ANTENNAS & WAVE PROPAGATION

by
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- In simple words, we can state the RECIPROCITY THEOREM as when the places of voltage and current source in any network are interchanged the amount or magnitude of current and voltage flowing in the circuit remains the same.

EQUALITY OF DIRECTION PATTERN

- Tansmitting antenna

- The directional pattern of transmitting antenna is polar characteristics that indicates the strength of radiated field at a fixed distance in different directions in space.

- Receiving antenna

- The directional pattern of receiving antenna is polar characteristics that indicates the response of antenna to unit field strength from different directions in space.

Equality of Directional Patterns. The directional pattern of a receiving antenna is identical with its directional pattern as a transmitting antenna.

Proof: This theorem results directly from an application of the reciprocity theorem. The directional pattern of a transmitting antenna is the polar characteristic that indicates the strength of the radiated field at a fixed distance in different directions in space. The directional pattern of a receiving antenna is the polar characteristic that indicates the response of the antenna to unit field strength from different directions. The pattern as a transmitting antenna could be measured as indicated in Fig. 11-2 by means of a short exploring dipole moved about on the surface of

CASE 1: TEST ANTENNA AS TRANSMITTING ANTENNA
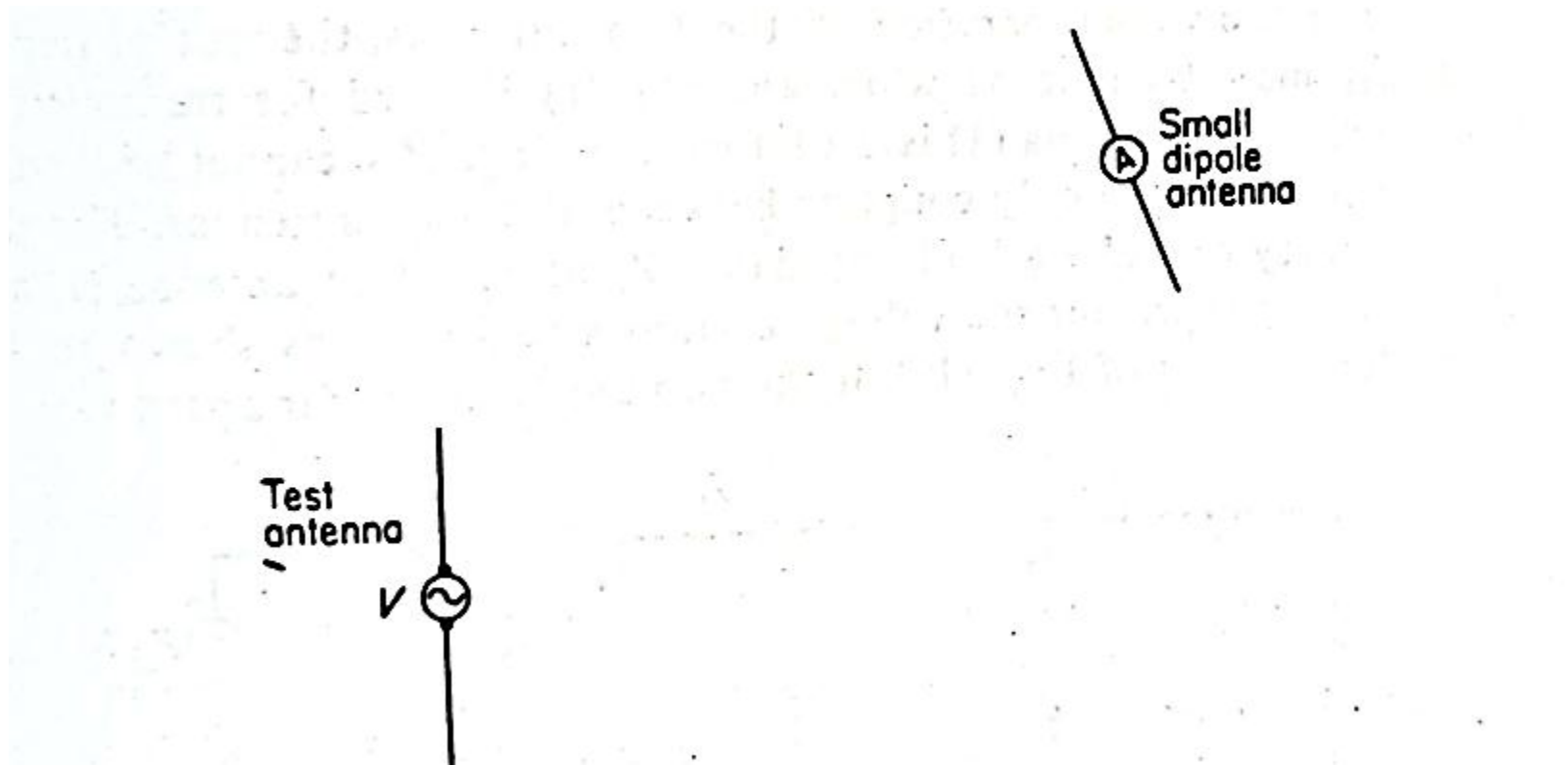
CASE 2: TEST ANTENNA AS RECEIVING ANTENNA
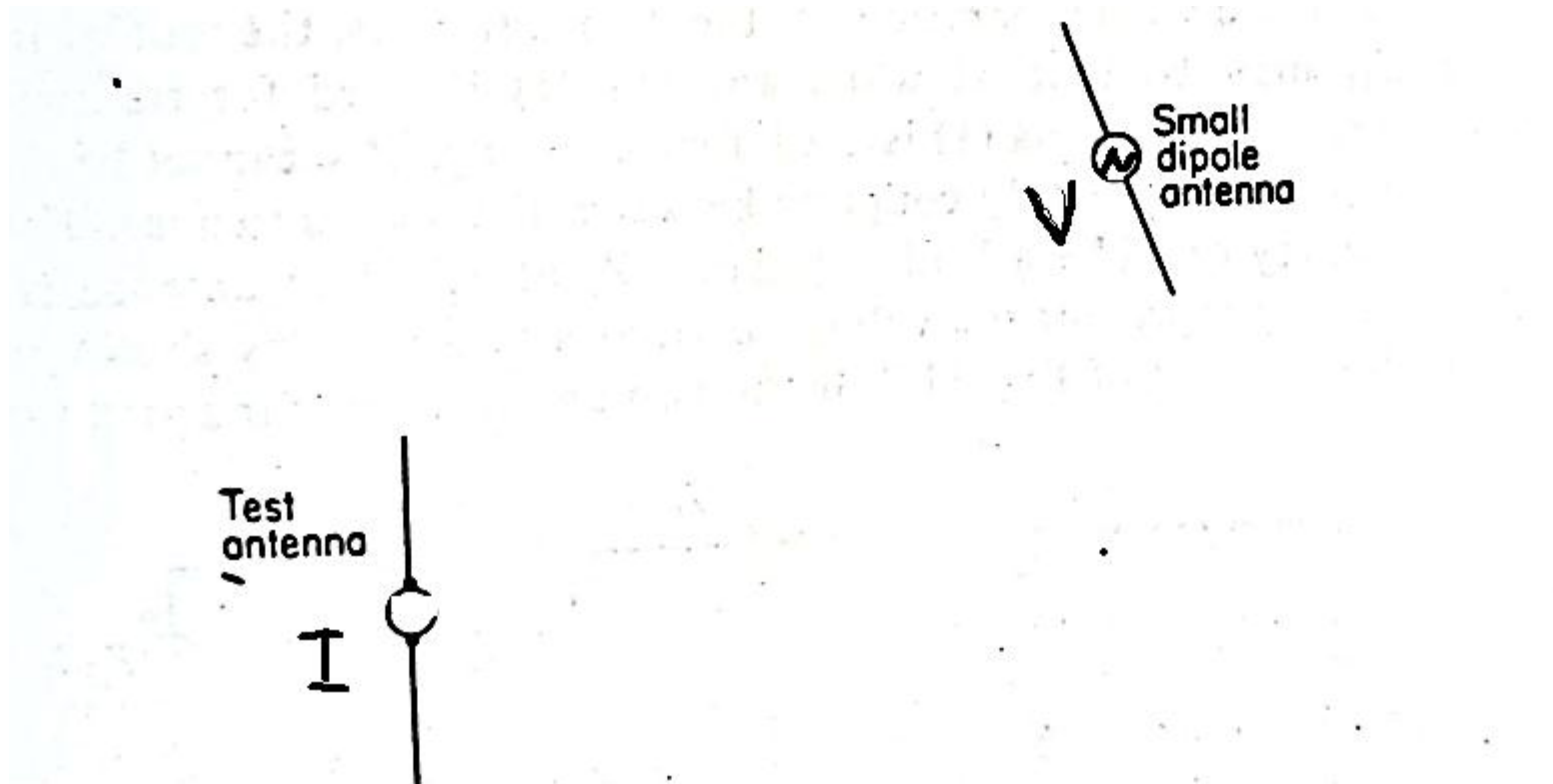
a large sphere centered at the antenna under test. (For the case of linear polarization, the exploring dipole is always oriented so as to be perpendicular to the radius vector and parallel to the electric vector.) A voltage V is applied to the test antenna, and the current I flowing in the short dipole antenna will be a measure of the electric field at the position of the dipole antenna. If then the voltage V is applied to the dipole and the test antenna current is measured, the receiving pattern of the test antenna can be obtained. But by the reciprocity theorem, for every location of the probe antenna, the ratio of V to I is the same as before. Therefore the radiation pattern as a receiving antenna will be identical with the pattern as a transmitting antenna.
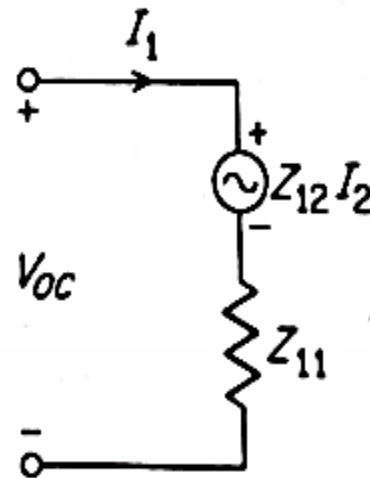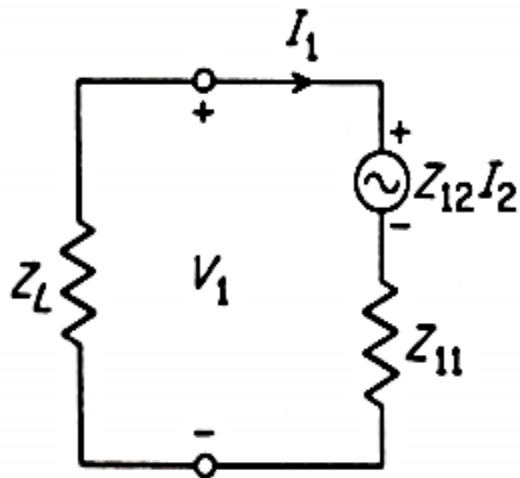
CASE 1: TEST ANTENNA AS TRANSMITTING ANTENNA

