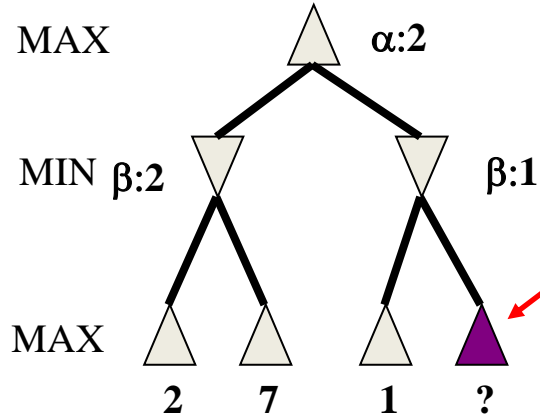


CMSC 471

Games : Part 2

Alpha-beta pruning

- Improve performance of the minimax algorithm through alpha-beta pruning
- *“If you have an idea that is surely bad, don't take the time to see how truly awful it is ” -Pat Winston (MIT)*



- We don't need to compute the value at this node
- No matter what it is, it can't affect value of the root node

Alpha-beta pruning

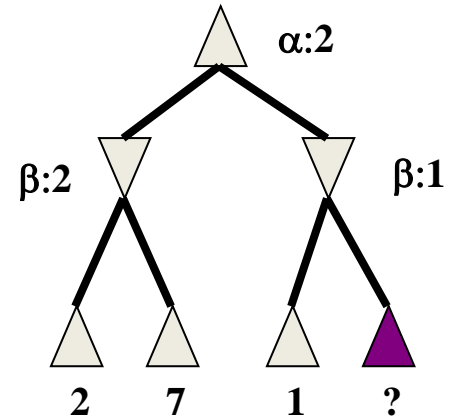
- Traverse tree in depth-first order
- At **MAX** node n , **alpha**(n) = max value found so far

Alpha values start at $-\infty$ and only increase

- At **MIN** node n , **beta**(n) = min value found so far

Beta values start at $+\infty$ and only decrease

- **Beta cutoff:** stop search below MAX node N (i.e., don't examine more descendants) if $\alpha(N) \geq \beta(i)$ for some MIN node ancestor i of N
- **Alpha cutoff:** stop search below MIN node N if $\beta(N) \leq \alpha(i)$ for a MAX node ancestor i of N



