

## Vanya and Cubes

time limit per test: 1 second  
memory limit per test: 256 megabytes

Vanya got  $n$  cubes. He decided to build a pyramid from them. Vanya wants to build the pyramid as follows: the top level of the pyramid must consist of 1 cube, the second level must consist of  $1 + 2 = 3$  cubes, the third level must have  $1 + 2 + 3 = 6$  cubes, and so on. Thus, the  $i$ -th level of the pyramid must have  $1 + 2 + \dots + (i - 1) + i$  cubes.

Vanya wants to know what is the maximum height of the pyramid that he can make using the given cubes.

## Input

The first line contains integer  $n$  ( $1 \leq n \leq 10^5$ ) — the number of cubes given to Vanya.

## Output

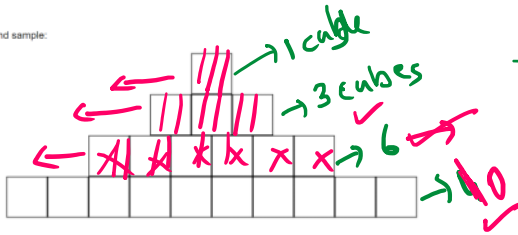
Print the maximum possible height of the pyramid in the single line.

## Examples

input	Copy
1	
output	Copy
1	
input	Copy
4	
output	Copy
4	

Note  
Illustration to the second sample:

Note  
Illustration to the second sample:



Math  $\rightarrow$  Basic  $\checkmark$  + logic  $\rightarrow$  800  
1000

n cubes  $\rightarrow$  how many level  
can be build

Pattern  
order  $\uparrow$   
1, 3, 6, 10  $\checkmark$   
1  $\rightarrow$  1+2  $\rightarrow$  1+2+3  $\rightarrow$  1+2+3+4  
 $\rightarrow$  sum of  $n$  element  
 $\frac{n(n+1)}{2}$  basic

$$\frac{3 \times (3+1)}{2} \rightarrow 6 \text{ cubes}$$

how many, require 3 levels as a whole

$$\sum (\text{required blocks of level}) = \text{total blocks}$$

$n = 25$  ?  $\rightarrow$  level

$$\begin{array}{l} 1 \rightarrow 1 \\ 2 \rightarrow 3 \rightarrow \frac{n(n+1)}{2} \\ 3 \rightarrow 6 \rightarrow 11 \\ 4 \rightarrow 10 \rightarrow \\ 5 \rightarrow 15 \end{array}$$

$$\begin{array}{l} 1+31 \checkmark \\ 4 < 25 \checkmark \\ 4+6 < 25 \checkmark \\ 10+10 < 25 \checkmark \\ 20+15 \not< 25 \times \end{array} \quad \left. \begin{array}{l} \\ \\ \\ \\ \end{array} \right\} 4 \text{ levels}$$

```

int testcases = 1;
while (testcases-- > 0) {

    int n = sc.nextInt();
    int total = 0;
    int sum = 0;
    int count = 0;
    for (int i = 1; i <= 10000; i++) {
        sum += i;
        total += sum;
        if (total > n) {
            break;
        }
        count++;
    }
    System.out.println(count);
}

```

totally sum → ✓

### C. Phoenix and Balance

time limit per test: 2 seconds  
memory limit per test: 256 megabytes

} no of coins is even

Math + Greedy better ✓

Phoenix has  $n$  coins with weights  $2^1, 2^2, \dots, 2^n$ . He knows that  $n$  is even.

He wants to split the coins into two piles such that each pile has exactly  $\frac{n}{2}$  coins and the difference of weights between the two piles is minimized. Formally, let  $a$  denote the sum of weights in the first pile, and  $b$  denote the sum of weights in the second pile. Help Phoenix minimize  $|a - b|$ , the absolute value of  $a - b$ .

#### Input

The input consists of multiple test cases. The first line contains an integer  $t$  ( $1 \leq t \leq 100$ ) — the number of test cases.

The first line of each test case contains an integer  $n$  ( $2 \leq n \leq 30$ ;  $n$  is even) — the number of coins that Phoenix has.