Equivalence of Transmitting and Receiving Antenna Impedances. The impedance of an isolated antenna when used for receiving is the same as when used for transmitting.

Proof: This theorem is particularly easy to prove for the case of two antennas which are widely separated. If antenna (2) is far from antenna (1), the self-impedance Z_{S1} of antenna (1) is given by

$$Z_{S1} = \frac{V_1}{I_1} = Z_{11}$$

Because of the wide separation of the two antennas, the mutual impedance Z_{12} may be ignored when antenna (1) is used for transmitting. However, when antenna (1) is used for receiving, Z_{12} cannot be ignored since it provides the only coupling between the two antennas. For this case one may consider a load impedance Z_L attached to antenna (1) and also one may represent the voltage $Z_{12} I_2$ as a generator as shown in the equivalent circuit of Fig. 11-3. If the two antennas are far apart, varying

CASE 2: TEST ANTENNA AS RECEIVING ANTENNA

$$V_{oc} = Z_{12} I_2$$
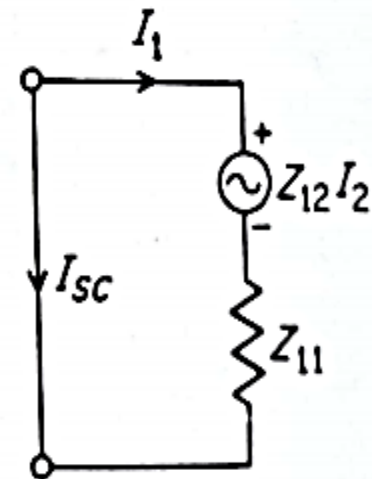
$$I_{sc} = \frac{Z_{12} I_2}{Z_{11}}$$

$$V_{oc} = Z_{11} I_{sc}$$

CASE 1: TEST ANTENNA AS TRANSMITTING ANTENNA

Equality of Effective Lengths. The *effective length, l_{eff},* of an antenna is a term used to indicate the effectiveness of the antenna as a radiator or collector of electromagnetic energy. The significance of the term as applied to transmitting antennas is illustrated in Fig. 11-4. The effective
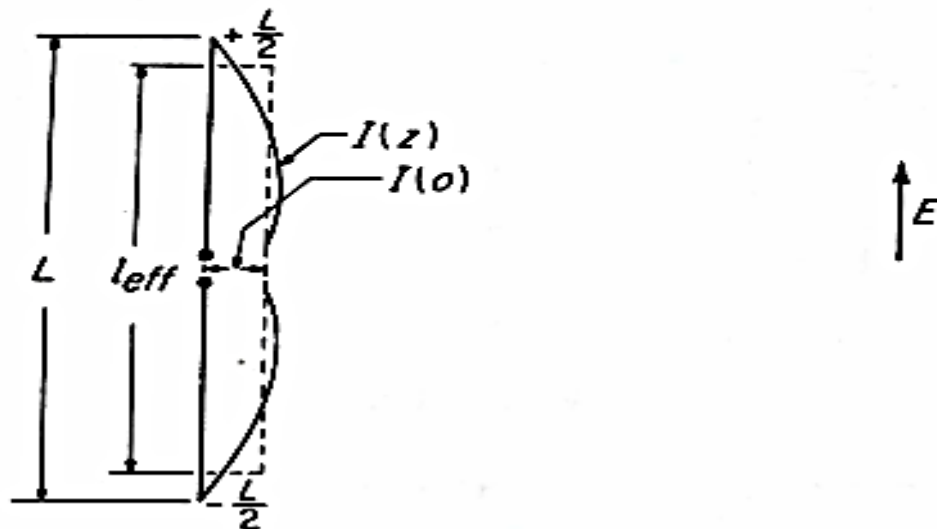

Figure 11-4.

length of a transmitting antenna is that length of an equivalent linear antenna that has a current $I(0)$ at all points along its length and that radiates the same field strength as the actual antenna in the direction perpendicular to its length. $I(0)$ is the current at the terminals of the actual antenna. That is, for transmitting

$$I(0)l_{eff}(\text{trans}) = \int_{-L/2}^{+L/2} I(z)\, dz$$

or

$$l_{eff}(\text{trans}) = \frac{1}{I(0)} \int_{-L/2}^{+L/2} I(z)\, dz \tag{11-3}$$

The effective length of a receiving antenna is defined in terms of the open-circuit voltage developed at the terminals of the antenna for a given received field strength E. That is, for receiving

$$l_{eff}(\text{rec}) = -\frac{V_{oc}}{E} \tag{11-4}$$

where V_{oc} is the open-circuit voltage at the antenna terminals produced

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CASE 2: TEST ANTENNA AS RECEIVING ANTENNA

by a uniform exciting field E volts per meter parallel to the antenna. [The minus sign is used in (4) to conform with conventional polarity markings (upper terminal positive).]