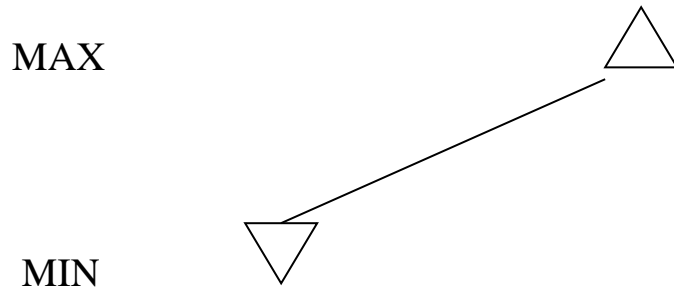
Another alpha-beta example

MAX

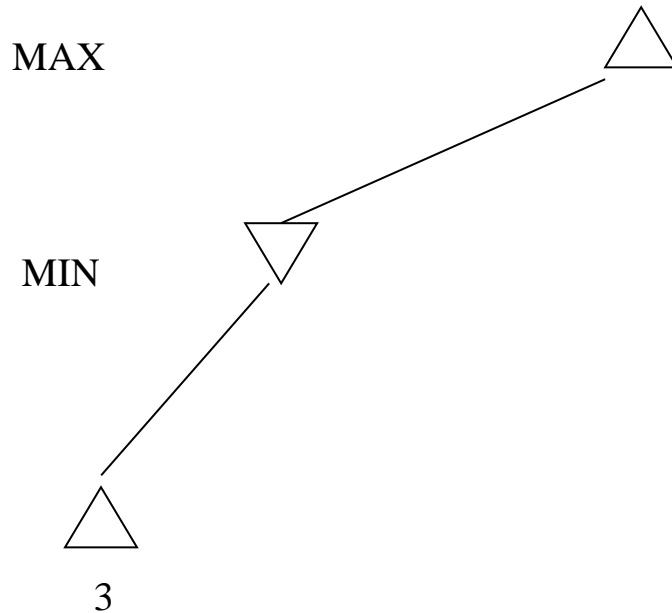


MIN

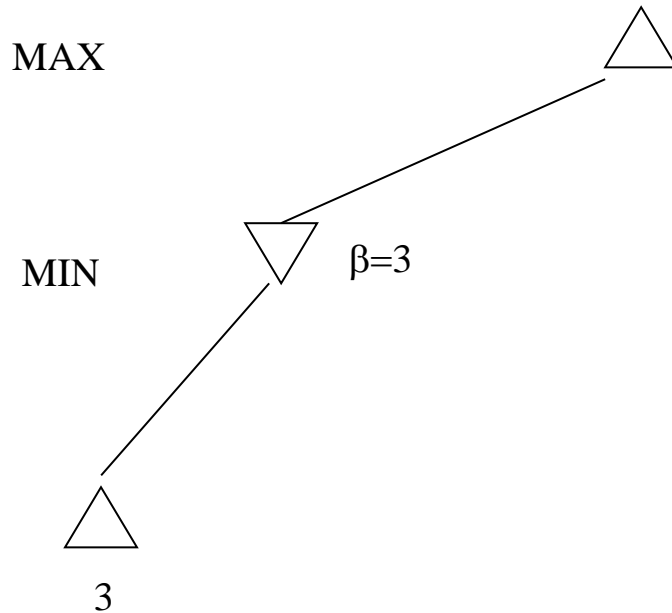
Another alpha-beta example



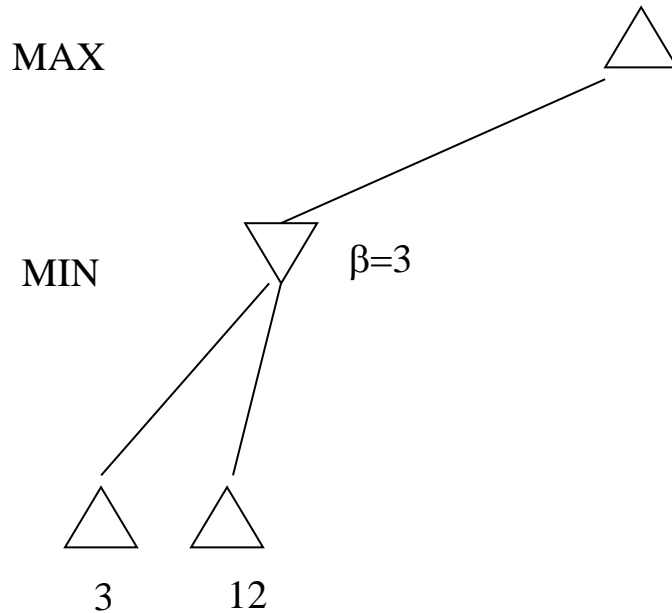
Another alpha-beta example



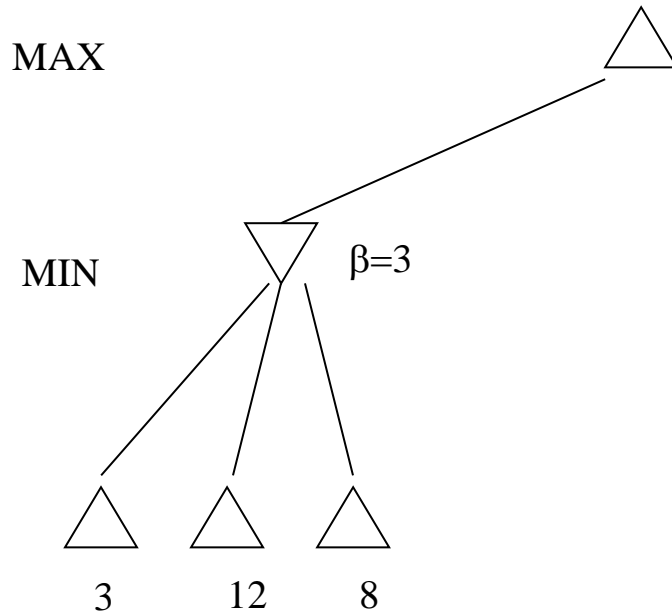
Another alpha-beta example



