

**A. Maximum GCD**  
time limit per test: 1 second  
memory limit per test: 256 megabytes

Let's consider all integers in the range from 1 to  $n$  (inclusive).

Among all pairs of distinct integers in this range, find the maximum possible greatest common divisor of integers in pair. Formally, find the maximum value of  $\gcd(a, b)$ , where  $1 \leq a < b \leq n$ .

The greatest common divisor,  $\gcd(a, b)$ , of two positive integers  $a$  and  $b$  is the biggest integer that is a divisor of both  $a$  and  $b$ .

**Input**  
The first line contains a single integer  $t$  ( $1 \leq t \leq 100$ ) — the number of test cases. The description of the test cases follows.

The only line of each test case contains a single integer  $n$  ( $2 \leq n \leq 10^9$ ).

**Output**  
For each test case, output the maximum value of  $\gcd(a, b)$  among all  $1 \leq a < b \leq n$ .

**Example**

input	output
2	1
3	1
4	2

**Note**  
In the first test case,  $\gcd(1, 2) = \gcd(2, 3) = \gcd(1, 3) = 1$ .  
In the second test case, 2 is the maximum possible value, corresponding to  $\gcd(2, 4)$ .

Maxi

$$1 \rightarrow n$$

$$(1, 2, 3, 4, 5, 6, \dots, n)$$

$$(1, 5)$$

$$\max(a, b)$$

$$a \rightarrow \text{mid} \rightarrow \frac{6}{a+1}$$

$$1, 2, 3$$

$$n/2$$

$$1, 2, 3, 4, 5, 6$$

$$\frac{b}{n} \times$$

$$\frac{b}{n} \times$$

$$\max = 1$$

idea

→

$$\frac{5}{2} \Rightarrow 2$$

$$(2, 4)$$

$$\frac{1}{2} \times 2$$

$$(1, 4) \rightarrow 1$$

$$1, 2, 3, 4, 5, 6, \dots$$

$$(1, 4) = 1 \rightarrow$$

$$(2, 4) = 2 \rightarrow$$

B)

**B. EhAb AnD gCd**  
time limit per test: 1 second  
memory limit per test: 256 megabytes

You are given a positive integer  $x$ . Find any such 2 positive integers  $a$  and  $b$  such that  $GCD(a, b) + LCM(a, b) = x$ .

As a reminder,  $GCD(a, b)$  is the greatest integer that divides both  $a$  and  $b$ . Similarly,  $LCM(a, b)$  is the smallest integer such that both  $a$  and  $b$  divide it.

It's guaranteed that the solution always exists. If there are several such pairs  $(a, b)$ , you can output any of them.

**Input**  
The first line contains a single integer  $t$  ( $1 \leq t \leq 100$ ) — the number of testcases.

Each testcase consists of one line containing a single integer,  $x$  ( $2 \leq x \leq 10^9$ ).

**Output**  
For each testcase, output a pair of positive integers  $a$  and  $b$  ( $1 \leq a, b \leq 10^9$ ) such that  $GCD(a, b) + LCM(a, b) = x$ . It's guaranteed that the solution always exists. If there are several such pairs  $(a, b)$ , you can output any of them.

**Example**

input	output
2	1 1
14	6 4

**Note**  
In the first testcase of the sample,  $GCD(1, 1) + LCM(1, 1) = 1 + 1 = 2$ .  
In the second testcase of the sample,  $GCD(6, 4) + LCM(6, 4) = 2 + 12 = 14$ .

