→ For a base band s/g Receiver, a 1-bit interval is represented by a pulse of 5 mV & a '0' bit is represented by a pulse of −5 mV with equal probability of occurence. Find P_e when noise is having a uniform PSD $\eta/2 = 10^{-9}$. Let bit rate = 9600 bits/sec.

b) If bit rate is doubled, calculate percentage increase in error.

Ans) <u>Given</u> :- $\eta/2 = 10^{-9}$, $S_1(t) = 5$ mV ; $S_2(t) = -5$ mV.

$f_b = 9600$ bits/sec $\Rightarrow T_b = \frac{1}{f_b} = \frac{1}{9600} = 0.000104$ sec

$$P_e = \frac{1}{2}\,\text{erfc}\left[\frac{E_s}{\eta}\right]^{1/2}$$

where $E_s = V^2 T = (5\times10^{-3})^2 \times \frac{1}{9600} = 2.6\times10^{-9}$

$$\therefore P_e = \frac{1}{2}\,\text{erfc}\left(\frac{2.6\times10^{-9}}{10^{-9}\times2}\right)^{1/2}$$

$$= \frac{1}{2}\,\text{erfc}\,(1.3)^{1/2} = \frac{1}{2}\,\text{erfc}\,(1.1402)$$

$\left\{\because \text{erfc}\,(1.1402) = 0.1068\right.$

$$= 0.0534$$

b) If f_b is doubled, then $T_b = \frac{1}{2f_b} = \frac{1}{2\times9600}$ {T_b is halved}

$\Rightarrow f_b \longrightarrow 2f_b$

$E_s = V^2 T = \frac{2.6\times10^{-9}}{2} = 1.3\times10^{-9}$

$$\therefore P_e = \frac{1}{2}\,\text{erfc}\left(\frac{1.3\times10^{-9}}{2\times10^{-9}}\right)^{1/2}$$

$$= \frac{1}{2}\,\text{erfc}\,(0.65)^{1/2}$$

$$= \frac{1}{2}\,\text{erfc}\,(0.8062)$$

$\left\{\because \text{erfc}\,(0.8062) = 0.2542\right.$

$$= 0.1271$$

∴ Percentage increase in error is

$$= \frac{0.1271 - 0.0534}{0.0534} \times 100\%$$

$$= 137.98\%$$

→ Optimum Filter realization using Matched filter :-

When the input noise is white, then the optimum filter can be treated as matched filter.

For optimum filter, $H(f) = \dfrac{K\, P^*(f)\, e^{-j2\pi fT}}{G_n(f)}$

when input noise is white, then $G_n(f) = \eta/2$

∴ For matched filter, $H(f) = \dfrac{K\, P^*(f)\, e^{-j2\pi fT}}{\eta/2} = \dfrac{2K}{\eta} P^*(f)\, e^{-j2\pi fT}$

If an unit strength of impulse is applied as input for this matched filter, then its impulse response is given by

$$h(t) = F^{-1}[H(f)]$$

$$= \int_{-\infty}^{\infty} \frac{2K}{\eta} P^*(f)\, e^{-j2\pi fT}\, e^{j2\pi ft}\, df$$

$$= \frac{2K}{\eta} \int_{-\infty}^{\infty} P^*(f)\, e^{j2\pi f(t-T)}\, df$$

For any physically realizable filter, the impulse response is always real. Hence $h(t) = h^*(t)$

$$\therefore\ h(t) = h^*(t) = \frac{2K}{\eta} \int_{-\infty}^{\infty} P(f)\, e^{-j2\pi f(t-T)}\, df.$$

$$= \frac{2K}{\eta} \int_{-\infty}^{\infty} P(f)\, e^{j2\pi f(T-t)}\, df$$

$$\boxed{h(t) = \frac{2K}{\eta} P(T-t)}$$

∵ Applying I.F.T

$p(t) = F^{-1}[P(f)]$

$= \int_{-\infty}^{\infty} P(f)\, e^{j2\pi ft}\, df.$

→ Probability of error for matched filter :-

$P_e = \frac{1}{2}\, \mathrm{erfc}\left(\frac{\nu^2}{8}\right)^{1/2}$ where $\nu = \dfrac{S_{o1}(T) - S_{o2}(T)}{\sigma_o} = \dfrac{P_o(T)}{\sigma_o}$

$$\nu^2_{max} = \int_{-\infty}^{\infty} \frac{|P(f)|^2}{G_n(f)}\, df = \frac{2}{\eta} \int_{-\infty}^{\infty} |P(f)|^2\, df$$