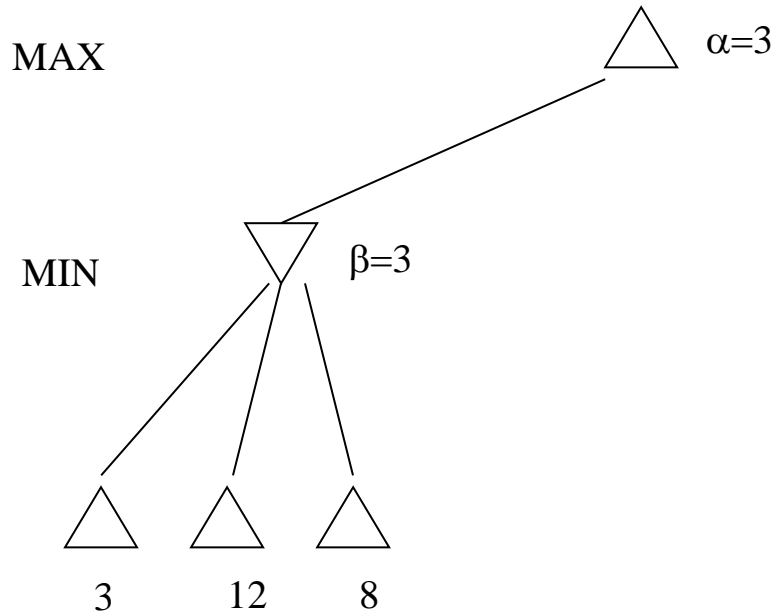
Another alpha-beta example



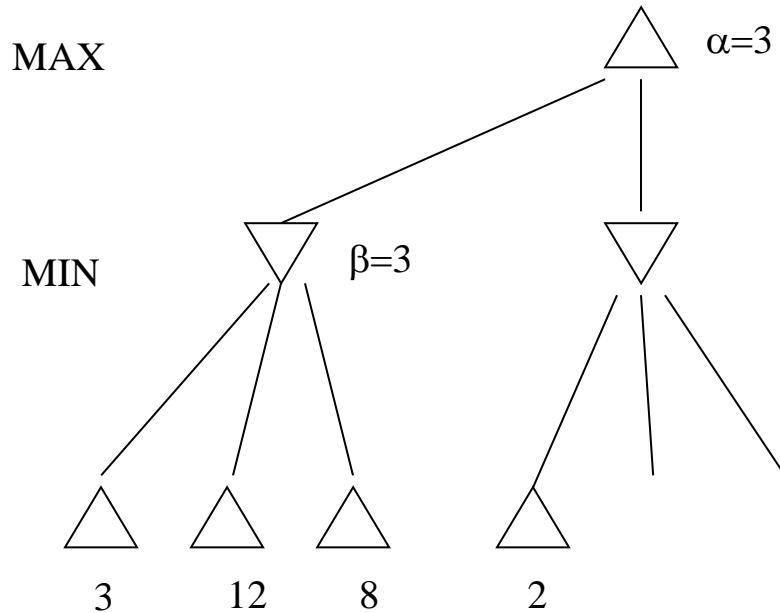
Another alpha-beta example



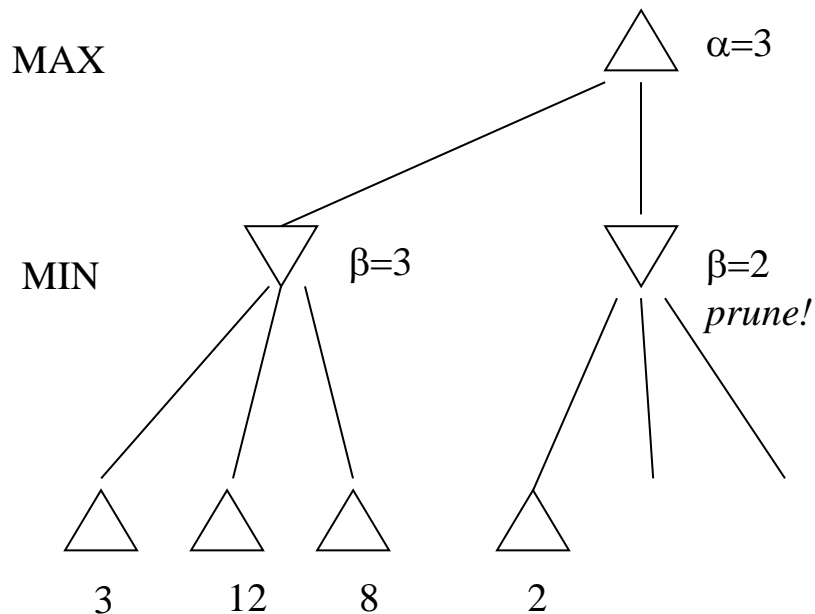
Another alpha-beta example



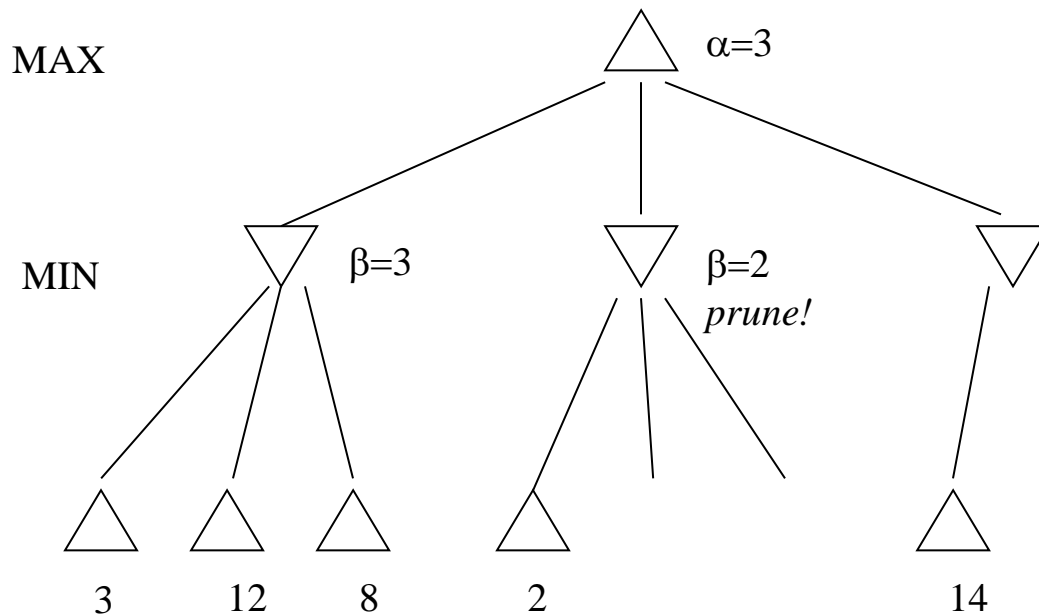
Another alpha-beta example



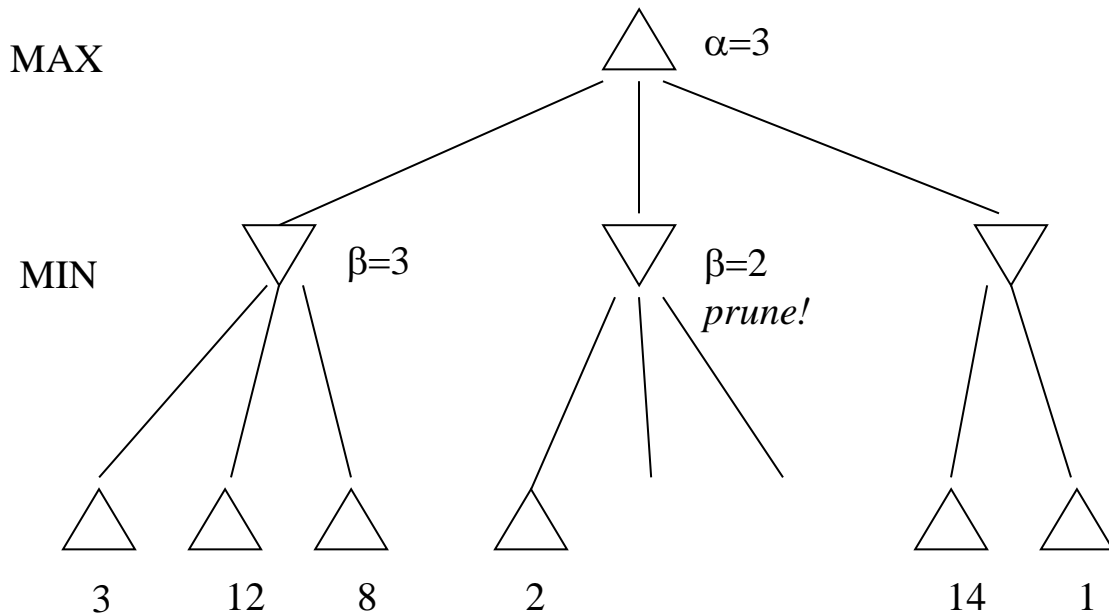