$$(a, b)$$

$$\gcd(a, b) +$$

$$LCM(a, b) = x$$

$$a \rightarrow b \rightarrow$$

$$1 \leq a, b \leq 10^9$$

$$a = b$$

$$\gcd(a, b) + LCM(a, b)$$

$$= x$$

$$a, b = 100$$

$$a, b = 100$$

$$a=5, b=5$$

$$a=b$$

$$\text{input} = x$$

$$q = \text{GCD}(a, b) + \text{LCM}(a, b)$$

$$(a, b) \rightarrow (a, b) = a, x - a - \text{LCM}$$

$$(1, x-1), (1, x-1)$$

$$\downarrow$$

$$1$$

$$\downarrow$$

$$(x-1)$$

$$2 \rightarrow 1+1$$

$$q \rightarrow \text{GCD}(a, b) + \text{LCM}(a, b)$$

$$\ast \text{Euclidean Algo} \rightarrow \text{GCD is } \max(a, a)$$

$$\rightarrow \text{GCD}(a/2, a/2) \rightarrow a/2$$

$$\rightarrow \text{LCM} = x - \text{GCD}$$

$$(a, a) = a$$

$$\rightarrow x - \text{GCD}$$

$$\textcircled{2} \rightarrow \text{GCD}(2, 2) = 2 + \text{LCM}(2, 2)$$

$$\downarrow$$

$$2 + 2 \Rightarrow 4$$

$$2 \rightarrow \text{GCD}(1, 2) + \text{LCM}(1, 2) \rightarrow \leq 2$$

$$\downarrow \quad \quad \downarrow$$

$$\text{GCD}(1) + 1 \quad 2 = 3 \quad \geq 2 \quad \textcircled{2}$$

0

$$2 \rightarrow \text{gcd}(1, 1) + \text{lcm}(1, 1)$$

$$1 + 1 \checkmark$$

$$3 \rightarrow \text{gcd}(1, 2) + \text{lcm}(1, 2)$$

$$1 + 2 = 3$$

$$4 \rightarrow \text{gcd}(2, 2) + \text{lcm}(2, 2)$$

$$5 \rightarrow \text{gcd}(1, 4) + \text{lcm}(1, 4)$$

$$a \quad 1 + b \quad 4 \Rightarrow 5$$

C)

**C. Madoka and Strange Thoughts**  
time limit per test: 1 second  
memory limit per test: 256 megabytes

Madoka is a very strange girl, and therefore she suddenly wondered how many pairs of integers  $(a, b)$  exist, where  $1 \leq a, b \leq n$ , for which  $\frac{\text{lcm}(a, b)}{\text{gcd}(a, b)} \leq 3$ .

In this problem,  $\text{gcd}(a, b)$  denotes the greatest common divisor of the numbers  $a$  and  $b$ , and  $\text{lcm}(a, b)$  denotes the smallest common multiple of the numbers  $a$  and  $b$ .

**Input**  
The input consists of multiple test cases. The first line contains a single integer  $t$  ( $1 \leq t \leq 10^4$ ) — the number of test cases. Description of the test cases follows.

The first and the only line of each test case contains the integer  $n$  ( $1 \leq n \leq 10^6$ ).

**Output**  
For each test case output a single integer — the number of pairs of integers satisfying the condition.

**Example**

input	output
6	1
1	4
2	7
3	10
4	11
5	20

**Note**  
For  $n = 1$  there is exactly one pair of numbers —  $(1, 1)$  and it fits.  
For  $n = 2$ , there are only 4 pairs —  $(1, 1)$ ,  $(1, 2)$ ,  $(2, 1)$ ,  $(2, 2)$  and they all fit.  
For  $n = 3$ , all 9 pairs are suitable, except  $(2, 3)$  and  $(3, 2)$ , since their  $\text{lcm}$  is 6, and  $\text{gcd}$  is 1, which doesn't fit the condition.

$$n \rightarrow 1 \text{ to } 10^6$$

$$n = 2$$

$$(1, 1) (2, 2) (2, 1) (1, 2)$$

$$2^2 \rightarrow$$

$$3 \rightarrow 2^3 \Rightarrow 8$$

$$n^2 \rightarrow \text{all combinations}$$

```

Output
For each test case output a single integer — the number of pairs of integers satisfying the condition.

Example
input
5
1
2
3
4
5
1000000000
output
1
4
7
10
11
266666666

```

**Note**  
For  $n = 1$  there is exactly one pair of numbers —  $(1, 1)$  and it fits.  
For  $n = 2$ , there are only 4 pairs —  $(1, 1)$ ,  $(1, 2)$ ,  $(2, 1)$ ,  $(2, 2)$  and they all fit.  
For  $n = 3$ , all 9 pairs are suitable, except  $(2, 3)$  and  $(3, 2)$ , since their  $\text{lcm}$  is 6, and  $\text{gcd}$  is 1, which doesn't fit the condition.

