Road Model Fusion by Multiple Cameras

Concept and Design Document

Authored By:

UDIT BHASKAR TALUKDAR

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Table of Contents

1. **INTRODUCTION** 4
2. **PROJECT SCOPE** 4
3. **DOCUMENT OVERVIEW** 4
4. **ASSUMPTIONS** 4
5. **PROBLEM DESCRIPTION** 5
5.1 Scenario Setup 5
5.2 Sensor Setup 6
5.3 Input Parameters 7
5.4 Required Output 8
6. **HIGH LEVEL DESIGN** 9
6.1 Part 1 : Association Of Line Measurements From Camera Sensors 9
6.2 Part 2 : Ego Lane Centre Line Estimation 10
6.3 Part 3 : Road Model Computation 11
7. **ROAD MODEL FUSION ARCHITECTURE** 12
8. **FUSION OF LANE LINE MEASUREMENTS FROM CAMERAS** 13
8.1 Data Flow/Block Diagram : Association Of Line Measurements 13
8.2 Execution Flow : Left Line & Right Line Fusion 14
8.3 Parameterization of Line Track 15
8.3.1 Clothoid Equations 15
8.3.2 Clothoid Approximations 16
8.4 Line Prediction 21
8.5 Line Measurement to Line Track Gating 23
8.5.1 High Level Design 23
8.5.2 Detailed Design 24
8.6 Line Measurements to Line Track Fusion 25
8.6.1 High Level Design - Step 1 : Kalman Filter State Updates with Gated Line Measurements 25
8.6.2 High Level Design - Step 2 : Line Track fusion by PDAF 26
8.6.3 High Level Design - Step 3 : Clothoid Stitch Concept 27
8.7 Detailed Design 28
8.7.1 Part 1 : Generation Of Multiple Line Hypothesis and Hypothesis merge 28
8.7.2 Part 2 : Clothoid Stitch 29
9. **EGO LANE WIDTH ESTIMATION** 30
9.1 Data Flow/Block Diagram 30
9.2 Detailed Design 31
9.2.1 Step 1 : Identification of Point Correspondences on Lane Boundary Lines 31
9.2.2 Step 2 : Width Computation For Each Pair of Point Correspondences 32
9.2.3 Plot : Ego Lane Width Before & After Correction 33
10. **EGO LANE CENTER LINE HYPOTHESIS** 34
10.1 Data Flow/Block Diagram 34
10.2 Detailed Design 35
10.2.1 Step 1 : Generate Points in Cartesian Coordinates along the Line 35
10.2.2 Step 2 : Ego Lane Centre Line Polynomial Fitting by Least Squares Estimation 36
10.2.3 Step 3 : Polynomial Coefficients to Clothoid Parameters Transformation 37

10.2.4 Step 4 : Ego Lane Centre Line Estimation by Multiple Hypothesis Merge 38
11. **ROAD MODEL COMPUTATION** 39
11.1 Data Flow/Block Diagram 39
11.2 High Level Design 40
11.2.1 Step 1 : Adjacent Lane Boundary Line Geometry Computation 40
11.2.2 Step 2 : Spatial Parameterization of Road Grid 41
11.2.3 Step 3 : Road Grid Computation and History Generation 42
11.3 Grid Plot in MATLAB 43
12. **PLOTS** 44
12.1 Clothoid Parameters as Line Measurement from Cameras 44
12.2 Ego Lane Boundary Measurement to Line Track Fusion 46
12.3 Ego Lane Boundary Line Estimate after Multi-hypothesis Fusion 48
12.4 Road Geometry 50
13. **CONCLUSION** 52
14. *References* 53

1. INTRODUCTION

This project '**Road Model Fusion by Multiple Cameras'** is the project 2 of a series of 4 Sensor Fusion Project.

The other 3 projects are as follows :

- Multi-Sensor Fusion for Autonomous Driving (AD) and Advance Driver Assistance System (ADAS) functions.
- Dynamic and Occupancy Grid Map Fusion of Multiple Radars and Cameras.
- Localization & Mapping by Multiple Radar and Camera Fusion.

2. PROJECT SCOPE

Scope of the project is to design & develop a proof of concept (POC) of Road Model fusion system, which fuses Lane Line information from multiple cameras to estimate the ego lane geometry and compute road model.

3. DOCUMENT OVERVIEW

This document provides the reader with a detailed description on system overview, mathematical formulations, implementation steps in the form of flowcharts and relevant plots for visualizations.

4. ASSUMPTIONS

The following assumptions are made in the **version 1** of the project. **(Current Version)**

- The number of camera sensors considered is 3, all of them are front facing
- The sensors are synchronized (Temporally aligned)
- The accuracy of the ego vehicle sensors (Vx, Vy, yaw Rate) is high.
- Only the clothoid line parameters are used as sensor measurements for fusion which includes initial lateral offset, initial heading, initial curvature, curvature rate and curve length
- The measurements from each of the sensors shall be with respect to its ego vehicle frame.
- The Fusion System shall be validated with simulated data.
- Ego Vehicle doesn't undergo any lane change.

The following shall be considered for **version 2** of the project **(Future Version)**

- The number of radars and cameras shall be kept as a variable.
- Multiple Surround Cameras shall be used for Road Model Fusion
- The Line measurements from each of the cameras shall be with respect to its own sensor frame.
- The ego vehicle parameters shall be considered as statistical parameters.
- The sensors shall be unsynchronized (Temporally misaligned)
- Lane Geometry shall also be hypothesised from radar measurements.
- HD Map shall be utilized for Road Model Fusion
- Particle filter based estimation shall be considered.
- Lane split, Lane merge, highway entry and exit ramp shall be supported.
- Reparameterization of Road Model during Ego Vehicle Lane Change event.

5. PROBLEM DESCRIPTION

5.1 Scenario Setup

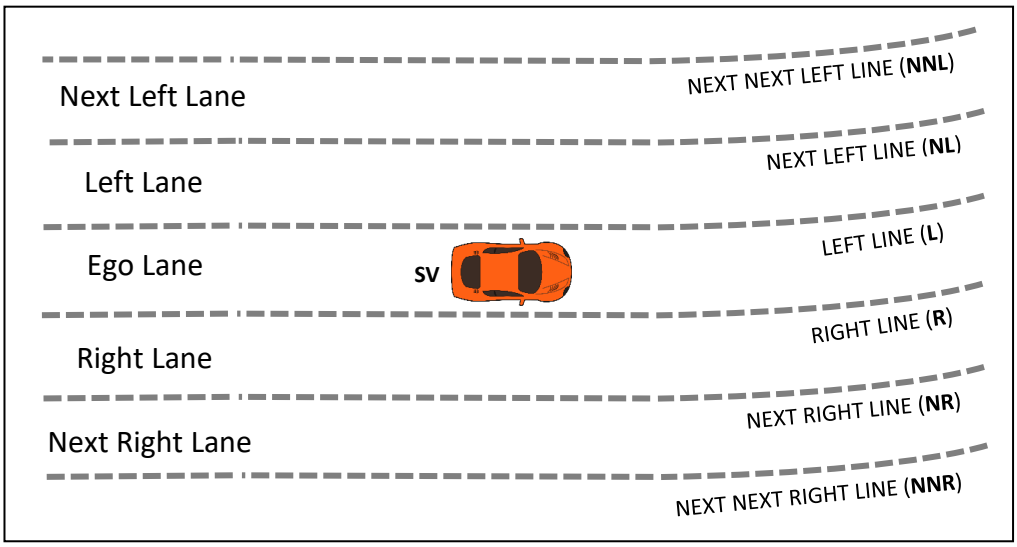


Figure : Subject Vehicle on a Multilane Road

Let us consider a scenario as depicted in the above figure, where the **subject vehicle (SV)** is travelling along a **multi lane highway**. Let the maximum number of lanes is **5**, the lanes and the corresponding borders are as follows :

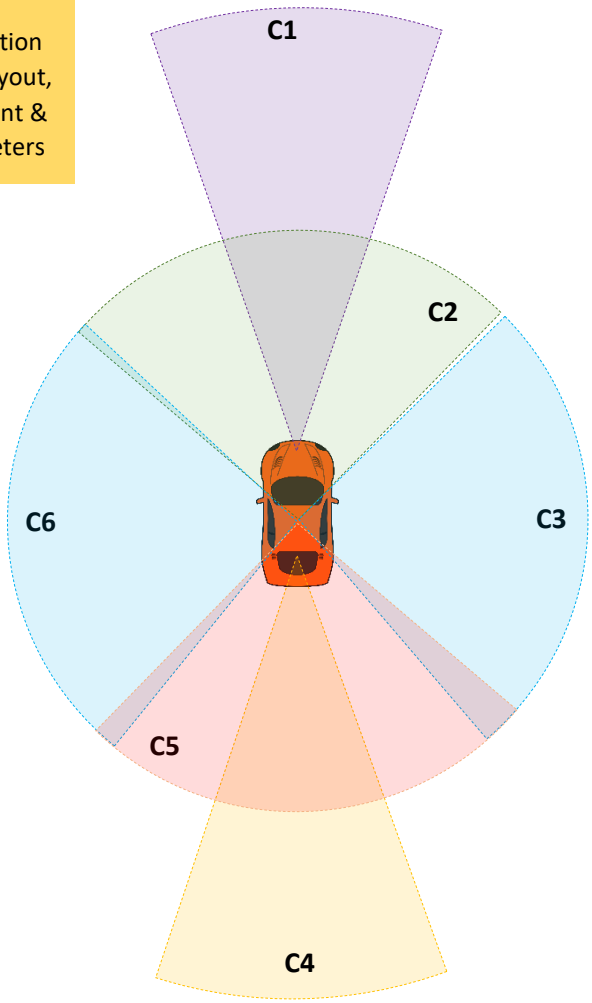
Lane No.	Lane Name	Border 1	Border 1 Notation	Border 2	Border 2 Notation
1	Ego Lane	Left Line	**L**	Right Line	**R**
2	Left Lane	Next left Line	**NL**	Left Line	**L**
3	Right Lane	Right Line	**R**	Next Right Line	**NR**
4	Next Left Lane	Next Next Left Line	**NNL**	Next left Line	**NL**
5	Next Right Lane	Next Right Line	**NR**	Next Next Right Line	**NNR**

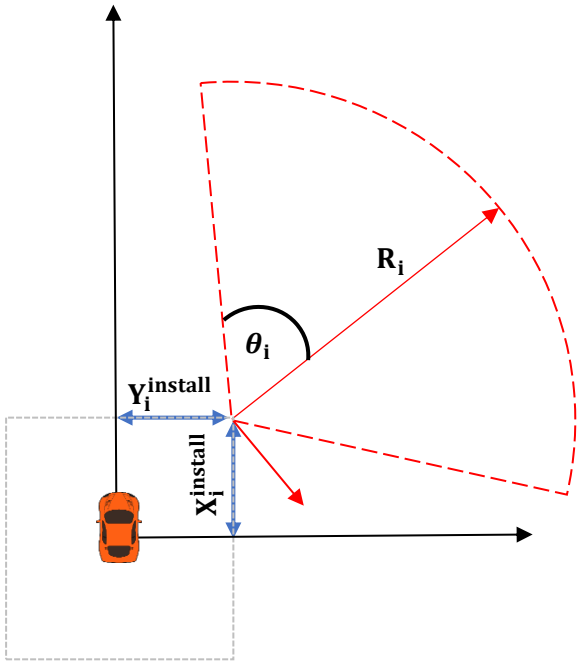
5.2 Sensor Setup

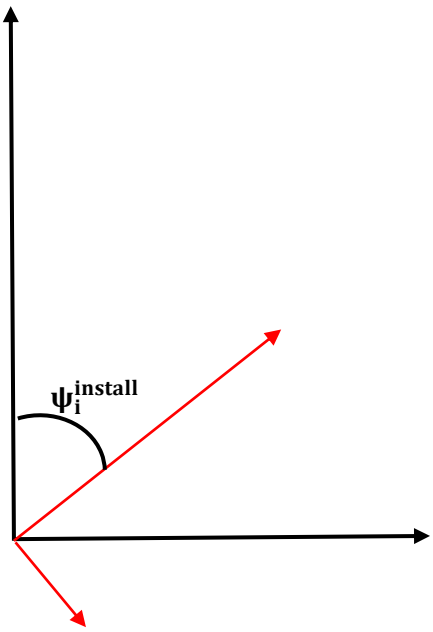
Let the sensor suite has **multiple cameras** as depicted in the below figure. The Sensor installation and Field of View (FoV) parameters are as follows :

Camera ID	Mount X	Mount Y	Mount Yaw	Maximum FoV Range	Maximum FoV Azimuth
C1	$X_1^{install}$	$Y_1^{install}$	$\psi_1^{install}$	R_1	θ_1
C2	$X_2^{install}$	$Y_2^{install}$	$\psi_2^{install}$	R_2	θ_2
C3	$X_3^{install}$	$Y_3^{install}$	$\psi_3^{install}$	R_3	θ_3
C4	$X_4^{install}$	$Y_4^{install}$	$\psi_4^{install}$	R_4	θ_4
C5	$X_5^{install}$	$Y_5^{install}$	$\psi_5^{install}$	R_5	θ_5
C6	$X_6^{install}$	$Y_6^{install}$	$\psi_6^{install}$	R_6	θ_6

Visual Representation of Sensor Layout, Sensor Mount & FoV parameters







$X_i^{install}$: longitudinal mount coordinate of the ith sensor w. r. t vehicle frame

$Y_i^{install}$: lateral mount coordinate of the ith sensor w. r. t vehicle frame

$\psi_i^{install}$: mount yaw angle of the ith sensor w. r. t vehicle frame

R_i : maximimum FoV range of ith sensor

θ_i : maximim FoV azimuth of ith sensor

5.3 Input Parameters

Each camera is capable is detecting **the road markings of the ego lane boundary** as depicted in the below figure.