#### Output

For each test case, output one integer — the minimum possible difference of weights between the two piles.

#### Example

input	Copy
2	
2	
4	
output	Copy
2	
6	

#### Note

In the first test case, Phoenix has two coins with weights 2 and 4. No matter how he divides the coins, the difference will be  $4 - 2 = 2$ .

In the second test case, Phoenix has four coins of weight 2, 4, 8, and 16. It is optimal for Phoenix to place coins with weights 2 and 16 in one pile, and coins with weights 4 and 8 in another pile. The difference is  $(2 + 16) - (4 + 8) = 6$ .

$n$  is even?  
→ total no of coins will be even

$$n=4$$

$2^1, 2^2, 2^3, 2^4 \Rightarrow$  equally

odd ✓  
 $n/2 \rightarrow$  2 2 differ

$$|2^1, 2^2 - 2^3, 2^4| = \text{minimum} \star$$

testcase 1 →  $2=n$

$2^1, 2^2 \rightarrow$  split it equally in 2 piles

$$\left| \textcircled{2^1} - \textcircled{2^2} \right| \Rightarrow |4 - \textcolor{green}{2}| = \textcolor{green}{2} = \text{minimize}$$

No we cant

test case - 2,  $n = 4$

$2^1, 2^2, 2^3, 2^4$  - All the coins

$$\left| \textcircled{2^1} \textcircled{2^2} - \textcircled{2^3} \textcircled{2^4} \right| = \text{minimized}$$

$$|2 + 4 - 8 + 16| \Rightarrow$$

$$|18 - 24| \Rightarrow \textcircled{18} \text{ value minimize } \checkmark$$

Yes

$$\textcircled{2^1} \textcircled{2^2} \textcircled{2^3} \textcircled{2^4}$$

$$\left| \textcircled{n/2} - \textcircled{n/2} \right| = \text{minimum}$$

$$|2^4 + 2^2 - 2^1 + 2^3| \Rightarrow |20 - 10| = \textcircled{10} \checkmark$$

Rest others

$$\left| 2^4 + 2^1 - 2^2 + 2^3 \right| = |18 - 12| = 6$$

Rest of



```
while (testcases-- > 0) {
    int coins = sc.nextInt();
    int req = coins/2;
    long sum1 = 0;
    long sum2 = 0;
    long cur = 2;

    for(int i = 1 ; i <= coins ; i++){
        if(i < req || i == coins){
            sum1 = sum1 + cur;
        }else{
            sum2 = sum2 + cur;
        }

        cur = cur * 2;
    }
    System.out.println(sum1 - sum2);
}
```

maximize  
↑  
minimum

3 = smallest + largest + smallest - n

smallest  
↓  
[n=6)  
(n=0)  
n

```
while (testcases-- > 0) {
    int coins = sc.nextInt();

    int req = coins/2;
    long sum1 = 0;
    long sum2 = 0;
    long cur = 2;

    for(int i = 1 ; i <= coins ; i++){
        if(i < req || i == coins){
            sum1 = sum1 + cur;
        }else{
            sum2 = sum2 + cur;
        }

        cur = cur * 2;
    }

    System.out.println(sum1 - sum2);
}
```

✓ → sum  
✓ → rear

## B. Queries about less or equal elements

time limit per test: 2 seconds ✓  
memory limit per test: 256 megabytes

You are given two arrays of integers  $a$  and  $b$ . For each element of the second array  $b_j$  you should find the number of elements in array  $a$  that are less than or equal to the value  $b_j$ .

### Input

The first line contains two integers  $n, m$  ( $1 \leq n, m \leq 2 \cdot 10^5$ ) — the sizes of arrays  $a$  and  $b$ .

The second line contains  $n$  integers — the elements of array  $a$  ( $-10^9 \leq a_i \leq 10^9$ ).

The third line contains  $m$  integers — the elements of array  $b$  ( $-10^9 \leq b_j \leq 10^9$ ).

### Output

Print  $m$  integers, separated by spaces: the  $j$ -th of which is equal to the number of such elements in array  $a$  that are less than or equal to the value  $b_j$ .