$2^n \rightarrow$   
 $3 \rightarrow 2^3 \Rightarrow 8$

$n^2 \rightarrow$  all combinations

$$\boxed{\frac{\text{LCM}(a,b)}{\text{GCD}(a,b)} \leq 3} \quad \left. \vphantom{\frac{\text{LCM}(a,b)}{\text{GCD}(a,b)}} \right\} \text{LCM}(a,b) \leq 3 \cdot \text{GCD}(a,b)$$

$a, b \rightarrow (1, \text{to } n)$

$\text{GCD} = \text{LCM}$   
 $(a=b)$

$1 \rightarrow \{1, 1\}$   
 $2 \rightarrow \{2, 2\}, \{1, 2\}, \{2, 1\}$   
 $3 \Rightarrow \{1, 3\}, \{3, 1\}, \{3, 3\}$   
 $4 \rightarrow \{2, 4\}, \{4, 2\}, \{4, 4\}$   
 $5 \rightarrow \{5, 5\}$

$\text{GCD}(2, 2) \Rightarrow 2$   
 $\text{LCM}(2, 2) \Rightarrow 2$

$\text{GCD}(2, 1) = 1$   
 $\text{LCM}(2, 1) = 2$   
 $\frac{2}{1} \leq 3 \checkmark$

$(1, 2) = (2, 1)$   
 $(2, 2)$   
 $(n, n) \rightarrow 1$

$(2, 3) \rightarrow \frac{6 = \text{LCM}}{1 = \text{GCD}}$

$6 \rightarrow \{2, 6\}, \{6, 2\}, \{3, 6\}, \{6, 3\}, \{6, 6\}$

$4, 6$   
 $\text{LCM} = 12$   
 $\text{GCD} = 2$

$$(n, n) + (n, \frac{n}{2}) (\frac{n}{2}, n) + (n, \frac{n}{3}) (\frac{n}{3}, n)$$

$(n, \frac{n}{2})$   
 $(10, 5) \rightarrow \checkmark$

$(6, \frac{6}{3})$   
 $(6, 2) \checkmark$

$\text{LCM} = 10$   
 $\text{GCD} = 2$   
 $2 \leq 3$

$$8 \rightarrow (8, 2) \rightarrow \text{LCM} \Rightarrow \underline{8} \quad (4)$$

$$\hookrightarrow \text{CD} \Rightarrow \cancel{2}$$



$$\left(n, \frac{n}{4}\right) \text{ NO}$$

$$7 \xrightarrow{n} (7, 7) \checkmark \quad n = 6$$

$$8 \rightarrow (8, 8) \checkmark \quad (8, 8)$$

$$9 \rightarrow (9) + \left(2 \times \left(\frac{n}{2} + \frac{n}{3}\right) - \frac{n}{6}\right)$$

$$\left. \begin{matrix} (6, 6) \\ (6, 6) \end{matrix} \right\} 2$$

$$\frac{n}{2} \quad (4) \rightarrow \left(\frac{n}{2}\right)$$

$$\left(\frac{n}{2}\right) \times 2$$

$$(1, 2) \rightarrow (2, 1)$$

$$\frac{n}{3} \quad (3) \rightarrow (n, n/3)$$

$$(n, n/3)^{\times 2}$$

$$\downarrow$$

$$(n/3, n)$$

$$\text{Order} \rightarrow n + 2^k \left(\frac{n}{2} + \frac{n}{3}\right)$$

} is it clear

#### D. Cat Cycle

time limit per test: 1 second  
memory limit per test: 256 megabytes

Suppose you are living with two cats: A and B. There are  $n$  napping spots where both cats usually sleep.