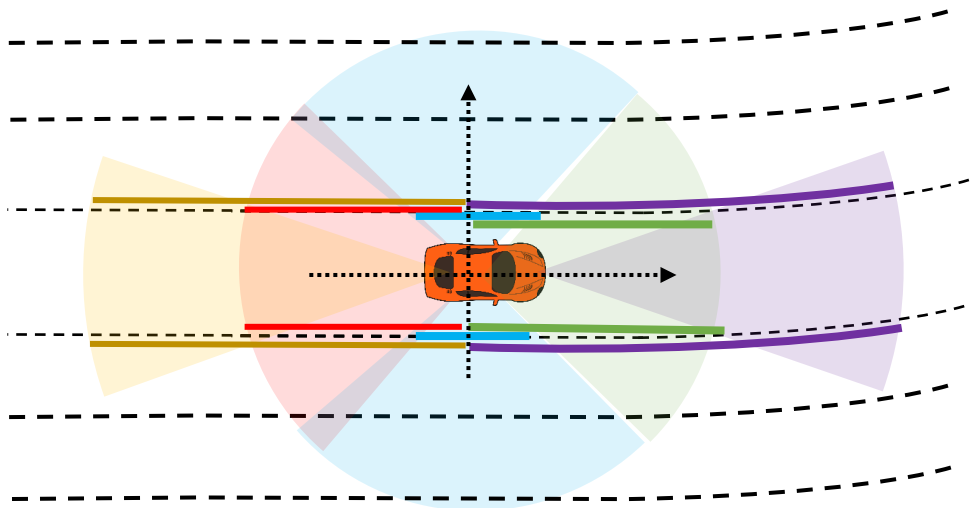
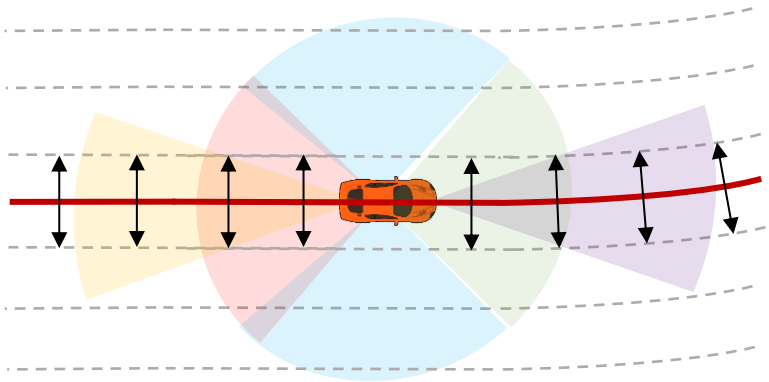
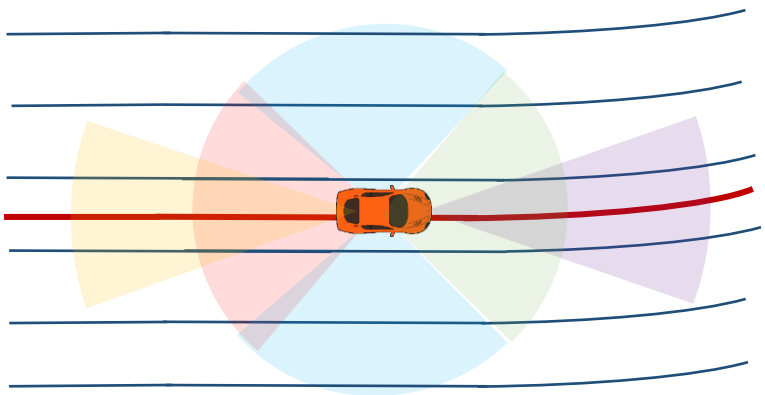
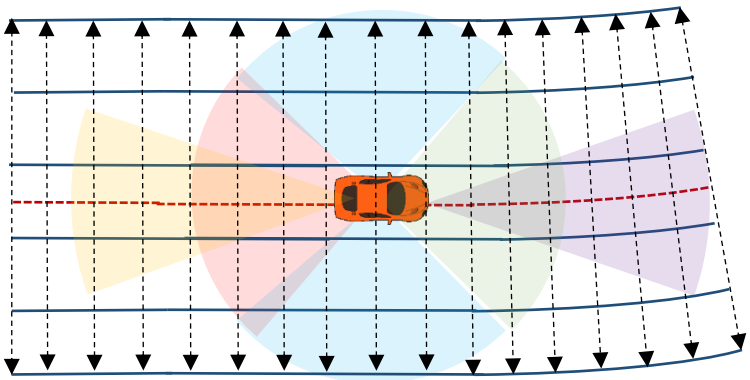


Figure : Line Measurements from camera sensors

Assuming the road boundary is represented as clothoid coefficient, we have the following set of measurements and the corresponding measurement parameters

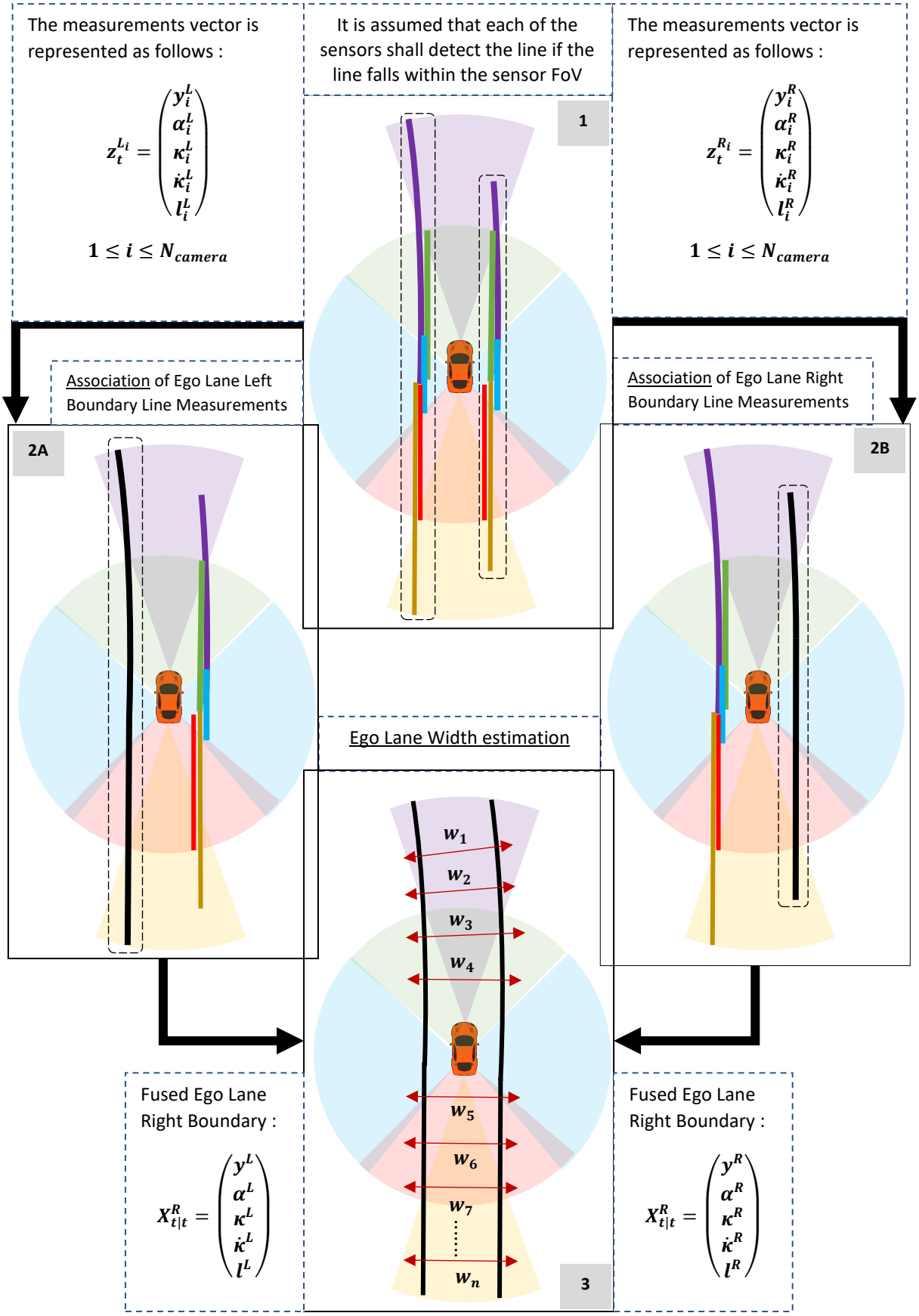
CAMERA	Ego Lane Left Border Line Clothoid Coefficients as Sensor Measurements					Ego Lane Right Border Line Clothoid Coefficients as Sensor Measurements				
	Lateral Offset	Initial alpha	Initial Kappa	Kappa Rate	Curve Length	Lateral Offset	Initial alpha	Initial Kappa	Kappa Rate	Curve Length
C1	y_1^L	α_1^L	κ_1^L	$\dot{\kappa}_1^L$	l_1^L	y_1^R	α_1^R	κ_1^R	$\dot{\kappa}_1^R$	l_1^R
C2	y_2^L	α_2^L	κ_2^L	$\dot{\kappa}_2^L$	l_2^L	y_2^R	α_2^R	κ_2^R	$\dot{\kappa}_2^R$	l_2^R
C3	y_3^L	α_3^L	κ_3^L	$\dot{\kappa}_3^L$	l_3^L	y_3^R	α_3^R	κ_3^R	$\dot{\kappa}_3^R$	l_3^R
C4	y_4^L	α_4^L	κ_4^L	$\dot{\kappa}_4^L$	l_4^L	y_4^R	α_4^R	κ_4^R	$\dot{\kappa}_4^R$	l_4^R
C5	y_5^L	α_5^L	κ_5^L	$\dot{\kappa}_5^L$	l_5^L	y_5^R	α_5^R	κ_5^R	$\dot{\kappa}_5^R$	l_5^R
C6	y_6^L	α_6^L	κ_6^L	$\dot{\kappa}_6^L$	l_6^L	y_6^R	α_6^R	κ_6^R	$\dot{\kappa}_6^R$	l_6^R

