Technische Universität Chemnitz Chemnitz, 29th January 2018
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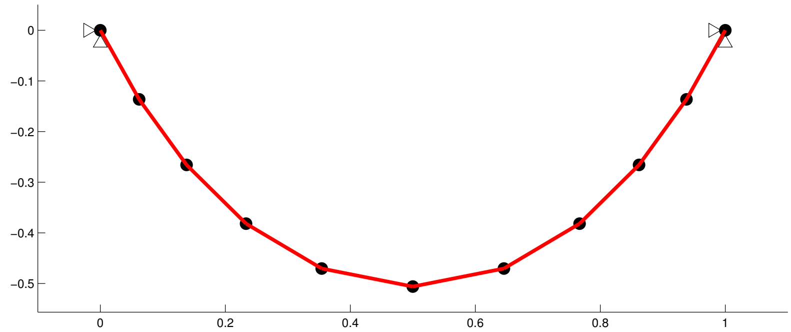
Optimization for Non-Mathematicians
Sheet 15

Exercise 35: Hanging chain

Several bodies are interconnected by ropes and form a hanging chain. We want to know the deflection of the chain, i.e., the positions of the bodies.



- The m bodies are considered as point masses. Let body i have the position (x_i, y_i) and the weight w_i. Some of the bodies are fixed, i.e., it holds $(x_i, y_i) = (\bar{x}_i, \bar{y}_i)$.
- The ropes between the bodies are assumed to be massless and are given by the edge set E. Rope $(i, j) \in E$ between bodies i and j has the length l_{ij}.
- The chain will be in a state of minimal potential energy. [1]

(a) How can we model this as an optimization problem?

(b) Solve the minimization problem using `fmincon`. On the homepage you will find the files `data_chain.m` and `data_mesh.m` with information about bodies, ropes and anchors.

Exercise 36: Route planning

In a landscape a new railway line from A to B is planned with preferably low building costs. The landscape is described by the graph of the function z, i.e., each point of the map has the height $z(x, y)$.

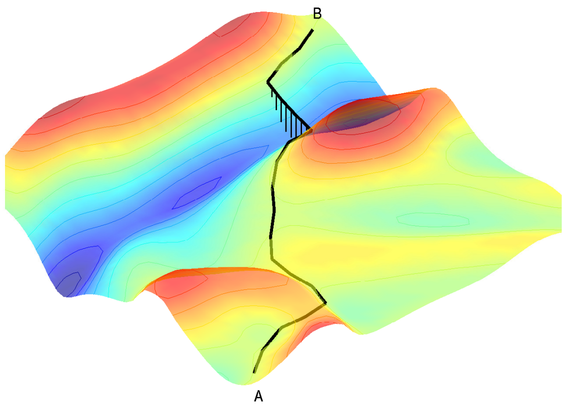
The railway line shall (for simplification) be located at a constant height of zero, because the railway cannot deal with large upward and downward slopes. For this reason, tunnels and bridges have to be built. The building costs depend on the height[2]

[1] With similar principles of energy minimization one can describe different phenomena in natural sciences, e.g. elastic deformations of structural elements or the folding of proteins.

[2] High bridges are expensive as well as tunnels which are built in deeper-lying lyers of rock.

of the landscape $z(x, y)$, the overall costs for a line of length $\mathrm{d}s$ are modeled by

$$(C_1 + C_2\, z^2)\, \mathrm{d}s.$$



The problem can be modeled as follows:

- The position of the points A and B are given by (x_A, y_A) and (x_B, y_B).
- The railway line is approximated by a polyline, i.e., by a sequence of n road points, which are interconnected. Let the position of road point i be (x_i, y_i).
- The costs for the line between point i and point $i+1$ are approximately (trapezoidal rule) given by

$$k_i = (C_1 + C_2\, z^2)\, \|(x_i, y_i) - (x_{i+1}, y_{i+1})\| \text{ with } z^2 = \frac{z(x_i, y_i)^2 + z(x_{i+1}, y_{i+1})^2}{2}.$$

- For the usability of this approximation, two neighbored points should not be too far apart. This can be achieved by the constraints $|x_i - x_{i+1}| \leq d$ and $|y_i - y_{i+1}| \leq d$.

(a) What is the corresponding optimization problem?

(b) Solve the problem using `fmincon` for the following data:

- cost parameters: $C_1 = 1$, $C_2 = 100$
- start and end: $(x_A, y_A) = (-1, -1)$, $(x_B, y_B) = (1, 1)$
- The landscape is given by the function `landscape.m` on the homepage. There you will also find the function `plot_route.m` to plot the solution.

- Find reasonable values for the number of road points n and the constraint d on your own.

(c) **Extra:** There exists a protected area where the railway line must not go through. The area is given by a circle with center $(0, -0.2)$ and the radius 0.3.