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BTECH
(SEM III) THEORY EXAMINATION 2022-23
MATHEMATICS IV

Time: 3 Hours ***Total Marks: 100***

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. **2 x 10 = 20**

(a) Find partial differential equation (PDE) by eliminating a and b from $z = ax + by + a^2 + b^2$.

(b) Solve the PDE, $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$.

(c) Classify the PDE, $u_{xx} + u_{yy} - u_{xy} = 0$.

(d) Write the wave equation in two dimensions.

(e) Find the arithmetic mean of the following frequency distribution:

x	1	2	3	4	5	5	7
f	5	9	12	17	14	10	6

(f) Write the formula of Karl Pearson correlation coefficient and write the range of correlation coefficient.

(g) If $P(A) = \frac{1}{4}, P(B) = \frac{1}{2}, P(A \cup B) = \frac{5}{8}$, then find the value of $P(A \cap B)$.

(h) Write probability mass function of binomial distribution with mean and variance of the distribution.

(i) Define "Null Hypothesis".

(j) Discuss (in brief) "Control Charts".

SECTION B

2. Attempt any *three* of the following: **10x3=30**

(a) Solve $\left(x^2D^2 - y^2D'^2\right) = xy$, where $D^2 = \frac{\partial^2}{\partial x^2}$, $D'^2 = \frac{\partial^2}{\partial y^2}$.

(b) A string is stretched and fastened to two points l meter apart. Motion is started by displacing the string in the form $u(x, 0) = A \sin\frac{\pi x}{l}$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is given by $u(x, t) = A \sin\frac{\pi x}{l}\cos\frac{\pi ct}{l}$.

(c) Fit a parabolic curve of second degree to the following data:

X:	0	1	2	3	4
Y:	1	1.8	1.3	2.5	6.3

(d) A bag contains 10 white and 15 black balls. If two balls are drawn in succession without replacement, then find the probability that the first ball is white and the second ball is black.

(e) The score of 10 candidates obtained in tests before and after attending some coaching classes are given below:

Before:	54	76	92	65	75	78	66	82	80	78
After:	60	80	86	72	80	72	66	88	82	73

Is the coaching for the test effective? Test at 5% level of significance.

SECTION C

3. Attempt any *one* part of the following: 10x1=10

(a) Solve, $(mz - ny)p + (nx - lz)q = ly - mx$, where $p = \frac{\partial z}{\partial x}$ & $q = \frac{\partial z}{\partial y}$.

(b) By Charpit's method, find the complete solution of PDE:

$$px + qy - pq = 0.$$

4. Attempt any *one* part of the following: 10x1=10

(a) Solve by the method of separation of variables, the heat equation $u_t = u_{xx}, 0 < x < 1, t > 0$subject to the initial and boundary conditions $u(x,0) = x - x^2, u(0,t) = u(1,t) = 0$.

(b) Solve the Laplace equation $u_{xx} + u_{yy} = 0 x \in (0,1), y \in (0,1)$ with the conditions $u(x,0) = u(x,1) = 0$ and $u(0,y) = 0, u(1,y) = f(y)$ by using the method of separation of variables.

5. Attempt any *one* part of the following: 10x1=10

(a) Calculate the correlation coefficient for the following heights (in inches) of fathers(X) and their sons (Y):

X:	65	66	67	67	68	69	70	72
Y:	67	68	65	68	72	72	69	71

(b) The first four moments of a distribution about the value 4 of the variables are $-1.5, 17, -30$ and 80. Find moments $\mu_1, \mu_2, \mu_3, \mu_4$ about mean. Also find β_1 and β_2.

6. Attempt any *one* part of the following: 10x1=10

(a) A random variable X has the following probability distribution values of X:

X:	0	1	2	3	4	5	6	7
$P(X)$:	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Then, evaluate $P(X \geq 6)$.

(b) For continuous random variable Xif

$$f(x) = \frac{3}{4}(x^2 + 1), 0 \leq x \leq 1.$$

then,

(i) Verify that $f(x)$ is a probability distribution function.

(ii) Find λ such that $P(X \leq \lambda) = P(X > \lambda)$.

7. Attempt any *one* part of the following: 10x1=10

(a) The values in two random samples are given below:

Sample1:	15	25	16	20	22	24	21	17	19	23		
Sample2:	35	31	25	38	26	29	32	34	33	27	29	31

Can we conclude that the two samples are drawn from the same population? Test at 5% level of significance.

(b) Discuss one way analysis of variance (ANOVA) with mathematical model and assumptions in the model.