5.4 Required Output

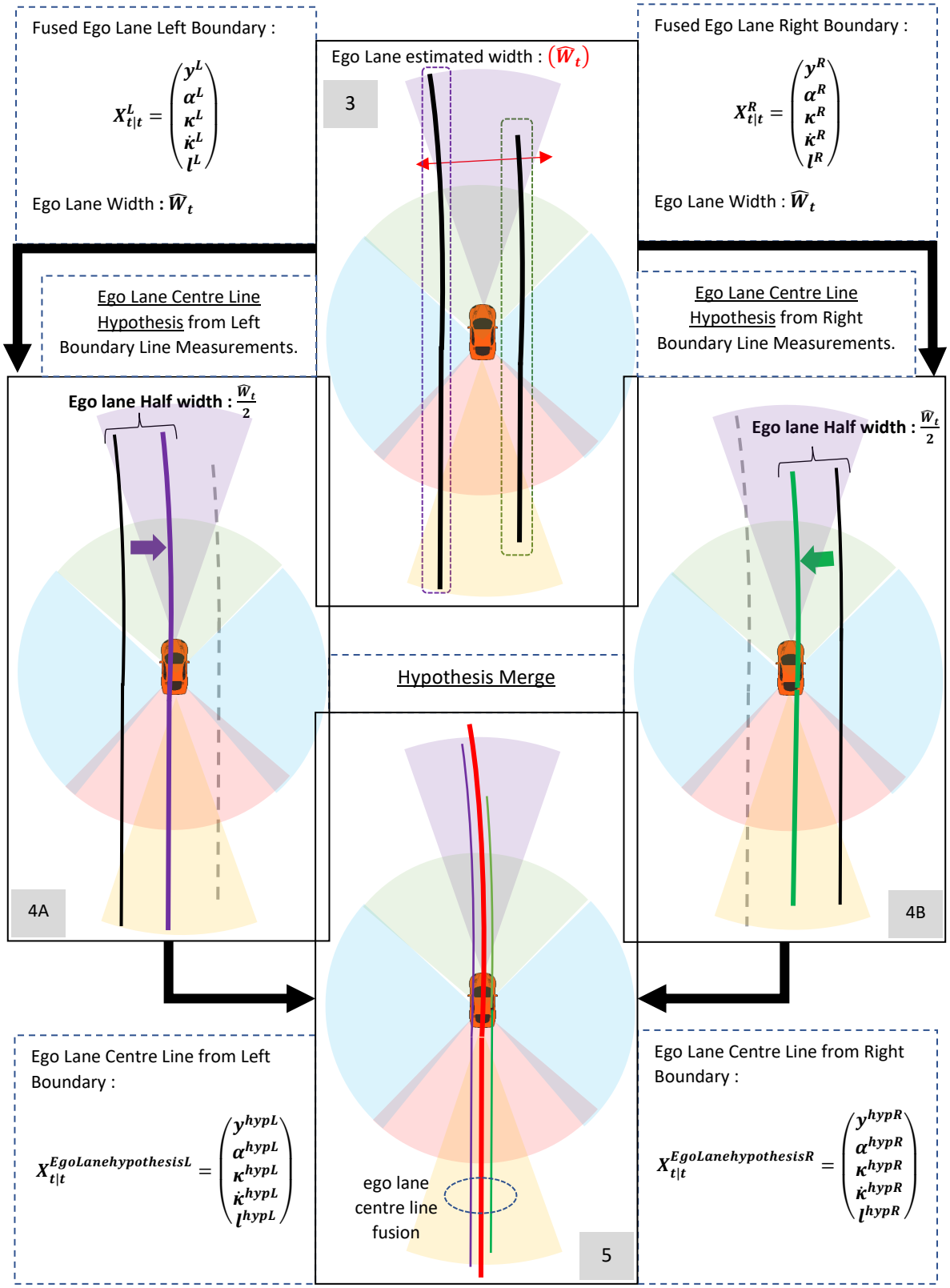
Estimated Ego Lane Geometry : 1. Clothoid Parameters • Lateral Offset • Initial Heading • Initial Curvature • Curvature Rate 2. Ego Lane Width	
ROAD MODEL Estimated Ego lane Centre Line Adjacent Lanes Border Geometry with respect to ego vehicle wheel base centre: 1. Ego Lane Left Line 2. Ego Lane Right Line 3. Next Left Line 4. Next of Next Left Line 5. Next Right Line 6. Next of Next Right Line 7. Ego Lane Centre Line	
ROAD GRID	

6. HIGH LEVEL DESIGN

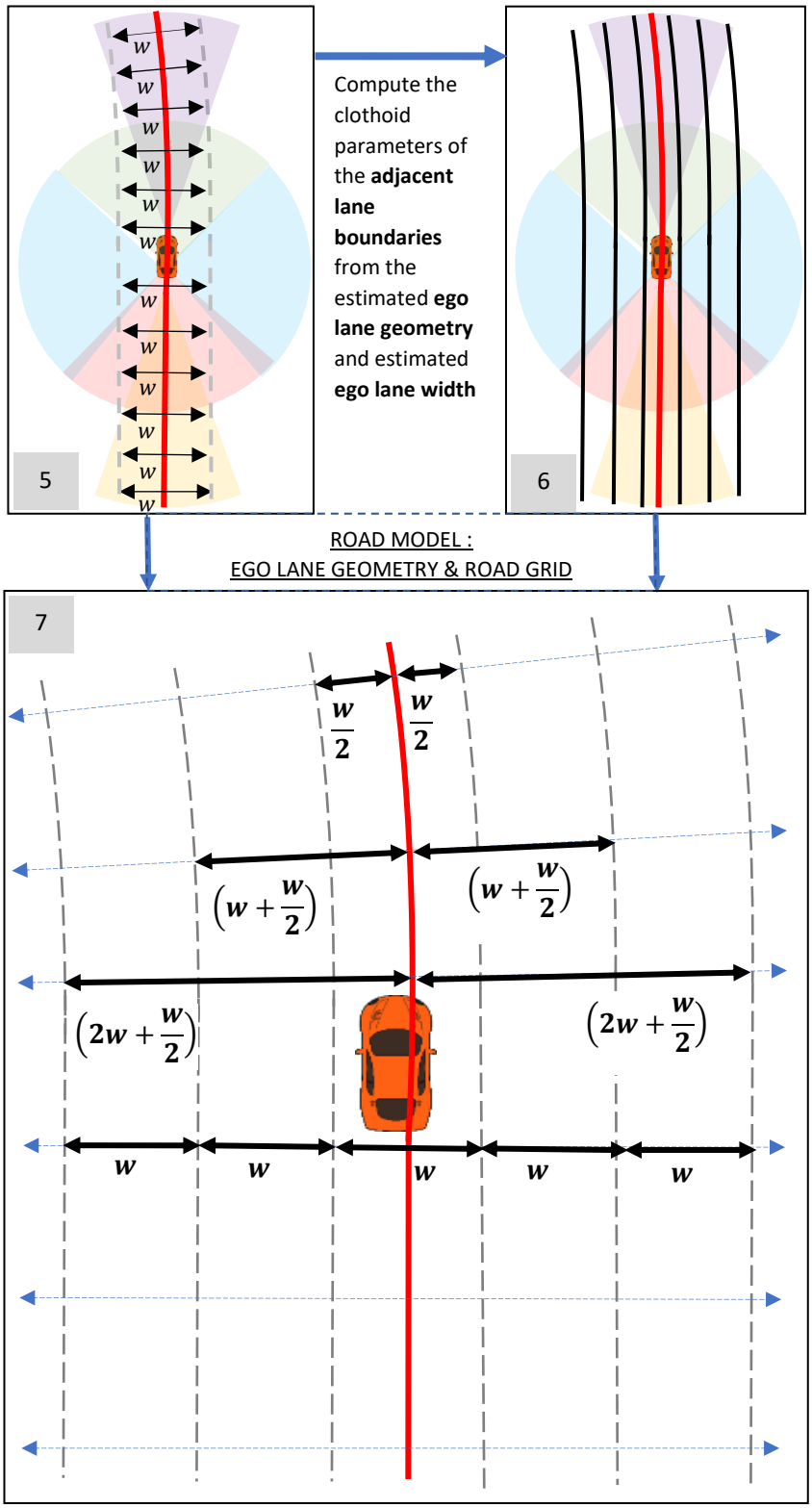
6.1 Part 1 : Association Of Line Measurements From Camera Sensors



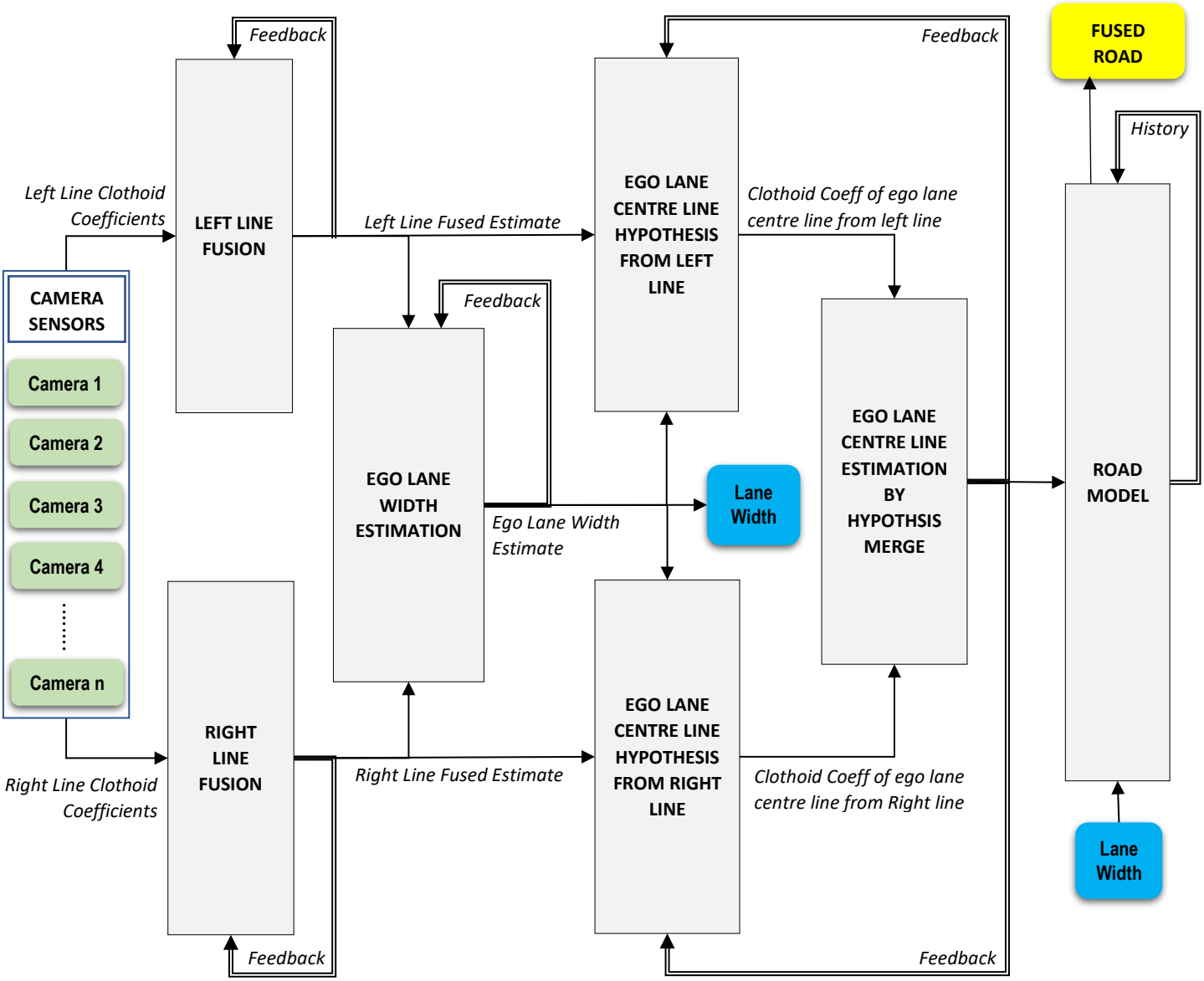
6.2 Part 2 : Ego Lane Centre Line Estimation



