsymbol{T}/2 = 2$ ms

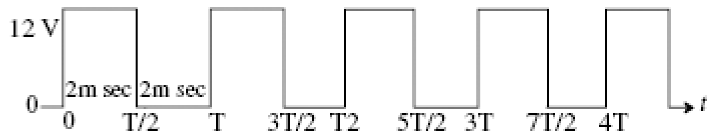

FIGURE 1p.5 Symmetric square wave as an input

6. A 20-Hz symmetric square wave, referenced to zero volts, and with a peak-to-peak amplitude of 10 V is fed to an amplifier through the coupling network shown in Fig. 1p.6. Calculate and plot the output waveform when the lower 3-dB frequency is: (i) 0.6 Hz, (ii) 6 Hz, (iii) 60 Hz.

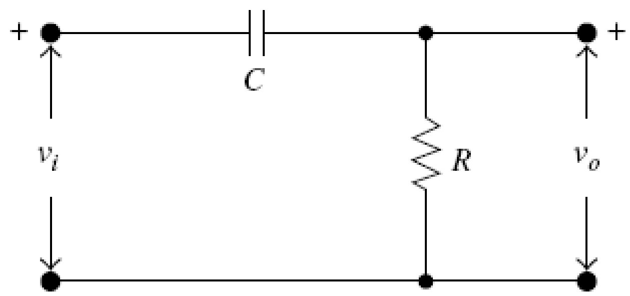

FIGURE 1p.6 The given coupling network

7. A square wave is applied as input to an amplifier through a coupling condenser of 10 μF. The amplifier has input resistance of 10 kΩ. Determine the lowest frequency if the tilt is not to exceed 10 per cent.
8. A pulse of 10 V amplitude and duration 1ms is applied to a high-pass $\boldsymbol{RC}$ circuit with $\boldsymbol{R}$ = 20 kΩ and $\boldsymbol{C}$ = 0.5 μF. Plot the output waveform to scale and calculate the per cent tilt in the output.
9. The input to the high-pass circuit shown in Fig. 1.1(a) is the waveform shown in Fig. 1p.7. Calculate and plot the output waveform to scale, given that $\boldsymbol{RC} = \boldsymbol{\tau} = 0.1$ ms.

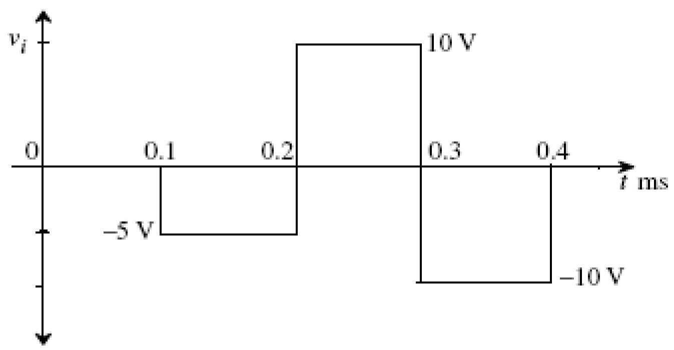

FIGURE 1p.7 Input to the high-pass circuit

10. A pulse of 10-Volt amplitude with a pulse width of 0.5 ms, as shown in Fig. 1p.8, is applied to a high-pass $\boldsymbol{RC}$ circuit of Fig. 1.1(a), having time constant 10 ms. Sketch the output waveform and determine the per cent tilt in the output.

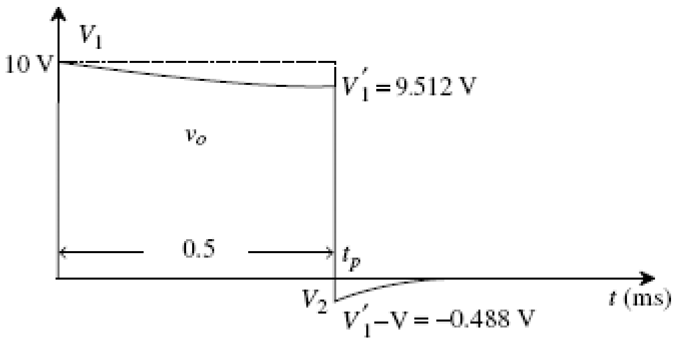

FIGURE 1p.8 Input and output of the high-pass circuit

11. A high-pass ***RC*** circuit is desired to pass a 3-ms sweep (ramp input) with less than 0.4 per cent transmission error. Calculate the highest possible value of the lower 3-dB frequency.
12. A symmetric square wave with ***f*** = 500 Hz shown in Fig. 1p.9 is fed to an ***RC*** high-pass network of Fig. 1.1(a). Calculate and plot the transient and the steady-state response if: (a) $\boldsymbol{\tau} = 5\,\boldsymbol{T}$ and (b) $\boldsymbol{\tau} = \boldsymbol{T}/20$.

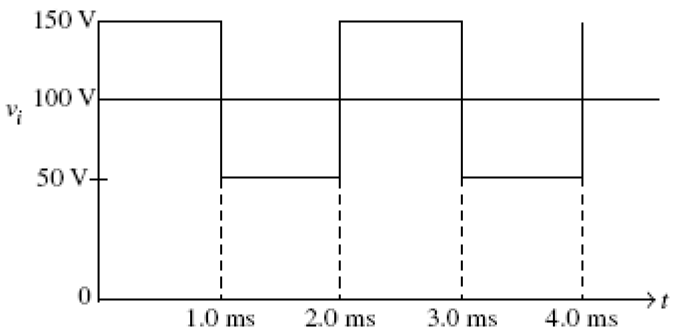

FIGURE 1p.9 Input to the coupling network

13. A current pulse of amplitude 5A in Fig. 1p.10 is applied to a parallel ***RC*** combination shown in Fig. 1p.11. Plot to scale the waveforms of the current flowing through the capacitor for the cases: (i) $\boldsymbol{t_p} = 0.1\,\boldsymbol{RC}$ (ii) $\boldsymbol{t_p} = \boldsymbol{RC}$, (iii) $\boldsymbol{t_p} = 5\boldsymbol{RC}$.

