Technische Universität Chemnitz Chemnitz, 15th January 2018
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Optimization for Non-Mathematicians

Sheet 13

Exercise 30: Geometrical shape optimization of a bottle

A bottle consists of a cylinder and a frustum on its top, it shall hold 1.5 liter. The bottle can be described by the radii r_1, r_2 and the heights h_1, h_2 (see Figure 1). The bottle opening is fixed to $r_2 = 0.8$ cm. In addition, the height of the bottle has to be at least 20 cm. How do we have to choose the dimensions to minimize the surface area? (A small surface area means e.g. low material requirements for the production and a low temperature loss into the environment.)

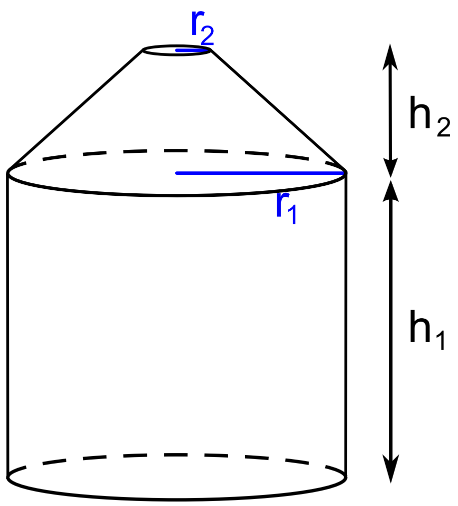

Figure 1: Dimensions of the bottle

(a) How can the volume and the surface area of the bottle be calculated?

(b) What is the optimization problem?

(c) Solve the minimization problem using `fmincon`.

Exercise 31: Evaluation of offers

A company wants to buy a certain amount M of a commodity and therefore requests offers from n supplying companies.

(a) Model an optimization problem to minimize the procurement costs.

(b) Solve the optimization problem using `fmincon` with the following data:

- $M = 1000$ units have to be bought.
- Company 1 can supply at most 400 units, with a cost of 0.80 € per unit.
- Company 2 can supply at most 500 units. The first 100 units cost 0.83 €. Beginning with 101st unit, a unit costs 0.78 €.
- Company 3 can supply at most 600 units with a basic cost of 0.85 €. The percentage discount for x_3 ordered units is $(\frac{x_3}{100})\%$.

(c) What can the Lagrange multipliers say about the additional costs for one extra unit?

Exercise 32: Distance between point and surface

Let a point $P = (\frac{1}{2}, 2, 2)$ and a surface, described by an equation $h(x) = 0$, be given in a three dimensional space. Find a point $x^\star$ which lies on the surface and has a minimal (Euclidian) distance to P.

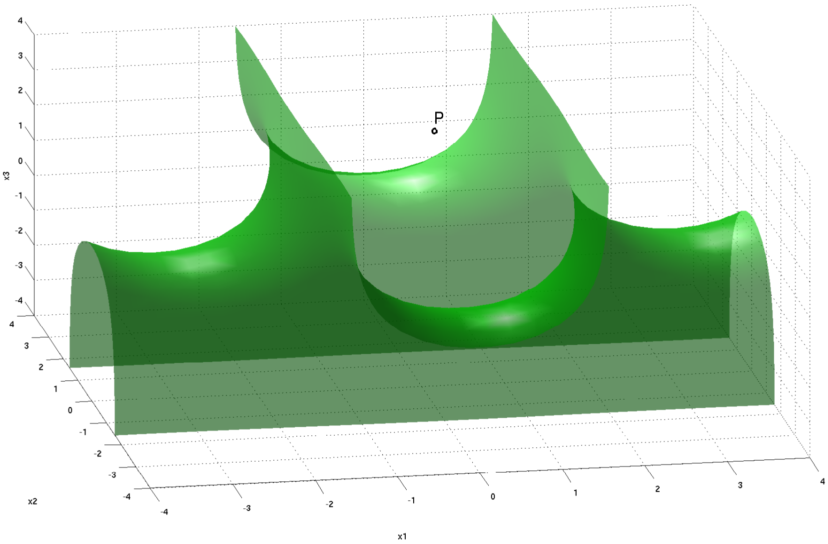

Figure 2: Determine the distance between the point P and e.g. Scherk's minimal surface

As examples we can use the following surfaces:

- $h(x) = 2x_1 - x_2 + x_3 - 4$
- $h(x) = x_1^2 + x_2^2 - x_3 - 4$

- $h(x) = (x_1 + 1)^2 + 9\,(x_2 + 1)^2 + \frac{1}{4}x_3^2 - 4$
- $h(x) = (x_1 + 1)^2 + (x_2 + 1)^2 - \frac{1}{4}x_3^2 - 2$
- $h(x) = (x_1 - x_3 - 1)^2 + (x_2 - x_3)^2 - 1$
- $h(x) = e^{x_3}\cos(x_1) + \cos(x_2)$

(a) What is the optimization problem?

(b) Solve the minimization problem using `fmincon`.

(c) Check the dependence of the last surface's solution of the initial point x_0.