UNIT - I

Problems

Errors measurement :-

1) The expected value of the voltage across a resistor is 80V. However the measurement gives a value of 79V. Calculate

(a) absolute error (c) relative accuracy

(b) % error (d) % of accuracy

Sol:-

(a) absolute error $\boxed{e = Y_n - X_n}$

$= 80 - 79$

$= 1V.$

(b) % Error $= \boxed{\frac{Y_n - X_n}{Y_n} \times 100}$

$= \frac{80-79}{80} \times 100 = 1.25\%$

(C) Relative accuracy

$A = 1 - \left|\frac{Y_n - X_n}{Y_n}\right|$

$= 1 - \left|\frac{80-79}{80}\right|$

$= 1 - \frac{1}{80}$

$= \frac{79}{80}$

$= 0.9875$

(d) % of accuracy $\boxed{a = 100 \times A}$

$= 100 \times 0.9875$

$= 98.75\%$

(a) $\boxed{a = 100\% - \%\text{ of error}}$

$= 100\% - 1.25\%$

$= 98.75\%$

2) The expected value of the current through a resistor is 20mA. However the measurement yields a current value of 18 mA. calculate (a) absolute error (c) relative accuracy

(b) % error (d) % accuracy

Sol:

(a) Absolute error:- $\boxed{e = Y_m - X_m}$

where e = error, Y_m = expected value, X_m = measured value

given $Y_m = 20mA$ & $X_m = 18mA$

$\therefore \quad e = Y_m - X_m = 20mA - 18mA = \boxed{2mA}$

(b) % error:- $\frac{Y_m - X_m}{Y_m} \times 100$

$= \frac{20mA - 18mA}{20mA} \times 100$

$= \frac{2mA}{20mA} \times 100$

$= \boxed{10\%}$

(c) Relative accuracy:- $A = 1 - \left|\frac{Y_m - X_m}{Y_m}\right|$

$= 1 - \left|\frac{20mA - 18mA}{20mA}\right|$

$= 1 - \frac{2}{20}$

$= 1 - 0.1$

$= 0.90$

d) % accuracy:-

$a = 100\% - \%\text{ error}$

$= 100\% - 10\%$

$\boxed{= 90\%}$

$a = A \times 100\%$

$= 0.90 \times 100\%$

$\boxed{= 90\%}$

(3) Below table gives the set of 10 measurement that were recorded in the laboratory calculate the precision of the measurement.

Measurement number	Measurement value x_n
1	98
2	101
3	102
4	97
5	101
6	100
7	103
8	98
9	106
10	99

Sol:- The average value for the set of measurement is given by

$$\bar{x}_n = \frac{\text{Sum of the 10 measurement values}}{10}$$

$$= \frac{1005}{10}$$

$$= 100.5$$

$$\text{precision} = 1 - \left|\frac{x_n - \bar{x}_n}{\bar{x}_n}\right|$$

For the 6th reading

$$\text{Precision} = 1 - \left|\frac{100 - 100.5}{100.5}\right|$$

$$= 1 - \frac{0.5}{100.5}$$

$$= \frac{100}{100.5} = \boxed{0.995}$$

4. For the following given data, calculate

(i) Arithmetic mean (iv) Average deviation

(ii) Deviation of each value (v) Standard deviation

(iii) Algebraic sum of the derivations

Given data $x_1 = 49.7$; $x_2 = 50.1$; $x_3 = 50.2$; $x_4 = 49.6$; $x_5 = 49.7$

Sol:- (i) Arithmetic mean:-

$$\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$$

$$= \frac{49.7 + 50.1 + 50.2 + 49.6 + 49.7}{5}$$

$$= \boxed{49.86}$$

(ii) Deviation of each value:-

$d_1 = x_1 - \bar{x} = 49.7 - 49.86 = -0.16$

$d_2 = x_2 - \bar{x} = 50.1 - 49.86 = +0.24$

$d_3 = x_3 - \bar{x} = 50.2 - 49.86 = +0.34$

$d_4 = x_4 - \bar{x} = 49.6 - 49.86 = -0.26$

$d_5 = x_5 - \bar{x} = 49.7 - 49.86 = -0.16$

(iii) The algebraic sum of the deviation is:-

$d_{total} = -0.16 + 0.24 + 0.34 - 0.26 - 0.16$

$= +0.58 - 0.58 = 0$

(iv) Average deviation:-

$$D_{av} = \frac{|d_1| + |d_2| + |d_3| + \cdots + |d_n|}{n}$$

$$= \frac{|-0.16| + |0.24| + |0.34| + |-0.26| + |-0.16|}{5}$$

$$= \frac{1.16}{5} = 0.232$$

(V) Standard deviation:-

Sol:
$$\sigma = \sqrt{\frac{d_1^2+d_2^2+d_3^2+\cdots+d_n^2}{n-1}}$$

$$= \sqrt{\frac{(-0.16)^2+(0.24)^2+(0.34)^2+(-0.26)^2+(-0.16)^2}{5-1}}$$

$$= \sqrt{\frac{0.0256+0.0576+0.1156+0.0676+0.0256}{4}}$$

$$= \sqrt{\frac{0.292}{4}} = \sqrt{0.073} = \boxed{0.27}$$

(5) A 500mA voltmeter is specified to be accurate with ±2%. Calculate the limiting error when instrument is used to measure 300mA.