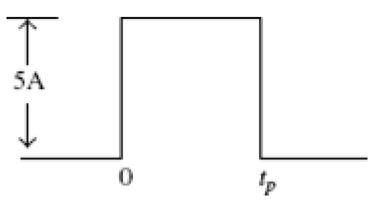

FIGURE 1p.10 The given input to the circuit

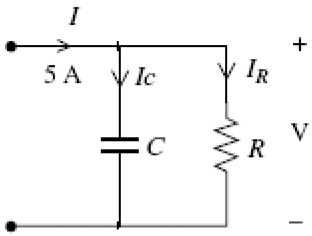

FIGURE 1p.11 The given circuit

14. Draw the output waveform if the waveform shown in Fig. 1p.12 is applied at the input of the ***RC*** circuit shown in Fig. 1p.13.

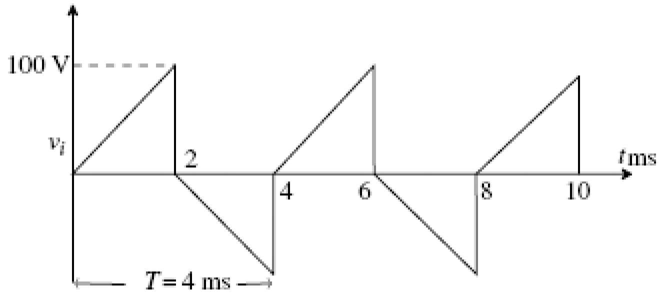

FIGURE 1p.12 The input to the high-pass circuit in Fig. 1p.13

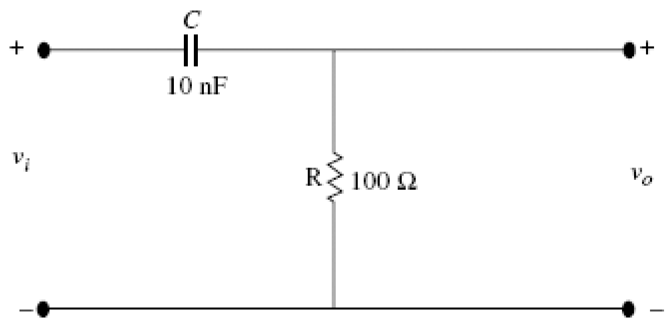

FIGURE 1p.13 The given high-pass circuit

RC LOW-PASS SUMMARY

- A low-pass circuit transmits low-frequency components and attenuates high frequencies.
- The cut-off frequency of a low-pass or high-pass circuit is the frequency at which the output is $1/\sqrt{2}$ (or 0.707) times the maximum output.
- The upper cut-off frequency of a low-pass circuit, f_2 is given by $1/2\pi RC$.
- f_2 is the bandwidth of the low-pass circuit.

- When a step is applied as input to a low-pass circuit, it takes a finite time for the output to reach the steady-state value. Rise time is defined as the time taken for the output to reach from 10 per cent of its final value to 90 per cent of its final value.
- The rise time t_r is given as 2.2τ $(= 0.35/f_2)$.
- If n stages of low-pass circuits having rise times $t_{r1}, t_{r2}, \ldots, t_{rn}$ are cascaded, then the rise time of the cascaded stages is given by $tr = \sqrt{t_{r1}^2 + t_{r2}^2 + \cdots + t_{rn}^2}$.
- A low-pass circuit behaves as an integrator if the time constant of the circuit is significantly greater than the time period of the input signal.
- Attenuators are essentially resistive networks employed to reduce the amplitude of the signal.
- An uncompensated attenuator is one in which the output depends on frequency.
- A compensated attenuator is one in which the output is independent of frequency.
- In an ideal attenuator, the output is independent of frequency.
- An attenuator is called a perfectly compensated attenuator if $v_o(0+) = v_o(\infty)$.
- An attenuator is treated as an over-compensated attenuator if $v_o(0+) > v_o(\infty)$.
- An attenuator in which $v_o(0+) < v_o(\infty)$ is called an under-compensated attenuator.
- At $t = 0+$, as capacitors behaves as short circuit, $v_o(0+)$ is calculated by considering capacitors in an attenuator.
- At $t = \infty$, as the capacitors are fully charged, they behave as open circuit for dc. Hence, $v_o(\infty)$ is calculated considering only the resistances in an attenuator.
- A ringing circuit is an RLC circuit which produces (nearly) undamped oscillations.

MULTIPLE CHOICE QUESTIONS

1. If two stages of identical low-pass RC circuits are cascaded, the rise time of the cascaded stage is:

1. $2t_r$
2. $\sqrt{2}t_r$
3. t_r^2
4. $0.2t_r$

1. A step input with certain pulse width is given to a low-pass RC circuit to transmit the same. To minimize the distortion:
   1. Time constant must be equal to the pulse width

2. Time constant must be larger than the pulse width

3. Time constant must be smaller than the pulse width

4. Time constant and pulse width are independent of one another

1. The general solution for a single time constant circuit having initial and final values $\boldsymbol{v_i}$ and $\boldsymbol{v_f}$ respectively is given by:

1. $\boldsymbol{v_o} = \boldsymbol{v_i} + (\boldsymbol{v_f} - \boldsymbol{v_i})\boldsymbol{e}^{-t/\tau}$

2. $\boldsymbol{v_o} = \boldsymbol{v_i} + (\boldsymbol{v_i} - \boldsymbol{v_f})\boldsymbol{e}^{-t/\tau}$

3. $\boldsymbol{v_o} = \boldsymbol{v_f} + (\boldsymbol{v_i} - \boldsymbol{v_f})\boldsymbol{e}^{-t/\tau}$

4. $\boldsymbol{v_o} = \boldsymbol{v_f} + (\boldsymbol{v_f} - \boldsymbol{v_i})\boldsymbol{e}^{-t/\tau}$

1. The following type of attenuator will faithfully reproduce the signal which appears at its input terminals.

1. Under-compensated

2. Over-compensated

3. Perfectly compensated attenuator

4. Uncompensated attenuator

1. The transmission error, $\boldsymbol{e_t}$ when a ramp input with $\boldsymbol{RC \ll T}$ is applied to a low-pass circuit is given by:

1. $\frac{T}{2RC}$

2. $\frac{T}{RC}$

3. $\frac{RC}{T}$

4. $\frac{RC}{2T}$

1. Integrators are invariably preferred over differentiators in analogue computer applications due to the following reason;