Another alpha-beta example



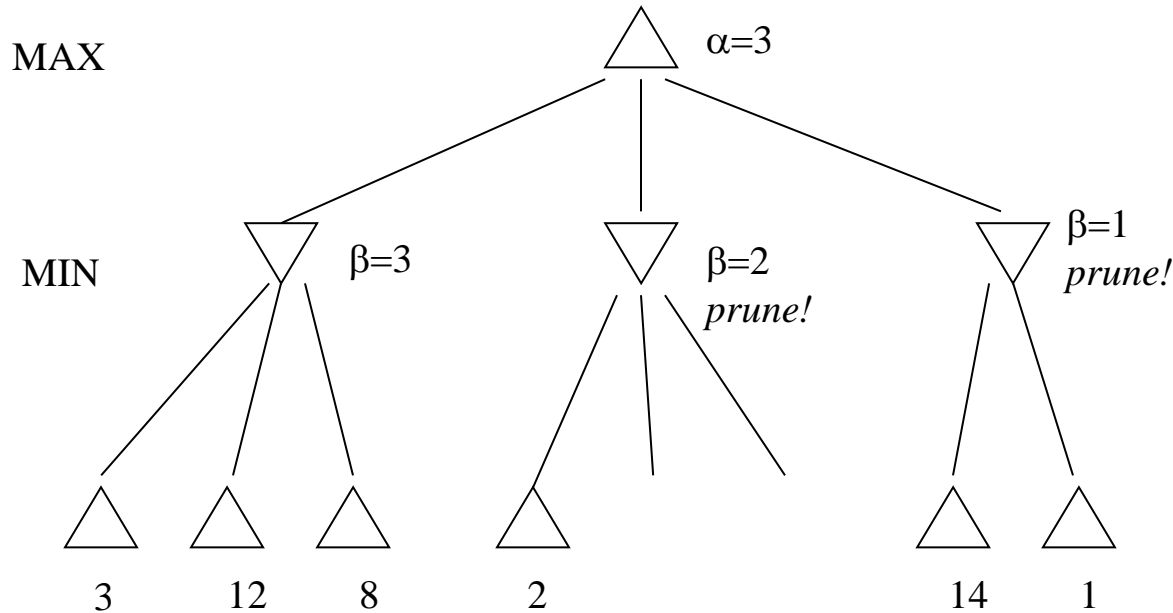
Another alpha-beta example



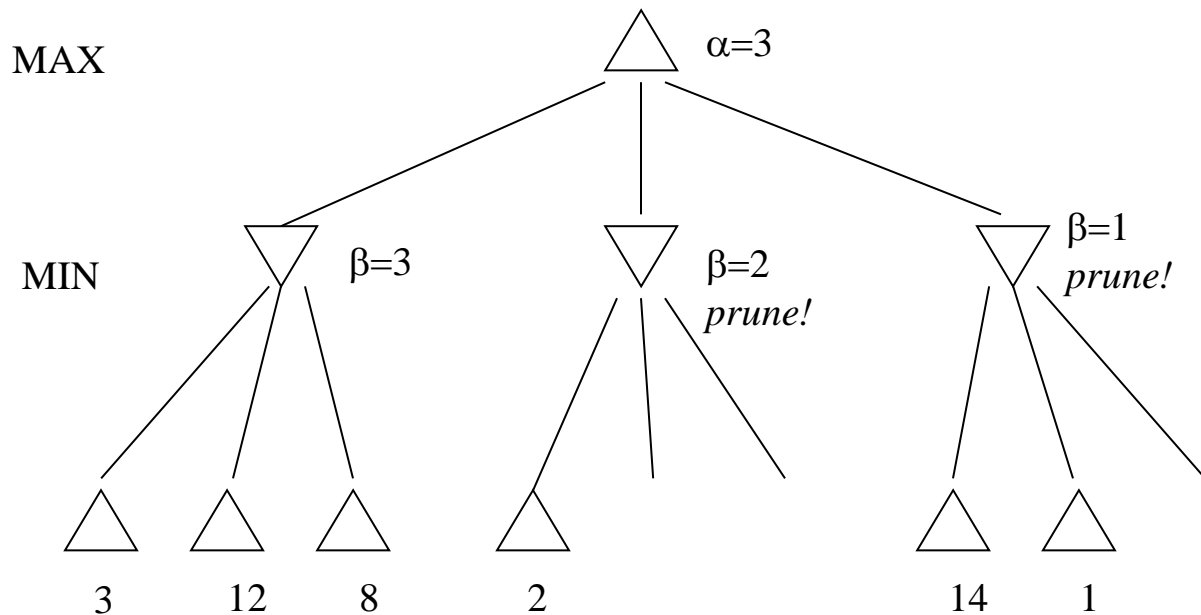
Another alpha-beta example



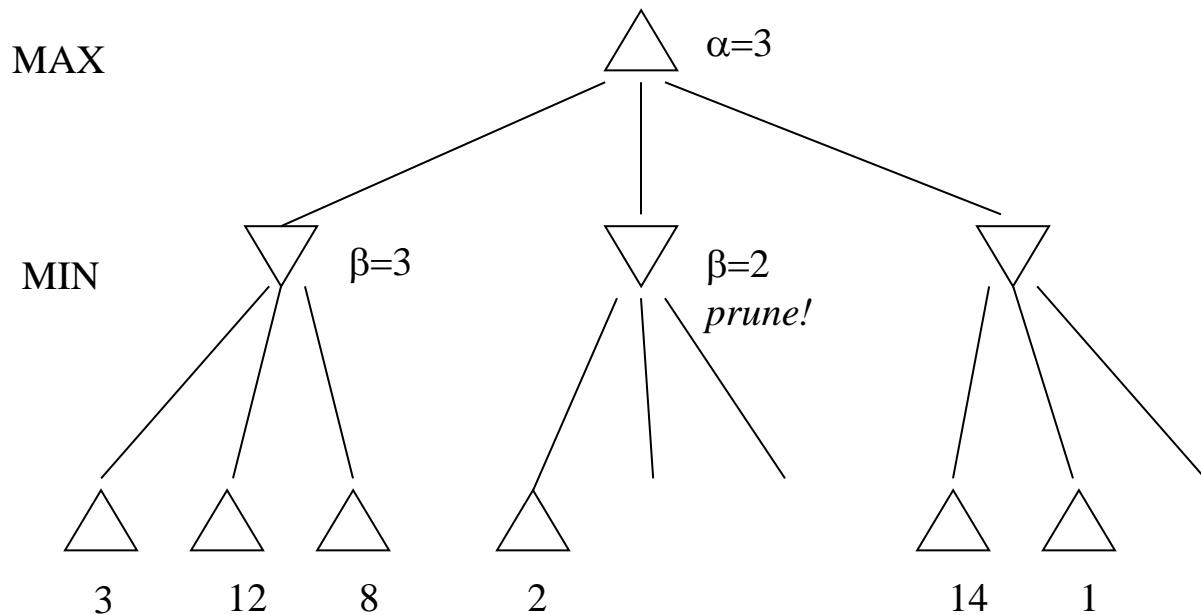
Another alpha-beta example

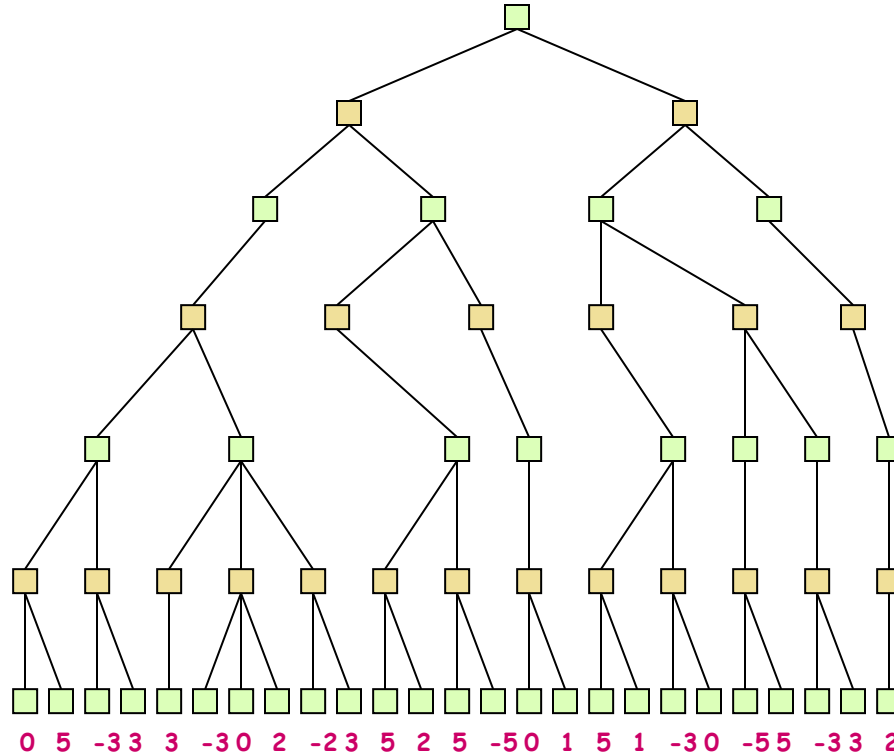


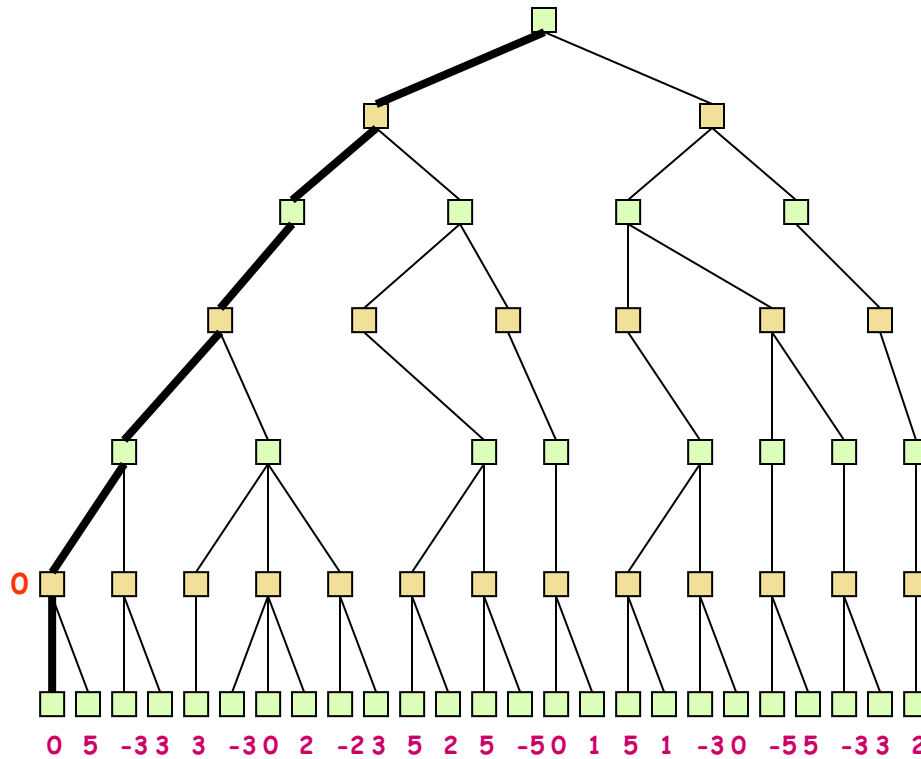
Another alpha-beta example

