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Chemnitz, 30th October 2017

Optimization for Non-Mathematicians

Sheet 4

Exercise 8: Finite differences vs. exact differentiation

Consider a quadratic multidimensional function

$$f(x) = \frac{1}{2} x^\top B\, x + g^\top x + c$$

where

$$B = \begin{pmatrix} 1 & 0 \\ 0 & 10^8 \end{pmatrix}, \quad g = \begin{pmatrix} -1 \\ -10^8 \end{pmatrix}, \quad c = 50\,000\,000.5$$

The function has a unique minimum at $x^\star = (1, 1)^\top$.

(a) Solve the minimization problem $f(x) \to \min$ with `fminunc`. Recommended settings for `fminunc` are `OptimalityTolerance` $= 10^{-9}$ and `StepTolerance` $= 10^{-9}$, $x_0 = (2, 2)^\top$.
Hint: Set the options with `optimoptions`.

(b) Calculate the gradient $\nabla f(x)$.

(c) Solve the minimization problem $f(x) \to \min$ again with `fminunc`, but now provide the gradient to `fminunc`. To this end, the routine for the objective has to return not only the function value $f(x)$ but also the gradient $\nabla f(x)$. Use `optimoptions` to set `SpecifyObjectiveGradient` to `true`.

Exercise 9: Different local minima

Consider the function

$$f(x_1, x_2) = (x_1^2 - 1)^2 + (x_2^2 - 1)^2 + \frac{x_1}{2} + \frac{x_2}{4}.$$

Solve the minimization problem $f(x) \to \min$ with `fminunc`. Use

$$\begin{aligned} x_a &= (0.128, 0.063) \\ x_b &= (0.128, 0.062) \\ x_c &= (0.127, 0.063) \\ x_d &= (0.127, 0.062) \end{aligned}$$

as starting points. What is the reason for the different results?

Exercise 10: Distance between a point and a parabola

Given is a point $P = (P_x, P_y)$ and a parabola $y = ax^2 + bx + c$ in the x-y plane. The aim is to find a point $(x^\star, y^\star)$ on the parabola which minimizes the (Euclidian) distance to P.

(a) Formulate the problem as an optimization problem with constraints.

(b) How can the constraint be eliminated? What is the new (reduced) optimization problem?

(c) Solve the reduced minimization problem $f(x) \to \min$ with MATLAB for the point $P = (2, 4)$ and the parabola $y = -x^2 + 2x$.

Hint: Write an objective function `DistancePointParabola` to calculate the distance to the parabola for an arbitrary point P.

(d) Illustrate the solution graphically.

Hint: The parabola can be drawn easily with `ezplot`. Additional points are drawn with `plot`.