6.3 Part 3 : Road Model Computation



7. ROAD MODEL FUSION ARCHITECTURE

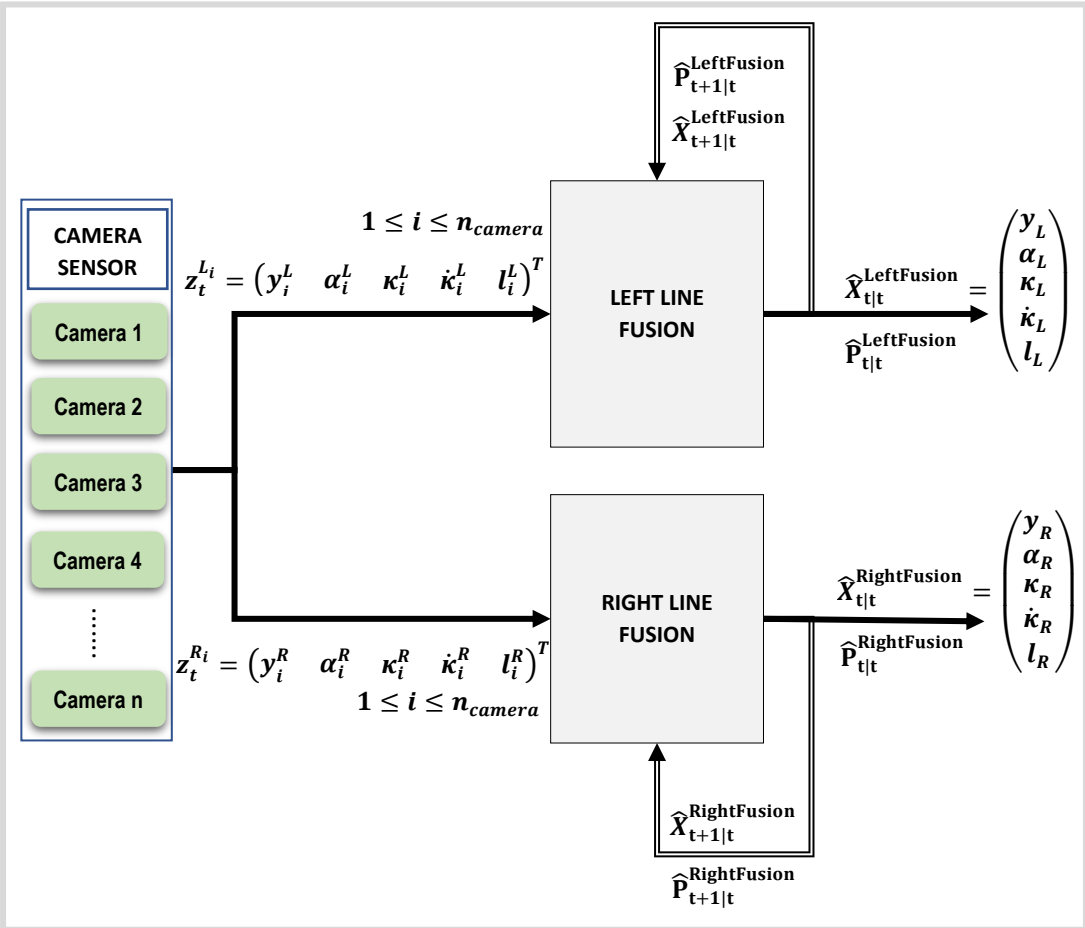


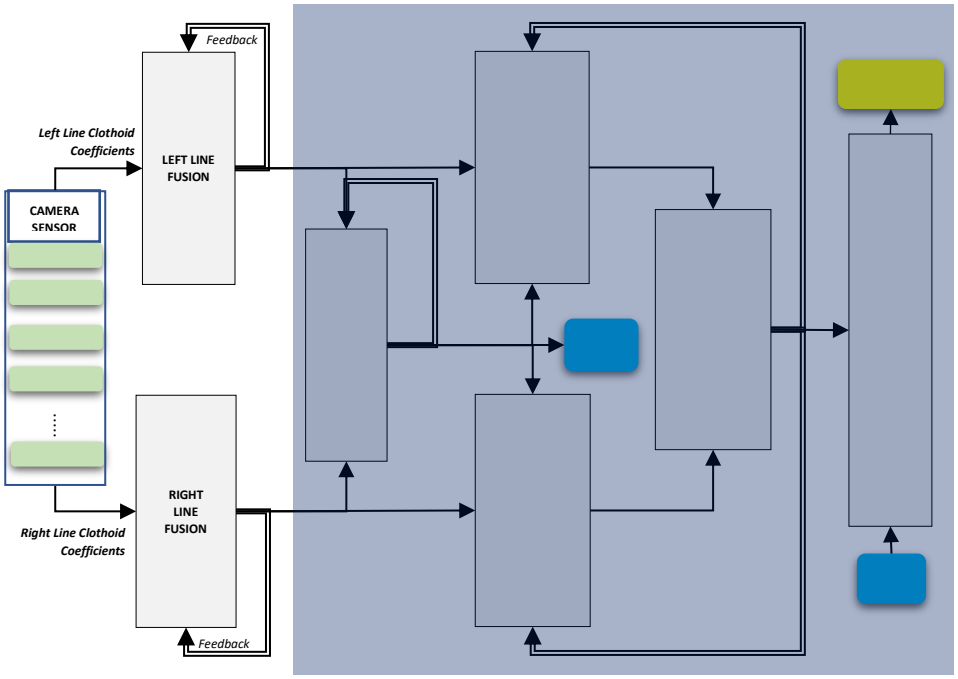
Functional Component Descriptions :

1. **LEFT LINE FUSION & RIGHT LINE FUSION** : Association of Ego Lane Boundary measurements. The ego lane boundaries are detected from each of the camera sensors. The Boundary lines are represented in terms of clothoid coefficients.
2. **EGO LANE WIDTH ESTIMATION** : Estimated width of the ego lane. The width is estimated from the distance between the fused left ego lane boundary and fused right ego lane boundary.
3. **EGO LANE CENTRE LINE HYPOTHESIS FROM LEFT LINE** : Ego lane centre line estimation from the left ego lane boundary line and the width of the ego lane.
4. **EGO LANE CENTRE LINE HYPOTHESIS FROM RIGHT LINE** : Ego lane centre line estimation from the right ego lane boundary line and the width of the ego lane.
5. **EGO LANE CENTRE LINE ESTIMATION BY HYPOTHESIS MERGE :** Estimation of ego lane centre line from multi hypothesis merge (hypothesis generated from 3 and 4)
6. **ROAD MODEL :** Road Model represented in terms of ego lane centre line, ego lane width, adjacent lane boundaries and road grid.

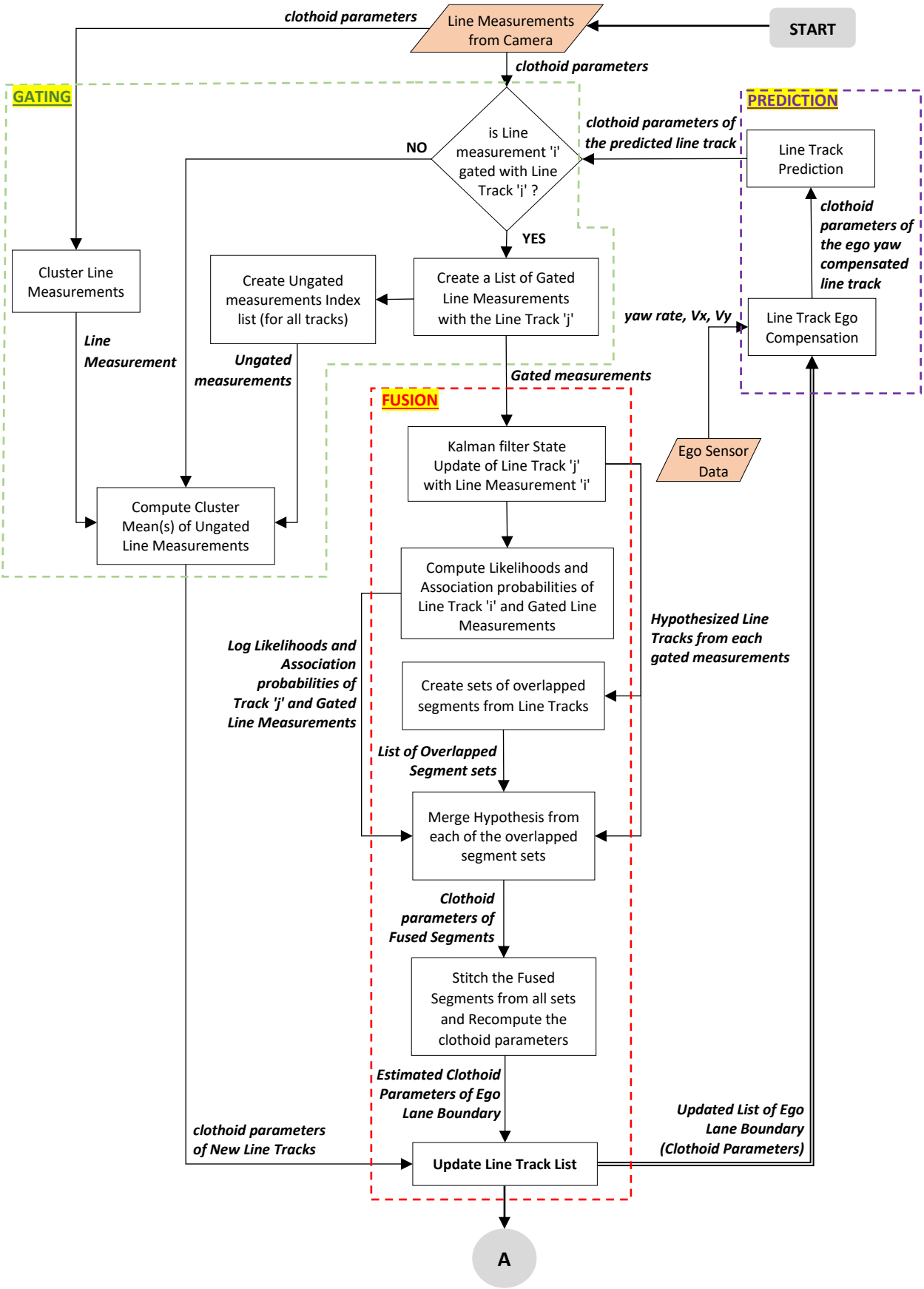
8. FUSION OF LANE LINE MEASUREMENTS FROM CAMERAS

8.1 Data Flow/Block Diagram : Association Of Line Measurements





8.2 Execution Flow : Left Line & Right Line Fusion



8.3 Parameterization of Line Track

The Line Track is represented in terms of clothoid parameters because the road is constructed as a clothoid

- A clothoid or an Euler spiral is a curve whose curvature changes linearly with its curve length.
- To travel along a circular path, an object needs to be subject to a centripetal acceleration. If a vehicle traveling on a straight path were to suddenly transition to a tangential circular path, it would require centripetal acceleration suddenly switching at the tangent point from zero to the required value, this would introduce a lot of jerk in the motion.
- A track transition curve is a mathematically-calculated curve on a section of highway, or railroad track, in which a straight section changes into a curve. It is designed to prevent sudden changes in lateral (or centripetal) acceleration.
- A road is typically modelled as a clothoid curve to improve driving safety and ride comfort.

8.3.1 Clothoid Equations

The Clothoid equations are formulated as follows :

$$\boldsymbol{\kappa}(\mathbf{s}) = \boldsymbol{\kappa}_0 + \dot{\boldsymbol{\kappa}}\mathbf{s}$$

$$\boldsymbol{\varphi}(\mathbf{s}) = \boldsymbol{\varphi}_0 + \boldsymbol{\kappa}_0\mathbf{s} + \frac{1}{2}\dot{\boldsymbol{\kappa}}\mathbf{s}^2$$

Heading and Curvature at different arc lengths (s) under the assumption of constant curvature

$$\mathbf{x}(\mathbf{s}) = \mathbf{x}_0 + \int_0^s \cos\big(\boldsymbol{\varphi}(\mathbf{t})\big)\,\mathbf{dt}$$

$$\mathbf{y}(\mathbf{s}) = \mathbf{y}_0 + \int_0^s \sin\big(\boldsymbol{\varphi}(\mathbf{t})\big)\,\mathbf{dt}$$

Clothoid to Cartesian coordinates

Where ,

$\mathbf{L} \rightarrow$ length of the curve

$\mathbf{s} \rightarrow$ length of a curve segment , $0 \leq s \leq L$

$\boldsymbol{\varphi}_0 \rightarrow$ initial curve heading

$\boldsymbol{\kappa}_0 \rightarrow$ initial curvature

$\dot{\boldsymbol{\kappa}} \rightarrow$ curvature rate (constant)

$\boldsymbol{\kappa}(\mathbf{s}) \rightarrow$ curvature at length s on the curve

$\boldsymbol{\varphi}(\mathbf{s}) \rightarrow$ curve heading at length s on the curve

$\mathbf{x}(\mathbf{s}) \rightarrow$ x cartesian coordinate at length s on the curve

$\mathbf{y}(\mathbf{s}) \rightarrow$ y cartesian coordinate at length s on the curve

8.3.2 Clothoid Approximations

- For computing the parametric cartesian coordinates as a function of arc lengths 's' , we need to compute the integral commonly referred as Fresnel integral.
- The Fresnel integral cannot be solved analytically, hence we resort to approximations.
- There are Different approximation techniques with varying degrees of computational complexity and approximation errors for converting clothoid to cartesian coordinates.
- An computational efficient method of clothoid approximation is needed for Road Geometry estimation for ADAS or Automated Driving Applications.

This section describes briefly the different approximation techniques and the enlists one technique that shall be used for approximation a clothoid. Clothoid curve formulation based on the third order Taylor series approximation shall be used for representing a line track.