1. The gain of an integrator decreases with frequency

2. It is easier to stabilize

3. It is more convenient to introduce initial conditions

4. All of the above

1. The output of an integrator to a square wave input is:

1. Triangular wave
2. Square wave
3. Quadratic response
4. Spikes

1. The time required for the capacitor to get charged completely is nearly __________ the time constant.
   1. five times
   2. one time
   3. equal to
   4. two times

1. When $\boldsymbol{R_1C_1 = R_2C_2}$, the attenuator is said to have achieved:
   1. Perfect compensation
   2. Over-compensation
   3. Under-compensation
   4. No compensation

1. When $\boldsymbol{R_1C_1 > R_2C_2}$, the attenuator is said to have achieved:
   1. Perfect compensation
   2. Over-compensation
   3. Under-compensation
   4. No compensation

1. When $\boldsymbol{R_1C_1 < R_2C_2}$, the attenuator is said to have achieved:
   1. Perfect compensation
   2. Over-compensation
   3. Under-compensation
   4. No compensation

SHORT ANSWER QUESTIONS

1. Obtain the expression for the bandwidth of a low-pass circuit.

2. The input to a low-pass $\boldsymbol{RC}$ circuit is a step of $\boldsymbol{V}$. Obtain the expression for the output voltage.
3. What is meant by the rise time of a pulse? Obtain the expression for the rise time of a low-pass $\boldsymbol{RC}$ circuit.
4. The input to a low-pass circuit is a pulse of duration $\boldsymbol{t_p}$ and magnitude $\boldsymbol{V}$. Plot its output when the time constant is very small and when the time constant is very large.
5. A symmetric square wave is applied as input to the low-pass circuit. Plot the output waveforms for different time constants.
6. A ramp is applied as input to a low-pass circuit. Derive the expression for the transmission error, $\boldsymbol{e_t}$.
7. Show that a low-pass circuit can be used as an integrator if the time constant of the circuit is significantly larger than the time period of the input signal.
8. What is an attenuator? Explain the drawbacks of an uncompensated attenuator.
9. How can an uncompensated attenuator be modified as a compensated attenuator?
10. Compare the responses of perfectly compensated, under-compensated and over-compensated attenuators.
11. Illustrate, with a suitable diagram, how a compensated attenuator can be used as a CRO probe.
12. Explain under which condition an $\boldsymbol{RL}$C circuit behaves as a ringing circuit.

LONG ANSWER QUESTIONS

1. If a symmetric square wave referenced to 0 voltage is the input to a low-pass circuit, derive the expression for the steady-state voltage levels at the output. Plot the typical waveforms.
2. A ramp $\boldsymbol{v_i} = \boldsymbol{\alpha t}$ is applied as input to a low-pass $\boldsymbol{RC}$ circuit. Derive the expression for the output voltage. Plot the typical waveforms.
3. An exponential signal $\boldsymbol{v_i} = \boldsymbol{V}(1 - \boldsymbol{e}^{-t/\tau_1})$ is the input to a low-pass $\boldsymbol{RC}$ circuit. Derive the expressions for the output when (i) $\boldsymbol{\tau} = \boldsymbol{\tau}_1$ and (ii) $\boldsymbol{\tau} \neq \boldsymbol{\tau}_1$.
4. What is an uncompensated attenuator and its major limitations? Implement a compensated attenuator. Explain the conditions under which this is called a perfectly compensated, over-compensated and under-compensated attenuator.
5. Obtain the response of an $\boldsymbol{RL}$C circuit under critically damped conditions.

SOLVED PROBLEMS

Example 1: An ideal pulse of amplitude 10 V is fed to an RC low-pass integrator circuit. The width of the pulse is 3 μs. Draw the output waveforms for the following upper 3-dB frequencies: (a) 30 MHz, (b) 3 MHz and (c) 0.3 MHz.

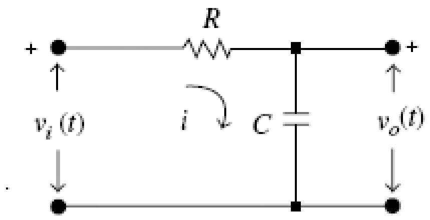

FIGURE 1.7(a) RC low-pass integrater circuit

Solution: Consider the low-pass circuit in Fig. 17(a).

1. At f_2 = 30 MHz

We know that $f_2 = 1/2\pi RC$

$$\tau = RC = \frac{1}{2\pi f_2} = \frac{1}{2\pi \times 30 \times 10^6} = 5.3 \text{ ns}$$

$t_r = 2.2\tau = 2.2 \times 5.3 \times 10^{-9} = 11.67$ ns

At $t = t_p$,

$$V_p = V(1 - e^{-t_p/\tau}) = 10(1 - e^{-3\times10-6/5.3\times10-9}) = 10 \text{ V}$$

The output is plotted in Fig. 1.7(b).

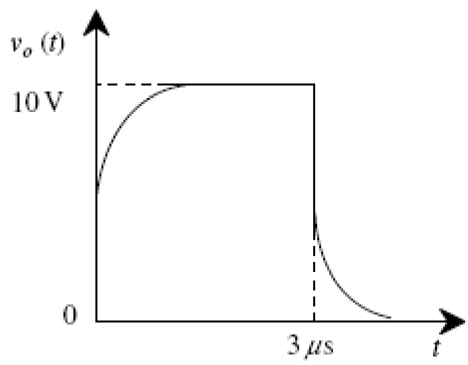

FIGURE 1.7(b) Output waveform at f_2 = 30 MHz

2. At f_2 = 3 MHz

$$\tau = RC = \frac{1}{2\pi f_2} = \frac{1}{2\pi \times 3 \times 10^6} = 53 \text{ ns}$$

$t_r = 2.2\tau = 2.2 \times 53 \times 10^{-9} = 116.6$ ns

At $t = t_p$,

$$V_p = V(1 - e^{-t_p/\tau}) = 10(1 - e^{-3\times10-6/53\times10-9}) = 10 \text{ V}$$

The output is plotted in Fig. 1.7(c).

