Unit-1 Problems

① 1000 bandwidth signals are TDM using natural sampling. Find the maximum pulse width 'τ' of the sample when the max band limiting freq is 5 KHz. Assume there is no guard band interval b/w successive samples.

Ans) $N = 1000$

$\tau = ?$

$f_M = 5\ KHz$

$\tau = \frac{T_s}{N}$ where $T_s = \frac{1}{2f_M}$ according to Nyquist sampling theorem

$\therefore T_s = \frac{1}{2f_M} = \frac{1}{2 \times 5 \times 10^3} = \frac{1}{10^4}$

$= 0.1\ msec$

$\therefore \tau = \frac{T_s}{N} = \frac{0.1 \times 10^{-3}}{1000} = 0.1 \times 10^{-6}$

$= 0.1\ \mu sec$

② (a) The s/g $v(t) = \cos 5\pi t + 0.5 \cos 10\pi t$ is sampled instantaneously. Find the maximum allowed value for 'T_s'

(b) If the sampling signal is $s(t) = 5 \sum_{k=-\infty}^{\infty} \delta(t - 0.1k)$

The sampled o/p $v_s(t) = v(t)\, s(t)$

$= \sum_{k=-\infty}^{\infty} I_k\, \delta(t - 0.1k)$

consists of a train of impulses whose strength is I_k. Find I_0, I_1, I_2, I_3 & also show that

$I_k = I_{k+4}$

(c) Find the minimum cut-off frequency required for a LPF to recover the base band s/g.

Ans) (a) $v(t) = \cos 5\pi t + 0.5 \cos 10\pi t$

$= \cos 2(2.5)\pi t + 0.5 \cos 2(5)\pi t$

$\therefore f_1 = 2.5$ Hz, $f_2 = 5$ Hz

$f_M = \max(f_1, f_2) = 5$ Hz

$\therefore T_s = \frac{1}{2f_M} = \frac{1}{2\times 5} = 0.1$ sec

(b) $V_s(t) = V(t)\, S(t)$

$\Rightarrow \sum_{k=-\infty}^{\infty} I_k\, \delta(t-0.1k) = \left[\cos 5\pi t + 0.5 \cos 10\pi t\right] \sum_{k=-\infty}^{\infty} \delta(t-0.1k)$

comparing L.H.S & R.H.S, we get

$$\boxed{I_k = 5\left[\cos 5\pi t + 0.5 \cos 10\pi t\right]} \longrightarrow (1)$$

At sample time, $\boxed{t = kT_s} = k(0.1) \Rightarrow$ at $t = 0.1k$

we get

$$I_k = 5\left[\cos 5\pi(0.1k) + 0.5 \cos 10\pi(0.1k)\right]$$

$$\Rightarrow \boxed{I_k = 5\left[\cos 0.5\pi k + 0.5 \cos \pi k\right]} \longrightarrow (2)$$

For $k=0$, $I_0 = 5\left[\cos 0.5\pi(0) + 0.5 \cos \pi(0)\right] = 7.5$

$\Rightarrow I_0 = 7.5$

$k=1$, $I_1 = -2.5$

$k=2$, $I_2 = -2.5$ } in radians

$k=3$, $I_3 = -2.5$

NOW, $I_{k+4} = 5\left[\cos 0.5\pi(k+4) + 0.5 \cos 10\pi(k+4)(0.1)\right]$

$= 5\left[\cos\left(2\pi + 0.5\pi k\right) + 0.5 \cos(4\pi + \pi k)\right]$

$= 5\left[\cos 0.5\pi k + 0.5 \cos \pi k\right]$ { From eq (2) }

i.e, at $t = (k+4)0.1 = (0.1)(k+4)$

$\Rightarrow \boxed{I_{k+4} = I_k}$

(c) LPF min cut-off frequency is the bandlimited freq of base band s/g $\Rightarrow f_M = 5$ Hz