8.3.2.1 Approximation Technique 1 : Small angle approximation

$$\boldsymbol{\kappa(s) = \kappa_0 + \dot{\kappa} s}$$

$$\boldsymbol{\varphi(s) = \varphi_0 + \kappa_0 s + \frac{1}{2}\dot{\kappa} s^2}$$

$$\boldsymbol{x(s) = x_0 + s}$$

$$\boldsymbol{y(s) = y_0 + \varphi_0 s + \frac{1}{2}\kappa_0 s^2 + \frac{1}{6}\dot{\kappa} s^2}$$

$$\begin{pmatrix} y(s) \\ \alpha(s) \\ \kappa(s) \\ \dot{\kappa}(s) \end{pmatrix} = \begin{pmatrix} 1 & s & \frac{s^2}{2} & \frac{s^3}{6} \\ 0 & 1 & s & \frac{s^2}{2} \\ 0 & 0 & 1 & s \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_0 \\ \alpha_0 \\ \kappa_0 \\ \dot{\kappa} \end{pmatrix}$$

for small angle $|\varphi(t)| \leq 15^o$

$$\cos(\varphi(t)) \approx 1$$

$$\sin(\varphi(t)) \approx \varphi(t)$$

$$x(s) = x_0 + \int_0^s 1\,dt \Rightarrow x(s) = x_0 + s$$

$$y(s) = y_0 + \int_0^s \varphi(t)\,dt$$

$$\Rightarrow y(s) = y_0 + \int_0^s \left(\varphi_0 + \kappa_0 t + \frac{1}{2}\dot{\kappa} t^2\right) dt$$

$$\Rightarrow y(s) = y_0 + \varphi_0 s + \frac{1}{2}\kappa_0 s^2 + \frac{1}{6}\dot{\kappa} s^3$$

8.3.2.2 Approximation Technique 2 : Numerical Computation of Fresnel Integral

$$\Delta l = \frac{\mathrm{s}}{\mathrm{N}-1}$$

$$\mathrm{s(n)} = (\mathrm{n}-1)\Delta l \qquad n \rightarrow 1,2,3,\ldots,N$$

$$\varphi(\mathrm{n}) = \varphi_0 + \kappa_0 \mathrm{s(n)} + \frac{1}{2}\dot{\kappa}\big(\mathrm{s(n)}\big)^2$$

$$\Rightarrow \varphi(\mathrm{n}) = \varphi_0 + \kappa_0(\mathrm{n}-1)\Delta l + \frac{1}{2}\dot{\kappa}\big((\mathrm{n}-1)\Delta l\big)^2$$

$$\mathrm{x(n)} = \mathrm{x}_0 + \sum_{k=1}^{n} \cos\big(\varphi(\mathrm{k})\big)$$

$$\Rightarrow \mathrm{x(n)} = \mathrm{x}_0 + \sum_{k=1}^{n} \cos\left(\varphi_0 + \kappa_0(\mathrm{k}-1)\Delta l + \frac{1}{2}\dot{\kappa}\big((\mathrm{k}-1)\Delta l\big)^2\right)$$

$$\mathrm{y(n)} = \mathrm{y}_0 + \sum_{k=1}^{n} \sin\big(\varphi(\mathrm{k})\big)$$

$$\Rightarrow \mathrm{y(n)} = \mathrm{y}_0 + \sum_{k=1}^{n} \sin\left(\varphi_0 + \kappa_0(\mathrm{k}-1)\Delta l + \frac{1}{2}\dot{\kappa}\big((\mathrm{k}-1)\Delta l\big)^2\right)$$

$$\boldsymbol{\Delta l = \frac{s}{N-1}} \qquad \mathrm{N} \rightarrow \text{max number of samples}$$

$$\mathbf{k(s)} = \boldsymbol{\kappa_0} + \dot{\boldsymbol{\kappa}}(\mathbf{N}-\mathbf{1})\Delta \boldsymbol{l}$$

$$\boldsymbol{\varphi(s)} = \boldsymbol{\varphi_0} + \boldsymbol{\kappa_0}(\mathbf{N}-\mathbf{1})\Delta \boldsymbol{l} + \frac{\mathbf{1}}{\mathbf{2}}\dot{\boldsymbol{\kappa}}\big((\mathbf{N}-\mathbf{1})\Delta \boldsymbol{l}\big)^2$$

$$\mathbf{x(s)} = \mathbf{x_0} + \sum_{k=1}^{N} \cos\left(\boldsymbol{\varphi_0} + \boldsymbol{\kappa_0}(\mathbf{k}-\mathbf{1})\Delta \boldsymbol{l} + \frac{\mathbf{1}}{\mathbf{2}}\dot{\boldsymbol{\kappa}}\big((\mathbf{k}-\mathbf{1})\Delta \boldsymbol{l}\big)^2\right)$$

$$\mathbf{y(s)} = \mathbf{y_0} + \sum_{k=1}^{N} \sin\left(\boldsymbol{\varphi_0} + \boldsymbol{\kappa_0}(\mathbf{k}-\mathbf{1})\Delta \boldsymbol{l} + \frac{\mathbf{1}}{\mathbf{2}}\dot{\boldsymbol{\kappa}}\big((\mathbf{k}-\mathbf{1})\Delta \boldsymbol{l}\big)^2\right)$$

8.3.2.3 Approximation Technique 3 : 3rd Order Taylor Series Approximation of Fresnel Integral

$$\sin\left(\boldsymbol{\varphi_0} + \boldsymbol{\kappa_0}\left(\frac{\mathbf{L}}{\mathbf{2}}\right) + \frac{\mathbf{1}}{\mathbf{2}}\dot{\boldsymbol{\kappa}}\left(\frac{\boldsymbol{L}}{\mathbf{2}}\right)^{\mathbf{2}}\right) \rightarrow \boldsymbol{S}$$

$$\cos\left(\boldsymbol{\varphi_0} + \boldsymbol{\kappa_0}\left(\frac{\mathbf{L}}{\mathbf{2}}\right) + \frac{\mathbf{1}}{\mathbf{2}}\dot{\boldsymbol{\kappa}}\left(\frac{\boldsymbol{L}}{\mathbf{2}}\right)^{\mathbf{2}}\right) \rightarrow \boldsymbol{C}$$

$$\boldsymbol{\kappa_0} + \dot{\boldsymbol{\kappa}}\left(\frac{\boldsymbol{L}}{\mathbf{2}}\right) \rightarrow \dot{\boldsymbol{\alpha}}$$

$$\begin{aligned}
\mathbf{x(s)} = \boldsymbol{x_0} &+ \left(\mathbf{C} + \left(\frac{\mathbf{L}}{\mathbf{2}}\right)\mathbf{S}\dot{\boldsymbol{\alpha}} - \left(\frac{\mathbf{L}}{\mathbf{2}}\right)^{\mathbf{2}}\left(\frac{\mathbf{C}\dot{\boldsymbol{\alpha}}^{\mathbf{2}} + \dot{\mathbf{k}}\mathbf{S}}{\mathbf{2}}\right) - \left(\frac{\mathbf{L}}{\mathbf{2}}\right)^{\mathbf{3}}\left(\frac{\mathbf{S}\dot{\boldsymbol{\alpha}}^{\mathbf{3}}}{\mathbf{6}} - \frac{\dot{\mathbf{k}}\mathbf{C}\dot{\boldsymbol{\alpha}}}{\mathbf{2}}\right)\right)\mathbf{s} \\
&+ \left(-\frac{\boldsymbol{S}\dot{\boldsymbol{\alpha}}}{\mathbf{2}} + \left(\frac{\mathbf{L}}{\mathbf{2}}\right)\left(\frac{\mathbf{C}\dot{\boldsymbol{\alpha}}^{\mathbf{2}} + \dot{\mathbf{k}}\mathbf{S}}{\mathbf{2}}\right) + \frac{\mathbf{3}}{\mathbf{2}}\left(\frac{\mathbf{L}}{\mathbf{2}}\right)^{\mathbf{2}}\left(\frac{\mathbf{S}\dot{\boldsymbol{\alpha}}^{\mathbf{3}}}{\mathbf{6}} - \frac{\dot{\mathbf{k}}\mathbf{C}\dot{\boldsymbol{\alpha}}}{\mathbf{2}}\right)\right)\boldsymbol{s}^{\mathbf{2}} \\
&+ \left(-\frac{\mathbf{1}}{\mathbf{3}}\left(\frac{\mathbf{C}\dot{\boldsymbol{\alpha}}^{\mathbf{2}} + \dot{\mathbf{k}}\mathbf{S}}{\mathbf{2}}\right) - \left(\frac{\mathbf{L}}{\mathbf{2}}\right)\left(\frac{\mathbf{S}\dot{\boldsymbol{\alpha}}^{\mathbf{3}}}{\mathbf{6}} - \frac{\dot{\mathbf{k}}\mathbf{C}\dot{\boldsymbol{\alpha}}}{\mathbf{2}}\right)\right)\boldsymbol{s}^{\mathbf{3}} \\
&+ \left(\left(\frac{\mathbf{1}}{\mathbf{4}}\right)\left(\frac{\mathbf{S}\dot{\boldsymbol{\alpha}}^{\mathbf{3}}}{\mathbf{6}} - \frac{\dot{\mathbf{k}}\mathbf{C}\dot{\boldsymbol{\alpha}}}{\mathbf{2}}\right)\right)\boldsymbol{s}^{\mathbf{4}}
\end{aligned}$$

$$\begin{aligned}
\mathbf{y(s)} = \boldsymbol{y_0} &+ \left(\mathbf{S} - \left(\frac{\mathbf{L}}{\mathbf{2}}\right)\mathbf{C}\dot{\boldsymbol{\alpha}} - \left(\frac{\mathbf{L}}{\mathbf{2}}\right)^{\mathbf{2}}\left(\frac{\mathbf{S}\dot{\boldsymbol{\alpha}}^{\mathbf{2}} - \dot{\mathbf{k}}\mathbf{C}}{\mathbf{2}}\right) + \left(\frac{\mathbf{L}}{\mathbf{2}}\right)^{\mathbf{3}}\left(\frac{\mathbf{C}\dot{\boldsymbol{\alpha}}^{\mathbf{3}}}{\mathbf{6}} + \frac{\dot{\mathbf{k}}\mathbf{S}\dot{\boldsymbol{\alpha}}}{\mathbf{2}}\right)\right)\mathbf{s} \\
&+ \left(\frac{\boldsymbol{C}\dot{\boldsymbol{\alpha}}}{\mathbf{2}} + \left(\frac{\mathbf{L}}{\mathbf{2}}\right)\left(\frac{\mathbf{S}\dot{\boldsymbol{\alpha}}^{\mathbf{2}} - \dot{\mathbf{k}}\mathbf{C}}{\mathbf{2}}\right) - \frac{\mathbf{3}}{\mathbf{2}}\left(\frac{\mathbf{L}}{\mathbf{2}}\right)^{\mathbf{2}}\left(\frac{\mathbf{C}\dot{\boldsymbol{\alpha}}^{\mathbf{3}}}{\mathbf{6}} + \frac{\dot{\mathbf{k}}\mathbf{S}\dot{\boldsymbol{\alpha}}}{\mathbf{2}}\right)\right)\boldsymbol{s}^{\mathbf{2}} \\
&+ \left(-\frac{\mathbf{1}}{\mathbf{3}}\left(\frac{\mathbf{S}\dot{\boldsymbol{\alpha}}^{\mathbf{2}} - \dot{\mathbf{k}}\mathbf{C}}{\mathbf{2}}\right) + \left(\frac{\mathbf{L}}{\mathbf{2}}\right)\left(\frac{\mathbf{C}\dot{\boldsymbol{\alpha}}^{\mathbf{3}}}{\mathbf{6}} + \frac{\dot{\mathbf{k}}\mathbf{S}\dot{\boldsymbol{\alpha}}}{\mathbf{2}}\right)\right)\boldsymbol{s}^{\mathbf{3}} \\
&- \left(\left(\frac{\mathbf{1}}{\mathbf{4}}\right)\left(\frac{\mathbf{C}\dot{\boldsymbol{\alpha}}^{\mathbf{3}}}{\mathbf{6}} + \frac{\dot{\mathbf{k}}\mathbf{S}\dot{\boldsymbol{\alpha}}}{\mathbf{2}}\right)\right)\boldsymbol{s}^{\mathbf{4}}
\end{aligned}$$

$\boldsymbol{note}$:

$\mathbf{L} \rightarrow$ length of the curve

$\mathbf{s} \rightarrow$ length of a curve segment , $0 \leq s \leq L$

$\boldsymbol{\varphi_0} \rightarrow$ initial curve heading

$\mathbf{k_0} \rightarrow$ initial curvature

$\dot{\boldsymbol{\kappa}} \rightarrow$ curvature rate (constant)

$\boldsymbol{\kappa}(\mathbf{s}) \rightarrow$ curvature at length s on the curve

$\boldsymbol{\varphi}(\mathbf{s}) \rightarrow$ curve heading at length s on the curve

$\mathbf{x}(\mathbf{s}) \rightarrow$ x cartesian coordinate at length s on the curve