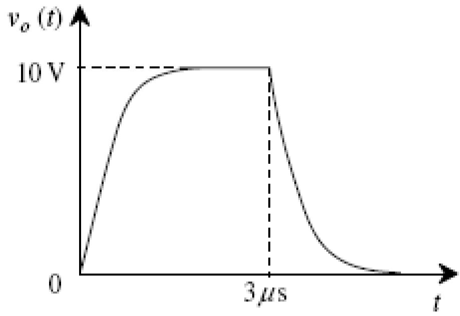

FIGURE 1.7(c) Output waveform at f_2 = 3 MHz

3. At f_2 = 0.3 MHz

$$\tau = RC = \frac{1}{2\pi f_2} = \frac{1}{2\pi \times 0.3 \times 10^6} = 530 \text{ ns}$$

$t_r = 2.2\tau = 2.2 \times 530 \times 10^{-9} = 1.166\ \mu$s

At $t = t_p$,

$$V_p = V(1 - e^{-t_p/\tau})$$

Therefore,

$$V_p = 10(1 - e^{-3\times10-6/530\times10-9}) = 9.96 \text{ V}$$

The output is plotted in Fig. 1.7(d).

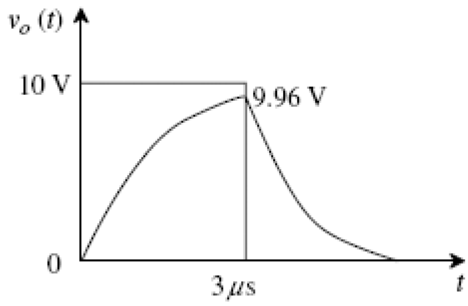

FIGURE 1.7(d) The output waveform at f_2= 0.3 MHz

Example 2: A symmetric square wave, whose peak-to-peak amplitude is 4 V and whose average value is zero is applied to a low-pass ***RC*** circuit shown in Fig. 1.7(a). The time constant equals the half-period of the square wave. Find the peak-to-peak output voltage of waveform.

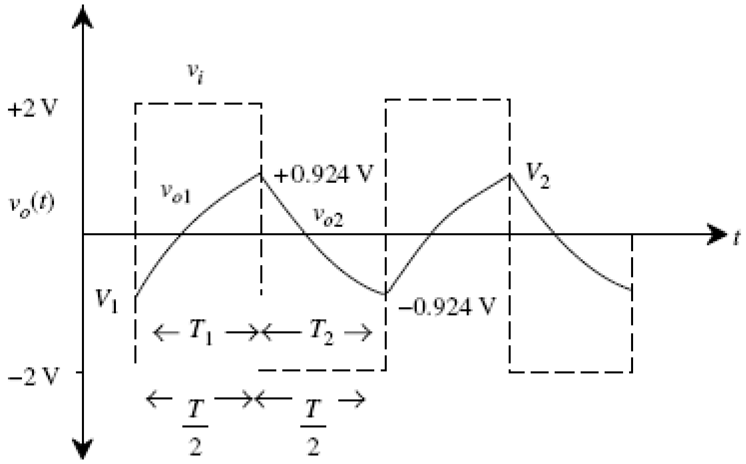

FIGURE 1.8 The input and output waveforms at $\boldsymbol{RC} = \boldsymbol{T}/2$

Solution: As the input is a symmetric square wave, we have:

$\boldsymbol{T}_1 = \boldsymbol{T}_2 = \frac{T}{2}$, $\boldsymbol{V}_1 = -\boldsymbol{V}_2$ and $\boldsymbol{V}' = \boldsymbol{V}'' = \frac{V}{2}$ $\quad \boldsymbol{v}_{o1} = \boldsymbol{V}' + (\boldsymbol{V}_1 - \boldsymbol{V}')\boldsymbol{e}^{-t/\tau}$

At $\boldsymbol{t} = \boldsymbol{T}_1$:

$$V_2 = V' + \left(V_1 - V'\right) e^{-T_1/\tau} = \frac{V}{2} + \left(-V_2 - \frac{V}{2}\right) e^{-(T/2)(2/T)}$$

where $\boldsymbol{\tau} = \boldsymbol{T}/2$.

$$V_2 = \frac{4}{2} + \left(-V_2 - \frac{4}{2}\right) e^{-1} = 2 + (-V_2 - 2)e^{-1} = 2 - 0.368\,V_2 - 0.736 = 0.924\text{ V}$$

The peak-to-peak output voltage = 2 × 0.924 = 1.848 V. $\boldsymbol{V}_2$ can also directly be calculated using Eq.

The input and output are plotted as shown in Fig. 1.8.

Example 3: The periodic waveform applied to an ***RC*** low-pass circuit in Fig. 1.7(a), is a square wave with $\boldsymbol{T}_1$ = 0.1 s, $\boldsymbol{T}_2$ = 0.2 s and time constant = 0.1 s, [see Fig. 1.9(a)]. Calculate the output voltages and draw the output waveform.

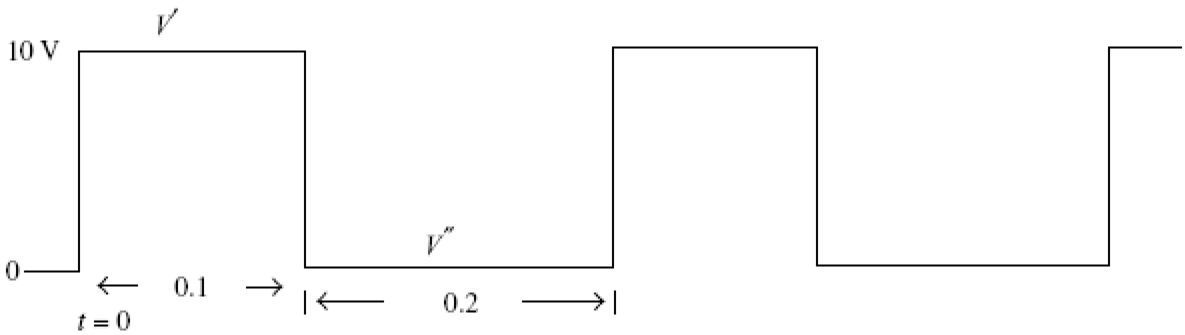

FIGURE 1.9(a) The given input waveform

Solution: The capacitor charges and discharges to the same level for each cycle.