The equality of transmitting and receiving effective lengths may be deduced by application of the reciprocity theorem to the two cases as shown in Fig. 11-5.
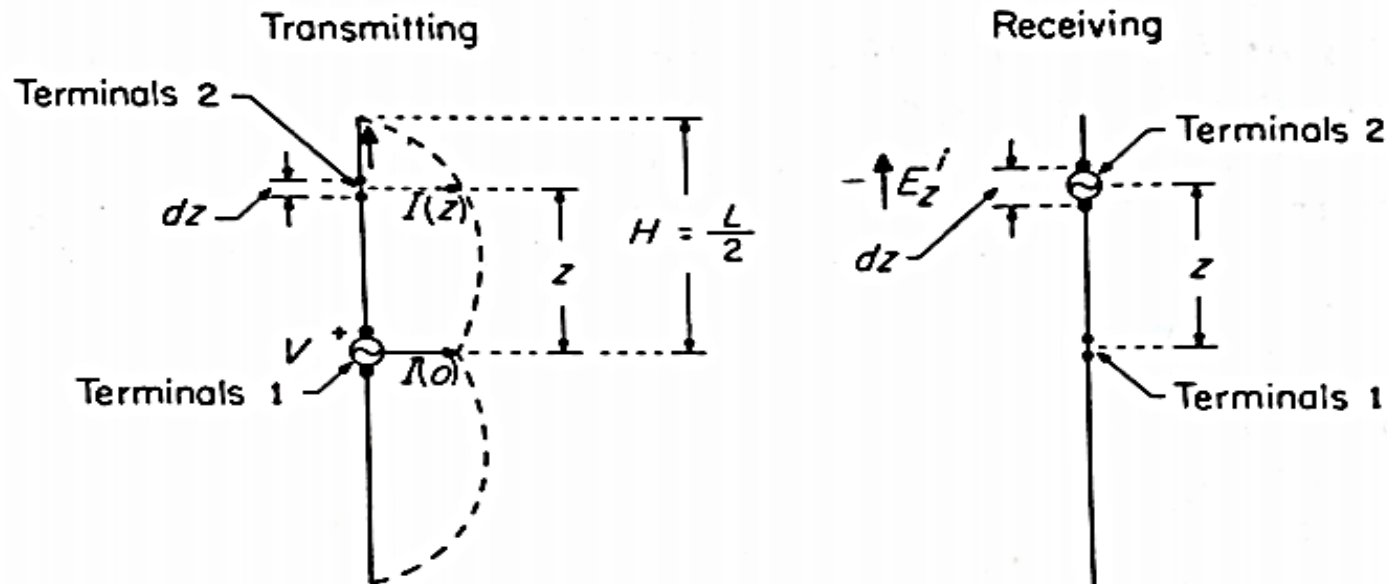
First consider the transmitting antenna case [Fig. 11-5(a)]. A voltage V applied at the terminals produces a current $I(0) = V/Z_a$ at the terminals and a current $I(z)$ at any point z along the antenna. Z_a is the antenna impedance measured at the terminals. This is the "prime" situation.

Next consider the same antenna for the receiving case [Fig. 11-5(b)] in which an electromagnetic field E_z^i is incident upon the antenna. As a result of this field a voltage $E_z^i\,dz$ is impressed at or induced in the element dz. Because this impressed voltage is independent of the current that flows in the antenna, it can be represented by a zero-impedance generator of voltage $E_z^i\,dz$ in series at the point z. With the antenna base terminals short-circuited, the voltage $E_z^i\,dz$ at z will produce a current dI_{sc} at the terminals. This is the "double-prime" situation.

Putting $V = V_1'$, $I(z) = I_2'$, $E_z^i\,dz = V_2''$, and $dI_{sc} = I_1''$,

the reciprocity theorem (eq. 11-2) gives

$$\frac{V}{I(z)} = \frac{E_z^i\,dz}{dI_{sc}} \qquad \text{or} \qquad dI_{sc} = \frac{E_z^i\,dz}{V}\,I(z)$$

By superposition the total short-circuit current produced at the antenna terminals by all of the differential voltages impressed along the entire length of the antenna will be

$$I_{sc} = \frac{1}{V} \int E_z^i I(z) \, dz \tag{11-5}$$

Knowing the short-circuit current, the open-circuit voltage at the terminals can be determined from Thevenin's theorem:

$$V_{oc} = -I_{sc} Z_a = -\frac{Z_a}{V} \int E_z^i I(z) \, dz = -\frac{1}{I(0)} \int E_z^i I(z) \, dz \tag{11-6}$$

where again the minus sign is chosen to conform with conventional polarity markings (upper terminal positive). For an incident field $E_z^i = E_z$ constant along the length of the antenna

$$V_{oc} = -\frac{E_z}{I(0)} \int I(z) \, dz$$

so that

$$\frac{V_{oc}}{E_z} = -\frac{1}{I(0)} \int I(z) \, dz$$

Therefore from (3) and (4) the effective length of an antenna for receiving is equal to its effective length as a transmitting antenna.

Antenna Gain. The ability of an antenna or antenna array to concentrate the radiated power in a given direction, or conversely to absorb effectively incident power from that direction, is specified variously in terms of its (antenna) gain, power gain, directive gain or directivity.
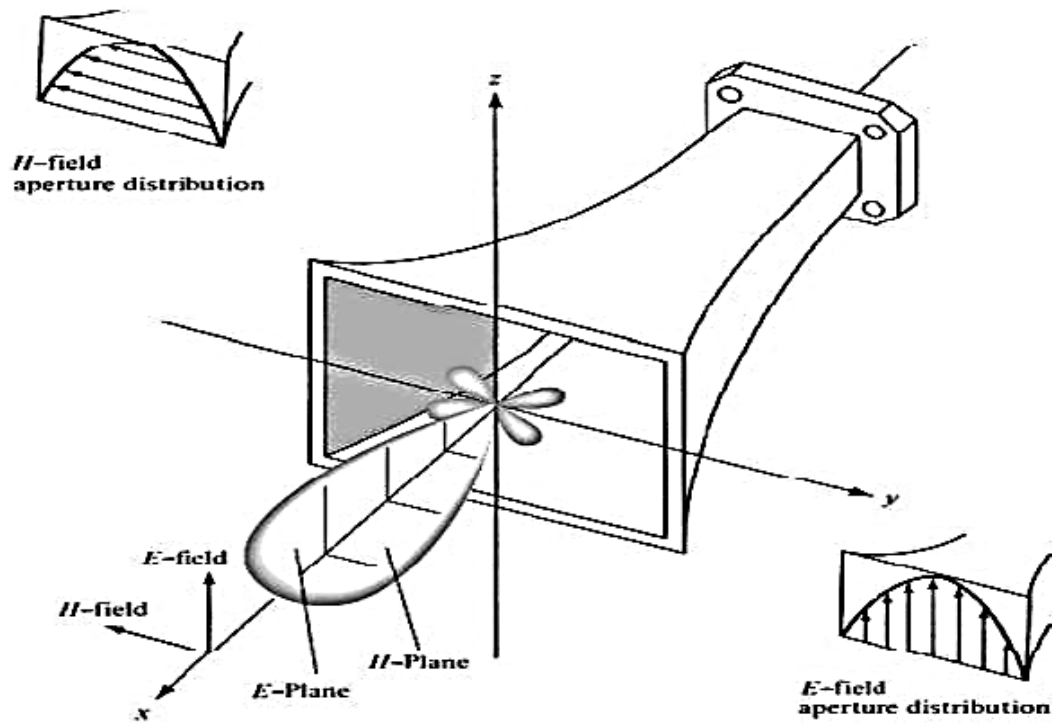
Fig. Principal E- and H-plane patterns for a pyramidal horn antenna

