CMSC 471

Resolution Refutation

Conjunctive Normal Form (CNF)

- Resolution works best when the formula is of the special form:
  - it is an ∧ of ∨s of (possibly negated, ¬) variables/literals
- This form is called a Conjunctive Normal Form, or CNF.
- Example:
  - $(y \lor \neg z) \land (\neg y) \land (y \lor z)$ is a CNF.
  - $(x \lor y \lor \neg z)$ is also a CNF.
  - $(x \land y \lor \neg z)$ is **not** a CNF.

Convert to CNF

- All statements in Propositional Logic can be converted to CNF.
- To convert to CNF:
  - Open up the implications to get ORs.
    - A -> B ≡ ¬ A V B
  - Get rid of double negations. Distribute Negations.
    - ¬ ¬ A ≡ A
    - ¬(A V B) ≡ ¬A ∧ ¬B
    - ¬(A ∧ B) ≡ ¬A V ¬B
- – Distribute Or over And:
  - AV(B∧C) to (AV B)∧ (AVC)

Convert to CNF: Example

- $A \rightarrow (B \land C)$

$\equiv \neg A \lor (B \land C)$

$\equiv (\neg A \lor B) \land (\neg A \lor C)$

Sound rules of inference

Examples of sound rules of inference

Each can be shown to be sound using a truth table

RULE	PREMISE	CONCLUSION
Modus Ponens	$A, A \rightarrow B$	B
And Introduction	A, B	$A \wedge B$
And Elimination	$A \wedge B$	A
Double Negation	$\neg\neg A$	A
Unit Resolution	$A \vee B, \neg B$	A
Resolution	$\mathbf{A \vee B, \neg B \vee C}$	$\mathbf{A \vee C}$

Resolution Refutation : Steps

Procedure tries to prove a goal **P**

1. Add negation of goal to the KB, **~P**
2. Convert all sentences in KB to CNF
3. Find pairs of sentences with complementary literals that have not yet been resolved.
    - Resolve using rules of Unit Resolution or Resolution
4. If there are no pairs stop else resolve each pair, adding the result to the KB and go to 2

- If we get an empty clause (i.e., a contradiction) then **P** follows from the KB
    - e.g., resolving **X** with **~X** results in an empty clause
- If not, conclusion can't be proved from the KB

Resolution Refutation: Example

- **Given the following statements, all of which are assumed to be true:**
  1. If you go swimming, you will get wet.
  2. If it is raining and you are outside, then you will get wet.
  3. If it is warm and there is no rain, then it is a pleasant day.
  4. You are not wet.
  5. You are outside.
  6. It is a warm day.

Resolution Refutation: Example

- **Convert these statements to propositional expressions.**
  1. swimming ⇒ wet
  2. (rain ∧ outside) ⇒ wet
  3. (warm ∧ ~ rain) ⇒ pleasant
  4. ~ wet
  5. outside
  6. warm

Resolution Refutation: Example

Convert these expressions into a single conjunctive normal form statement.

1. (~ swimming V wet)
2. (~ rain V ~ outside V wet)
3. (~ warm V rain V pleasant)
4. (~ wet)
5. (outside)
6. (warm)

Proof using Resolution Refutation

- **Prove, using resolution that "It is not raining".**

To proof: ~rain
Assume:
~ (~rain) ≡ rain

Knowledge Base(KB)	
1	(~ swimming V wet)
2	~ rain V ~ outside V wet
3	~ warm V rain V pleasant
4	~ wet
5	outside
6	Warm
7	Rain

Action	Result
Resolve 2, 7	~ outside V wet (8)
Resolve 5, 8	wet (9)
Resolve 4, 9	⊥

This is what we assume.

Hence proved, "It is not raining."