$RC = \tau = 0.1$ s, $T_1 = 0.1$ s, $T_2 = 0.2$ s

At $0 < t < 0.1$ s, $v_f = V' = 10$ V and $v_i = V_2$

$v_{o1} = 10 - (10 - V_2)e^{-t/\tau}$

At $t = 0.1$ s:

$$v_{o1} = V_1 = 10 - (10 - V_2)e^{-0.1/0.1} = 10 - (10 - V_2)0.368$$

$$V_1 = 6.32 + 0.368V_2 \qquad (1)$$

For $0.1 < t < 0.3$ s, $v_f = V'' = 0$ V and $v_i = V_1$

$v_{o2} = 0 - (0 - V_1)e^{-(t-0.1)/\tau}$

At $t = 0.3$ s:

$$v_{o2} = V_2 = 0 - (0 - V_1)e^{-0.2/0.1} = 0 - (0 - V_1)0.135$$
$$V_2 = 0.135V_1 \qquad (2)$$

Substitute Eq. (2) in Eq. (1) to get the values of V_1:

$V_1 = 6.32 + 0.368 \times 0.135V_1 \quad V_1(1 - 0.05) = 6.32 \quad 0.95V_1 = 6.32 \quad V_1 = 6.65$ V

From Eq. (2),

$V_2 = 0.135 \times 6.65 = 0.898$ V

The input and output waveforms are plotted in Fig. 1.9(b).

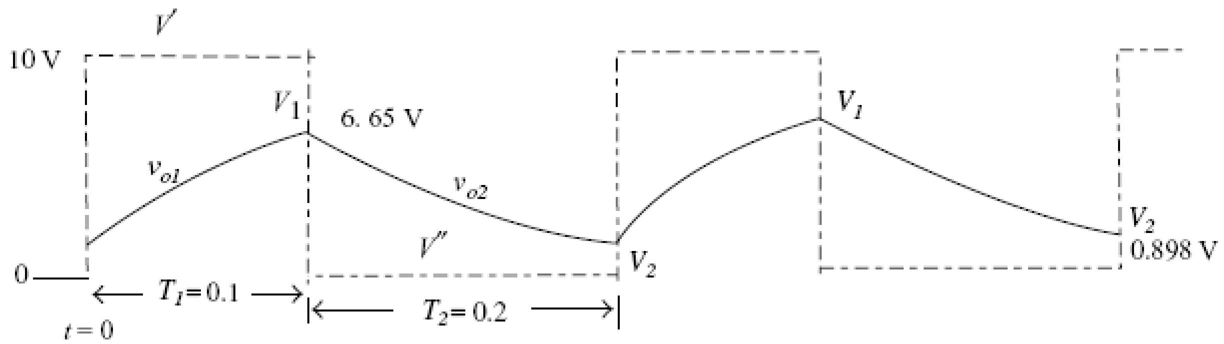

FIGURE 1.9(b) The input and output waveforms of the low-pass circuit

Example 4: A limited ramp, shown Fig. 1.10(a) (the pulse rises linearly and reaches V at T and remains at V beyond T), is applied to the low-pass RC circuit in Fig. 1.7(a). Plot the output waveforms when (a) $T = \tau$, (b) $T = 0.3\tau$ and (c) $T = 6\tau$.

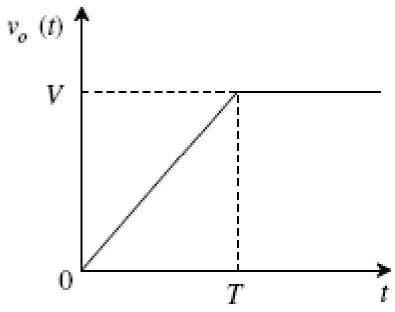

FIGURE 1.10(a) The given input waveform

Solution: For a low-pass circuit, $v_o(t) = \alpha(t - \tau) + \alpha\ \tau\ e^{-t/\tau}$, where $\alpha = V/T$.

At $t = T$, $v_o(T) = \alpha(T - \tau) + \alpha\ \tau\ e^{-T/\tau}$

1. $T = \tau$

$$v_o(T) = \alpha(\tau - \tau) + \alpha\ \tau\ e^{-\tau/\tau} = \alpha\ \tau\ e^{-1}$$

$$v_O(T) = \frac{V}{\tau}\tau \times 0.368 = 0.368V$$

Beyond T, the output varies exponentially and reaches V

The output is as shown in Fig. 1.10(b).

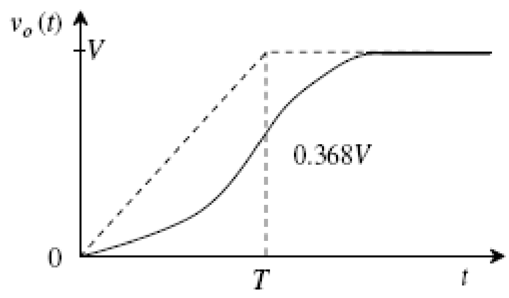

FIGURE 1.10(b) The output waveform for $T = \tau$

2. $T = 0.3\tau$

$$v_O(T) = \alpha(0.3\tau - \tau) + \alpha\ \tau\ e^{-0.3\tau/\tau}$$

$$= \frac{V}{0.3\tau}(-0.7\tau) + \frac{V}{0.3\tau}\tau\ e^{-0.3}$$

$$= \frac{-0.7}{0.3}V + \frac{0.74}{0.3}V$$

$$v_O(T) = 0.133V$$

The output is as shown in Fig. 1.10(c).

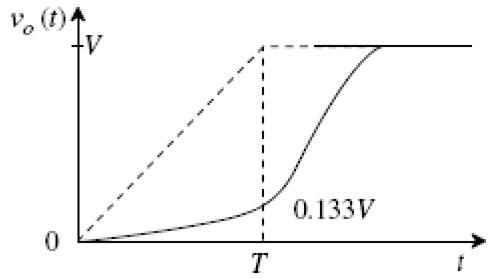

FIGURE 1.10(c) The output waveform for $\boldsymbol{T} = 0.3\,\boldsymbol{\tau}$

3. $\boldsymbol{T} = 6\boldsymbol{\tau}$

$$v_o(T) = \alpha(6\tau - \tau) + \alpha\,\tau\,e^{-6\tau/\tau}$$

$$= \frac{V}{6\tau}(5\tau) + \frac{V}{6\tau}\tau\,e^{-6}$$

$$v_o(T) = 0.833\ V$$

The output is as in Fig. 1.10(d).

