→ Find the sampled o/p $v_s(t)$ for the base band s/g $v(t) = \cos 5\pi t + \cos 10\pi t$ i.e, flat topped sampled at Nyquist rate. Also prove that it is periodic

Ans] Flat topped sampled o/p $V_s(t) = V(t)\, s(t)$

where $s(t)$ is a pulse train, $s(t) = A \Sigma P(t - KT_s)$

let $A = 1$ then $s(t) = \Sigma P(t - KT_s)$

Given : $v(t) = \cos 5\pi t + \cos 10\pi t$

$\Rightarrow v(t) = \cos 2\pi(2.5)t + \cos 2\pi(5)t$

$\therefore f_1 = 2.5 \text{ Hz}$; $f_2 = 5 \text{ Hz}$

$f_M = f_{max}(f_1, f_2) = f_{max}(2.5, 5)$

$= 5 \text{ Hz}$

Nyquist rate, $f_s = 2f_M$

$= 2 \times 5 = 10$

$\therefore$ Sampling Time, $T_s = \frac{1}{f_s} = \frac{1}{10} = 0.1 \text{ sec}$

$\therefore s(t) = \Sigma P(t - KT_s) = \Sigma P(t - K(0.1))$

$= \Sigma P(t - 0.1K)$

$\therefore$ sampled o/p, $V_s(t) = v(t)\, s(t)$

$= \left[\cos 5\pi t + \cos 10\pi t\right]\left[\Sigma P(t - 0.1K)\right]$

$\therefore V_s(t + 0.4) = \left[\cos 5\pi(t + 0.4) + \cos 10\pi(t + 0.4)\right]$

$\left\{\Sigma P(t + 0.4 - 0.1K)\right\}$

$= \left[\cos(2\pi + 5\pi t) + \cos(4\pi + 10\pi t)\right]$

$\left\{\Sigma P(t + 0.4 - 0.1K)\right\}$

$= \left[\cos 5\pi t + \cos 10\pi t\right]\left\{\Sigma P(t + 0.4 - 0.1K)\right\}$

$\Rightarrow V_s(t) = v(t)\; \Sigma P(t + 0.4 - 0.1K)$

Hence proved $V_s(t)$ is periodic.

problem

Consider a sig having probability density function

$$f(v) = Ke^{-|v|} \; ; \; -4 \le v \le 4$$

find

a, The value of K

b, Find step size 's' if there are '4' quantization levels. & design a quantizer indicating various quantization levels

c, Calculate the variance of quantization Error when there are four quantization levels. Donot assume f(v) is constant over each level. Compare your result with $\bar{e^2} = S^2/12$

Sol :- a, $\int_{-\infty}^{\infty} f(v)\,dv = 1$

$$\Rightarrow \int_{-4}^{4} Ke^{-|v|}\,dv = 1$$

$$\Rightarrow K\int_{-4}^{0} e^{+v}\,dv + \int_{0}^{4} Ke^{-v}\,dv = 1$$

$$\Rightarrow K\left[(e^{0} - e^{-4}) + \left(\frac{e^{-4} - e^{0}}{(-1)}\right)\right] = 1$$

