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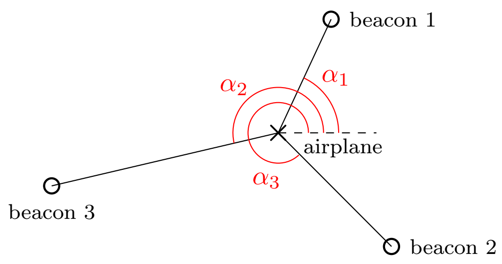
Optimization for Non-Mathematicians

Sheet 7

Exercise 15: Positioning



The position (x, y) of an airplane should be determined by radio bearing. Therefore the directions towards several radio beacons is measured from the airplane. The positions (x_i, y_i) of the beacons are known and the directions are measured as angles α_i to the x-axis:[1]

beacon i	position (x_i, y_i)	angle α_i
1	$(8, 6)$	38°
2	$(-3, -3)$	220°
3	$(1, 0)$	222°
4	$(8, -3)$	300°

The aim is to find the position (x, y) of the airplane for which the differences between measured and real angles are as small as possible (in the least squares sense).

(a) Previously, we used `lsqcurvefit` and provided a model function and a measurement vector. In this example, this may not be the most convenient approach. Is there another way to calculate the residuals $r_i(x)$ directly so that you can use `lsqnonlin`?

(b) Solve the problem using `lsqnonlin` with the Levenberg-Marquardt algorithm.

(c) Extra: The measured data for the angles α_i are usually perturbed. Generate a scatter plot by performing the parameter identification for different (randomly) perturbed angle data. Which of the four beacons is particularly important for the positioning result in this example?

[1] For clarity you see only three beacons in the picture.

Exercise 16: Parameter identification for an ODE model

A mass mounted on a spring under the influence of damping moves according to the following ordinary differential equation (ODE),

$$m\ddot{y} + r\dot{y} + ky = 0,$$

with known initial conditions $y(0) = 1$ and $\dot{y}(0) = 0$. In an experiment, the deflection y_i is measured at different points in time t_i, $i = 1, \ldots, N$. The goal is to determine the unknown damping and spring constants r and k. The mass $m = 2$kg is known.

(a) Describe the functionality of the model function when the dependent variables are given implicitly by an ODE, as is the case here.

(b) Solve the problem using `lsqcurvefit` with the Levenberg-Marquardt algorithm.

(c) Generate a plot showing the measurements and the solution of the ODE with optimal parameters.

Hints:

- On our homepage you will find the file `schwingung.txt` with the measurements (t_i, y_i) and the file `ode_rhs.m`, which describes the ODE.
- The ODE can be solved with

```
ode = @(t,y) ode_rhs(t,y,m,r,k);
[T,Y] = ode23(ode,tData,Y0);
```

  `tData` should contain the measuring points t_i, and the corresponding function values $y(t_i)$ are returned in `Y`. The first column contains the deflections and the second column contains the corresponding velocity values. The initial conditions $(y(0), \dot{y}(0))^\top = (1, 0)^\top$ are passed in `Y0`.