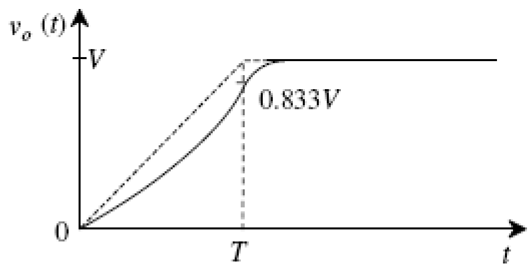

FIGURE 1.10(d) The output waveform for $\boldsymbol{T} = 6\,\boldsymbol{\tau}$

Example 5: Calculate the output voltages and draw the waveforms when (a) $\boldsymbol{C}_1$ = 75 pF, (b) $\boldsymbol{C}_1$ = 100 pF, (c) $\boldsymbol{C}_1$ = 50 pF for the circuit shown in Fig. 1.11(a). The input step voltage is 50 V.

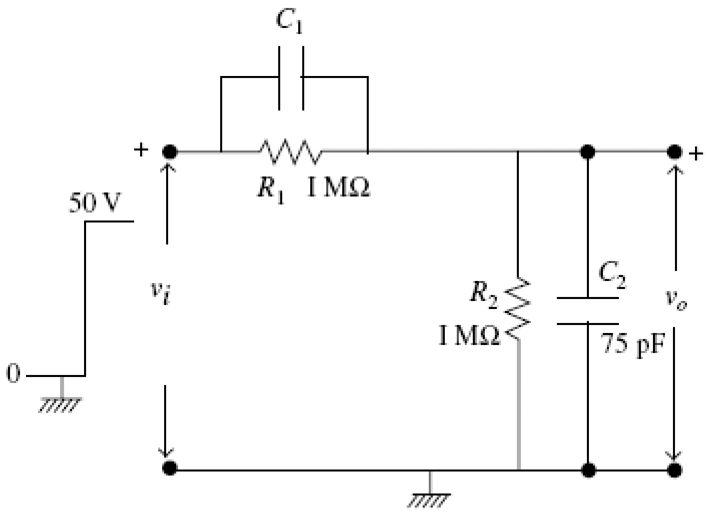

FIGURE 1.11(a) The given attenuator circuit

Solution: For perfect compensation, $R_1C_1 = R_2C_2$. Here $R_1 = R_2$.

1. When $C_1 = 75$ pF, then the attenuator is perfectly compensated. The rise time of the output waveform is zero.

Attenuation, $\alpha = \dfrac{R_2}{R_1 + R_2} = \dfrac{1}{1+1} = 0.5$

$v_o(0+) = v_o(\infty) = \alpha v_i = 0.5 \times 50 = 25$ V

2. When $C_1 = 100$ pF, then the attenuator is over-compensated, hence $v_o(0^+) > v_o(\infty)$.

The output at $t = 0+$,

$$v_o(0+) = v_i \times \frac{C_1}{C_1 + C_2} = 50 \times \frac{100}{100 + 75} = 28.6 \text{ V}$$

The output at $t = \infty$,

$$v_o(\infty) = v_i \times \frac{R_2}{R_1 + R_2} = 50 \times \frac{1}{1+1} = 25 \text{ V}$$

From Fig. 3.21(b):

$R = \dfrac{R_1 R_2}{R_1 + R_2}$ and $C = C_1 + C_2$

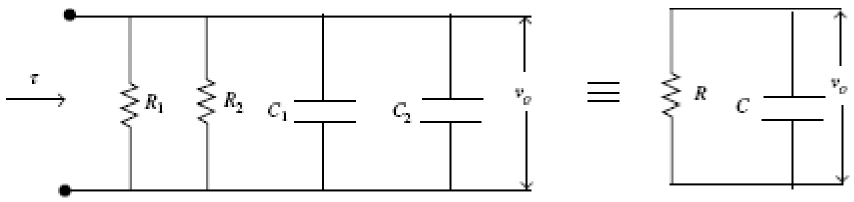

FIGURE 1.11(b) The equivalent circuit to get the time constant for the decay of the overshoot

Time constant τ_1 with which the overshoot at $t = 0^+$ decays to the steady-state value is:

$$\tau_1 = \frac{R_1 R_2}{R_1 + R_2}(C_1 + C_2) = \frac{1 \times 1}{1+1} \times 10^6 \times (100 + 75) \times 10^{-12} = 87.5\ \mu\text{s}$$

Fall time $t_f = 2.2\,\tau_1 = 2.2 \times 87.5 \times 10^{-6} = 192.5\ \mu$s

3. When $C_1 = 50$ pF, then the attenuator is under-compensated.

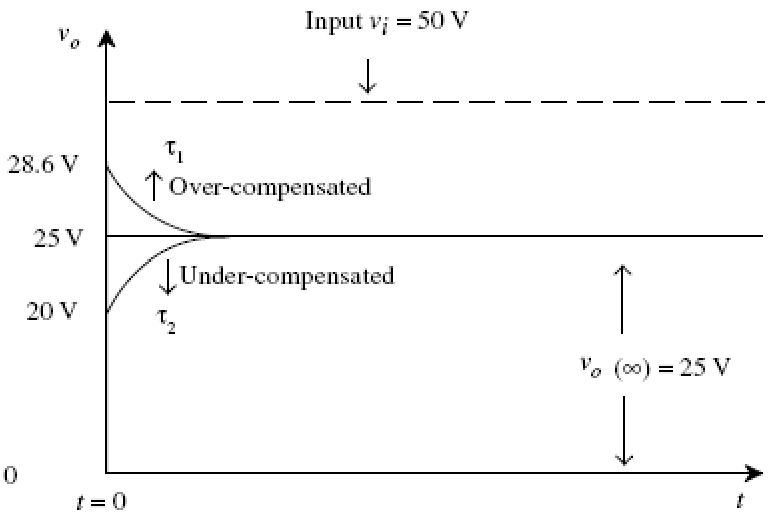

FIGURE 1.11(c) The input and output responses

The output at t = 0+:

$$v_o(0^+) = v_i \times \frac{C_1}{C_1 + C_2} = 50 \times \frac{50}{50 + 75} = 20\text{ V}$$

The output at $t = \infty$:

$$v_o(\infty) = v_i \times \frac{R_2}{R_1 + R_2} = 50 \times \frac{1}{1 + 1} = 25\text{ V}$$

The time constant, τ_2, with which the output rises to the steady-state value is:

$$\tau_2 = \frac{R_1 R_2}{R_1 + R_2}(C_1 + C_2) = \frac{1 \times 1}{1 + 1} \times 10^6 \times (50 + 75) \times 10^{-12} = 62.5\ \mu\text{s}$$

Rise time, $t_r = 2.2\ \tau_2$

$t_r = 2.2 \times 62.5 \times 10^{-6} = 137.5\ \mu$s

The output responses are plotted in Fig. 1.11(c).