$$\Rightarrow K[2 - 2e^{-4}] = 1$$

$$\Rightarrow 2K[1 - e^{-4}] = 1$$

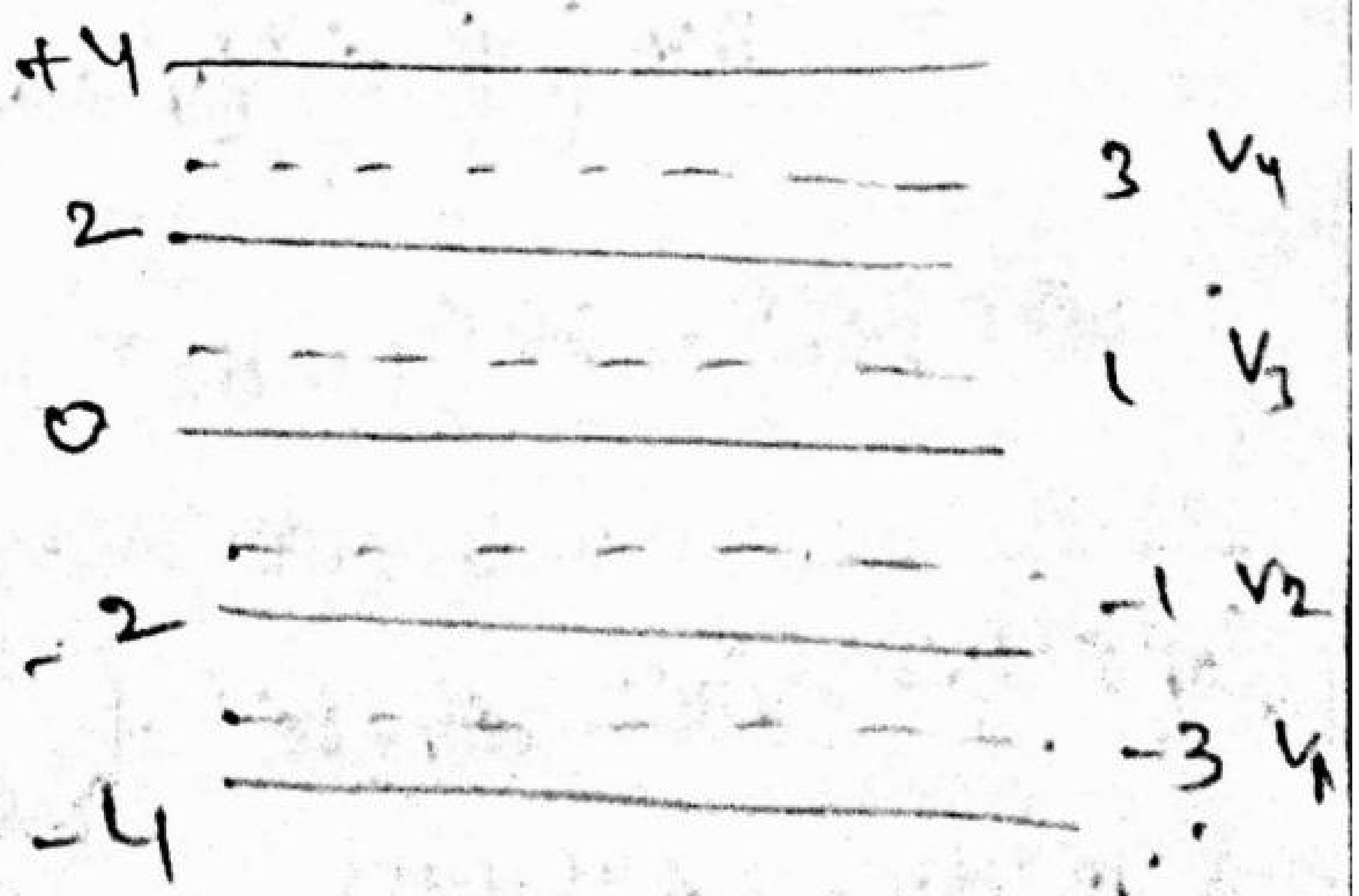
$$\Rightarrow K = \frac{1}{2(1-e^{-4})}$$

b, $S = \frac{V_H - V_L}{M}$

$V_H = 4$

$V_L = -4$, $M = 4$

$$S = \frac{4-(-4)}{4} = 2$$



c, Assuming $f(u)$ is not constant, over each level

$$\text{var } e[v] = \bar{e}^2 = \sum_{k=1}^{M=4} \int_{m_k - s/2}^{m_k + s/2} (m-m_k)^2 f(m)\, dm$$

$$= \int_{u_1 - s/2}^{u_1+s/2} (u-u_1)^2 f(u)\, du + \int_{u_2 - s/2}^{u_2+s/2} (v-v_2)^2 f(u)\, du +$$

$$\int_{v_3 - s/2}^{v_3+s/2} (u-u_3)^2 f(u)\, dv + \int_{u_4 - s/2}^{u_4+s/2} (v-u_4)^2 f(u)\, du.$$