$\mathbf{y}(\mathbf{s}) \rightarrow$ y cartesian coordinate at length s on the curve

8.3.2.4 Derivation of x(s) for approximation technique 3

For the below equation:

$$k(s) = k_0 + \dot{k}s \qquad \varphi(s) = \varphi_0 + k_0 s + \frac{1}{2}\dot{k}s^2$$

$$x(s) = x_0 + \int_0^s \cos\left(\varphi_0 + k_0 l + \frac{1}{2}\dot{k}l^2\right) dl$$

$$y(s) = y_0 + \int_0^s \sin\left(\varphi_0 + k_0 l + \frac{1}{2}\dot{k}l^2\right) dl$$

Approximate:

$$\cos\left(\varphi_0 + k_0 s + \frac{1}{2}\dot{k}s^2\right) \text{ at } s = s_0$$

$$\sin\left(\varphi_0 + k_0 s + \frac{1}{2}\dot{k}s^2\right) \text{ at } s = s_0$$

$$f(x) = f(x_0) + \frac{(x - x_0)}{1!}\frac{df(x_0)}{dx} + \frac{(x - x_0)^2}{2!}\frac{d^2f(x_0)}{dx^2} + \frac{(x - x_0)^3}{3!}\frac{d^3f(x_0)}{dx^3} + \ldots$$

$$\Rightarrow \mathbf{f(x)} = \sum_{k=0}^{\infty} \frac{(\mathbf{x} - \mathbf{x_0})^{\mathbf{k}}}{\mathbf{k}!}\frac{\mathbf{d^k f(x_0)}}{\mathbf{dx^k}}$$

Taylor Series

Taylor series approximation of :

$$\cos\left(\varphi_0 + k_0 s + \frac{1}{2}\dot{k}s^2\right) \text{ at } s = s_0$$

$$f(s) = f(s_0) + \frac{(s - s_0)}{1!}\frac{df(s_0)}{ds} + \frac{(s - s_0)^2}{2!}\frac{d^2f(s_0)}{ds^2} + \frac{(s - s_0)^3}{3!}\frac{d^3f(s_0)}{ds^3}$$

$$f(s_0) = \cos\left(\varphi_0 + k_0 s_0 + \frac{1}{2}\dot{k}s_0^2\right) = C \qquad (k_0 + \dot{k}s_0) \to \dot{\alpha} \qquad \cos\left(\varphi_0 + k_0 s_0 + \frac{1}{2}\dot{k}s_0^2\right) \to C$$

$$\frac{df(s_0)}{ds} = -(k_0 + \dot{k}s_0)\sin\left(\varphi_0 + k_0 s_0 + \frac{1}{2}\dot{k}s_0^2\right) = -\dot{\alpha}S \qquad sin\left(\varphi_0 + k_0 s_0 + \frac{1}{2}\dot{k}s_0^2\right) \to S$$

$$\frac{d^2f(s_0)}{ds^2} = -\dot{k}.\sin\left(\varphi_0 + k_0 s_0 + \frac{1}{2}\dot{k}s_0^2\right) - (k_0 + \dot{k}s)^2.\cos\left(\varphi_0 + k_0 s_0 + \frac{1}{2}\dot{k}s_0^2\right) = -\left(\dot{k}S + \dot{\alpha}^2 C\right)$$

$$\frac{d^3f(s_0)}{ds^3} = -3\dot{k}(k_0 + \dot{k}s).\cos\left(\varphi_0 + k_0 s_0 + \frac{1}{2}\dot{k}s_0^2\right) + (k_0 + \dot{k}s)^3.\sin\left(\varphi_0 + k_0 s_0 + \frac{1}{2}\dot{k}s_0^2\right) = \dot{\alpha}^3 S - 3\dot{k}\dot{\alpha}C$$

$$\cos(\varphi(t)) = C - (t - s_0)\dot{\alpha}S - \frac{(t - s_0)^2}{2}\left(\dot{k}S + \dot{\alpha}^2 C\right) + \frac{(t - s_0)^3}{6}\left(\dot{\alpha}^3 S - 3\dot{k}\dot{\alpha}C\right)$$

$$\Rightarrow \cos(\varphi(t)) = C - (t - s_0)\dot{\alpha}S - \frac{(t^2 - 2s_0 t + s_0^2)}{2}\left(\dot{k}S + \dot{\alpha}^2 C\right) + \frac{(t^3 - 3s_0 t^2 + 3s_0^2 t - s_0^3)}{6}\left(\dot{\alpha}^3 S - 3\dot{k}\dot{\alpha}C\right)$$

$$\Rightarrow \cos(\varphi(t)) = C + s_0\dot{\alpha}S - \frac{s_0^2}{2}\left(\dot{k}S + \dot{\alpha}^2 C\right) - \frac{s_0^3}{6}\left(\dot{\alpha}^3 S - 3\dot{k}\dot{\alpha}C\right) + \left(-S\dot{\alpha} + s_0\left(\dot{k}S + \dot{\alpha}^2 C\right) + \frac{s_0^2}{2}\left(\dot{\alpha}^3 S - 3\dot{k}\dot{\alpha}C\right)\right)t \ldots$$

$$+ \left(-\frac{1}{2}\left(\dot{k}S + \dot{\alpha}^2 C\right) - \frac{s_0}{2}\left(\dot{\alpha}^3 S - 3\dot{k}\dot{\alpha}C\right)\right)t^2 + \left(\frac{1}{6}\left(\dot{\alpha}^3 S - 3\dot{k}\dot{\alpha}C\right)\right)t^3$$

$$\mathbf{x(s)} = x_0 + \left(C + \left(\frac{L}{2}\right)S\dot{\alpha} - \left(\frac{L}{2}\right)^2\left(\frac{C\dot{\alpha}^2 + \dot{k}S}{2}\right) - \left(\frac{L}{2}\right)^3\left(\frac{S\dot{\alpha}^3}{6} - \frac{\dot{k}C\dot{\alpha}}{2}\right)\right)s \qquad \text{let } s_0 \to \frac{L}{2}$$

$$+ \left(-\frac{S\dot{\alpha}}{2} + \left(\frac{L}{2}\right)\left(\frac{C\dot{\alpha}^2 + \dot{k}S}{2}\right) + \frac{3}{2}\left(\frac{L}{2}\right)^2\left(\frac{S\dot{\alpha}^3}{6} - \frac{\dot{k}C\dot{\alpha}}{2}\right)\right)s^2$$

$$+ \left(-\frac{1}{3}\left(\frac{C\dot{\alpha}^2 + \dot{k}S}{2}\right) - \left(\frac{L}{2}\right)\left(\frac{S\dot{\alpha}^3}{6} - \frac{\dot{k}C\dot{\alpha}}{2}\right)\right)s^3 + \left(\left(\frac{1}{4}\right)\left(\frac{S\dot{\alpha}^3}{6} - \frac{\dot{k}C\dot{\alpha}}{2}\right)\right)s^4$$

8.3.2.5 Derivation of y(s) for approximation technique 3

Taylor series approximation of :

$$\sin\left(\varphi_0 + k_0 s + \frac{1}{2}\dot{k}s^2\right) \text{ at } s = s_0$$

$$f(s) = f(s_0) + \frac{(s-s_0)}{1!}\frac{df(s_0)}{ds} + \frac{(s-s_0)^2}{2!}\frac{d^2f(s_0)}{ds^2} + \frac{(s-s_0)^3}{3!}\frac{d^3f(s_0)}{ds^3}$$

$$f(s_0) = \sin\left(\varphi_0 + k_0 s_0 + \frac{1}{2}\dot{k}s_0^2\right) = S$$

$$\left(\frac{df(s)}{ds}\right)_{s=s_0} = (k_0 + \dot{k}s_0)\cos\left(\varphi_0 + k_0 s_0 + \frac{1}{2}\dot{k}s_0^2\right) = \dot{\alpha}C$$

$$\left(\frac{d^2f(s)}{ds^2}\right)_{s=s_0} = \dot{k}.\cos\left(\varphi_0 + k_0 s_0 + \frac{1}{2}\dot{k}s_0^2\right) - (k_0 + \dot{k}s)^2.\sin\left(\varphi_0 + k_0 s_0 + \frac{1}{2}\dot{k}s_0^2\right) = \dot{k}C - \dot{\alpha}^2 S$$

$$\left(\frac{d^3f(s)}{ds^3}\right)_{s=s_0} = -3\dot{k}(k_0 + \dot{k}s).\sin\left(\varphi_0 + k_0 s_0 + \frac{1}{2}\dot{k}s_0^2\right) - (k_0 + \dot{k}s)^3.\cos\left(\varphi_0 + k_0 s_0 + \frac{1}{2}\dot{k}s_0^2\right) = -(\dot{\alpha}^3 C + 3\dot{k}\dot{\alpha}S)$$

$$\sin(\varphi(t)) = S + (t-s_0)\dot{\alpha}C + \frac{(t-s_0)^2}{2}(\dot{k}C - \dot{\alpha}^2 S) - \frac{(t-s_0)^3}{6}(\dot{\alpha}^3 C + 3\dot{k}\dot{\alpha}S)$$

$$\Rightarrow \sin(\varphi(t)) = S + (t-s_0)\dot{\alpha}C + \frac{(t^2 - 2s_0 t + s_0^2)}{2}(\dot{k}C - \dot{\alpha}^2 S) - \frac{(t^3 - 3s_0 t^2 + 3s_0^2 t - s_0^3)}{6}(\dot{\alpha}^3 C + 3\dot{k}\dot{\alpha}S)$$

$$\Rightarrow \sin(\varphi(t)) = S - s_0\dot{\alpha}C + \frac{s_0^2}{2}(\dot{k}C - \dot{\alpha}^2 S) + \frac{s_0^3}{6}(\dot{\alpha}^3 C + 3\dot{k}\dot{\alpha}S) + \left(C\dot{\alpha} - s_0(\dot{k}C - \dot{\alpha}^2 S) - \frac{s_0^2}{2}(\dot{\alpha}^3 C + 3\dot{k}\dot{\alpha}S)\right)t \ \ldots$$

$$+ \left(\frac{1}{2}(\dot{k}C - \dot{\alpha}^2 S) + \frac{s_0}{2}(\dot{\alpha}^3 C + 3\dot{k}\dot{\alpha}S)\right)t^2 - \left(\frac{1}{6}(\dot{\alpha}^3 C + 3\dot{k}\dot{\alpha}S)\right)t^3$$

let $s_0 \rightarrow \frac{L}{2}$

$$\mathbf{y(s)} = y_0 + \left(S - \left(\frac{L}{2}\right)C\dot{\alpha} - \left(\frac{L}{2}\right)^2\left(\frac{C\dot{\alpha}^2 - \dot{k}S}{2}\right) + \left(\frac{L}{2}\right)^3\left(\frac{C\dot{\alpha}^3}{6} + \frac{\dot{k}S\dot{\alpha}}{2}\right)\right)s$$

$$+ \left(\frac{C\dot{\alpha}}{2} + \left(\frac{L}{2}\right)\left(\frac{S\dot{\alpha}^2 - \dot{k}C}{2}\right) - \frac{3}{2}\left(\frac{L}{2}\right)^2\left(\frac{C\dot{\alpha}^3}{6} + \frac{\dot{k}S\dot{\alpha}}{2}\right)\right)s^2$$

$$+ \left(-\frac{1}{3}\left(\frac{S\dot{\alpha}^2 - \dot{k}C}{2}\right) + \left(\frac{L}{2}\right)\left(\frac{C\dot{\alpha}^3}{6} + \frac{\dot{k}S\dot{\alpha}}{2}\right)\right)s^3$$

$$- \left(\left(\frac{1}{4}\right)\left(\frac{C\dot{\alpha}^3}{6} + \frac{\dot{k}S\dot{\alpha}}{2}\right)\right)s^4$$

8.4 Line Prediction

- The Left and Right Line Track of the ego lane boundaries are predicted using constant curvature process model.
- Taylor series approximations are used for the prediction of initial lateral offset of the curve.
- The process model describes our belief of how a line Track evolves over time as the ego vehicle moves.
- Particle filter based density approximations shall be done in subsequent versions of the project

INPUTS : Previous State Estimates of ith Line Track $\quad \boldsymbol{i} \in \{\text{Left}, \text{Right}\}$

1) state vector from the previous cycle : $\widehat{\boldsymbol{X}}^{\boldsymbol{i}}_{\boldsymbol{t-1|t-1}}$

$$\widehat{\boldsymbol{X}}^{\boldsymbol{i}}_{\boldsymbol{t-1|t-1}} = \begin{pmatrix} \boldsymbol{y_0} \\ \boldsymbol{\alpha_0} \\ \boldsymbol{\kappa_0} \\ \boldsymbol{\dot{\kappa}} \end{pmatrix}$$