Your cats like to sleep and also like all these spots, so they change napping spot each hour cyclically:

- Cat A changes its napping place in order:  $n, n-1, n-2, \dots, 3, 2, 1, n, n-1, \dots$ . In other words, at the first hour it's on the spot  $n$  and then goes in decreasing order cyclically;
- Cat B changes its napping place in order:  $1, 2, 3, \dots, n-1, n, 1, 2, \dots$ . In other words, at the first hour it's on the spot 1 and then goes in increasing order cyclically.

The cat B is much younger, so they have a strict hierarchy: A and B don't lie together. In other words, if both cats'd like to go in spot  $x$  then the A takes this place and B moves to the next place in its order (if  $x < n$  then to  $x+1$ , but if  $x = n$  then to 1). Cat B follows his order, so it won't return to the skipped spot  $x$  after A frees it, but will move to the spot  $x+2$  and so on.

Calculate, where cat B will be at hour  $k$ ?

Input

The first line contains a single integer  $t$  ( $1 \leq t \leq 10^4$ ) — the number of test cases.

The first and only line of each test case contains two integers  $n$  and  $k$  ( $2 \leq n \leq 10^9$ ;  $1 \leq k \leq 10^9$ ) — the number of spots and hour  $k$ .

Output

A, B

n

The first line contains a single integer  $t$  ( $1 \leq t \leq 10^4$ ) — the number of test cases.

The first and only line of each test case contains two integers  $n$  and  $k$  ( $2 \leq n \leq 10^9$ ;  $1 \leq k \leq 10^9$ ) — the number of spots and hour  $k$ .

### Output

For each test case, print one integer — the index of the spot where cat B will sleep at hour  $k$ .

#### Example

##### input

```
7
2 1
2 2
3 1
3 2
3 3
5 5
69 1337
```

##### output

```
1
2
1
3
2
2
65
```

#### Note

In the first and second test cases  $n = 2$ , so:

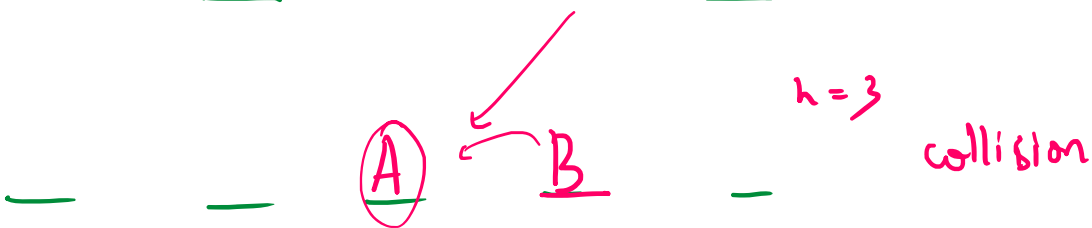
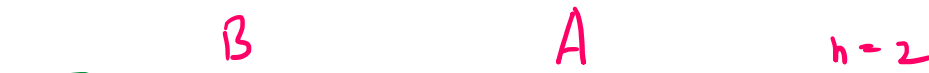
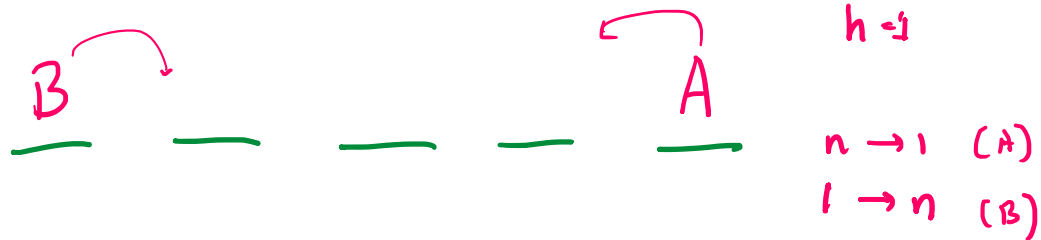
- at the 1-st hour, A is on spot 2 and B is on 1;
- at the 2-nd hour, A moves to spot 1 and B — to 2.