The power radiated per unit area in any direction is given by the Poynting vector **P**. For the distant or radiation field for which **E** and **H** are orthogonal in a plane normal to the radius vector, and for which $E = \eta_v H$, the power flow per unit area is given by†

$$P = \frac{E^2}{\eta_v} \qquad \text{watts/sq m} \quad (11\text{-}17)$$

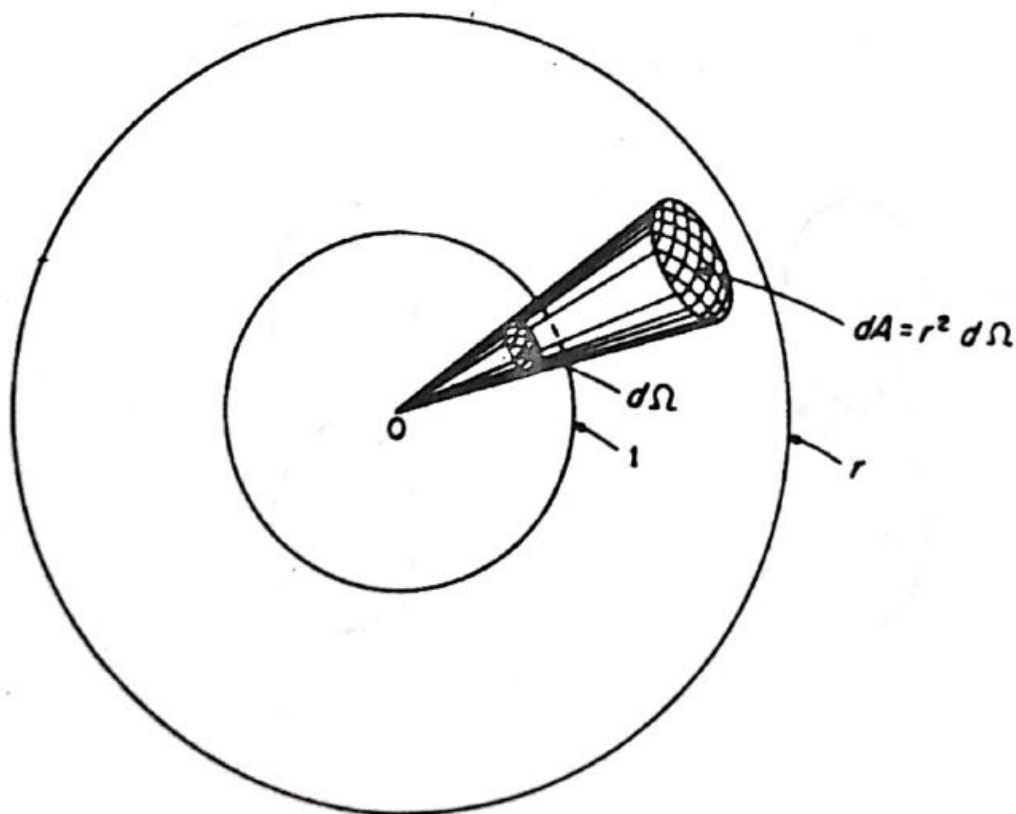Referring to Fig. 11-24, noting that there are r^2 square meters of

Figure 11-24.

(a) Radian

(b) Steradian

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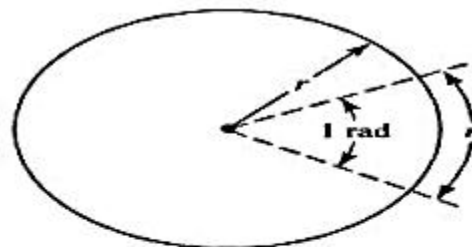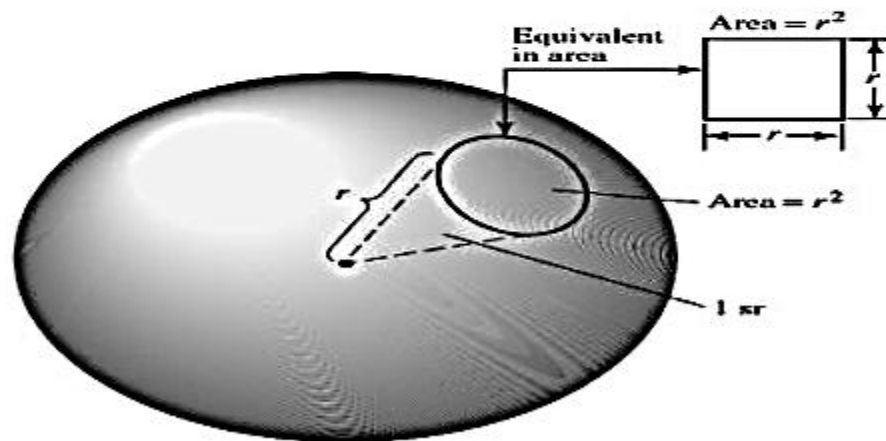
surface area per unit solid angle (or sterradian), and defining the *radiation intensity* $\Phi(\theta, \phi)$ in a given direction as the power per unit solid angle in that direction, we see that

$$\Phi(\theta, \phi) = r^2 P = \frac{r^2 E^2}{\eta_v} \qquad \text{watts/unit solid angle} \quad (11\text{-}18)$$

It should be noted that radiation intensity is independent of r. The total power radiated is

$$W_r = \int \Phi \, d\Omega \qquad \text{watts}$$

and since there are 4π sterradians in the total solid angle, the average power radiated per unit solid angle is

$$\frac{W_r}{4\pi} = \Phi_{av} \qquad \text{watts/sterradian}$$

Φ_{av} represents the radiation intensity that would be produced by an *isotropic* radiator (one that radiates uniformly in all directions) radiating the same total power W_r.

The *directive gain* g_d, *in a given direction*, is defined as the ratio of the radiation intensity in that direction to the average radiated power. That is,

$$g_d(\theta, \phi) = \frac{\Phi(\theta, \phi)}{\Phi_{av}} = \frac{\Phi(\theta, \phi)}{W_r/4\pi} = \frac{4\pi\Phi(\theta, \phi)}{W_r} \qquad (11\text{-}19a)$$

$$= \frac{4\pi\Phi(\theta, \phi)}{\int \Phi \, d\Omega} \qquad (11\text{-}19b)$$

When expressed in decibels the directive gain is denoted by G_d where

$$G_d = 10 \log_{10} g_d \qquad (11\text{-}20)$$

The *directivity, D*, of an antenna is its maximum directive gain. Whereas the directive gain is a function of angles (θ, ϕ) which should be specified, the directivity is a constant, having been specified for a particular direction.

Efficiency of the antenna

If total input power W_t is used in expression (11-19a) instead of radiated power W_r, the result is power gain rather than directive gain. The *power gain*, g_p, is defined by

$$g_p = \frac{4\pi\Phi}{W_t} \qquad (11\text{-}21)$$

where $W_t = W_r + W_l$, W_l being the ohmic losses in the antenna. It

$$\frac{g_p}{g_d} = \frac{W_r}{W_r + W_l} \qquad (11\text{-}22)$$

is a measure of the efficiency of the antenna. For many well-constructed antennas the efficiency is nearly 100 per cent, so that power gain and directive gain are nearly equal, a fact which has led to the rather loose use of the term *antenna gain* (designated as g without subscript) for either g_d or g_p. For electrically small antennas and for super-directive antennas, power gain may be very much less than directive gain, and in these circumstances careful distinction between the two should be made.