UNSOLVED PROBLEMS

1. A pulse with zero rise time, an amplitude of 10 V and duration 10 μs is applied to an amplifier through a low-pass coupling network shown in Fig. 1p.14. Plot the output waveform to scale under the following conditions:

1. f_2 = 20 MHz,
2. f_2 = 0.2 MHz.

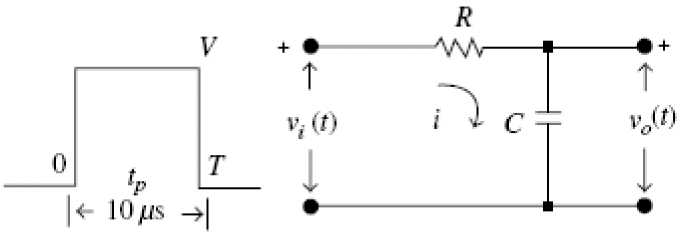

FIGURE 1p.14 The given coupling network with input

1. A ramp shown in Fig. 1p.15 is applied to a low-pass RC circuit. Draw to scale the output waveform for the cases:

$T = 0.2RC$,
$T = 10\ RC$.

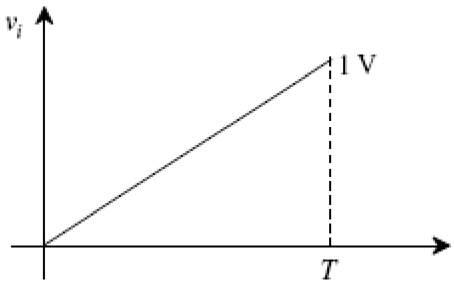

FIGURE 1p.15 The given input to the low-pass RC circuit

1. The input to a low-pass ***RC*** circuit is periodic and trapezoidal as shown in Fig. 1p.16. Find and sketch the steady-state output if $RC = 10\,T_1 = 10\,T_2$.

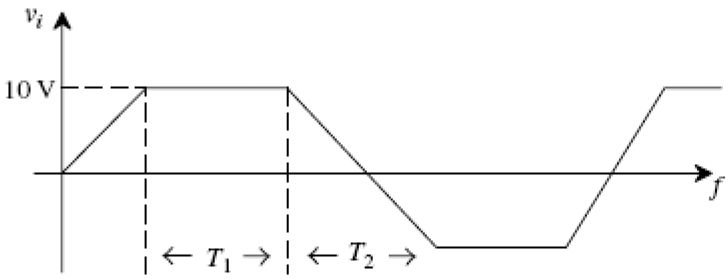

FIGURE 1p.16 The given input to the low-pass ***RC*** circuit

2. A 1 kHz symmetrical square wave of peak-to-peak voltage 20 V, as shown in Fig. 1p.17, is applied to a low-pass ***RC*** circuit with R = 100 Ω, C = 1 μF. Sketch the output waveform to scale by determining the corner voltages.

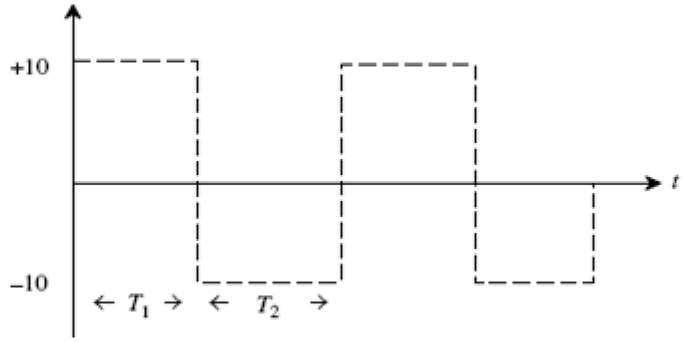

FIGURE 1p.17 The given input to the low-pass ***RC*** circuit

3. The input waveform shown in Fig. 1p.18 is applied to a low-pass ***RC*** network. Sketch the waveform of the voltage across the capacitor to scale for two cycles. The time constant of the ***RC*** circuit is 0.11 ms.

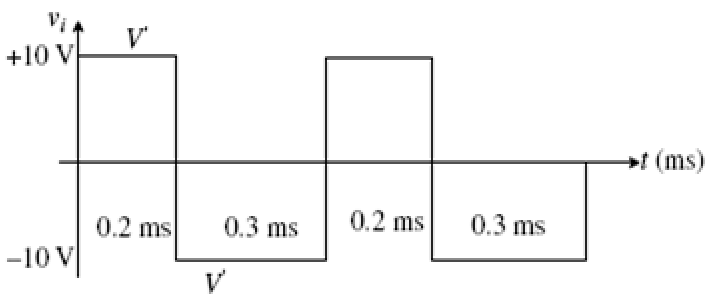

FIGURE 1p.18 The given input to a low-pass ***RC*** circuit

4. The periodic waveform shown in Fig. 1p.19 is applied to an ***RC*** integrating network whose time constant is 15 μs. Sketch the output and calculate the maximum and minimum values of output voltage with respect to the ground.

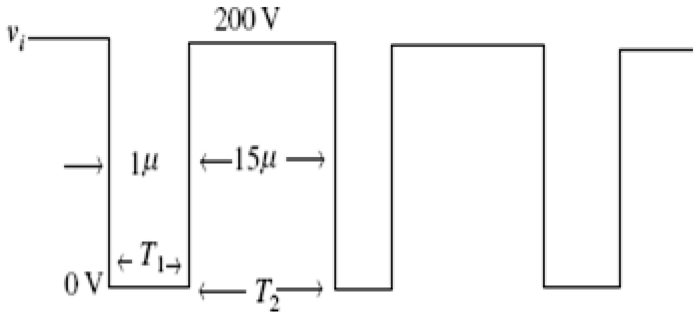

FIGURE 1p.19 The given input to a low-pass $\boldsymbol{RC}$ circuit

5. The waveform shown in Fig. 1p.20 is applied to a low-pass $\boldsymbol{RC}$ circuit. Sketch the output waveform to scale by determining the corner voltages for the following cases: (1) $\boldsymbol{RC} = 20\,\boldsymbol{T}$, (2) $\boldsymbol{RC} = \boldsymbol{T}/20$.