If  $n = 3$  then:

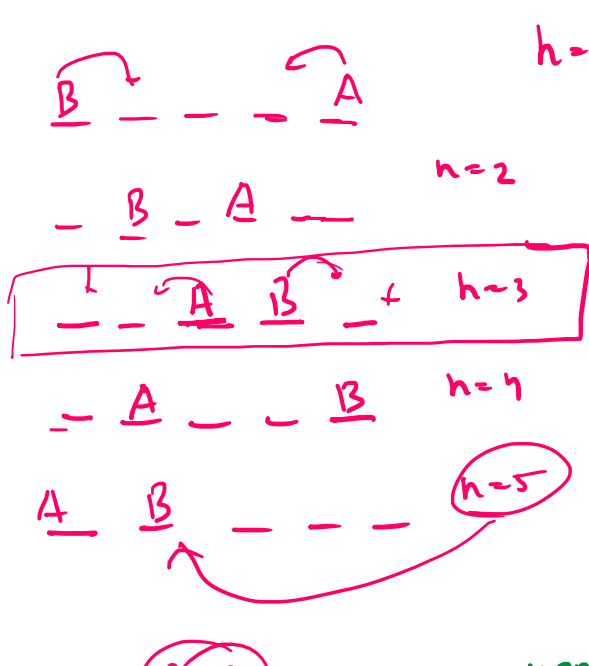
- at the 1-st hour, A is on spot 3 and B is on 1;
- at the 2-nd hour, A moves to spot 2; B'd like to move from 1 to 2, but this spot is occupied, so it moves to 3;
- at the 3-rd hour, A moves to spot 1; B also would like to move from 3 to 1, but this spot is occupied, so it moves to 2.

In the sixth test case:

- A's spots at each hour are [5, 4, 3, 2, 1];
- B's spots at each hour are [1, 2, 4, 5, 2].



$k \rightarrow$  cat B will  
be where



problem statement

$n = 4$

$h = 4$



$n = 2$

$B = 10 \Rightarrow$  ✓

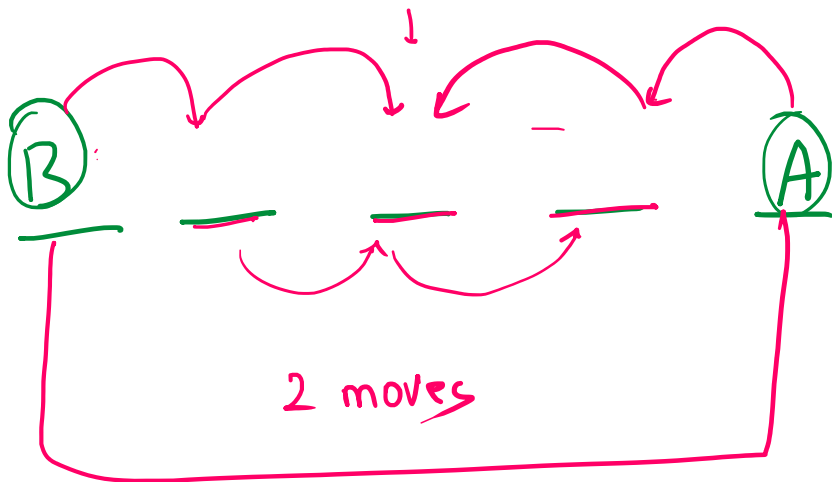
$$n \rightarrow 10$$

lyen

     B A       $h=2$   
           ↓  
     A B       $h=3$   
A           B  $h=4$   
B           A  $h=5$