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$$\text{var } e[v] = \int_{-4}^{-2} Ke^{-|v|}(v+3)^2 dv + \int_{-2}^{0} Ke^{-|v|}(v+1)^2 du + \int_{0}^{2} Ke^{-v}(v-1)^2 du$$
$$+ \int_{2}^{4} Ke^{-v}(v-3)^2 dv.$$

$$= \int_{-4}^{-2} Ke^{v}(v+3)^2 dv + \int_{-2}^{0} Ke^{v}(v+1)^2 dv + \int_{0}^{2} Ke^{-v}(v-1)^2 du + \int_{2}^{4} Ke^{-v}(v-3)^2 dv$$

W.K.T, $\int_{-a}^{-b} f(x)\,dx = \int_{b}^{a} f(-x)\,dx$ & also $(a-b)^2 = (b-a)^2$

$$= \int_{2}^{4} Ke^{-v}(-v+3)^2 dv + \int_{0}^{2} Ke^{-v}(-v+1)^2 dv + \int_{0}^{2} Ke^{-v}(v-1)^2 dv +$$
$$\int_{2}^{4} Ke^{v}(v-3)^2 dv$$

$$= 2\int_{2}^{4} Ke^{-v}(v-3)^2 dv + 2\int_{0}^{2} Ke^{-v}(v-1)^2 du$$

$$= 2K\left[\int_{2}^{4} e^{-v}(v^2+9-6v)\,dv + \int_{0}^{2} e^{-v}(v^2-2v+1)\,dv\right]$$

$$\therefore \text{var}(ev) = 2K\left[(1-5e^{-2}) + (e^{2}-5e^{-4})\right]$$
$$= 0.379$$

