Optimization for Non-Mathematicians

Sheet 9

Exercise 20: Chebyshev center of a polyhedron

Let $\mathcal{P}$ be a convex polyhedron (e.g. a triangle in 2D), i.e., the intersection of m halfspaces. The halfspaces are described by the inequalities

$$a_i^\top x \leq b_i, \ i = 1, \ldots, m$$

with $a_i \in \mathbb{R}^n$ and $b \in \mathbb{R}$ and therefore

$$\mathcal{P} := \left\{ x \in \mathbb{R}^n : a_i^\top x \leq b_i, \ i = 1, \ldots, m \right\}$$

holds. The aim is to find a ball as large as possible, which lies completely inside the polyhedron $\mathcal{P}$. Its center is called **Chebyshev center**, i.e. a point with maximal distance to all faces.

(a) How can the distance of a point to a plane be calculated using the Hessian normal form?

(b) What are the conditions for a ball with center x and radius r to lie completely inside $\mathcal{P}$?

(c) How can the problem of finding the Chebyshev center be formulated as a linear optimization problem?

(d) Solve the problem using `linprog` when the following halfspaces in 2D are given:

halfspace i	a_i	b_i
1	$(0, -1)$	0
2	$(-1, 0)$	0
3	$(1, -1)$	1
4	$(1, 2)$	4

Exercise 21: Time variable demand

An producer of icecream is planning her production for the coming year. From experience one expects the following demand in the several months:
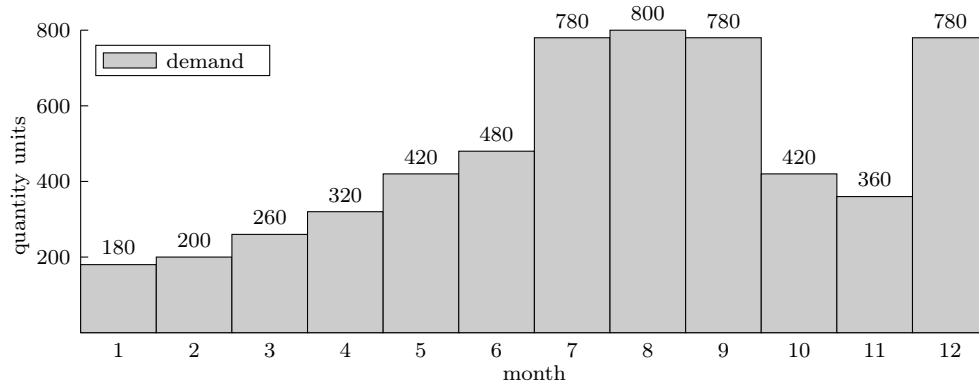
The following aspects are important for the planning:

- **Production output:** The production of one quantity unit (1 QU = 1000 kg) costs 300 €. In every month, no more than 600 QU can be produced (factory at full capacity).

- **Storage:** Up to 600 QU can be stored. This storage costs 30 € per month and QU.

- **Worker:** A worker (employed for the whole year) can produce 10 QU per month on average and receives a wage of 3000 €. A worker can also work overtime and thus produce an additional 5 QU per month and receive an extra wage of 2400 €.

Find a production plan for one business year (including production output per month, storage, number of workers and overtime per month) which minimizes the overall costs.

(a) What are the optimization variables? Are there constant quantities?

(b) What inequalities and equalities exist?

(c) What is the objective function?

(d) Solve the problem using `linprog` with the (dual) simplex algorithm.

(e) Generate a bar plot containing demand, production and stock.

(f) What is the meaning of the multipliers λ pertaining to the upper bounds with respect to the production plan?

Hints:

- Read the help for `eye` and `zeros` commands for generating an identity or zero matrix, respectively, which can be used for a simpler generation of the coefficient matrix.