Optimization for Non-Mathematicians Sheet 8

Exercise 17: Maximum profit for given resources

A manufacturer of tents produces 3 different types. The materials used are polyester (outer part of the tent), nylon (inner part of the tent and bottom) and aluminium (rods). The amount of available resources and how many is used for the different types of tents, as well as the sales profit are stated in the table below:

tent type i	1	2	3	available
polyester p_i in m ²	5	12	25	500
nylon n_i in m ²	5	15	34	500
aluminium a_i in m	4	16	40	520
profit g_i in \in	6	20	47	

The manufacturer wants to maximize his profit with the given amount of resources.

- (a) Which are the optimization variables? What are constant quantities?
- (b) Which inequalities exist?
- (c) What is the objective function?
- (d) Solve the problem using linprog with the (dual) simplex method.

Exercise 18: Modeling of a diet problem

The following example can be found in George B. Dantzig (inventor of the simplex method), Linear Programming and Extensions, Princeton University Press, 1998 (reprint of the classic from 1963), page 117:

Formulate as a linear programming problem: Suppose six foods listed below have calories, amounts of protein, calcium, vitamin A, and costs per pound purchased as shown. In what amounts should these foods be purchased in order to meet exactly the daily equivalent per person shown in the last column at minimum cost? How is the model modified if the daily requirements may be exceeded; if the requirements except for calories may be exceeded?

	Contents and Costs Per Pound Purchased					Ι	Daily	
	Bread	Meat	Potatoes	Cabbage	Milk	Gelatin	Requ	irements
Calories	1254	1457	318	46	309	1725	3000	
Protein	39	73	8	4	16	43	70	(grams)
Calcium	418	41	42	141	536	_	800	(mg.)
Vitamin A	_	_	70	860	720	_	500	(I.U.)
Cost	\$ 0.30	\$ 1.00	\$ 0.05	\$ 0.08	\$ 0.23	\$ 0.48	Mir	nimum

Exercise 19: Alloy Composition

An alloy can be composed of different metals, for example

metal i	1	2	3	4
density ρ_i	6500	5800	6200	5900
carbon ratio c_i in %	0.2	0.35	0.15	0.11
phosphor ratio p_i in%	0.05	0.015	0.065	0.1
$cost k_i in \in /kg$	2.00	2.50	1.50	2.00

With ratio we mean the volume ratio, not the mass ratio. The aim is to find an alloy as cost-efficient as possible, which fulfills the following requirements:

	density ρ_i	carbon ratio $\%$	phosphor ratio in $\%$
minimal	5950	0.1	0.045
maximal	6050	0.3	0.055

- (a) Which are the optimization variables? What are constant quantities?
- (b) Which inequalities and equalities exist?
- (c) What is the objective function?
- (d) Solve the problem using linprog with the (dual) simplex method.