$\int v^2 e^{-v} dv = -v^2 e^{-v} - 2e^{-v}v - 2e^{-v}$

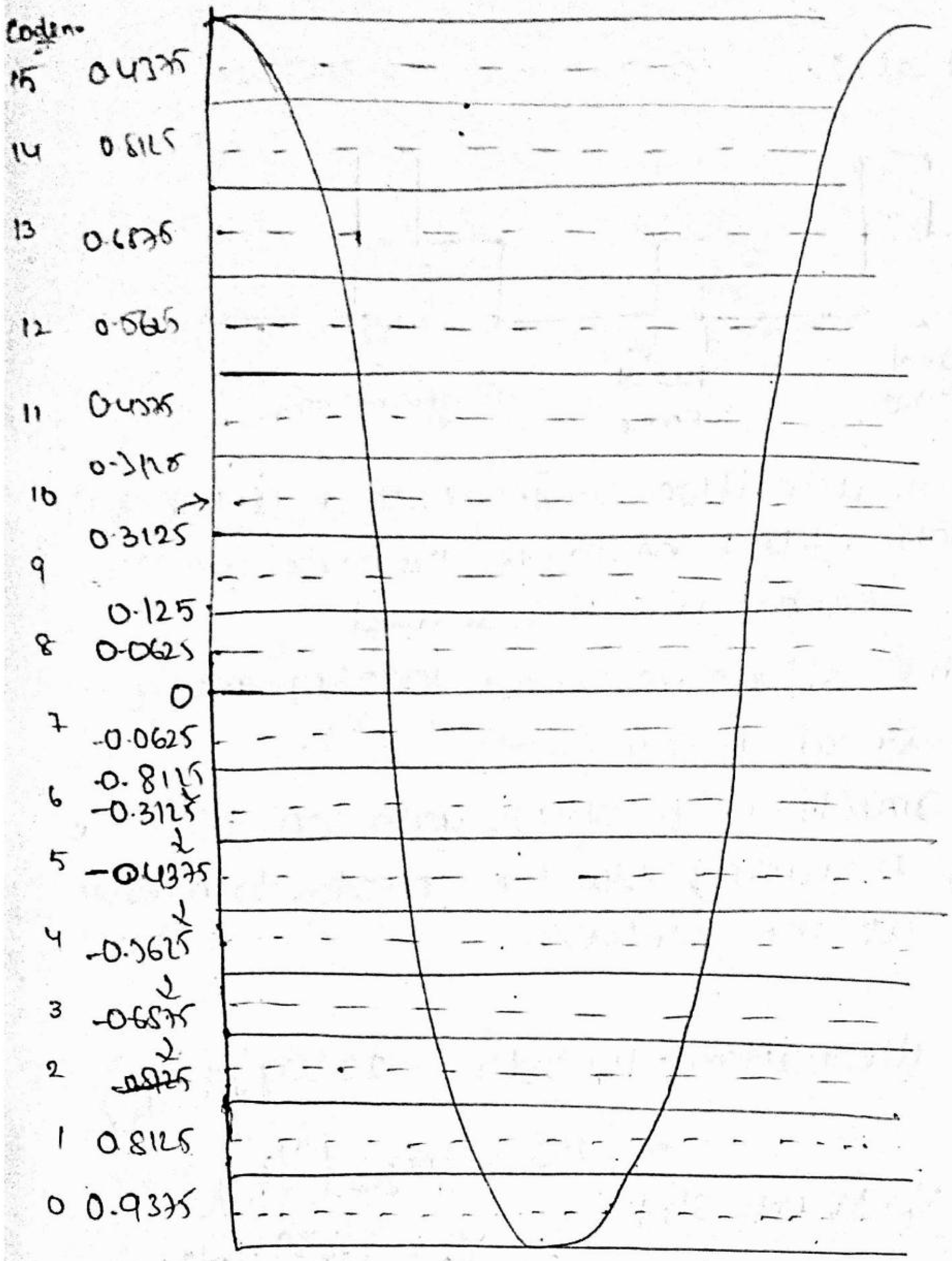
$\int ve^{-v} dv = -e^{-v}(v+1)$

$\left\{ \because K = \frac{1}{2(1-e^{-4})} \right\}$

Assuming $f(v)$ is constant over each level then

$$\bar{e}^2 = \frac{s^2}{12} = \frac{2^2}{12} + \frac{4^2}{12} = 1/3 = 0.333$$

* Consider a sig $\cos 2\pi t$ is quantized into 16 levels! the sampling rate is 4Hz. The sampling sig consists of pulses having unit height and duration dt. The pulses occur every $t = K/4$ sec $(-\infty < K < \infty)$ find out the code numbers corresponding to each sample value for a pcm system. How many bits are required for a sample & sketch the binary signal representing each sample voltage.