 $k=1, k=5$ 

$k=1, k=5$   
 1 2 3 4 5 6 7  
B A B A B B B

$$B = \underbrace{k \text{ steps}} + \underbrace{\text{No of collisions}}$$


$n \rightarrow \text{odd}$   
↓  
collide

3 hop

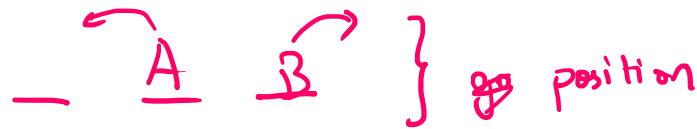
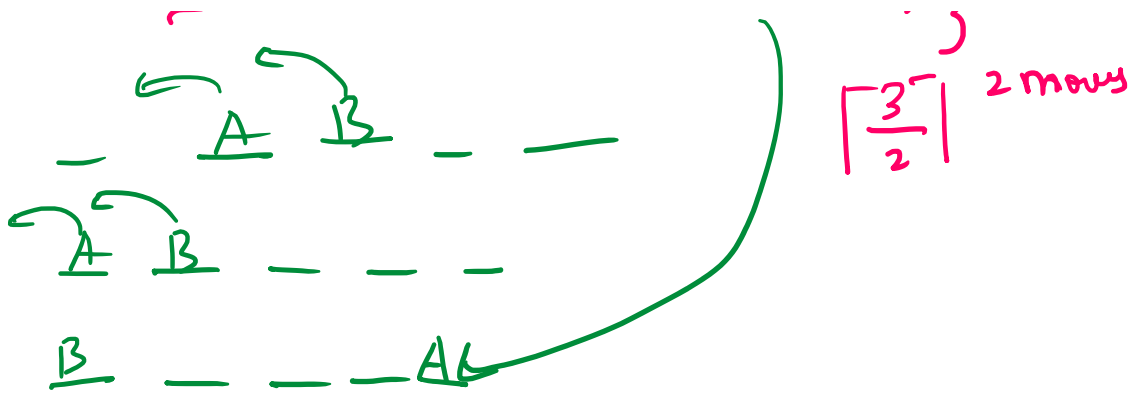
5 groups

$\rightarrow \left\lceil \frac{3}{2} \right\rceil \Rightarrow 2$

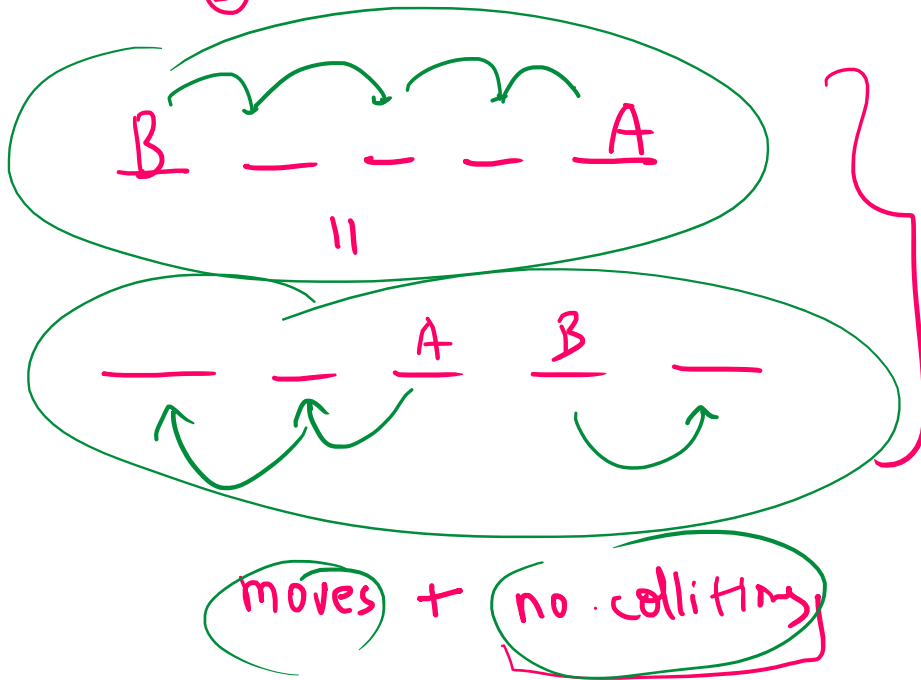


why 0?