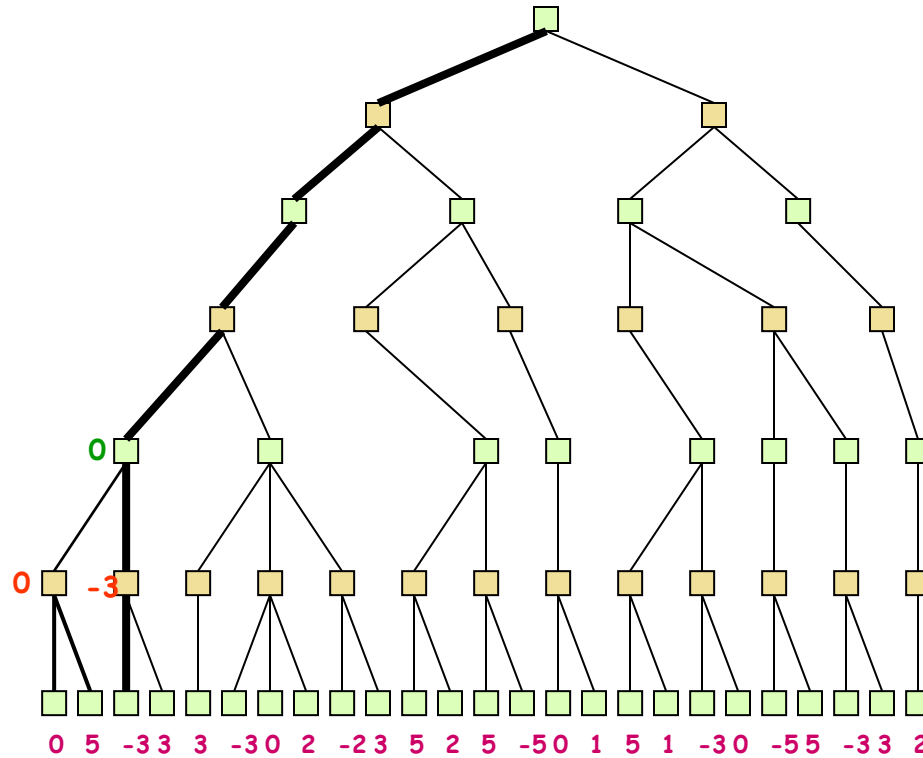


Another alpha-beta example



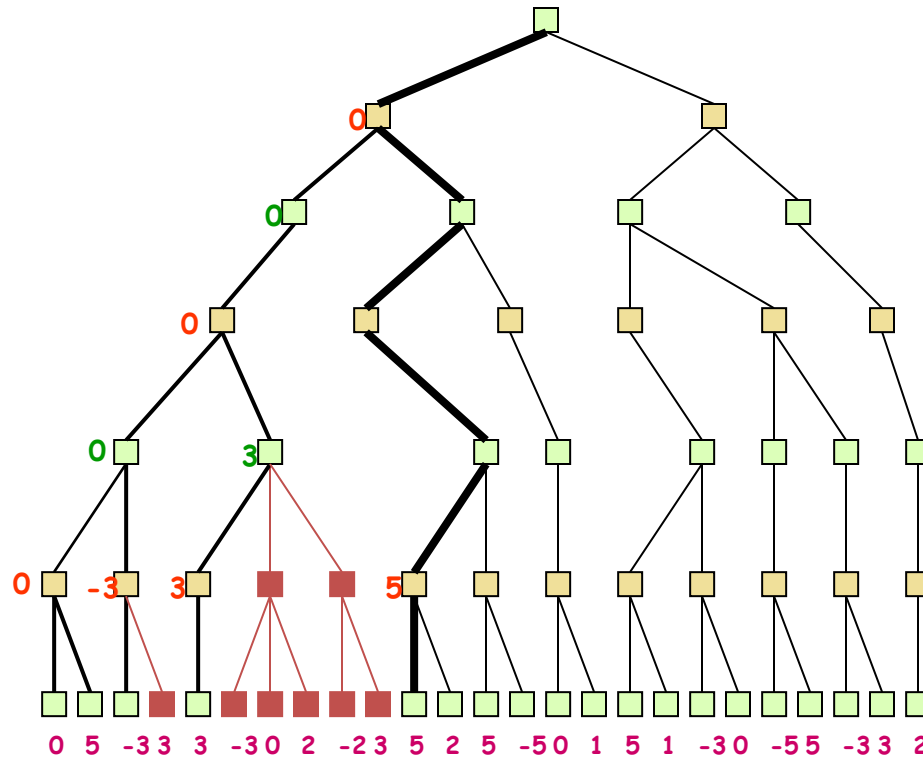


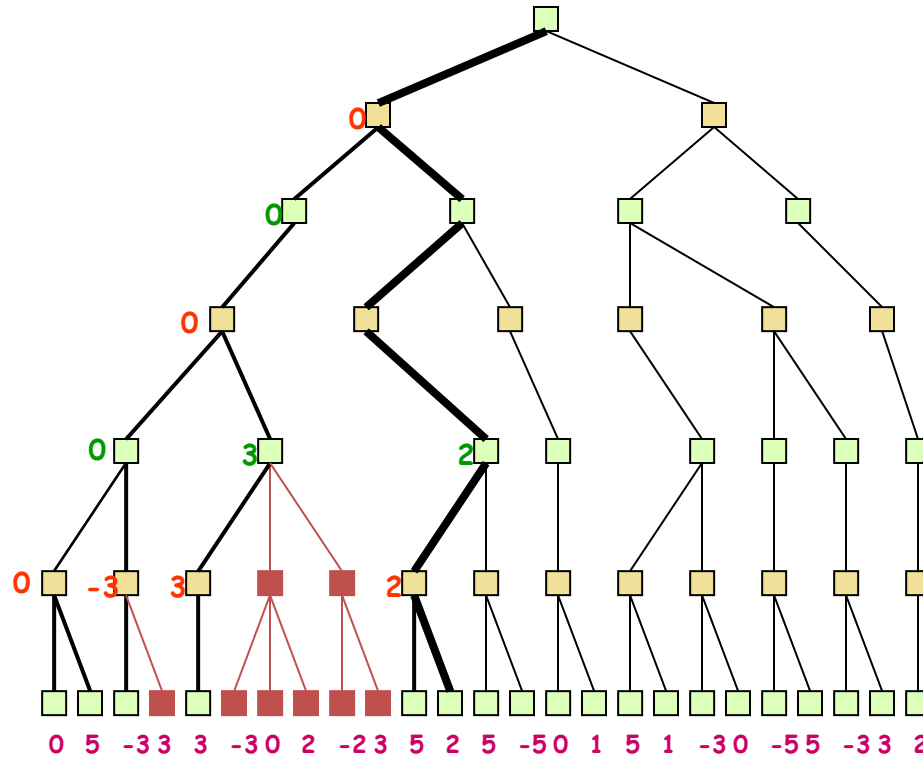


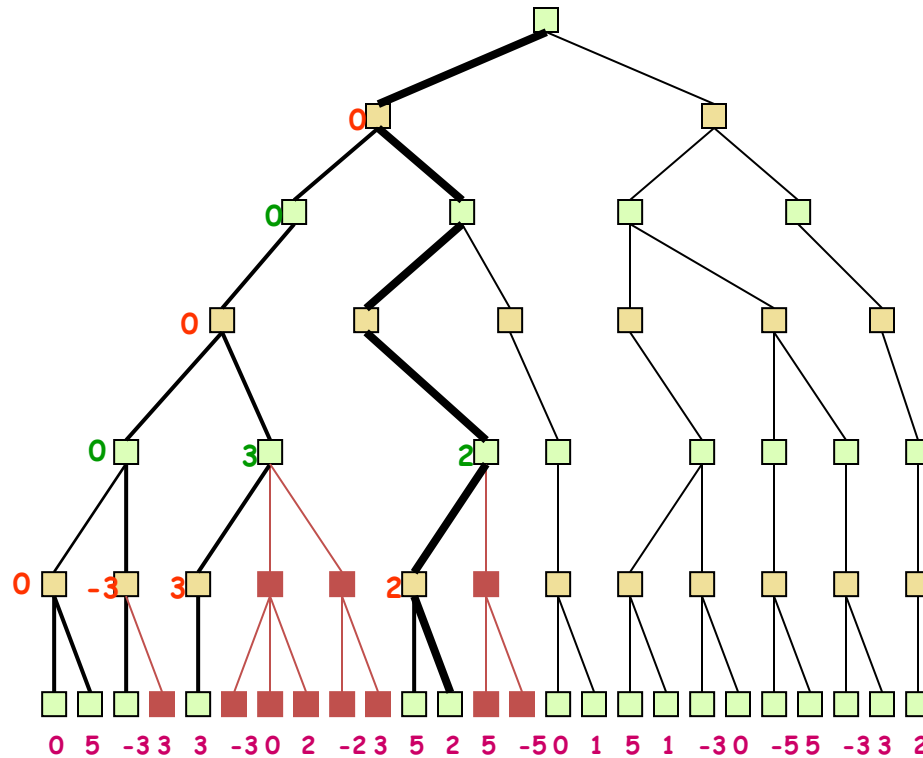


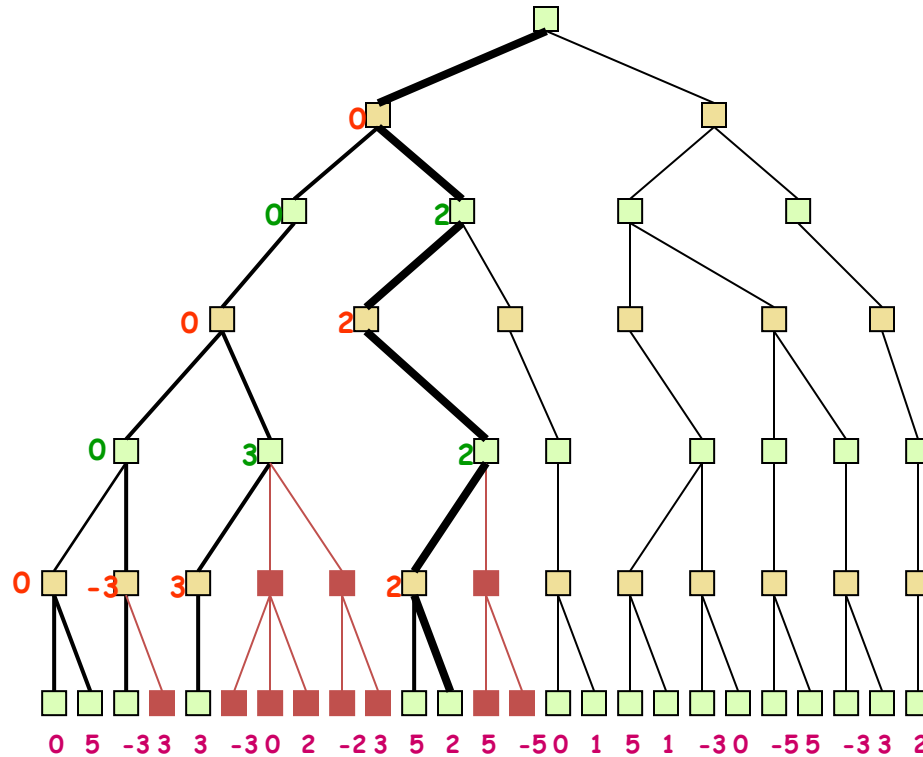


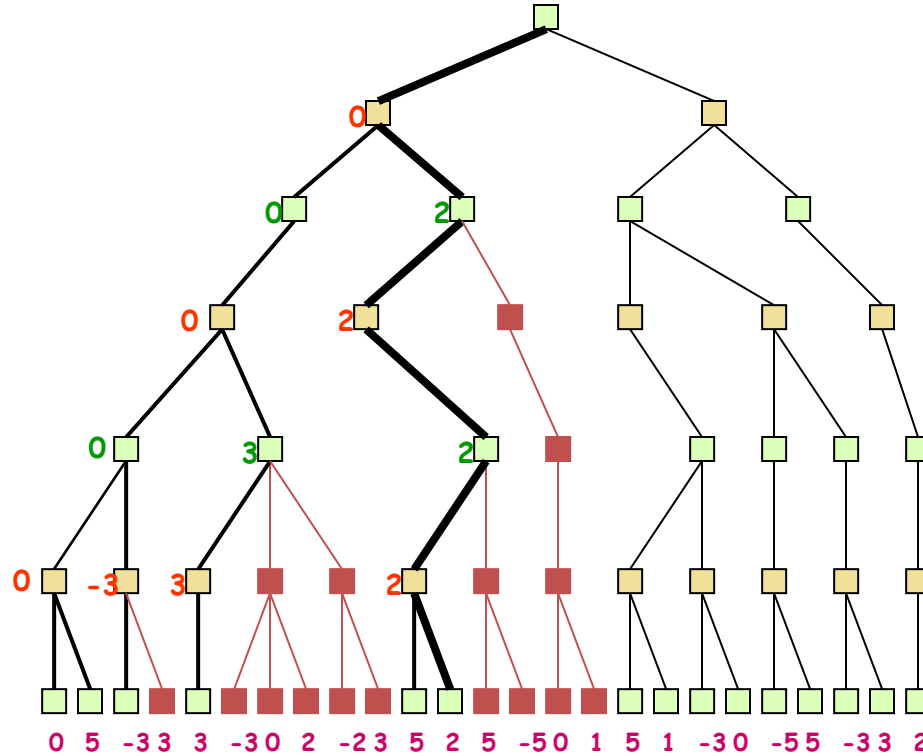




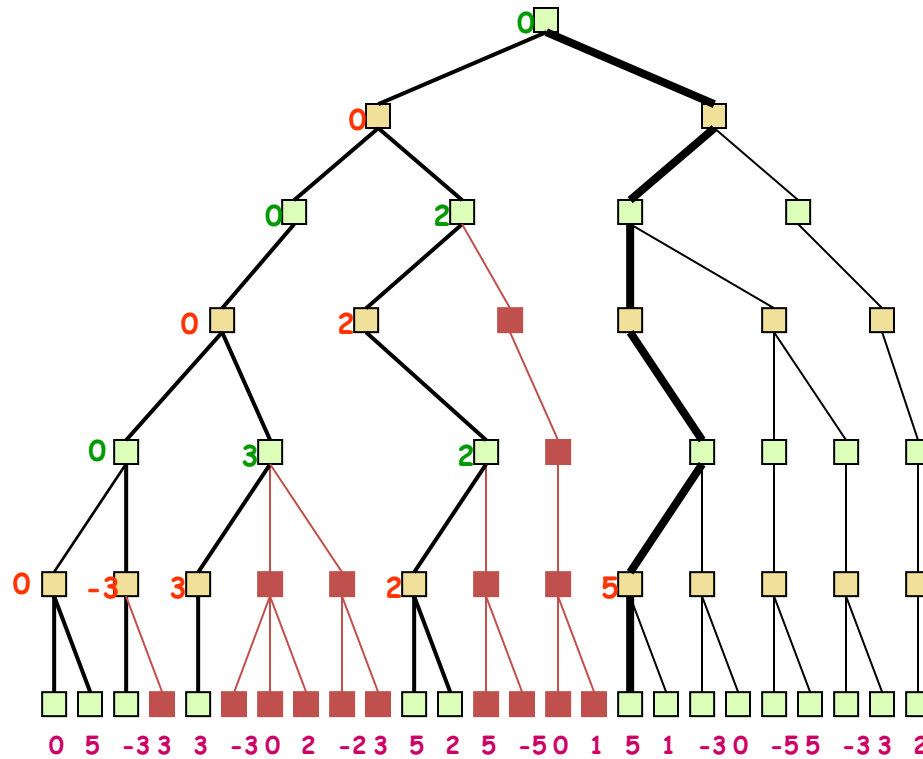