$M = 16$

$\text{step size} = \frac{V_H - V_L}{M}$

$= \frac{1-(-1)}{16}$

$= 0.125$

$m(t) = \cos 2\pi t$

at ~~k~~ $t = k/4$, $m(k/4) = \cos 2\pi (k/4) = \cos \frac{\pi k}{2}$

for $k = 0 \Rightarrow \cos 0 = 1$

$k = 1 \Rightarrow \cos \pi/2 = 0$

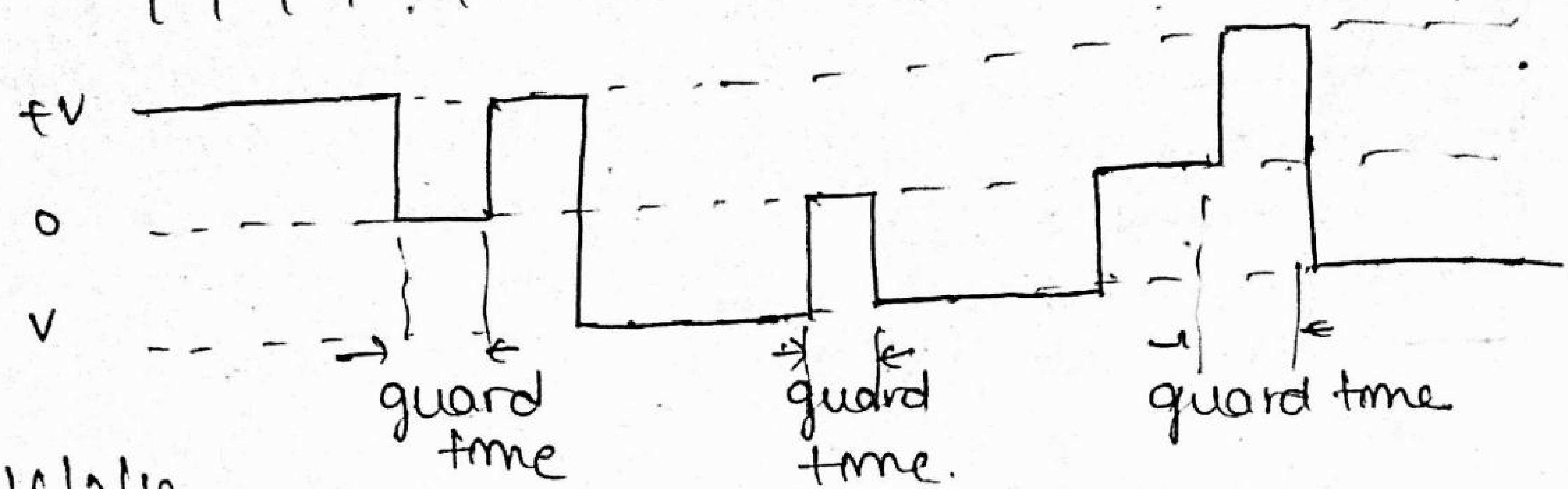
$k = 2 \Rightarrow \cos \pi = -1$

$k = 3 \Rightarrow \cos 3\pi/2 = 0$

Sample value	1	0	-1	0
Nearest quantization level	0.9375	0.0675 (or) -0.0675	-0.9375	0.0675 (or) -0.0675
Code no	15	8 (or) 7	0	8 (or) 7
Binary Equivalent	1111	1000 (or) 0111	0000	1000 (or) 0111

Scanned by TapScanner

1111 . 1000 0000 . 1000.



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Consider a communication system with a distance of 1000 Km in which a telephone cable produces an attenuation of 1 decibal per Km is used

i, Calculate, the voltage received 1000 Km away when 1V Rms signal is transmitted

II Using an amplifier of voltage gain 100, calculate no. of repeaters & spacing b/w the repeaters to receive 1V rms signal at the receiver

Sol: i, Total attenuation = 1000 db = $-20 \log_{10}\left(\frac{V_2}{V_1}\right)$

$\Rightarrow -50 = \log_{10}\left(\frac{V_2}{V_1}\right)$

[∵ given V_1 = 1V Rms Sig]

$\Rightarrow V_2 = 10^{-50}$ Volts

ii, Gain = 100 ⇒ $[20 \log_{10}(100)]$ dB

$= 20 \log_{10} 10^2$

= 40 db.

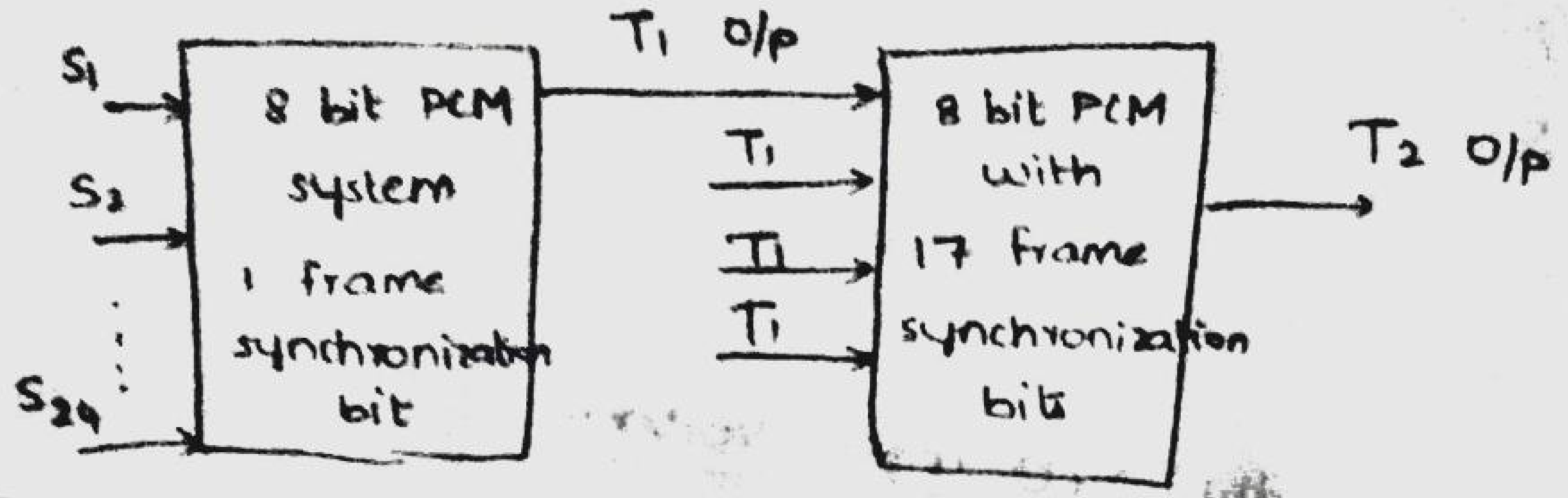
The same 1V sig can be received, if the gain of amplifier = attenuation (Equal to 40dB)