### Examples

input	Copy
5 4	
1 3 5 7 9	
6 4 2 8	
output	Copy
3 2 1 4	

✓ } Ans

input	Copy
5 5	
1 2 1 2 5	
3 1 4 1 5	
output	Copy
4 2 4 2 5	

## 2 Arrays

$a = [1, 3, 5, 7, 9]$   $O(n)$   
 $b = [6, 4, 2, 8]$   $m$   
Time ✓

3 2 1 4 → ti

problem with this approach is time

Most →

$$n = 2 \times 10^5$$

$$m = 2 \times 10^5$$

$$m \times n \Rightarrow 2 \times 10^5 \times 2 \times 10^5$$

$$\Rightarrow 4 \times 10^{10} / 2 \text{ seconds}$$

$$4 \times 10^{10} \gg 2 \times 10^9 \quad \downarrow \quad 2 \times 10^9$$

\* Searching → Binary Search ✓?

5 4  
1 3 5 7 9  
6 4 2 8  
↓  
[1 2 4 5 6 7 8] sorted  
↓  
log n  
6 4 2 8  
→ 6 4 2 8

$\log n \ll n \rightarrow m \log n \gg m \times n$  ✓ Binary

5 4  
① ③ ⑤ 7 9  
⑥ ④ 2 8  
log n → time complexity  
≤ b\_j  
B.S ★  
upper bound → learn this  
lower bound → learn this

① ③ ⑤ ⑦ ⑨  
⑥ ④ 2 8

upper bound  
lower bound → learn this

upper bound → Exactly }

↳ how many are less than or equal to the current element

\* C++ upperbound (start ind, end ind, arr) ✓

\* Java → upperbound — have —

\* python → left bisection / Right bisection — have —

overall → Sort the array + Search

$n \log n + m \log n$  } put <<<  $m \times n$   
Binary search

```
int testcases = 1;
while (testcases-- > 0) {
    int n = sc.nextInt();
    int m = sc.nextInt();

    int[] arr = inputArray(sc, n);
    int[] queries = inputArray(sc, m);
    Arrays.sort(arr);

    for (int i = 0; i < queries.length; i++) {
        int ans = upperBound(arr, queries[i]);
        out.print(ans + " ");
    }
    out.println("");
}
out.close();
```

# E. AvtoBus

time limit per test: 1 second  
memory limit per test: 256 megabytes

Spring has come, and the management of the AvtoBus bus fleet has given the order to replace winter tires with summer tires on all buses.

You own a small bus service business and you have just received an order to replace  $n$  tires. You know that the bus fleet owns two types of buses: with two axes (these buses have 4 wheels) and with three axes (these buses have 6 wheels).

You don't know how many buses of which type the AvtoBus bus fleet owns, so you wonder how many buses the fleet might have. You have to determine the minimum and the maximum number of buses that can be in the fleet if you know that the total number of wheels for all buses is  $n$ .

## Input

The first line contains an integer  $t$  ( $1 \leq t \leq 1\,000$ ) — the number of test cases. The following lines contain description of test cases.

The only line of each test case contains one integer  $n$  ( $1 \leq n \leq 10^{18}$ ) — the total number of wheels for all buses.

## Output

For each test case print the answer in a single line using the following format.

Print two integers  $x$  and  $y$  ( $1 \leq x \leq y$ ) — the minimum and the maximum possible number of buses that can be in the bus fleet.

If there is no suitable number of buses for the given  $n$ , print the number  $-1$  as the answer.

## Example

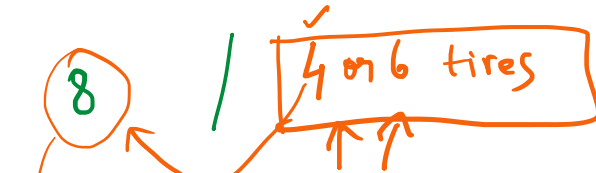
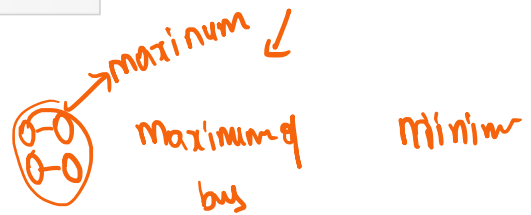
input	Copy
4	
4	
7	
24	
998244353998244352	
output	Copy
1 1	
-1	
4 5	
166374058999707392 249561088499561088	