$y_0 \rightarrow$ lateral offset of Line Track i w. r. t the ego vehicle WBC
$\alpha_0 \rightarrow$ Initial heading angle of Line Track i w. r. t the ego vehicle WBC
$\kappa_0 \rightarrow$ Initial curvature of Line Track i w. r. t the ego vehicle WBC
$\dot{\kappa} \rightarrow$ curvature rate of the Line Track i

2) error covariance of the previous state estimate : $\widehat{\boldsymbol{P}}^{\boldsymbol{i}}_{\boldsymbol{t-1|t-1}}$

INPUTS : Process Model Parameters

3) Small angle approximated State Transition Model : $\boldsymbol{A_t}$

$$\Delta S = \left(\sqrt{v_x^2 + v_y^2}\right) dT$$

$$\boldsymbol{A_t} = \begin{pmatrix} 1 & \Delta S & \frac{1}{2}\Delta S^2 & \frac{1}{6}\Delta S^3 \\ 0 & 1 & \Delta S & \frac{1}{2}\Delta S^2 \\ 0 & 0 & 1 & \Delta S \\ 0 & 0 & 0 & 1 \end{pmatrix}_t$$

$\Delta S \rightarrow$ distance travelled by the ego vehicle during dT
$dT \rightarrow$ sample time
$v_x \rightarrow$ longitudinal ego vehicle velocity w.r.t body frame
$v_y \rightarrow$ lateral ego vehicle velocity w.r.t body frame

4) Process Noise Covariance : $\boldsymbol{Q_t}$

if $\Delta S \geq \varepsilon$

$$Q_t = \begin{pmatrix} \boldsymbol{\sigma^2_{y0}} & 0 & 0 & 0 \\ 0 & \boldsymbol{\sigma^2_{\alpha 0}} & 0 & 0 \\ 0 & 0 & \boldsymbol{\sigma^2_{\kappa 0}} & 0 \\ 0 & 0 & 0 & \boldsymbol{\sigma^2_{\dot{\kappa}}} \end{pmatrix}_t$$

$\boldsymbol{\sigma^2_{y0}} \rightarrow$ error (variance) in lateral offset prediction
$\boldsymbol{\sigma^2_{\alpha 0}} \rightarrow$ error (variance) in initial heading angle prediction
$\boldsymbol{\sigma^2_{\kappa 0}} \rightarrow$ error (variance) in initial curvature prediction
$\boldsymbol{\sigma^2_{\dot{\kappa}}} \rightarrow$ error (variance) in curvature rate prediction

if $\Delta S < \varepsilon$

$$Q_t = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

OUTPUTS : Predicted State Estimates of ith Line Track $\boldsymbol{i} \in \{\text{Left, Right}\}$

1) **Predicted state vector** : $\widehat{\boldsymbol{X}}^{\boldsymbol{i}}_{\boldsymbol{t|t-1}}$

$$\widehat{\boldsymbol{X}}^{\boldsymbol{i}}_{\boldsymbol{t|t-1}} = \begin{pmatrix} \boldsymbol{y_0} \\ \boldsymbol{\alpha_0} \\ \boldsymbol{\kappa_0} \\ \boldsymbol{\dot{\kappa}} \end{pmatrix}$$

$y_0 \rightarrow$ predicted lateral offset of Line Track i
$\alpha_0 \rightarrow$ predicted Initial heading angle of Line Track i
$\kappa_0 \rightarrow$ predicted Initial curvature of Line Track i
$\dot{\kappa} \rightarrow$ predicted curvature rate of the Line Track i

2) **error covariance** of the predicted state estimate : $\widehat{\boldsymbol{P}}^{\boldsymbol{i}}_{\boldsymbol{t|t-1}}$

EQUATIONS :

1) **State Prediction**

$$\boldsymbol{y_{t|t-1}} = \mathbf{f_{taylor}}\left(\boldsymbol{y_{t-1|t-1}}, \boldsymbol{\alpha_{t-1|t-1}}, \boldsymbol{\kappa_{t-1|t-1}}, \boldsymbol{\dot{\kappa}_{t-1|t-1}}, \boldsymbol{\Delta S}\right)$$

$$\boldsymbol{\alpha_{t|t-1}} = \boldsymbol{\alpha_{t-1|t-1}} + \boldsymbol{\kappa_{t-1|t-1}}\boldsymbol{\Delta S} + \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{\dot{\kappa}_{t-1|t-1}}\boldsymbol{\Delta S^2}$$

$$\boldsymbol{\kappa_{t|t-1}} = \boldsymbol{\kappa_{t-1|t-1}} + \boldsymbol{\dot{\kappa}_{t-1|t-1}}\boldsymbol{\Delta S}$$

$$\boldsymbol{\dot{\kappa}_{t|t-1}} = \boldsymbol{\dot{\kappa}_{t-1|t-1}}$$

$$\boldsymbol{l_{t|t-1}} = \boldsymbol{l_{t-1|t-1}} - \boldsymbol{\Delta S}$$

$\left(y_{t-1|t-1}, \alpha_{t-1|t-1}, \kappa_{t-1|t-1}, \dot{\kappa}_{t-1|t-1}\right) \rightarrow$ previous clothoid coefficients at time $t-1$

$\left(y_{t|t-1}, \alpha_{t|t-1}, \kappa_{t|t-1}, \dot{\kappa}_{t|t-1}\right) \rightarrow$ predicted clothoid coefficients at time t

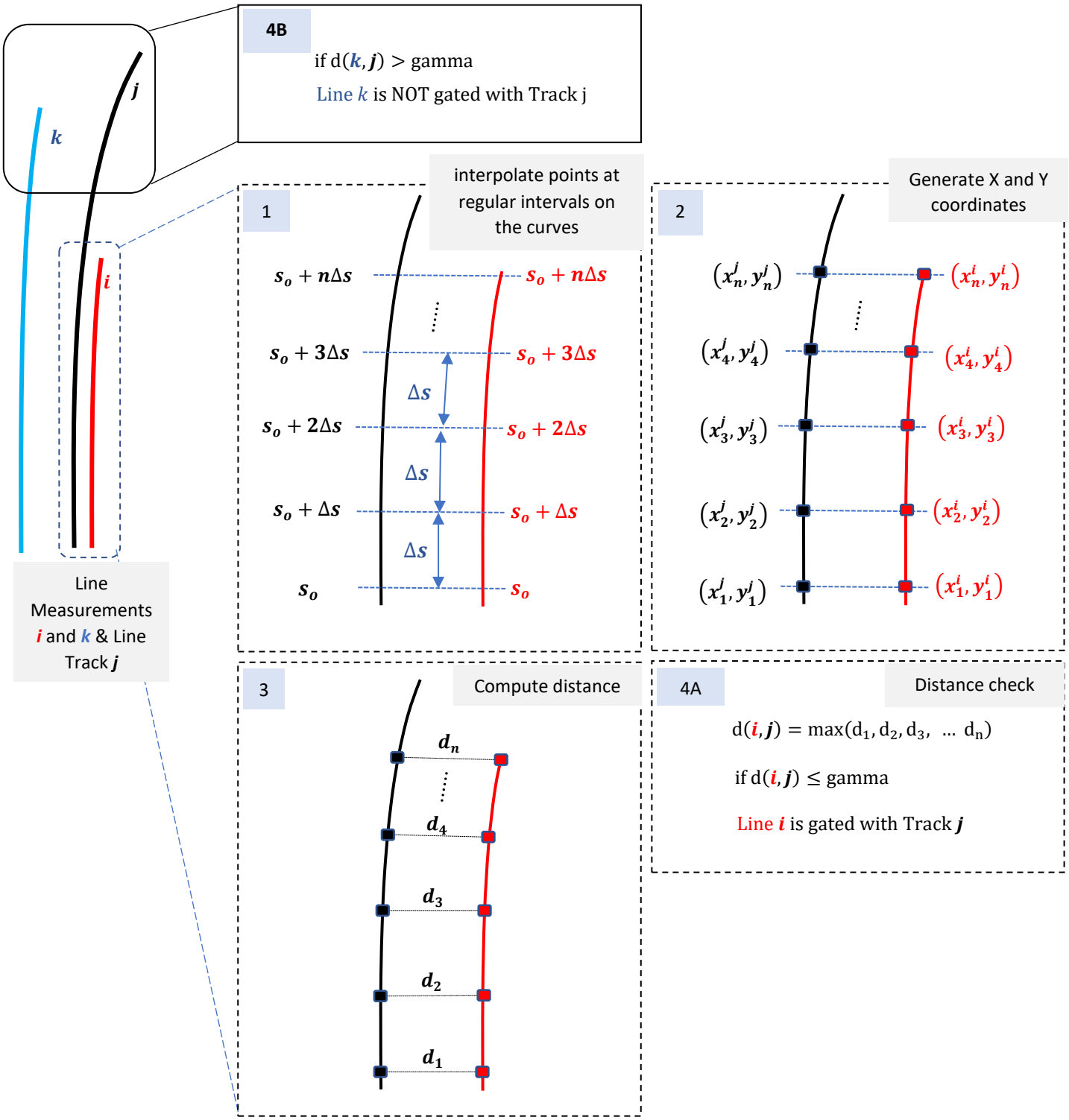
2) **Error Covariance Projection** predicted error covariance is approximated as gaussian by considering the small angle approximated state transition matrix

$$\widehat{\boldsymbol{P}}^{\boldsymbol{i}}_{\boldsymbol{t|t-1}} = \boldsymbol{A_t}\widehat{\boldsymbol{P}}^{\boldsymbol{i}}_{\boldsymbol{t-1|t-1}}\boldsymbol{A_t^T} + \boldsymbol{Q_t}$$

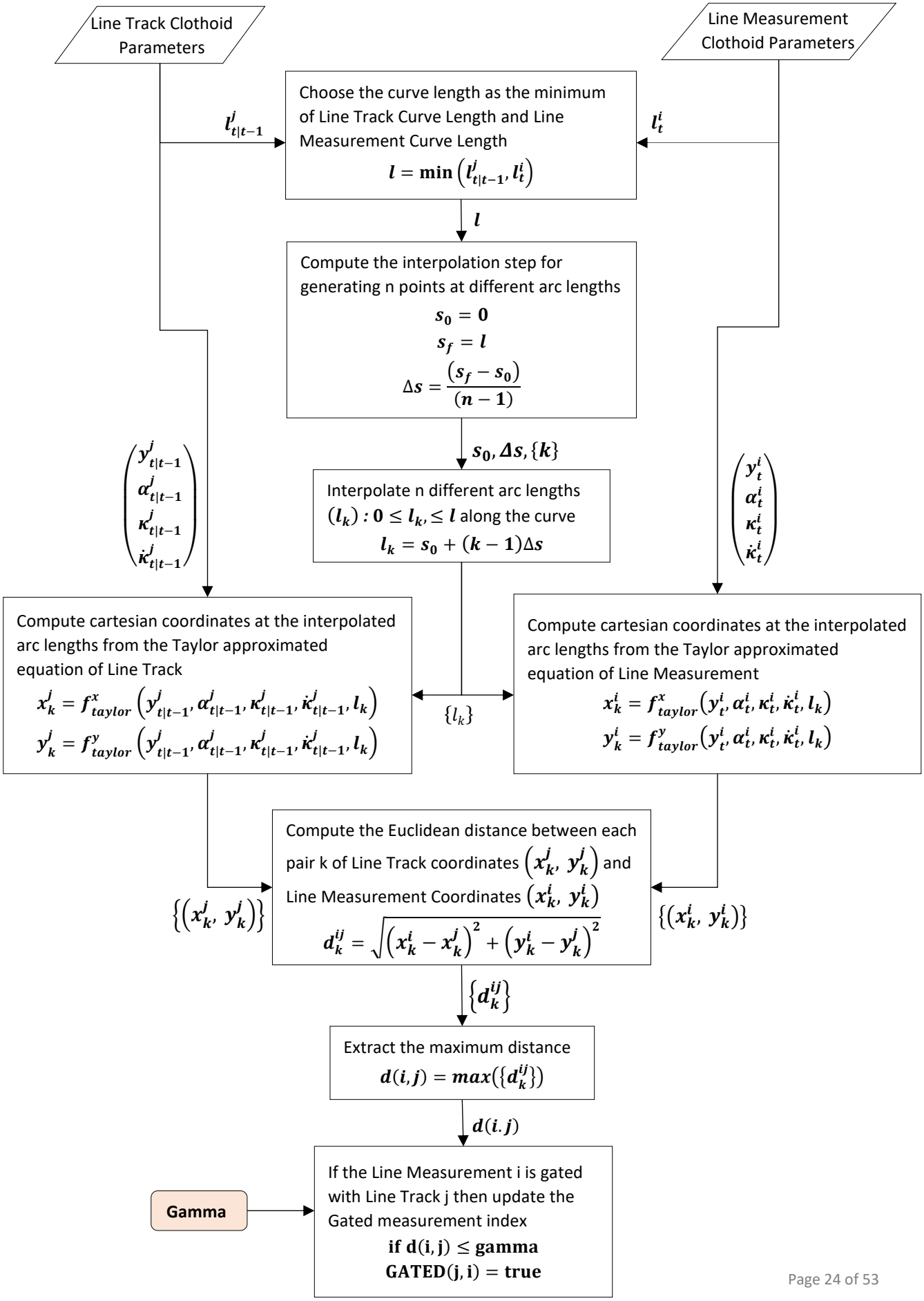
$$A_t \rightarrow \begin{pmatrix} 1 & \Delta S & \frac{1}{2}\Delta S^2 & \frac{1}{6}\Delta S^3 \\ 0 & 1 & \Delta S & \frac{1}{2}\Delta S^2 \\ 0 & 0 & 1 & \Delta S \\ 0 & 0 & 0 & 1 \end{pmatrix}_t$$