Therefore keeping a repeater after 40Km gain = attenuation = 40dB. Therefore no of repeaters required = 1000/40 = 25. ∴ The spacing b/w the repeaters to receive 1V RMS sig at the receiver is 40Km

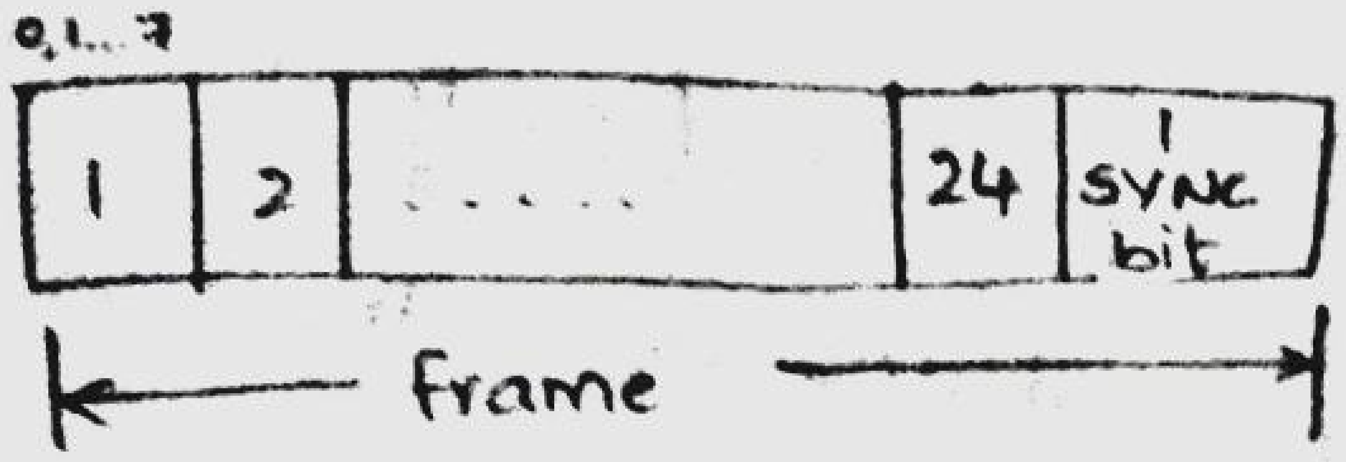
→ Problem :

24 s/gs have been TDM using 8-bit PCM system. Design a T_1 digital system to transmit these 24 s/gs when they are sampled at 8 KHz, consider one frame synchronization bit & calculate the bit rate on T_1 channel. Design a T_2 system when four similar channels have been multiplexed with 17 frame synchronization bits. Calculate bit rate on T_2 channel.

Ans)



f_s = 8 KHz



{ 8-bit PCM :- so that each word of a s/g is of 8-bits i.e. (0,1,...7)

At T_1 Digital system, frame time $= T_s = \frac{1}{f_s} = \frac{1}{8 \times 10^3} = 125\ \mu sec$

Total no. of bits in frame $= (24 \times 8) + 1 = 193$

Bit rate, $f_b = \frac{193}{125\ \mu sec} = 1.544 \times 10^6$ bits/sec

$= 1.544$ Mb/sec

{ 193 → 125 μsec
? → 1 sec

At T_2 Digital system,

frame time $= T_s = \frac{1}{f_s} = \frac{1}{8 \times 10^3} = 125\ \mu sec$

No. of bits in a frame $= (4 \times 193) + 17 = 789$

Bit rate, $f_b = \frac{789}{125\ \mu sec} = 6.312$ Mb/sec