7 → tie  
↪  $n = 7$

2 types of bus

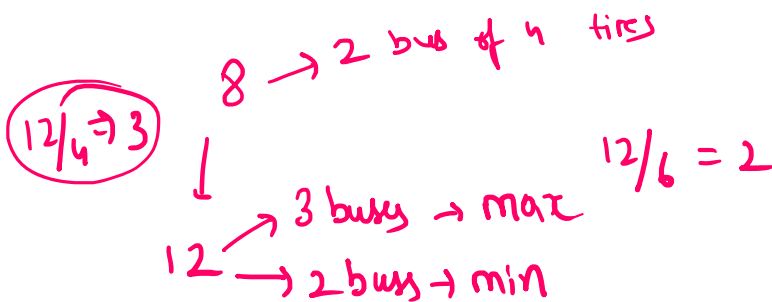


$n$  = number of tires



$$8/4 \Rightarrow 2 \text{ buses}$$

$$8/6 \Rightarrow 8-6 \Rightarrow 2 \text{ tires} \times \text{only bus} \times$$

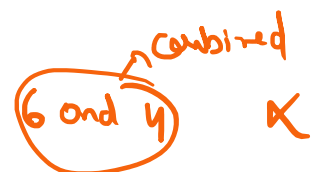


problem statement

24

$$\text{max} \rightarrow 24/4 \Rightarrow 6 \rightarrow \text{max}$$

$$\text{min} \rightarrow 24/6 \Rightarrow 4 \rightarrow \text{min}$$



26 tires  $\rightarrow$  Is it possible  
Is it not

#### Note

In the first test case the total number of wheels is 4. It means that there is the only one bus with two axes in the bus fleet.

In the second test case it's easy to show that there is no suitable number of buses with 7 wheels in total.

In the third test case the total number of wheels is 24. The following options are possible:

- Four buses with three axes.
- Three buses with two axes and two buses with three axes.
- Six buses with two axes.

So the minimum number of buses is 4 and the maximum number of buses is 6.

$$\frac{24}{6}$$

4 and 6

\* 7 tires  $\rightarrow$  buses  $\times$

$$4 + 6 = 10 \rightarrow \times$$

$\rightarrow$  odd is wrong  $\times$

\* 3 tires / 2 / 1  $\times \rightarrow$  -1

edge cases

26  $\rightarrow$  fit the axles

(4, 6)  $\rightarrow$  combined can add upto any positive number  $\geq 4$  ✓

$$\left. \begin{array}{l} 26 \% 4 == 2 \\ 26 \% 6 == 2 \end{array} \right\} \text{not divisible}$$

$$26 - 4 - 4 - 4 - 4 - 4 \Rightarrow 22 \rightarrow 6 \rightarrow \checkmark$$

$$26 \% 4 \neq 2 \text{ remainder}$$



$$26 \% 4 \neq 2 \text{ remainder}$$

$$\frac{26}{4} \Rightarrow \text{X } 5 \quad 2+4 \Rightarrow 6 \rightarrow \text{remainder}$$

26  $\rightarrow$  maximum possible divides !

$$26 - 4 \times 5 \Rightarrow 6 \rightarrow 1+5 \Rightarrow 6$$

$$\frac{26}{4} \Rightarrow \text{Ans } \checkmark \rightarrow 6$$

$$26 - \underline{20} + 6 \rightarrow \frac{26}{4}$$

$\uparrow 6$

$\frac{26}{4}$  is true  $\rightarrow$  maximum

$$26 - 6 \Rightarrow 20 - 6 \Rightarrow 14 - 6 \Rightarrow 8/4$$

4, 6 are magical together

\* Max number of 6

$$\begin{array}{l} 1) \quad 26 \% 6 = 2 \rightarrow \frac{x-8}{6} + 2 \quad \checkmark \text{ sach } \checkmark \\ 2) \quad x \% 6 = 4 \rightarrow \frac{x-4}{6} + 1 \quad \checkmark \end{array}$$

