→ P_e for QPSK :

In QPSK, one of the 4 possible waveforms is transmitted during each interval T. These waveforms are

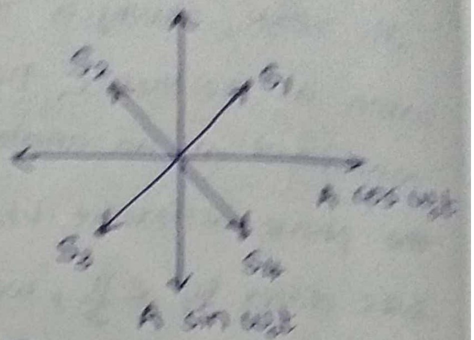


fig ① : Phasor diagram representation of s/gs in QPSK.

$$S_i(t) = A \cos\left[\omega_0 t + (2m-1)\pi/4\right]$$

where $m = 1, 2, 3, 4$

$0 \le t \le T_s = 2T$ → bit duration

↓ symbol duration

A correlation Rxer for QPSK :

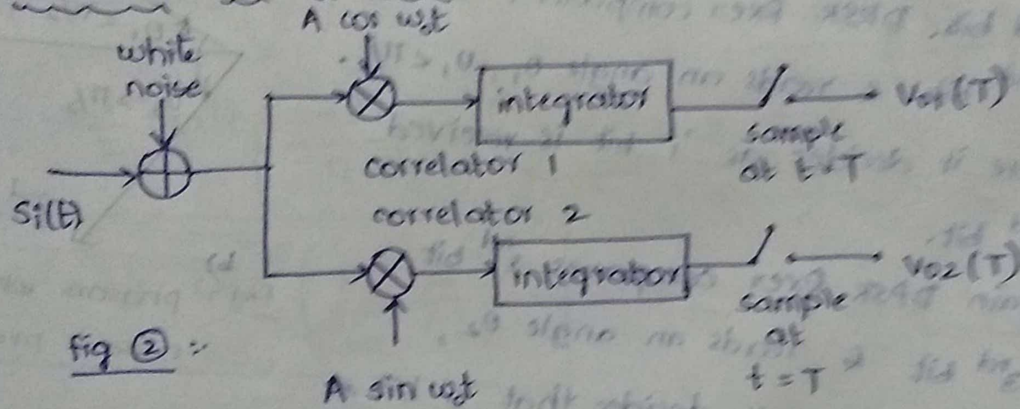


fig ② :

let in the absence of noise, s/g $S_1(t)$ is Rxed. let us use the symbol V_0 to represent the corresponding o/p of correlator 1.

$\therefore V_{01}(T) = V_0$ when $S_1(t)$ is Rxed

o/p of correlator 1, $V_{01}(T) = \frac{1}{\tau} \int_0^T [S_i(t) + n(t)]\, A \cos \omega_0 t \, dt$

let $\tau = 1$ & absence of noise

$$\therefore V_{01}(T) = \int_0^T S_i(t)\, A \cos \omega_0 t \, dt$$

$$= \int_0^T A \cos\left(\omega_0 t + (2m-1)\pi/4\right) A \cos \omega_0 t \, dt$$

$$= \frac{A^2}{2} \int_0^T \left[\cos\left(2\omega_0 t + (2m-1)\pi/4\right) + \cos (2m-1)\pi/4 \right] dt$$

$$= \frac{A^2}{2} \left[\frac{\sin\left(2\omega_0 t + (2m-1)\pi/4\right)}{2\omega_0} + t \cos (2m-1)\pi/4 \right]_0^T$$

$\Rightarrow V_{01}(T) = \frac{A^2}{2}\left[T\cos(2m-1)\pi/4\right]$

$= \frac{A^2T}{2}\cos(2m-1)\pi/4 = \sqrt{2}V_0 \cos(2m-1)\pi/4$

where $V_0 = \frac{A^2T}{2\sqrt{2}}$

$\therefore \boxed{V_{01}(T) = \sqrt{2}V_0\cos(2m-1)\pi/4}$ where $V_0 = \frac{A^2T}{2\sqrt{2}}$

lairly o/p of correlator - 2, $V_{02}(T) = \frac{1}{T}\int_0^T (s_i(t)+n(t))\,A\sin\omega_0 t\,dt$

let $\tau = 1$ & absence of noise,

then $V_{02}(T) = \int_0^T A\cos\left(\omega_0 t + (2m-1)\pi/4\right) A\sin\omega_0 t\,dt$

$= \frac{A^2}{2}\left[\int_0^T \sin\left(2\omega_0 t + (2m-1)\pi/4\right) + \sin\left(-(2m-1)\pi/4\right)\right]dt$

$= \frac{A^2}{2}\int_0^T \left(\sin\left(2\omega_0 t + (2m-1)\pi/4\right) - \sin(2m-1)\pi/4\right)dt$

$= \frac{A^2}{2}\left[\frac{-\cos\left(2\omega_0 t + (2m-1)\pi/4\right)}{2\omega_0} - t\sin(2m-1)\pi/4\right]_0^T$

$= \frac{A^2}{2}\left[-T\sin(2m-1)\pi/4\right]$

$\Rightarrow V_{02}(T) = -\frac{A^2}{2}T\sin(2m-1)\pi/4 = -\sqrt{2}V_0\sin(2m-1)\pi/4$

where $V_0 = \frac{A^2T}{2\sqrt{2}}$

$\therefore \boxed{V_{02}(T) = -\sqrt{2}V_0\sin(2m-1)\pi/4}$ where $V_0 = \frac{A^2T}{2\sqrt{2}}$

$\therefore$ Correlator 1 o/p $\quad V_{01}(T) = \sqrt{2}V_0\cos(2m-1)\pi/4$

correlator 2 o/p $\quad V_{02}(T) = -\sqrt{2}V_0\sin(2m-1)\pi/4$

for $V_0 = \frac{A^2T}{2\sqrt{2}}$

$\therefore$ The o/p of correlators according to each of the 4 possible s/gs are as follows :

o/p	signals $s_1(t)$	$s_2(t)$	$s_3(t)$	$s_4(t)$
$V_{01}(T)$	$+V_0$	$-V_0$	$-V_0$	$+V_0$
$V_{02}(T)$	$-V_0$	$-V_0$	$+V_0$	$+V_0$

But in the presence of noise, there will be some probability that an error will occur by one or both correlators.