∵ for white noise $G_n(f) = \eta/2$

By using Parseval's theorem,

$$\boxed{\int_{-\infty}^{\infty} |P(f)|^2 \, df = \int_{-\infty}^{\infty} p^2(t)\, dt}$$

$$\therefore \gamma^2_{max} = \frac{2}{\eta} \int_{-\infty}^{\infty} |P(f)|^2 \, df = \frac{2}{\eta} \int_{-\infty}^{\infty} p^2(t)\, dt$$

$$= \frac{2}{\eta} \int_{-\infty}^{\infty} \left(s_1(t) - s_2(t)\right)^2 dt$$

$$= \frac{2}{\eta} \int_{-\infty}^{\infty} \left[s_1^2(t) + s_2^2(t) - 2 s_1(t) s_2(t)\right] dt$$

For optimum filter, $s_1(t) = -s_2(t)$

$$\therefore \gamma^2_{max} = \frac{2}{\eta} \int_{-\infty}^{\infty} \left[s_1^2(t) + s_1^2(t) + 2 s_1^2(t)\right] dt$$

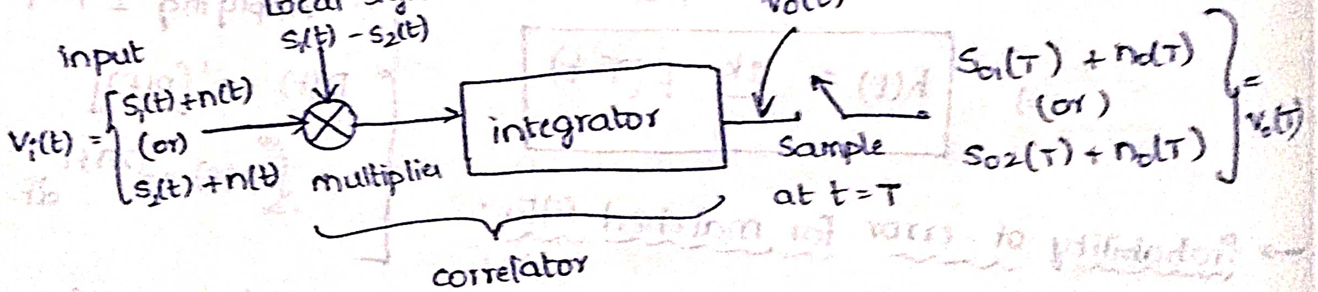
$$= \frac{2}{\eta} \int_{-\infty}^{\infty} 4\, s_1^2(t)\, dt$$

$$\Rightarrow \gamma^2_{max} = \frac{8}{\eta} \int_{-\infty}^{\infty} s_1^2(t)\, dt = \boxed{\frac{8}{\eta} E_s} \quad \left\{\because \int_{-\infty}^{\infty} s^2(t)\, dt = E_s\right\}$$

$$\therefore P_e = \frac{1}{2} \operatorname{erfc} \left(\frac{\gamma^2}{8}\right)^{1/2} = \frac{1}{2} \operatorname{erfc} \left(\frac{8}{\eta} \frac{E_s}{8}\right)^{1/2}$$

$$\Rightarrow \boxed{P_e = \frac{1}{2} \operatorname{erfc} \left(\frac{E_s}{\eta}\right)^{1/2}}$$

→ Optimum filter realization using correlator :



For correlator Receiver,

$$\left.\begin{aligned} S_0(T) &= \frac{1}{\tau} \int_0^T s_i(t) \left[s_1(t) - s_2(t)\right] dt \\ n_0(T) &= \frac{1}{\tau} \int_0^T n(t) \left[s_1(t) - s_2(t)\right] dt \end{aligned}\right\} \rightarrow (1)$$

where τ is constant.

Apply the binary waveforms for $s_1(t)$ or $s_2(t)$ along with noise $n(t)$ to a multiplier circuit, the s/g is multiplied by a local s/g which is in the form of $s_1(t) - s_2(t)$ & the resulting s/g is applied as i/p to the integrator, the o/p is sampled for every 'T' sec where 'T' is bit duration.