The *effective area* or *effective aperture* of an antenna is defined in terms of the directive gain of the antenna through the relation

$$A = \frac{\lambda^2}{4\pi} g_d \qquad (11\text{-}26)$$

Using this relation it can be shown (problem 3) that the effective area is the ratio of power available at the antenna terminals to the power per unit area of the *appropriately polarized* incident wave. That is

$$W_R = PA \qquad \text{watts} \qquad (11\text{-}27)$$

where W_R is the received power and P is the power flow per square meter for the incident wave. When directive gain g_d is used in (11-26) it is assumed that all of the available power is delivered to the load.

Basic Antenna Elements

he basic antenna elements are

1. Alternating current element or Hertzian dipole
2. Short dipole
3. Short monopole
4. Half-wave dipole
5. Quarter-wave monopole

1. **Alternating Current Element or Hertzian Dipole**   It is a short linear antenna in which the current along its length is assumed to be constant.

2. **Short Dipole**   It is a linear antenna whose length is less than $\frac{\lambda}{4}$ and the current distribution is assumed to be triangular.

3. **Short Monopole**   It is a linear antenna whose length is less than $\frac{\lambda}{8}$ and the current distribution is assumed to be triangular.

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The current distribution for dipole of length $l \ll \lambda$
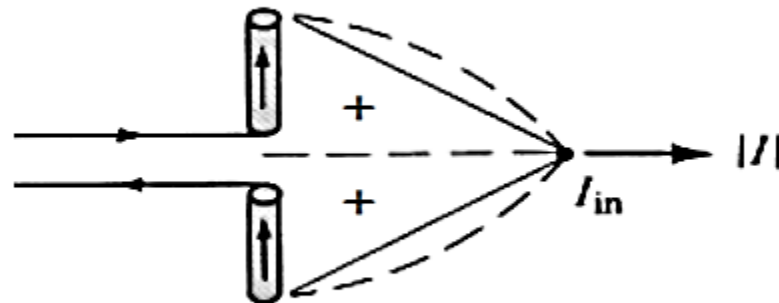
(a) $l \ll \lambda$

4. **Half-wave Dipole**   It is a linear antenna whose length is $\frac{\lambda}{2}$ and the current distribution is assumed to be sinusoidal. It is usually centre-fed.

5. **Quarter-wave Monopole**   It is a linear antenna whose length is $\frac{\lambda}{4}$ and the current distribution is assumed to be sinusoidal. It is fed at one end with respect to earth.
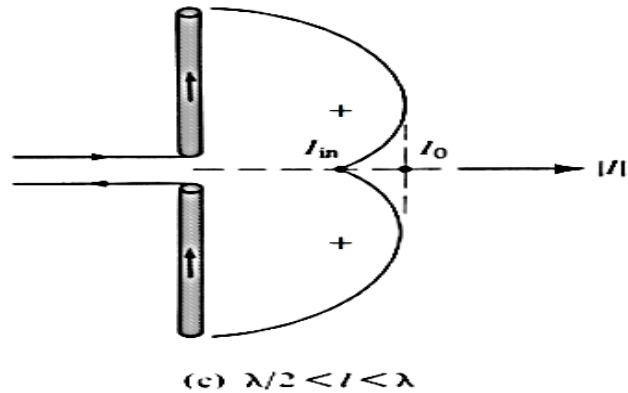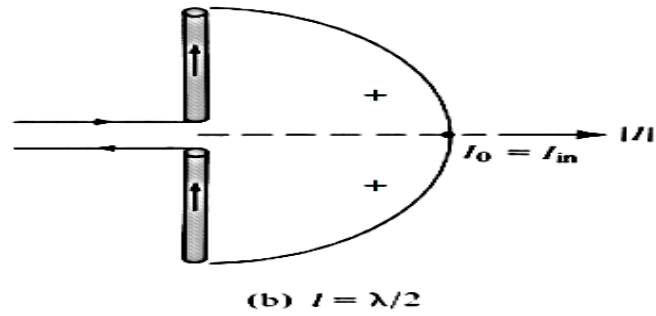
For $l = \lambda/2$

(b) $l = \lambda/2$

(c) $\lambda/2 < l < \lambda$

RADIATION FIELDS OF ALTERNATING CURRENT ELEMENT (OR OSCILLATING ELECTRIC DIPOLE)

The concept of an alternating current element, $I\,dl \cos \omega t$ is of theoretical interest. But the theory developed for this can be extended to practical antennas. The concept of retarded vector magnetic potential A is very useful to derive radiation fields of antenna elements including current element.

Derivation of radiation fields consists of the following steps:

1. Write expression for retarded vector magnetic potential.

2. Write expressions for the components of A in Cartesian coordinates.

3. Express A in components of spherical coordinate system.

4. Obtain the components of H from $\mu H = \nabla \times A$.

5. Obtain the components of E from

$$\dot{E} = \frac{\partial E}{dt} = \frac{1}{\epsilon} \nabla \times H \qquad \text{(as } J = 0 \text{ for free space)}$$

6. $E = \dfrac{1}{\epsilon} \int (\nabla \times H)\, dt.$

7. Identify radiation and near-field terms.

An alternating current element at the origin of a spherical coordinate system is shown in Fig. 3.6.

The retarded vector magnetic potential, $A\,(r,\ t)$ is given by

$$A\,(r,\ t) = \dfrac{\mu}{4\pi} \int_{v} \dfrac{J\,(r,\ t - r/v_0)}{r}\, dv \qquad \ldots(3.1)$$

As the element is z-directed, A is also z-directed. $\dfrac{r}{v_0}$ is the delay time.

Figure 10-1.

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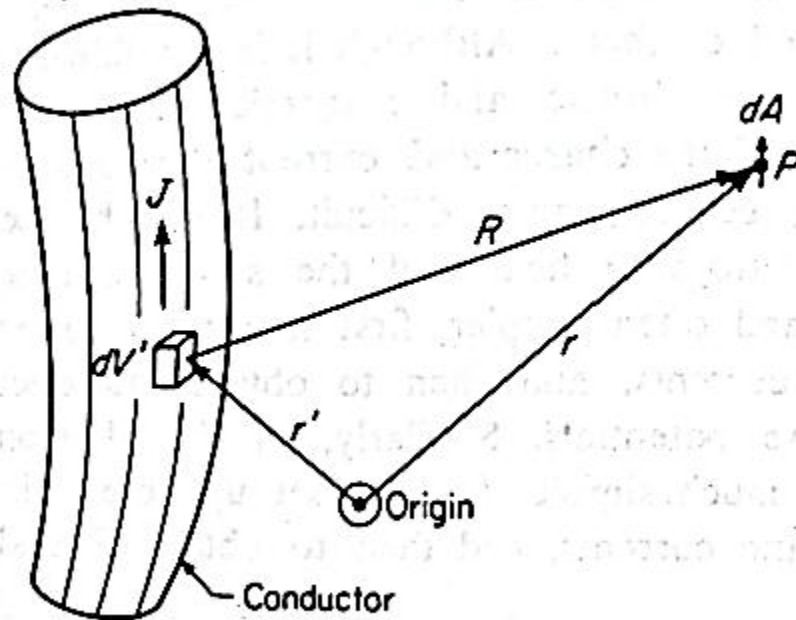
The sources of the *electromagnetic* field are current and charge distributions that vary with time, so it is reasonable to try these same potentials, generalized for time, variations. That is, we might expect to be able to write

$$A(r, t) = \frac{\mu}{4\pi} \int \frac{J(r', t)}{R} dV' \qquad (10\text{-}1)$$

$$V(r, t) = \frac{1}{4\pi\epsilon} \int \frac{\rho(r', t)}{R} dV' \qquad (10\text{-}2)$$

in which $R = |r - r'|$. As written above the strengths of these potentials vary instantaneously with the strengths of the sources. However, the principal result of the time-varying theory (wave theory) discussed so far is the appearance of a finite propagation time for electromagnetic waves. It is logical that the potentials from which the fields are to be

derived should also display finite propagation time. This can be readily accomplished by modifying the above relations to give

$$A(\mathbf{r}, t) = \frac{\mu}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}', t - R/v)}{R} \, dV' \qquad (10\text{-}3)$$

$$V(\mathbf{r}, t) = \frac{1}{4\pi\epsilon} \int \frac{\rho(\mathbf{r}', t - R/v)}{R} \, dV' \qquad (10\text{-}4)$$