→ 3  
[2] 2 moves



①



K hours

1) collision happens  $\rightarrow \lceil \frac{n}{2} \rceil$

$K / \lceil \frac{n}{2} \rceil \rightarrow$  colliding



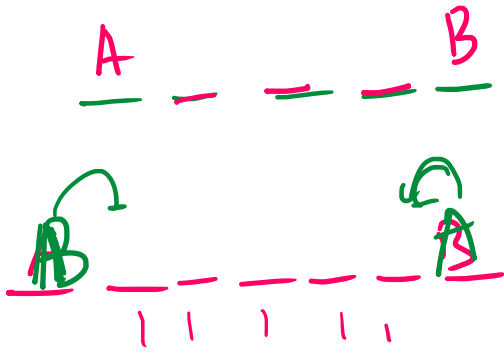
moves  $\rightarrow$  K hours  
 $\downarrow$   
 (K-1) moves

$\downarrow$   
 colliding  $\rightarrow$

$n-2 \Rightarrow \lceil 3 \rceil$



collisions  $\rightarrow$



$$n-2 \Rightarrow \left\lceil \frac{3}{2} \right\rceil$$

$$\left\lceil \frac{n-2}{2} \right\rceil$$

$\rightarrow$  hours

$\downarrow$   
 $(k-1)$   
 $\downarrow$   
moves

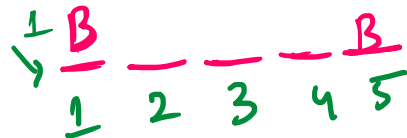
$(k-1) - m$

$\frac{k-1}{\left\lceil \frac{n-2}{2} \right\rceil} \rightarrow$  no of collisions

total B moves  $\rightarrow$  moves + #

$$\Rightarrow (k-1) + \left\lceil \frac{k-1}{\left\lceil \frac{n-2}{2} \right\rceil} \right\rceil \left\{ \begin{array}{l} \text{collisions} \\ \text{infinite} \end{array} \right.$$

$$(k-1) + \frac{(k-1)}{\left\lceil \frac{n-2}{2} \right\rceil}$$



$k=5$

6 moves  
 $\downarrow$  5th

$$4 + \frac{(4)}{\left\lceil \frac{5-2}{2} \right\rceil} \Rightarrow 4 + \frac{4}{2}$$

$\swarrow$  length + some position

$$k = 5 \% 5 = 0$$

$\rightarrow$  4 moves  $\rightarrow$  5 moves

$\rightarrow$  6 moves

$\downarrow$  m move it's coming back

$k \% n = \text{my position}$

$$\left\lceil 5 \% 5 = 0 \right\rceil + 1$$

$$6 \% 5 = 1 + 1 \Rightarrow 2 \text{ nd}$$

$$\rightarrow \text{Rotations} \Rightarrow \left\{ \frac{\text{total moves}}{\text{length}} \right\}$$

the position  $\Rightarrow (\text{total moves} \% \text{length}) + 1$

↓  
moves B  $\rightarrow$  %

5) T-prime

$\rightarrow n \rightarrow \text{T-prime} \rightarrow 7 \rightarrow 1 \times 7$   
 $\rightarrow 1 \times 7 \times 7 \Rightarrow 49$

↓  
(1, 7, 49)