$$26 \% 6 = 2 \Rightarrow 8 \rightarrow 4 \times 2$$

$$26 \rightarrow 6 \times 3 \Rightarrow 18 \quad 24$$

$$\begin{array}{r} 26 \\ -24 \\ \hline \end{array}$$

$$\begin{array}{r} 26 \\ -18 \\ \hline \end{array}$$

$$(3)$$

$$22 \% 6 = 4$$

$$\frac{22}{6} \Rightarrow 3+1 \Rightarrow 4$$

$$22 \% 6 \Rightarrow 4$$

$$\begin{array}{r} 26 \\ -24 \\ \hline 2 \end{array} \text{ times } \times$$

$$\begin{array}{r} 26 \\ -18 \\ \hline 8 \end{array} \rightarrow 2$$

$$6 \\ 22 \div 6 \Rightarrow 4$$

```
int testcases = sc.nextInt();
while (testcases-- > 0) {
    long n = sc.nextLong();
    if (n < 4 || (n & 1) == 1) {
        out.println(-1);
    } else {
        long min = n / 6;
        if (n % 6 != 0) {
            min++;
        }
        long max = n / 4;
        out.println(min + " " + max);
    }
}
```

how this works.

### D. Maximum Sum

time limit per test: 2 seconds  
memory limit per test: 256 megabytes

You are given an array  $a_1, a_2, \dots, a_n$ , where all elements are different.

You have to perform exactly  $k$  operations with it. During each operation, you do exactly one of the following two actions (you choose which to do yourself):

- find two minimum elements in the array, and delete them;
- find the maximum element in the array, and delete it.

You have to calculate the maximum possible sum of elements in the resulting array.

#### Input

The first line contains one integer  $t$  ( $1 \leq t \leq 10^4$ ) — the number of test cases.

Each test case consists of two lines:

- the first line contains two integers  $n$  and  $k$  ( $3 \leq n \leq 2 \cdot 10^5$ ;  $1 \leq k \leq 99999$ ;  $2k < n$ ) — the number of elements and operations, respectively.
- the second line contains  $n$  integers  $a_1, a_2, \dots, a_n$  ( $1 \leq a_i \leq 10^9$ ; all  $a_i$  are different) — the elements of the array.

Additional constraint on the input: the sum of  $n$  does not exceed  $2 \cdot 10^5$ .

#### Output

For each test case, print one integer — the maximum possible sum of elements in the resulting array.

For each test case, print one integer — the maximum possible sum of elements in the resulting array.

#### Example

input	output
6	21
5 1	11
2 5 1 10 6	3
5 2	62
2 5 1 10 6	46
3 1	3999999986
1 2 3	
6 1	
15 22 12 10 13 11	
6 2	
15 22 12 10 13 11	
5 1	
999999996 999999999 999999997 999999998 999999995	

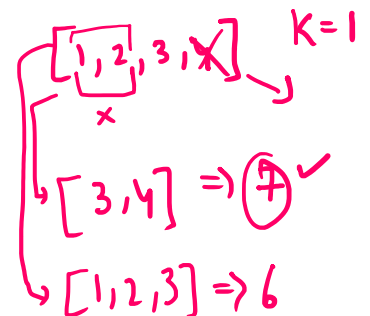
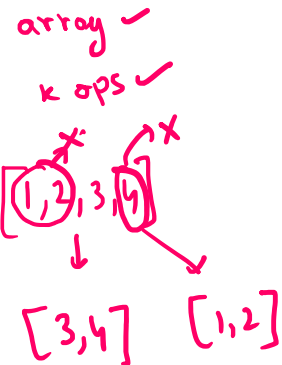
#### Note

In the first testcase, applying the first operation produces the following outcome:

- two minimums are 1 and 2; removing them leaves the array as  $[5, 10, 6]$ , with sum 21;
- a maximum is 10; removing it leaves the array as  $[2, 5, 1, 6]$ , with sum 14.

21 is the best answer.

