Solutions:

2. The dimension of row space and column space are equal and it is equal to r. The column space of A is equal to the row space of A^{+}. So,

$$\text{rank of } A = \text{rank of } A^{+}$$

If A is square,

$$Ax = \lambda x \quad \text{with } \lambda \neq 0$$

$$\Rightarrow A^{+}x = \frac{1}{\lambda}x$$

The eigenvectors of A and A^{+} are same. but the eigen values are $\frac{1}{\lambda_n}$ (except for $\lambda = 0$ - is same for both)

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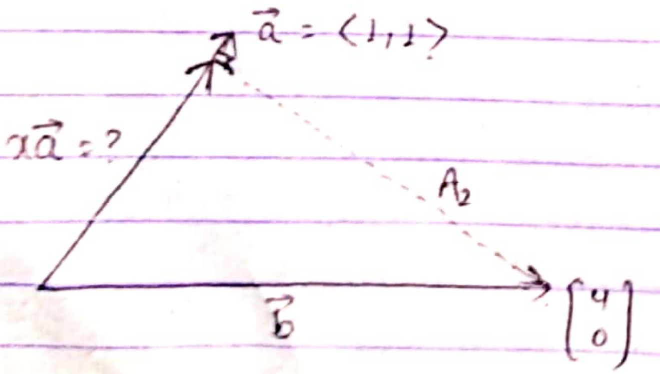

$x\vec{a} + \vec{A_2} = \vec{b}$

$x\vec{a} = \vec{b} - \vec{A_2}$

Dot with $\vec{a}$

$x\|a\|^2 = \vec{a}\cdot\vec{b}$

$x = \frac{\vec{a}\cdot\vec{b}}{\|a\|^2}$ $\quad q = \frac{4}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$

So, we must subtract $\left(\frac{\vec{a}\cdot\vec{b}}{\|a\|^2}\right)\vec{a}$ from $\vec{a}$ to produce A_2

9. Graham-Schmidt

$$q_1 = \frac{a}{\|a\|} = \frac{\begin{bmatrix}1\\1\end{bmatrix}}{\sqrt{2}} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$$

$$\vec{A_2} = \vec{b} - x\vec{a}$$

$$= \begin{bmatrix}4\\0\end{bmatrix} - \frac{4}{2}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}4\\0\end{bmatrix} - \begin{bmatrix}2\\2\end{bmatrix} = \begin{bmatrix}2\\-2\end{bmatrix}$$

$$\frac{\vec{A_2}}{\|A_2\|} = \frac{\begin{bmatrix}2\\-2\end{bmatrix}}{2\sqrt{2}} = \frac{2}{2\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$$

$$\therefore \begin{bmatrix}1 & 4\\1 & 0\end{bmatrix} = \begin{bmatrix}1/\sqrt{2} & 1/\sqrt{2}\\1/\sqrt{2} & -1/\sqrt{2}\end{bmatrix}\begin{bmatrix}\sqrt{2} & 2\sqrt{2}\\0 & 2\sqrt{2}\end{bmatrix}$$