Sub Code:KAS103

Paper Id: **199103** **Roll No:**

B. TECH.
(SEM I) THEORY EXAMINATION 2019-20
MATHEMATICS-I

Time: 3 Hours ***Total Marks: 100***

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions.

Q. No.	Question	Marks	CO
a.	Show that vectors (1, 6, 4), (0, 2, 3) and (0, 1, 2) are linearly independent.	2	1
b.	Define Lagrange's mean value theorem.	2	2
c.	If $u = x(1-y), v = xy$, find $\frac{\partial(u,v)}{\partial(x,y)}$.	2	3
d.	Show that vector $\vec{V} = (x+3y)\hat{\imath} + (y-3z)\hat{\jmath} + (x-2z)\hat{K}$ is solenoidal.	2	5
e.	Find the value of 'b' so that rank of $A = \begin{bmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & b \end{bmatrix}$ is 2.	2	1
f.	Evaluate $\int_0^2 \int_0^1 (x^2 + 3y^2)\,dydx$.	2	4
g.	Find grad $\emptyset$ at the point (2, 1, 3) where $\emptyset = x^2 + yz$	2	5
h.	If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.	2	3
i.	Find $\frac{du}{dt}$ if $u = x^3 + y^3, x = a\cos t, y = b\sin t$.	2	3
j.	Find the area lying between the parabola $y = 4x - x^2$ and above the line $y = x$.	2	4

SECTION B

2. Attempt any *three* of the following:

Q. No.	Question	Marks	CO
a.	Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 4 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$ and hence find A^{-1}.	10	1
b.	If $y = e^{m\cos^{-1}x}$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$. Hence find y_n when $x = 0$.	10	2
c.	If $u^3 + v^3 + w^3 = x + y + z$, $u^2 + v^2 + w^2 = x^3 + y^3 + z^3$ and $u + v + w = x^2 + y^2 + z^2$, then show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{(x-y)(y-z)(z-x)}{(u-v)(v-w)(w-u)}$.	10	3
d.	Evaluate the integral by changing the order of integration: $I = \int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$.	10	4
e.	Verify Stoke's theorem for the vector field $\vec{F} = (x^2 - y^2)\hat{\imath} + 2xy\hat{\jmath}$ integrated round the rectangle in the plane $z = 0$ and bounded by the lines $x = 0, y = 0, x = a, y = b$.	10	5

SECTION C

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3. Attempt any *one* part of the following:

Q. No.	Question	Marks	CO
a.	For what values of λ and μ the system of linear equations: $x+y+z=6$, $x+2y+5z=10$, $2x+3y+\lambda z=\mu$ has (i) a unique solution (ii) no solution (iii) infinite solution. Also find the solution for $\lambda=2$ and $\mu=8$.	10	1
b.	Find the rank of the matrix $A=\begin{bmatrix}1&3&4&2\\2&-1&3&2\\3&-5&2&2\\6&-3&8&6\end{bmatrix}$ by reducing it to normal form.	10	1

4. Attempt any *one* part of the following:

Q. No.	Question	Marks	CO
a.	Verify the Cauchy's mean value theorem for the function e^x and e^{-x} in the interval$[a,b]$. Also show that 'c' is the arithmetic mean between a and b.	10	2
b.	Trace the curve $r^2=a^2\cos 2\theta$.	10	2

5. Attempt any *one* part of the following:

Q. No.	Question	Marks	CO
a.	If $u=f(2x-3y,3y-4z,\ 4z-2x)$, prove that $\frac{1}{2}\frac{\partial u}{\partial x}+\frac{1}{3}\frac{\partial u}{\partial y}+\frac{1}{4}\frac{\partial u}{\partial z}=0$.	10	3
b.	Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.	10	3

6. Attempt any *one* part of the following:

Q. No.	Question	Marks	CO
a.	Evaluate $\iint(x+y)^2dx\,dy$, where R is the parallelogram in the xy-plane with vertices $(1,0),(3,1),(2,2),(0,1)$ using the transformation $u=x+y,\ v=x-2y$.	10	4
b.	Find the volume of the region bounded by the surface $y=x^2,\ x=y^2$ and the planes $z=0,\ z=3$.	10	4

7. Attempt any *one* part of the following:

Q. No.	Question	Marks	CO
a.	Verify the divergence theorem for $\vec{F}=4xz\hat{\imath}-y^2\hat{\jmath}+yz\hat{K}$ taken over the rectangular parallelepiped $0\le x\le 1,\ 0\le y\le 1,\ 0\le z\le 1$.	10	5
b.	Find the directional derivative of $\emptyset(x,y,z)=x^2yz+4xz^2$ at (1, -2, 1) in the direction of $2\hat{\imath}-\hat{\jmath}-2\hat{k}$. Find also the greatest rate of increase of $\emptyset$.	10	5