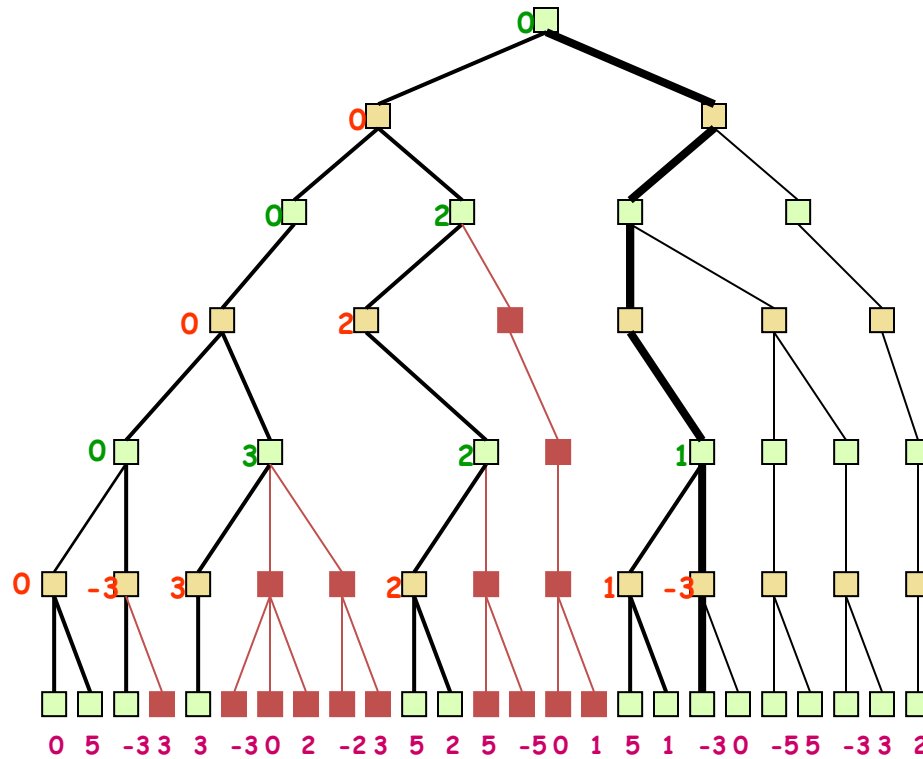


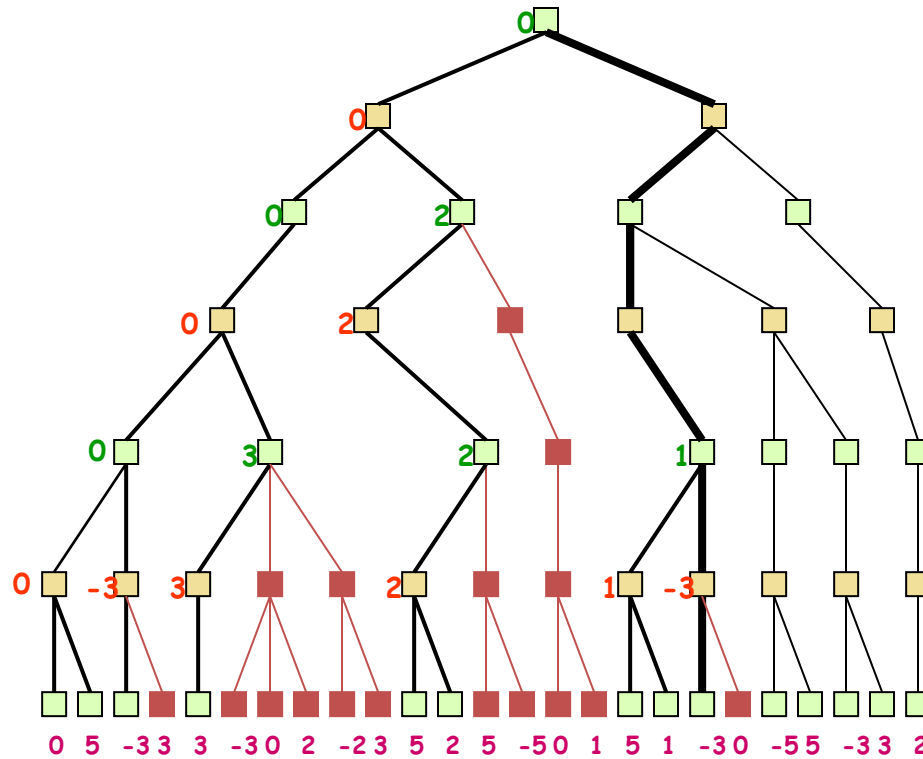


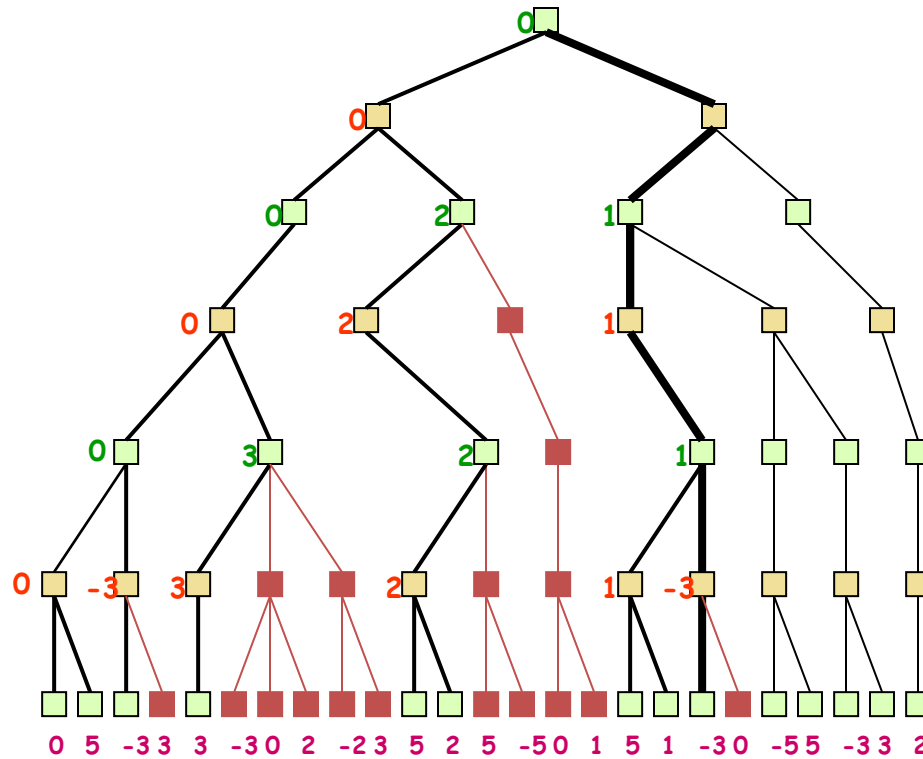


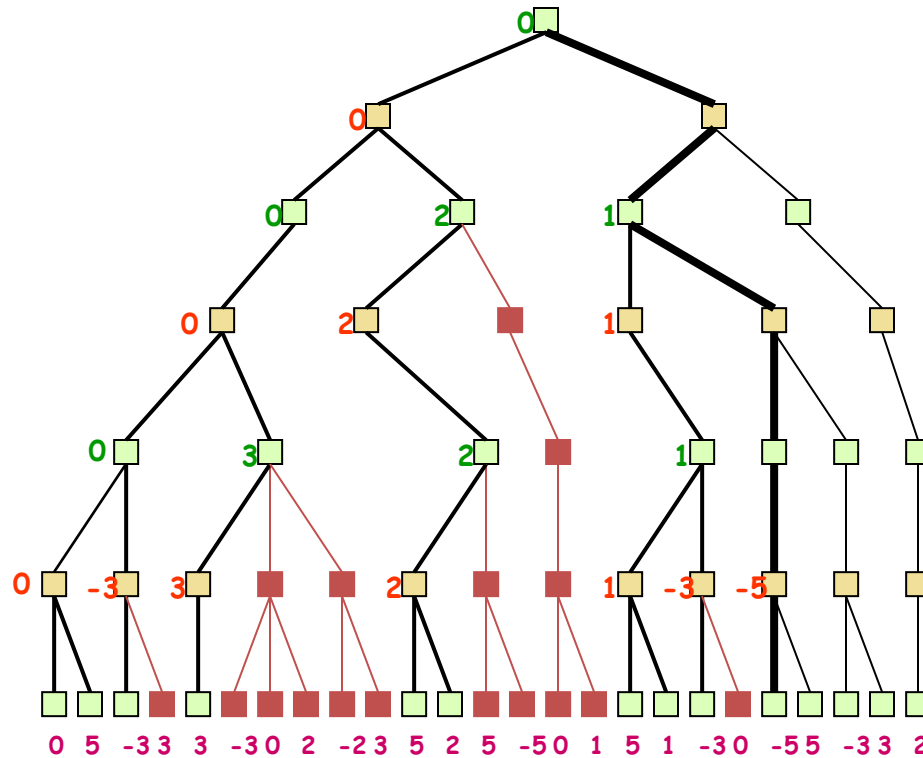


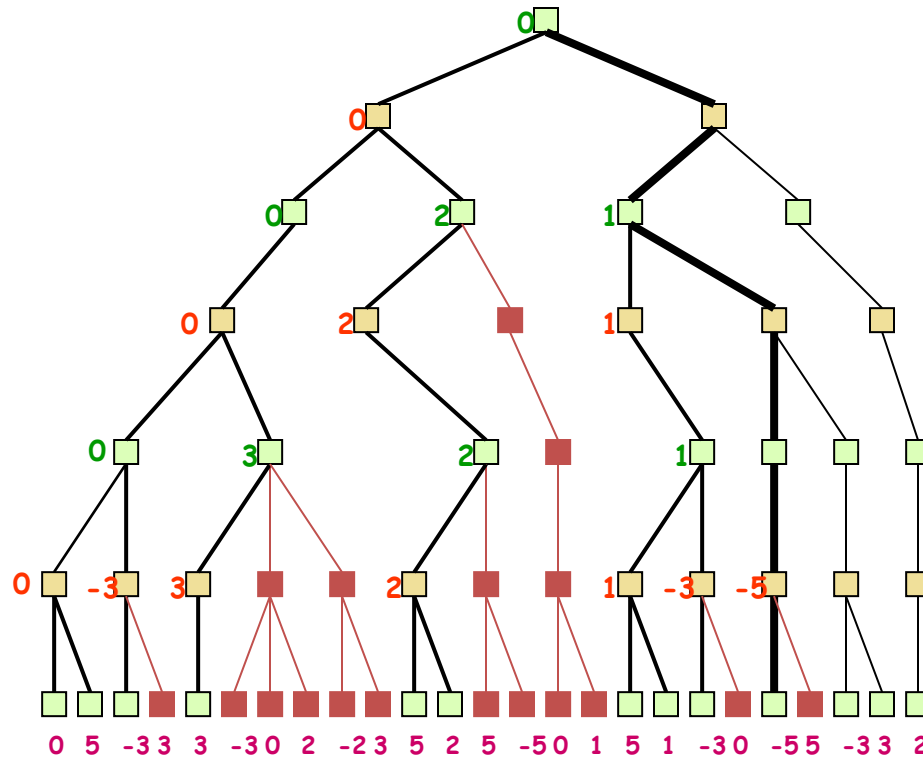


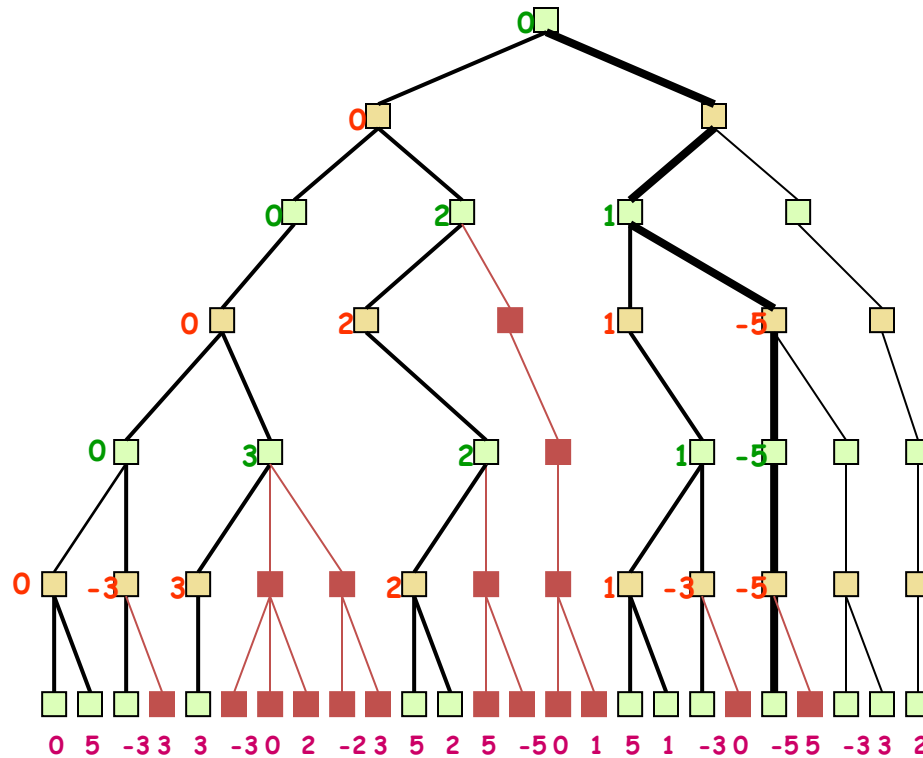


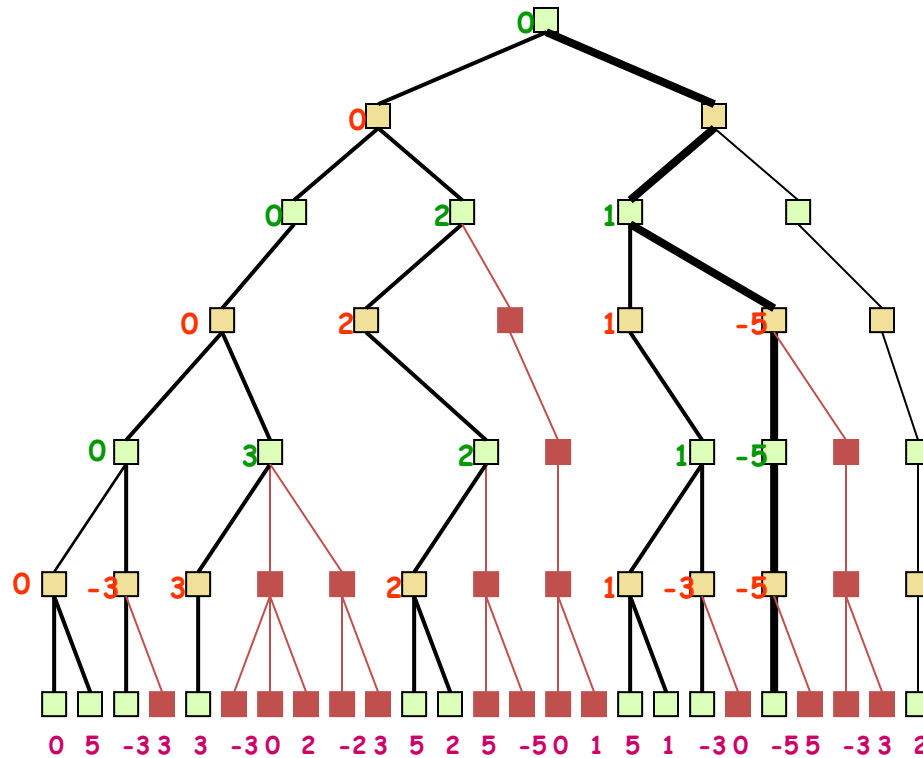


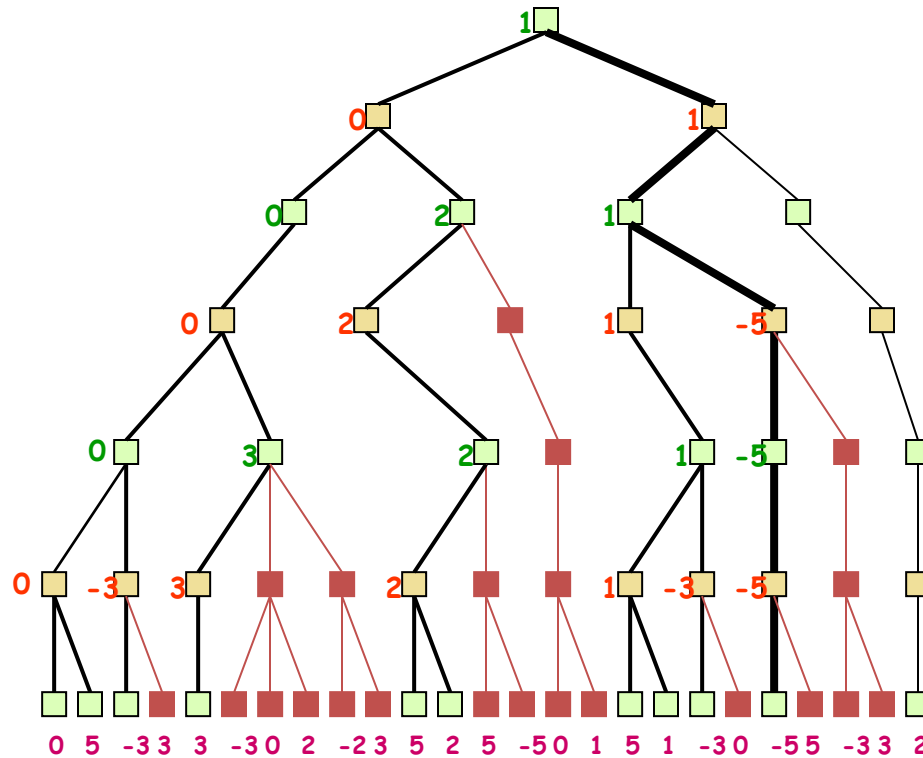