Sol: Given accuracy of 0.02 = ±2%.

The magnitude of limiting error is

$500mA \times 0.02$

$= 10mA$

∴ The limiting error at 300mA $= \frac{10mA}{300mA} \times 100\%$

$\boxed{= 3.33\%}$

(6) A voltmeter reading 70V on its 100V range & an ammeter reading 80mA on its 150mA range are used to determine the power dissipa-ted in a resistor. Both these instruments are guaranteed to be acc-urate within ±1.5% at full scale deflection. Determine the limiting error of the power.

Sol. The magnitude of the limiting error for the voltmeter is

$0.015 \times 100 = \boxed{1.5V}$

The limiting error at 70V is

$\frac{1.5}{70} \times 100 = \boxed{2.143\%}$

The magnitude of limiting error of the ammeter is

$$0.015 \times 150\,mA = \boxed{2.25\,mA}$$

The limiting error at 80mA is

$$\frac{2.25\,mA}{80\,mA} \times 100 = \boxed{2.813\%}$$

∴, the limiting error for the power calculation is the sum of the individual limiting errors involved.

$$\therefore \text{limiting error} = 2.143\% + 2.813\%$$

$$\boxed{= 4.956\%}$$

Basic meter movement:-

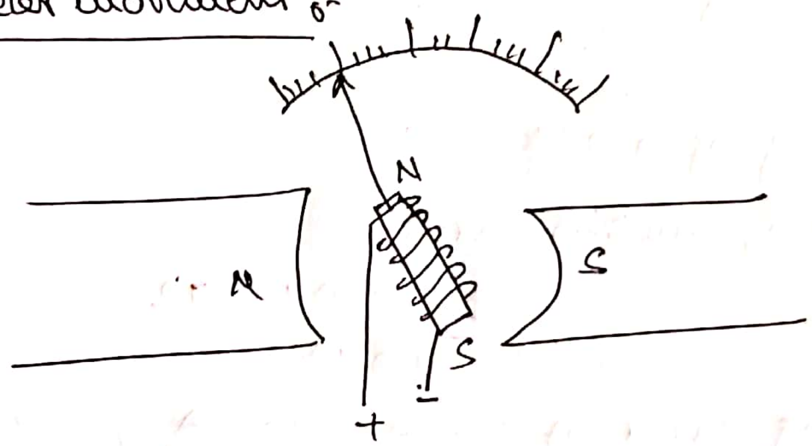

fig:. D'Arsonval principle

$$\tau = B \times A \times I \times N$$

where,

τ = Torque, Newton-meter

B = flux density in the air gap, wb/m^2

A = effective coil area (m^2)

N = no. of turns of wire of the coil

I = current in the movable coil (amperes)

① A moving coil instrument has the following dat

No. of Turns = 100

width of the Coil = 20 mm

Depth " " " = 30 mm

flux density in the gap = 0.1 wb/m^2

Calculate the deflecting Torque when carrying a current of 10 mA. Also calculate the deflection, if the control spring constant is 2×10^{-6} Nm/degree.

Sol:: The deflecting Torque is given by

$$\boxed{\tau_d = B\times A\times N\times I}$$

$$= 0.1\times30\times10^{-3}\times20\times10^{-3}\times100\times10\times10^{-3}$$

$$= 600\times1000\times0.1\times10^{-9}$$

$$= 600\times1000\times10^{-10}$$

$$= 60\times10^{-6}\,Nm$$

The spring control provides a restoring torque, i.e. $\boxed{\tau_c = K\theta}$

where K is the spring constant

As deflecting Torque = restoring torque

$$\therefore \tau_c = 6\times10^{-5}\,Nm = K\theta$$

$$\therefore \theta = \frac{6\times10^{-5}}{2\times10^{-6}}$$

$$= 3\times10$$

$$\boxed{= 30^\circ}$$

∴, The deflection is 30°

(2) A moving coil instrument has the following data

No. of turns = 100

Width of the coil = 20mm

Depth of the coil = 30mm

Flux density in the gap = 0.1 wb/m^2

The deflection torque = 30×10^{-6} Nm

calculate the current through the moving coil

sol:

$$\boxed{\tau_d = B\times A\times N\times I}$$

$$30\times10^{-6} = 0.1\times30\times10^{-3}\times20\times10^{-3}\times100\times I$$

$$I = \frac{30\times10^{-6}}{0.1\times30\times10^{-3}\times20\times10^{-3}\times100}$$

$$I = \frac{30\times10^{-6}}{0.1\times600\times10^{-6}\times100}$$

$$\boxed{I = 5mA}$$