After sampling, all the energy storing elements must be discharged. Since the binary s/gs along with noise is correlated with a local s/g, this type of Rxer can be termed as correlator Rxer.

→ Comparing performance of matched filter & correlator Rxer :-

For matched filter, we have impulse response, $h(t) = \frac{2k}{\eta} P(T-t)$

By using convolution theorem,

$$v_0(t) = \int_{-\infty}^{\infty} v_i(\tau)\, h(t-\tau)\, d\tau$$

$$\therefore \text{o/p } v_0(t) = \int_0^T v_i(\lambda)\, h(t-\lambda)\, d\lambda$$

$$\therefore h(t-\lambda) = \frac{2k}{\eta} P(T-(t-\lambda)) = \frac{2k}{\eta} P(T-t+\lambda)$$

$$= \frac{2k}{\eta}\left[s_1(T-t+\lambda) - s_2(T-t+\lambda)\right]$$

$$\{\because P(t) = s_1(t) - s_2(t)\}$$

$$\therefore \text{o/p } v_0(t) = \int_0^T v_i(\lambda)\, \frac{2k}{\eta}\left[s_1(T-t+\lambda) - s_2(T-t+\lambda)\right] d\lambda$$

$$= \frac{2k}{\eta}\int_0^T v_i(\lambda)\left[s_1(T-t+\lambda) - s_2(T-t+\lambda)\right] d\lambda$$

At $t = T$ sec, for every T sec, we collect the sampled values

$$\therefore v_0(T) = \frac{2k}{\eta}\int_0^T v_i(\lambda)\left[s_1(\lambda) - s_2(\lambda)\right] d\lambda$$

Let the o/p $v_0(T)$ consists of o/p due to s/g $s_0(T)$ & o/p due to noise $n_0(T)$. Hence, we can write,

$$v_0(T) = s_0(T) + n_0(T)$$

$$\left[\begin{array}{l} v_i(\lambda) = s_i(\lambda) + n(\lambda) \\ \& \\ v_0(\lambda) = s_0(\lambda) + n_0(\lambda) \end{array}\right.$$

$$\therefore\ s_o(T) = \frac{2K}{\eta}\int_0^T s_i(\lambda)\left[s_1(\lambda) - s_2(\lambda)\right] d\lambda$$

$$n_o(T) = \frac{2K}{\eta}\int_0^T n(\lambda)\left[s_1(\lambda) - s_2(\lambda)\right] d\lambda$$

$\Big\} \rightarrow$ ② Eqs for matched filter

$\therefore$ By comparing 1^{st} & 2^{nd} set of eqs, the performance of a correlator & matched filter is identical & these two systems are two different techniques which yields the same performance.

→ <u>Optimal of Coherent Reception</u> : <u>PSK, FSK, QPSK</u>

(or)

<u>Comparison of digital modulation techniques with P_e</u>

(or)

<u>P_e for various Data Txion techniques</u> :-

→ <u>BPSK system</u> :- It provides minimum P_e because it satisfies optimum condition $s_1(t) = -s_2(t)$

For BPSK system, $\left.\begin{aligned} s_1(t) &= A\cos\omega_0 t \\ s_2(t) &= -A\cos\omega_0 t \end{aligned}\right\}\ 0 \le t \le T$

$\therefore\ s_1(t) = -s_2(t)$ ✓

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\frac{E_s}{\eta}\right)^{1/2}$$

$$E_s = \int_0^T s_1^2(t)\,dt = \int_0^T s_2^2(t)\,dt = \int_0^T (A\cos\omega_0 t)^2\,dt$$

$$= A^2\int_0^T \frac{(1+\cos 2\omega_0 t)}{2}\,dt$$

$$= \frac{A^2}{2}\left[t + \frac{\sin 2\omega_0 t}{2\omega_0}\right]_0^T$$

$$\Rightarrow E_s = \frac{A^2}{2}T.$$

$$\boxed{\therefore\ P_e = \frac{1}{2}\,\mathrm{erfc}\left[\frac{A^2T}{2\eta}\right]^{1/2}}$$

→ <u>BPSK in a correlator (imperfect phase synchronization)</u>

$$s_o(T) = \frac{1}{\tau}\int_0^T s_i(t)\left[s_1(t) - s_2(t)\right] dt$$

let $s(t)$ is Rxed by correlator, for BPSK system, the sampled o/p is