In these expressions a time delay of R/v seconds has been introduced, so that now the potentials have been delayed or retarded by this amount. For this reason they are often called *retarded potentials.*

Figure 10-2 shows an alternating-current element $I\,dl\cos\omega t$ located at the origin of a spherical co-ordinate system. The problem is to calculate the electromagnetic field at an arbitrary point P.
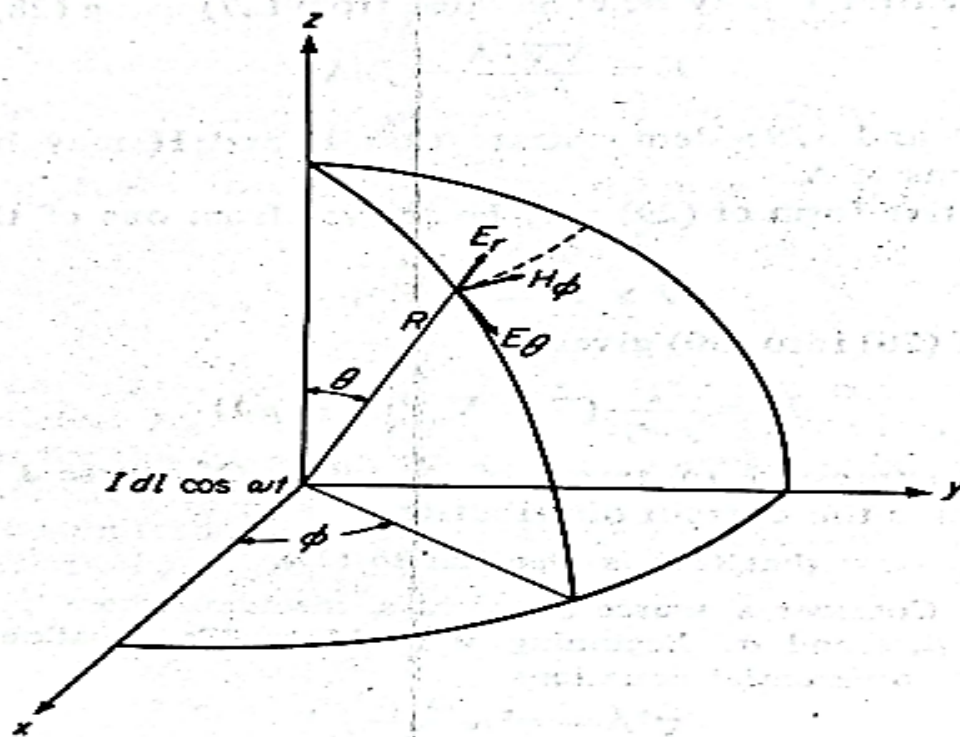
Figure 10-2. A current element at the center of a spherical co-ordinat...

The first step is to obtain the vector potential **A** at *P*. The general expression for **A** is given by

$$A(r) = \frac{\mu}{4\pi} \int \frac{J\left(t - \frac{r}{v}\right)}{R} \, dV' \qquad (10\text{-}34)$$

The integration over the volume in (34) consists of an integration over the cross-sectional area of the wire and an integration along its length. The current density **J**, integrated over the cross-sectional area of the wire, is just the current *I*, and because this is assumed to be constant

along the length, integration over the length gives $I\,dl$. Therefore in this simple example the expression for vector potential becomes*

$$A_z = \frac{\mu}{4\pi} \frac{I\,dl\,\cos\omega\left(t - \frac{r}{v}\right)}{r} \qquad (10\text{-}35)$$

The vector potential has the same direction as the current element, in this case the z direction, and is retarded in time by r/v seconds. The magnetic field strength $\mathbf{H}$ is obtained through the relation

$$\mu\mathbf{H} = \nabla \times \mathbf{A}$$

Reference to the expressions in chap. 1, showing the curl in spherical coordinates, gives the components of $\mathbf{H}$ in terms of A_r, A_θ, and A_ϕ. From Fig. 10-2 it is seen that for this case

$$A_r = A_z \cos\theta; \qquad A_\theta = -A_z \sin\theta; \qquad A_\phi = 0 \qquad (10\text{-}38)$$

here are

etated

potentials

Vector and vector delay

$A_r = \vec{A_2} \cdot \vec{a_r}$

$= A_z \cos\theta$

$A_\theta = -A_2 \sin\theta \quad \{\vec{A_2} \cdot \vec{a_\theta}\}$

$\{A_2 (1) \cos(90+\theta)$

$A_\phi = 0$

$\Rightarrow H = \frac{1}{\mu} (\nabla \times A)$

z and ϕ are $\perp$ in

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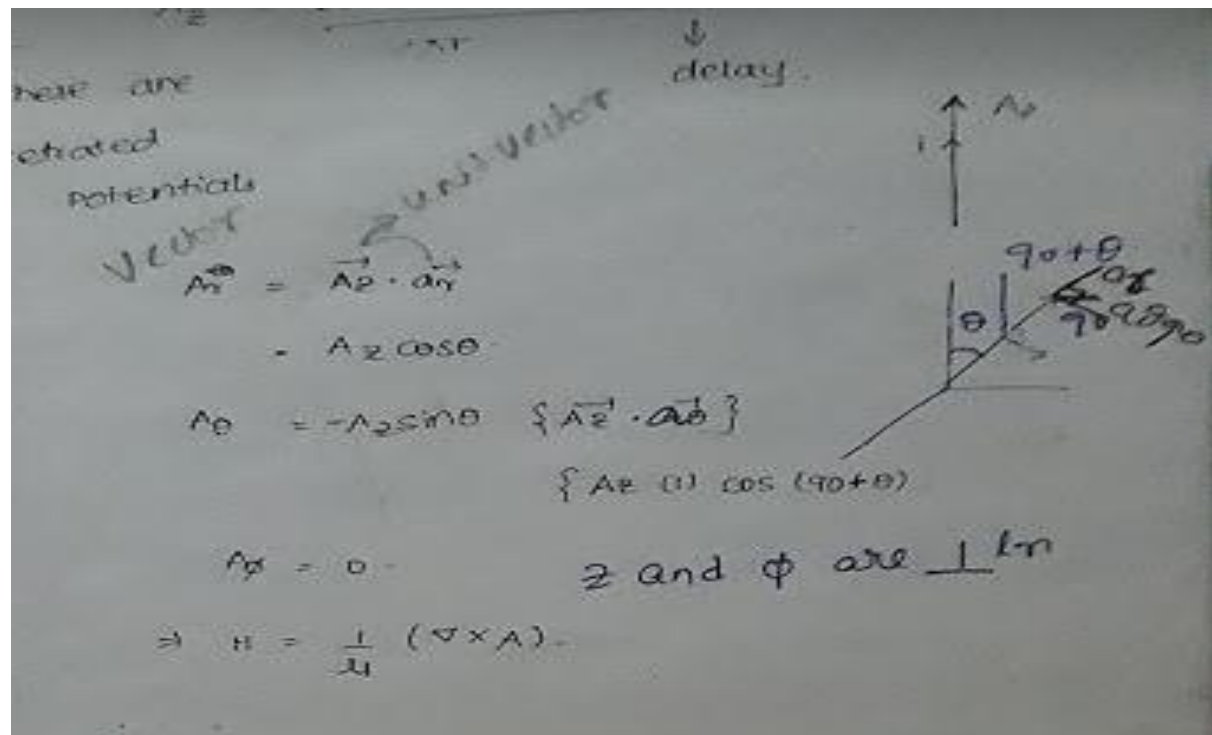
Then from expressions (1-39a, b, and c) on page 15, and noting that because of symmetry, $\partial/\partial\phi = 0$,