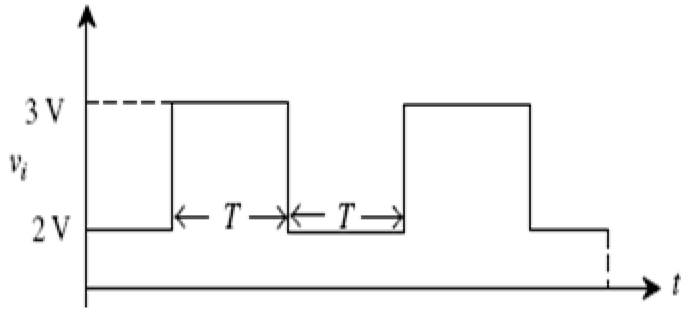

FIGURE 1p.20 The given input to the low-pass $\boldsymbol{RC}$ circuit

6. A square wave extends ±0.5V with respect to ground. The duration of the positive section is 0.1s and the negative duration is 0.2 s, as shown in Fig. 1p.21. This waveform is applied as a input to a low-pass $\boldsymbol{RC}$ circuit whose time constant is 0.2 s, sketch the steady-state output waveform to scale, and find the maximum and minimum values of the output.

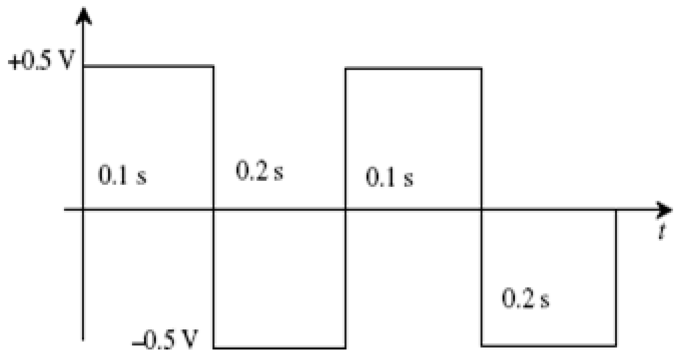

FIGURE 1p.21 The given input to the low-pass $\boldsymbol{RC}$ circuit

7. A pulse of amplitude 10 V and pulse width of 10 μs, as shown in Fig. 1p.22, is applied to an $\boldsymbol{RC}$ circuit with $\boldsymbol{R}$ = 100 kΩ and $\boldsymbol{C}$ = 0.1 μF. Sketch the capacitor voltage waveform to scale.

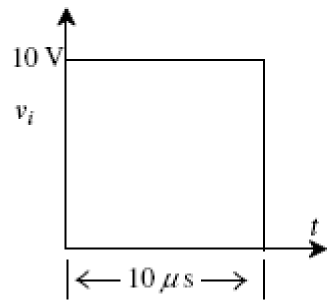

FIGURE 1p.22 The given input to the low-pass ***RC*** circuit

8. An oscilloscope has an input impendence of 10 MΩ in parallel with 25 pF. Design a 10:1 voltage divider for the oscilloscope. Determine the values of the input resistance and the capacitance for the compensated divider.
9. The attenuator shown Fig. 1p.23 has $\boldsymbol{R}_1 = \boldsymbol{R}_2 =$ 1 MΩ, $\boldsymbol{C}_2 =$ 50 pF. Find the magnitudes of the initial and final responses and draw the output waveform to scale for $\boldsymbol{C}_1 =$ 50 pF, 75 pF and 25 pF.

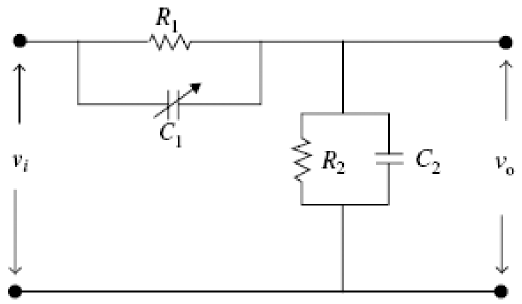

FIGURE 1p.23 The given attenuator circuit

10. The periodic ramp with $\boldsymbol{T}_1 = \boldsymbol{T}_2 = 2\boldsymbol{\tau}$, shown in Fig. 1p.24, is applied to a low-pass ***RC*** circuit. Find the equations to determine the steady-state output waveform. The initial voltage on the condenser is $\boldsymbol{V}_1$. Find the maximum and minimum value of the voltage and plot the waveform.

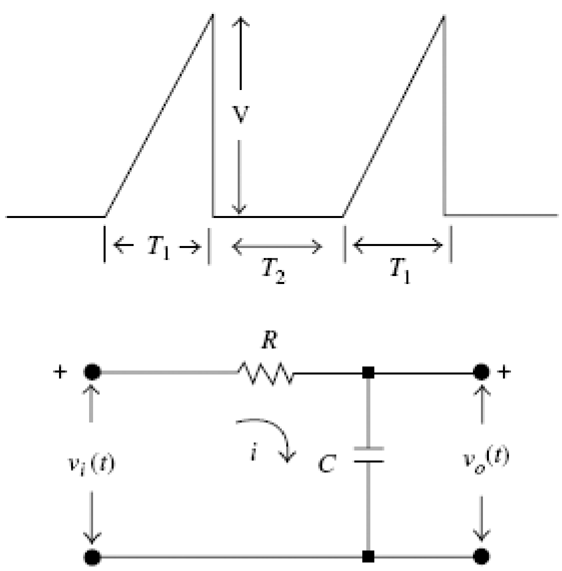

FIGURE 1p.24 The given periodic ramp input and the low-pass circuit

11. For a low-pass $\boldsymbol{RC}$ circuit, it is desired to pass a 3 ms sweep for a ramp input with less than 0.4 per cent transmission error. Calculate the upper3-dB frequency.
12. A step input of 20 V is applied to an $\boldsymbol{RC}$ integrating circuit. Calculate the upper 3-dB frequency and the value of resistance, if the rise time and capacitor values are 100 $\boldsymbol{\mu}$s and 0.28 $\boldsymbol{\mu}$F, respectively.