In the second testcase, it's optimal to first erase two minimums, then a maximum.



Greedy strategy.

\* This will fail

6  $k=2$



6 k=2

max  
min

15 2 2 12 10 13 11

6 2

① → remove

~~10~~ + ~~11~~  
↓  
21

12 13 15 22  
↓  
22

k=1 → one move

[~~12, 13~~], 15, [~~22~~]  
↓  
25 > 22

[12, 13, 15] ⇒ 40

[10 + 11 12 13 15 ~~22~~]  
↓ 21 < 22

[10, 11, 12, 13, ~~22~~]

[10, 11, 12, 13] ⇒ 46

\* Greedy fails

dp → (MLE)/TLE ✓

X All combination

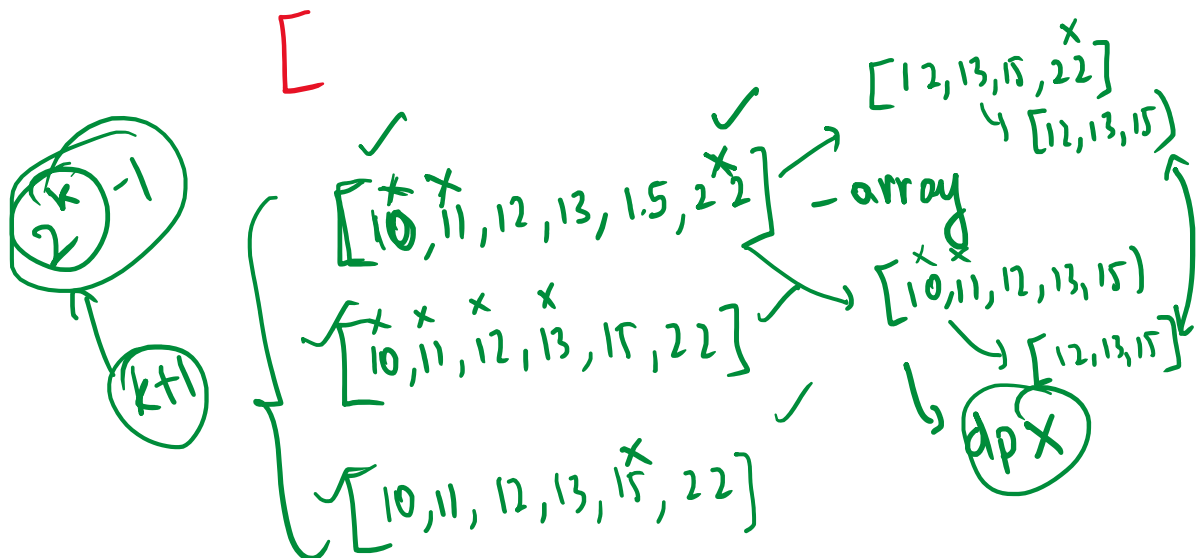
\* here is all combination  $\rightarrow$  dp all combos

$k = 2$

	Op1	Op2
→	✓	✓
→	✓✓	✗
→	✗	✓✓✓

→ } - k1  
ho  
0

$k+1$  combinations  $\rightarrow$  possibilities  $\rightarrow 2^k \times$



crux of the problem  $\rightarrow k+1$



- \* We need to take the max of all the possible combinations (i.e k+1)
- \* We can calculate the remaining sum with  $O(1)$  time complexity using prefix sums

```
int testcases = sc.nextInt();
while (testcases-- > 0) {

    int n = sc.nextInt();
    int k = sc.nextInt();
    int[] arr = new int[n];
    long[] prefix = new long[n + 1];

    for (int i = 0; i < n; i++) {
        arr[i] = sc.nextInt();
    }

    Arrays.sort(arr);
    for (int i = 0; i < n; i++) {
        if (i == 0) {
            prefix[i + 1] = arr[i];
        } else {
            prefix[i + 1] = arr[i] + prefix[i];
        }
    }

    printArray(prefix, out);
}

long max = 0;
for (int i = 0; i <= k; i++) {
    max = Math.max(max, prefix[n - i] - prefix[2 * (k - i)]);
}

System.out.println(max);
```