$$\mu H_r = (\nabla \times \mathbf{A})_r = 0$$

$$\mu H_\theta = (\nabla \times \mathbf{A})_\theta = 0$$

$$H_\phi = \frac{1}{\mu r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}\right]$$

$$= \frac{I\,dl}{4\pi r}\left\{\frac{\partial}{\partial r}\left[-\sin\theta\cos\omega\left(t - \frac{r}{v}\right)\right] - \frac{\partial}{\partial\theta}\left[\frac{\cos\theta}{r}\cos\omega\left(t - \frac{r}{v}\right)\right]\right\}$$

$$= \frac{I\,dl\,\sin\theta}{4\pi}\left[\frac{-\omega\sin\omega\left(t - \frac{r}{v}\right)}{rv} + \frac{\cos\omega\left(t - \frac{r}{v}\right)}{r^2}\right] \qquad (10\text{-}39)$$

The electric field strength $\mathbf{E}$ can be obtained from $\mathbf{H}$ through Maxwell's first equation, which at the point P (in free space) is

$$\nabla \times \mathbf{H} = \epsilon\dot{\mathbf{E}}$$

$$\mathbf{E} = \frac{1}{\epsilon}\int \nabla \times \mathbf{H}\,dt \qquad (10\text{-}40)$$

Taking the curl of (39) and then integrating with respect to time (the order is immaterial) gives for the components of $\mathbf{E}$,

$$E_\theta = \frac{I\,dl\,\sin\theta}{4\pi\epsilon}\left(-\frac{\omega\sin\omega t'}{rv^2} + \frac{\cos\omega t'}{r^2 v} + \frac{\sin\omega t'}{\omega r^3}\right) \qquad (10\text{-}41)$$

Changing Cartesian components to spherical coordinate components, we get

$$
\left.\begin{aligned}
A_r &= A_z \cos \theta \\
A_\theta &= - A_z \sin \theta \\
A_\phi &= 0
\end{aligned}\right\} \qquad \qquad \text{...(3.3)}
$$

But we know
$$
\mathbf{H} = \frac{1}{\mu_0} \nabla \times \mathbf{A} \qquad \qquad \text{...(3.4)}
$$

$B = \mu H$

$\nabla \times A = \mu H$

So
$$
H_r = \frac{1}{\mu_0} (\nabla \times \mathbf{A})_r
$$

$$
H_\theta = \frac{1}{\mu_0} (\nabla \times \mathbf{A})_\theta
$$

$$
H_\phi = \frac{1}{\mu_0} (\nabla \times \mathbf{A})_\phi
$$

$$H_r = \frac{1}{\mu_0\, r \sin \theta} \left[\frac{\partial}{\partial \theta} (\sin \theta\, A_\phi) - \frac{\partial A_\theta}{\partial \phi} \right]$$

As $\qquad A_\phi = 0$ and $A_\theta \neq f(\phi)$, $H_r = 0$

Similarly, $\qquad H_\theta = \frac{1}{\mu_0} \left[\frac{1}{r \sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{1}{r} \frac{\partial (r A_\phi)}{\partial r} \right] = 0 \qquad\qquad$ [as $A_r \neq f(\phi)$]

$$H_\phi = \frac{1}{\mu_0\, r} \left[\frac{\partial}{\partial r} (r A_\theta) - \frac{\partial A_r}{\partial r} \right] \qquad\qquad ...(3.5)$$

Here $\qquad A_\theta = - A_z \sin \theta$

Here
$$A_\theta = - A_z \sin \theta$$

$$= - \frac{\mu_0}{4\pi} \frac{I\,dl \cos \omega\,(t - r/v_0)}{r} \sin \theta \qquad \ldots(3.6)$$

$$A_r = A_z \cos \theta$$

$$= - \frac{\mu_0}{4\pi} \frac{I\,dl \cos \omega\,(t - r/v_0)}{r} \cos \theta \qquad \ldots(3.7)$$

$$H_\phi = \frac{I\,dl \sin \theta}{4\pi} \left[\frac{-\omega \sin \omega \left(t - \dfrac{r}{v_0} \right)}{r v_0} - \frac{\cos \omega \left(t - \dfrac{r}{v_0} \right)}{r^2} \right] \qquad \ldots(3.8)$$

From Maxwell's first equation, we have

$$\nabla \times \mathbf{H} = \dot{\mathbf{D}} = \epsilon_0 \dot{\mathbf{E}}$$

or

$$\mathbf{E} = \frac{1}{\epsilon_0} \int (\nabla \times \mathbf{H})\, dt \qquad \qquad \qquad \text{...(3.9)}$$

From Equations (3.6) and (3.9), we get E_r, E_θ and E_ϕ

But $\qquad\qquad\qquad E_\phi = 0 \qquad\qquad\qquad\qquad\qquad\qquad$ [as $\mathbf{H} = H_\phi \, \mathbf{a}_\phi$]

On simplification of Equation (3.9), we get

$$E_\theta = \frac{I\, dl \sin \theta}{4\pi \epsilon_0} \left[-\frac{\omega \sin \omega t_d}{r v_0^2} + \frac{\cos \omega t_d}{r^2 v_0} + \frac{\sin \omega t_d}{\omega r^3} \right] \qquad \text{...(3.10)}$$

and

$$E_r = \frac{2I\, dl \cos \theta}{4\pi \epsilon_0} \left[\frac{\cos \omega t_d}{r^2 v_0} + \frac{\sin \omega t_d}{\omega r^3} \right] \qquad\qquad \text{...(3.11)}$$

where

$$t_d = (t - r/v_0) \qquad\qquad\qquad\qquad\qquad \text{...(3.12)}$$

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The **resultant field components** of an alternating current element are

$$H_\phi = \frac{I\,dl\,\sin\theta}{4\pi}\left[-\frac{\omega\sin\omega t_d}{r v_0} + \frac{\cos\omega t_d}{r^2}\right]$$

$$E_\theta = \frac{I\,dl\,\sin\theta}{4\pi\,\epsilon_0}\left[-\frac{\omega\sin\omega t_d}{r v_0^2} + \frac{\cos\omega t_d}{r^2\,v_0} + \frac{\sin\omega t_d}{\omega r^3}\right]$$

$$E_r = \frac{2I\,dl}{4\pi\,\epsilon_0}\cos\theta\left[\frac{\cos\omega t_d}{r^2 v_0} + \frac{\sin\omega t_d}{\omega r^3}\right]$$

$$H_r = 0, \quad H_\theta = 0, \quad E_\phi = 0.$$

$\frac{\partial}{\partial \phi} = 0 \quad$ for symmetry $\qquad$ ④

$A_r \, A_\phi = 0 \qquad \frac{\partial}{\partial \phi} = 0$

$$\nabla \times A = \begin{vmatrix} \overrightarrow{a_r} & \overrightarrow{a_\theta} & \overrightarrow{a_\phi} \\ r\sin\theta & r\sin\theta & r \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi}(o) \\ A_r & r A_\theta & r\sin\theta A_\phi \end{vmatrix}$$