4  
 1 2 4  $\rightarrow \{1, p, p^2\}$  —

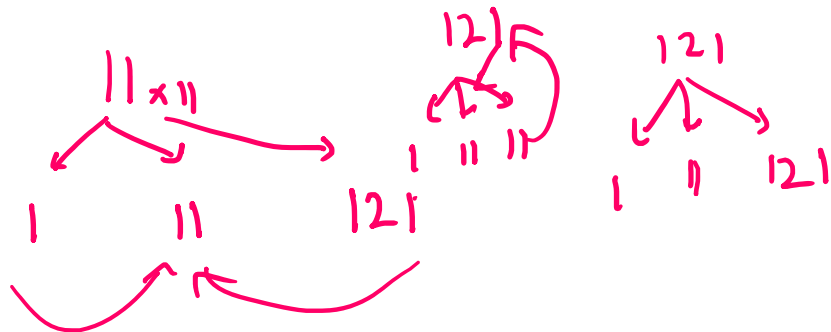
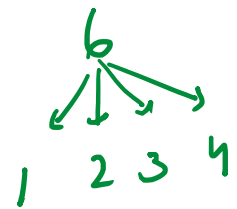
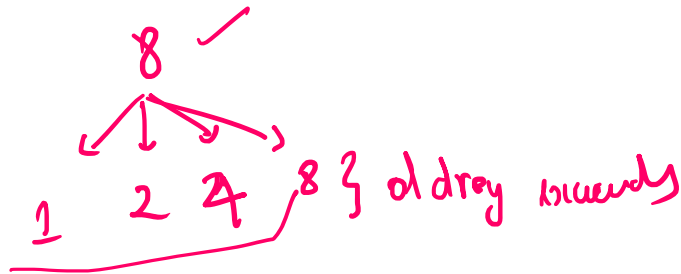
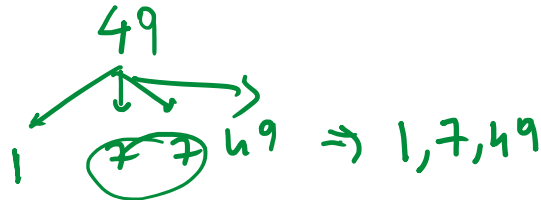
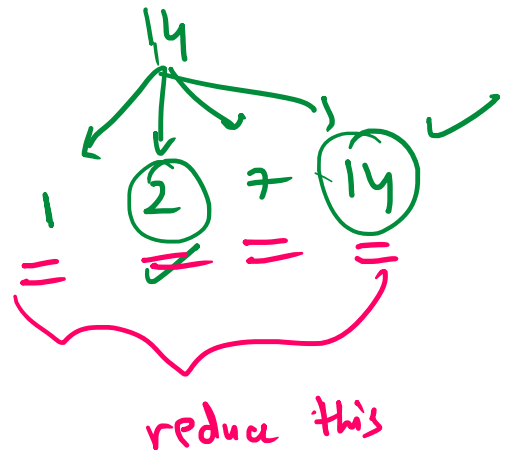
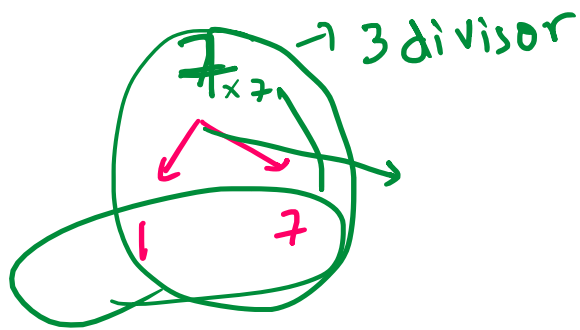
$n \Rightarrow 10 \rightarrow$  T prime or not

10  
 1 10 2 5  
 — — — —  
 4, 3 div  
 No

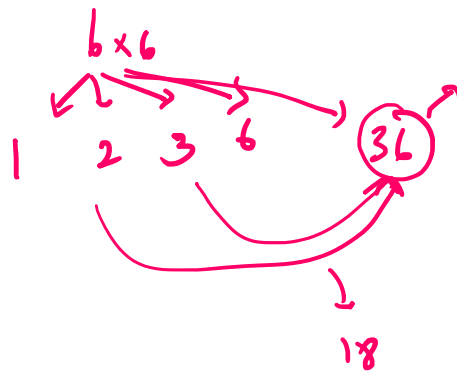
4  
 1 2 5  
 — — — —  
 Yes

1  $\rightarrow$  3 divisor

14



\* Why does work → 6x6



→  $n \rightarrow T_{\text{prime}}$

$$\rightarrow n \rightarrow T_{\text{prime}}$$