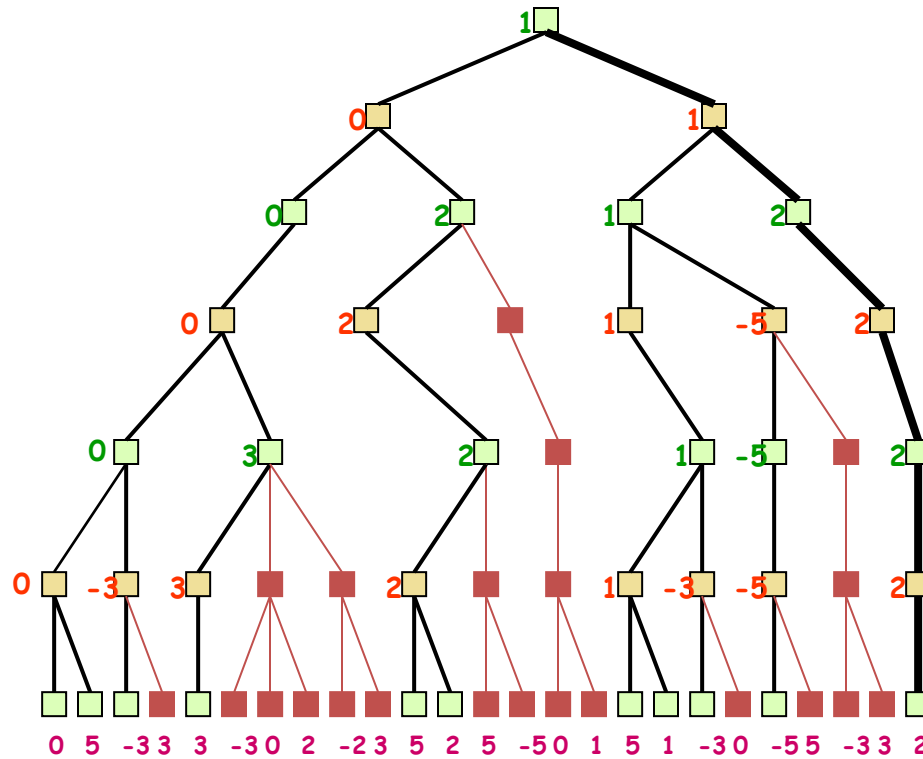




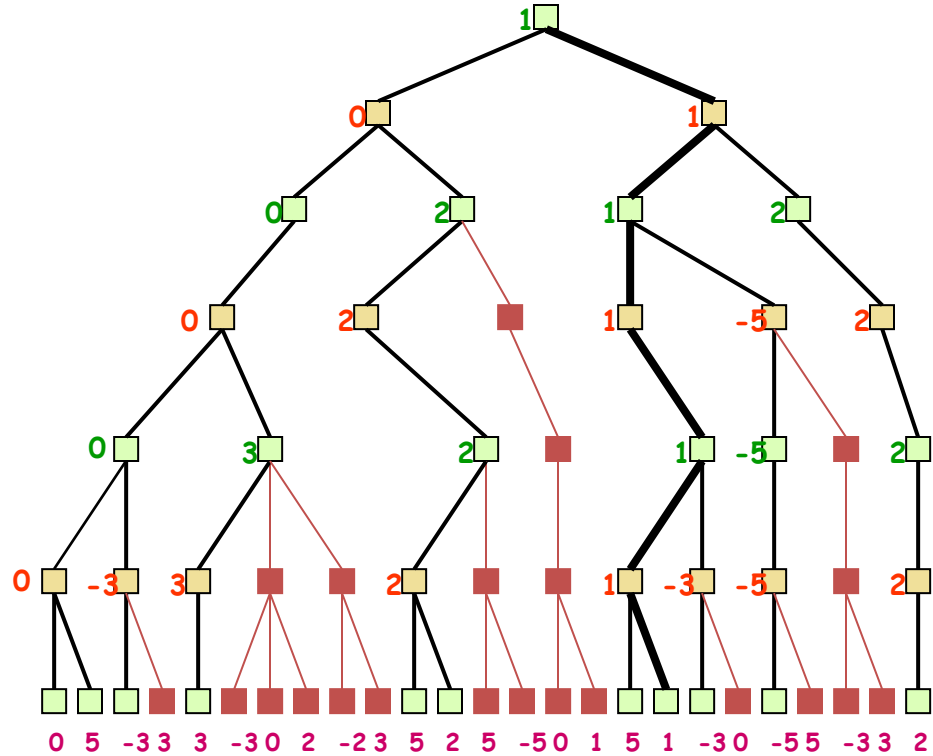








With alpha-beta we avoided computing a static evaluation metric for 14 of the 25 leaf nodes



Effectiveness of alpha-beta

- Alpha-beta guaranteed to compute same value for root node as minimax, but with \leq computation
- **Worst case:** no pruning, examine b^d leaf nodes, where nodes have b children & d -ply search is done
- **Best case:** examine only $(2b)^{d/2}$ leaf nodes
 - You can search twice as deep as minimax!
 - **Occurs if each player's best move is 1st alternative**
- In [Deep Blue](#), alpha-beta pruning reduced effective branching factor from ~ 35 to ~ 6