$H_r = \frac{1}{\mu}\left[\nabla \times A\right]_r = 0$

$H\theta = \frac{1}{\mu}\left[\nabla \times A\right]_\theta = 0$

$H_\phi = \frac{1}{\mu}\left[\frac{1}{r}\left\{\frac{\partial}{\partial r} r A_\theta - \frac{\partial}{\partial \theta} A_r\right\}\right]$

$= \frac{1}{\mu r}\left\{\frac{\partial}{\partial r} - \frac{r \mu I dl \cos\omega(t-r/v)\sin\theta}{4\pi r} \quad \frac{\partial}{\partial \theta}\frac{\mu I dl \cos\theta \cos\omega(t-r/v)}{4\pi r}\right\}$

$= \frac{I dl}{4\pi r}\left[\frac{\partial}{\partial r}\frac{\cos\omega(t-r/v)\,\omega\sin\theta}{v} + \frac{\sin\theta \cos\omega(t-r/v)}{r}\right]$

$= \frac{I dl \sin\theta}{4\pi}\left[\frac{-\sin(t-r/v)\,\omega}{rv} + \frac{\cos\omega(t-r/v)}{r^2}\right]$

$= \frac{I dl \sin\theta}{4\pi}\left[\frac{-\omega\sin t\omega}{rv} + \frac{\cos\omega t}{r^2}\right]$

Source: E.M. waves and Radiating Systems
by E. C. JORDAN and K. G. Balmain – PHI ,
New Delhi

Propagation in free space $\quad J = 0$

$$\nabla \times H = \epsilon \dot{E}$$

$$E = \frac{1}{\epsilon} \int (\nabla \times H)\, dt$$

$$= \frac{1}{\epsilon} \int (\nabla \times H)\, dt$$

$$E_r = \int (\nabla \times H)_r\, dt$$

$H_r = 0 \qquad H_\theta = 0 \qquad H_\phi \neq 0$

$$\nabla \times H = \begin{vmatrix} \dfrac{\overline{a_r}}{r^2 \sin\theta} & \dfrac{\overline{a_\theta}}{r \sin\theta} & \dfrac{\overline{a_\phi}}{r} \\[2mm] \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial \phi} \\[2mm] 0 & 0 & r\sin\theta\, H_\phi \end{vmatrix}$$

$$(\nabla \times H)_r = \frac{1}{r^2 \sin\theta} \left[\frac{\partial}{\partial \theta}\, r \sin\theta\, H_\phi \right]$$

$$= \frac{r}{r^2 \sin\theta} \left[\frac{\partial}{\partial \theta} \left(\overset{u}{\sin\theta}\ \overset{v}{H_\phi} \right) \right]$$

$$= \frac{1}{r \sin\theta} \left[\sin\theta\, \frac{\partial H_\phi}{\partial \theta} + H_\phi\, \frac{\partial}{\partial \theta} \sin\theta \right]$$

$$= \frac{1}{r \sin\theta} \left[\sin\theta\cos\theta + \sin\theta\cos\theta \right] \times \frac{Idl}{4\pi} \left[\frac{-\sin\omega t'}{\mu} + \frac{\omega \omega t'}{r^2} \right]$$

$$\frac{\sin\theta\,\omega\theta}{\gamma\,\sin\theta} \quad \frac{Idl}{2\pi\epsilon} \left[\frac{-\omega\sin\omega t'}{\gamma\theta\sigma} + \frac{\cos\omega t'}{\gamma^2} \right] \qquad \text{⑤}$$

$$\frac{Idl\cos\theta}{2\pi\gamma^2} \left[\frac{-\omega\sin\omega t'}{\dfrac{\upsilon}{2}} + \frac{\cos\omega t'}{\gamma} \right]$$

$$E_\gamma = \frac{Idl\cos\theta}{2\pi\epsilon\gamma^2} \left[\frac{-\omega}{\upsilon}\int\frac{\sin\omega t'}{}\,dt + \frac{1}{\gamma}\int\frac{\cos\omega t'}{}\,dt \right] \qquad t' = t - \gamma/\upsilon$$

$$= \frac{Idl\cos\theta}{2\pi\epsilon\gamma^2} \left[\frac{\omega}{\upsilon}\,\frac{\cos\omega t'}{\omega\gamma} + \frac{1}{\gamma}\cdot\frac{\sin\omega t'}{\omega} \right]$$

$$= \frac{Idl\cos\theta}{2\pi\epsilon} \left[\frac{\cos\omega t'}{\upsilon\gamma^2} + \frac{\sin\omega t'}{\omega\gamma^3} \right]$$

$$E_\theta = \frac{1}{\epsilon}\int(\nabla\times H)_\theta\,dt$$

$$(\nabla\times H)_\theta = \frac{-1}{\gamma\sin\theta}\left[\frac{\partial}{\partial\gamma}\left[\gamma\sin\theta\,H_\phi\right] - 0 \right] \qquad t' = t - \gamma/\upsilon$$

$$= \frac{-1}{\gamma\sin\theta}\left[\frac{\partial}{\partial\gamma}\gamma\left[\frac{Idl\sin\theta}{4\upsilon}\left[\frac{-\omega\sin\omega t'}{\gamma\upsilon} + \frac{\cos\omega t'}{\gamma^2} \right] \right] \right]$$

$$= \frac{-Idl\sin\theta}{4\pi\gamma}\left[\frac{\partial}{\partial\gamma}\left[\frac{-\omega\sin\omega t'}{\upsilon} + \frac{\cos\omega t'}{\gamma} \right] \right]$$

$$= + A\left[\frac{-\omega\cos\omega t'\,\omega\cdot\frac{-1}{\upsilon}}{\upsilon} + \frac{-\gamma\sin\omega t'\,\omega\left(\frac{-1}{\upsilon}\right) - \cos\omega t'}{\gamma^2} \right]$$

Source :EM waves and Radiating systems –
by E. C. JORDAN and K. G. Balmain – PHI ,
New Delhi

$$+A\left[\frac{\omega^2\cos\omega t'}{v^2} + \frac{\omega\sin\omega t'}{vr} - \frac{\cos\omega t'}{r^2}\right]$$

$$E_\theta = \frac{1}{\epsilon}\int (\nabla\times H)_\theta \, dt$$

$$= \frac{+A}{\epsilon}\left[\frac{\omega^2}{v^2}\int\cos\omega t' + \frac{\omega}{vr}\int\sin\omega t' - \frac{1}{r^2}\int\cos\omega t'\right]$$

$$= \frac{+A}{\epsilon}\left[\frac{\omega^2}{v^2}\frac{\sin\omega t'}{\omega} + \frac{\omega}{vr}\frac{\cos\omega t'}{\omega} - \frac{1}{r^2}\frac{\sin\omega t'}{\omega}\right]$$

$$= \frac{+A}{\epsilon}\left[\frac{\omega\sin\omega t'}{v^2} - \frac{\cos\omega t'}{vr} - \frac{\sin\omega t'}{r^2\omega}\right]$$

$$= \frac{1}{\epsilon}\frac{Idl\sin\theta}{4\pi}\left[\frac{-\omega\sin\omega t'}{rv^2} + \frac{\cos\omega t'}{vr^2} + \frac{\sin\omega t'}{r^3\omega}\right]$$

$$E_\phi = \frac{1}{\epsilon}\int (\nabla\times H)_\phi \, dt$$

$$E_\phi = 0.$$

wave motion $\rightarrow E_r, E_\theta, H_\phi$