From fig ②, the reference waveform of correlator - 1 is at an angle of $\phi = 45°$ to the axes of orientation to all of the 4 possible s/gs. Hence probability that correlator - 1 makes an error is $P'_{e_1} = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_s \cos^2\phi}{\eta}}$

$\because \phi = 45° \Rightarrow \cos\phi = \cos 45° = \frac{1}{\sqrt{2}}$

$\Rightarrow \cos^2 45° = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$

$$= \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_s}{2\eta}}$$

$$\Rightarrow P'_{e_1} = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{A^2 T}{2\eta} \times \frac{1}{2}} \quad \left\{ \because E_s = \frac{A^2 T}{2} \right\}$$

Now similarly the probability that correlator 2 makes an error is $P'_{e_2} = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{A^2 T}{4\eta}}$

let $P'_{e_1} = P'_{e_2} = P'_e$

$\therefore$ The probability that QPSK Rxer will correctly identify the Txed s/g is $P_c = \underbrace{(1 - P'_e)}_{\text{probability that correlator - 1 yields correct result}} \underbrace{(1 - P'_e)}_{\text{probability that correlator - 2 yields correct result}}$

$$\Rightarrow P_c = (1 - P'_e)(1 - P'_e)$$

$$= 1 + P'^2_e - 2P'_e$$

Now $P'_e << 1 \Rightarrow P'^2_e \simeq 0$

$$\therefore P_c = 1 - 2P'_e$$

$$\Rightarrow \boxed{P_c = 1 - 2P'_e}$$

Probability that QPSK system detects an error,
ie, Probability of error, P_e

$$\boxed{P_e = 1 - P_c}$$

$$= 1 - [1 - 2P'_e]$$

$$\Rightarrow P_e = 2P'_e$$

$$\therefore P_e = 1 - P_c = 1 - [1 - 2P_e'] = 2P_e'$$

$$= 2 \times \frac{1}{2} \operatorname{erfc}\sqrt{\frac{A^2 T}{4\eta}}$$

$$\Rightarrow \boxed{P_e = \operatorname{erfc}\sqrt{\frac{A^2 T}{4\eta}}}$$

→ Note :-

$$P_e|_{BPSK} = \frac{1}{2} \operatorname{erfc}\sqrt{\frac{E_s}{\eta}} \quad ; \quad P_e|_{QPSK} = \operatorname{erfc}\sqrt{\frac{A^2 T}{4\eta}} = \operatorname{erfc}\sqrt{\frac{E_s}{2\eta}}$$

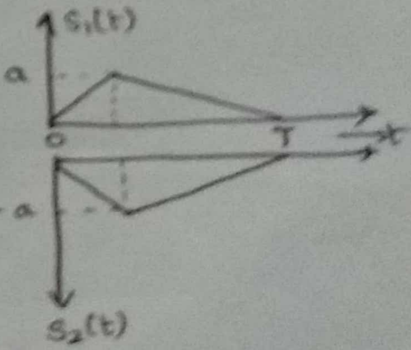
when $E_s = 0 \Rightarrow \operatorname{erfc}(0) = 1$, then

$$P_e|_{BPSK} = \frac{1}{2} \times 1 = \frac{1}{2} = 0.5 \quad ; \quad P_e|_{QPSK} = 1 \times 1 = 1 = 1$$

$$\therefore \boxed{P_e|_{QPSK} >> P_e|_{BPSK}}$$

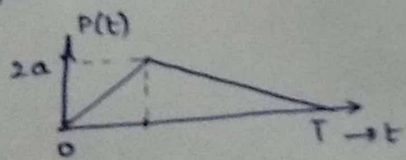
But QPSK has the advantage, that using QPSK, we can transmit 2 bits simultaneously.

→ Let $s_1(t)$ and $s_2(t)$ are triangular waveforms as shown in fig. which are satisfying optimum condition $s_1(t) = -s_2(t)$ then find P(t) and by rotating P(t) over the ordinate obtain P(-t) and by delaying this by '1' bit interval T sec. Obtain P(T-t) which is impulse response for matched filter

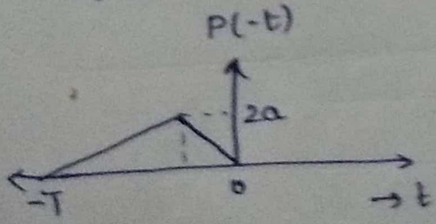


Ans) $P(t) = s_1(t) - s_2(t)$

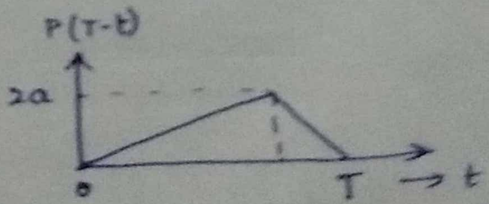
$$P(T-t) = P[-(t-T)]$$



$\therefore$ P(-t)



$$P(T-t) = P[-(t-T)]$$



P(T-t) which is the impulse response for a matched filter.