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Chemnitz, 18th December 2017

# Optimization for Non-Mathematicians

## Sheet 11

### Exercise 25: Shortest path

In large wireless networks data are often sent from a transmitter to a receiver along paths across several nodes. The reliability of such a path plays an important role, because the data package has to be sent again if an error occurs. The individual edges between the nodes have a known probability for an error-free transfer of a data packet (see Figure 1). The individual transfers over an edge within a path are considered to be independent events. For given transmitter and receiver nodes, find a path in the network with a minimal probability of errors.

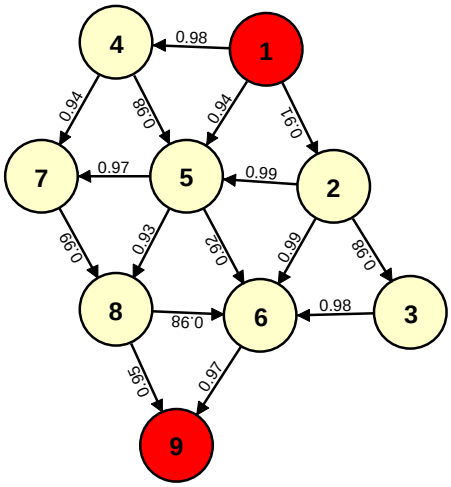


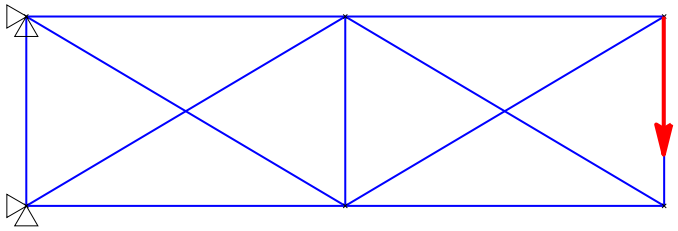

Figure 1: Graph of the wireless network with edges and their probabilities of an error-free transfer. The transmitter is node #1 and the receiver is node #9.

(a) Model a mathematical optimization problem. Reformulate the objective function to get a linear problem.

(b) Solve the problem in MATLAB using `linprog`. Details about the wireless network can be found in the file `data_wireless_network.m` on the homepage. Plot the solution using `Graphviz`.

**Exercise 26: Structural optimization**

A planar ideal structure is to be optimized with regard to its weight. Suppose that the structure is described by a graph (joints of the structure = nodes, rods = edges). The positions $p_1, \ldots, p_m \in \mathbb{R}^2$ of the joints and the edge set $E$ are given. In addition, the external forces acting on some of the nodes are given, and the nodes which are mounted are known as well.



- The force in a rod between the nodes $i$ and $j$ is denoted as $x_{ij} \in \mathbb{R}$ and it can only act along the rod. Positive $x_{ij}$ correspond to tensile loads and negative $x_{ij}$ to pressure loads.
- In every node, a balance of forces regarding to the $x$ and $y$ component has to be fulfilled. This balance includes the forces of all rods connected to the joint as well as the external forces acting at the joint.
- For clamped (mounted) nodes, the balance of forces can be omitted, because the balance is automatically fulfilled by the clamping.
- The weight of a rod between the joints $i$ and $j$ is (for fixed length) proportional to the surface area $F_{ij}$ of its cross section. For simplification, the weight is only included in the objective but not in the balances of forces.
- The surface area $F_{ij}$ of each rod is chosen in such a way that only a given maximal stress works on the rod. This leads to the proportionality $|x_{ij}| = \lambda F_{ij}$, where we set the factor $\lambda = 1$ for simplification.

(a) Formulate the balance of forces for each node.

(b) What is the objective function modeling the overall weight of the structure?

(c) How can we reformulate $|x_{ij}|$ to get a linear problem?

(d) Solve the problem in MATLAB using `linprog`. With the script `structure_example.m` on the homepage you can load the joint positions `pos`, the edge set `E`, the external forces `b` and the clamped nodes `clamp`. To draw the structure with rod forces in a vector `x` you can use the function `plot_structure(pos,E,solution,clamp,b)`.

(e) On the homepage you will find more examples with significantly larger numbers of possible edges: `structure_beam.m` and `structure_bridge.m`.