8.5 Line Measurement to Line Track Gating

8.5.1 High Level Design

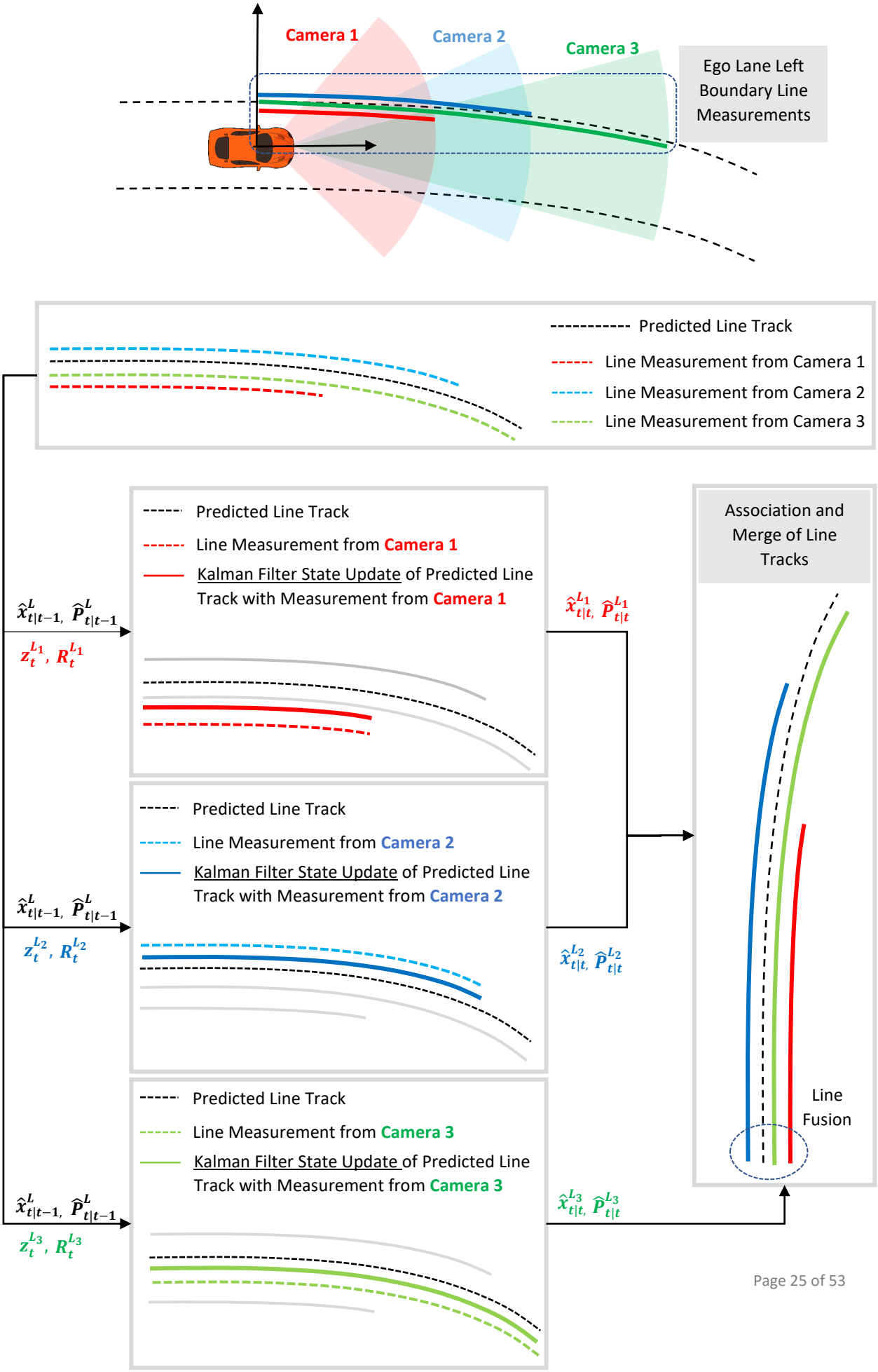


8.5.2 Detailed Design

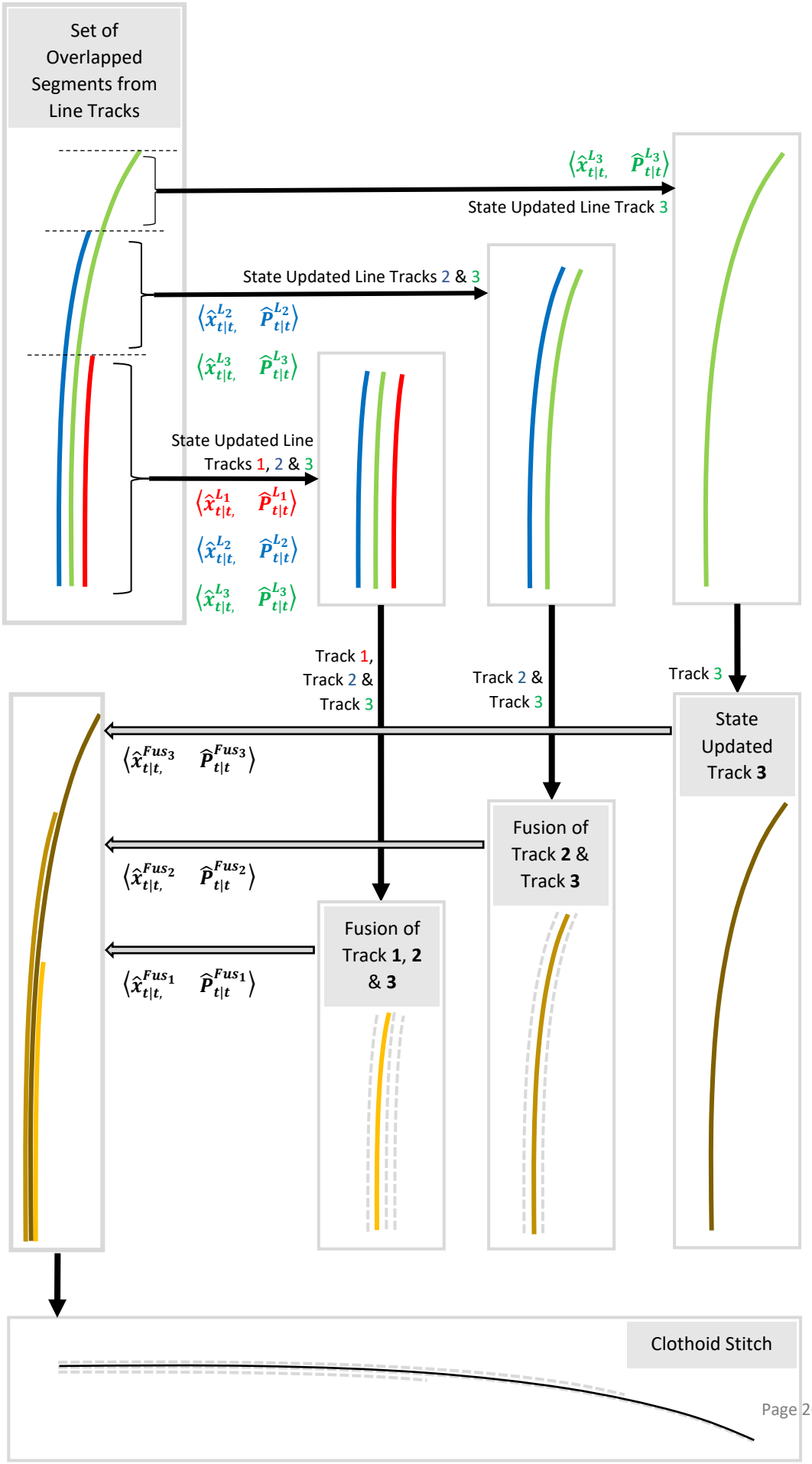


8.6 Line Measurements to Line Track Fusion

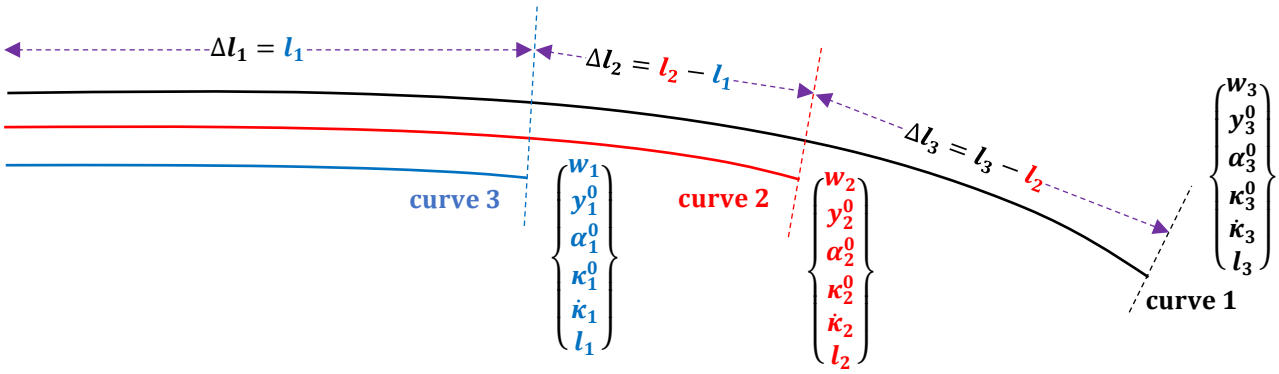
8.6.1 High Level Design - Step 1 : Kalman Filter State Updates with Gated Line Measurements



8.6.2 High Level Design - Step 2 : Line Track fusion by PDAF



8.6.3 High Level Design - Step 3 : Clothoid Stitch Concept



$$\left.\begin{aligned} y^0_{\text{stitch}} &= w_1 y_1^0 + w_2 y_2^0 + w_3 y_3^0 \\ \alpha^0_{\text{stitch}} &= w_1 \alpha_1^0 + w_2 \alpha_2^0 + w_3 \alpha_3^0 \\ \kappa^0_{\text{stitch}} &= w_1 \kappa_1^0 + w_2 \kappa_2^0 + w_3 \kappa_3^0 \end{aligned}\right\}$$

weighted average of the initial clothoid parameters $\{y^0_{\text{stitch}}, \alpha^0_{\text{stitch}}, \kappa^0_{\text{stitch}}\}$

weighted average of the curvature rates at different overlapped segments $\{\Delta l_1, \Delta l_2, \Delta l_3\}$

$$\dot{\kappa}_{\Delta l_i} = \begin{cases} w_1\dot{\kappa}_1 + w_2\dot{\kappa}_2 + w_3\dot{\kappa}_3 & i = 1 \quad \text{curvature rate for } 0 \le l \le \Delta l_1 \\ \left(\frac{w_2}{w_2 + w_3}\right)\dot{\kappa}_2 + \left(\frac{w_3}{w_2 + w_3}\right)\dot{\kappa}_3 & i = 2 \quad \text{curvature rate for } \Delta l_1 < l \le \Delta l_2 \\ \dot{\kappa}_3 & i = 3 \quad \text{curvature rate for } \Delta l_2 < l \le \Delta l_3 \end{cases}$$

INPUTS

let $i = \{1,2,3\}$

$w_i \rightarrow$ normalized weitage for curve i

such that, $\sum_{i=1}^{3} w_i = 1$

$y_i^0 \rightarrow$ initial lateral offset of curve i

$\alpha_i^0 \rightarrow$ initial heading angle of curve i

$\kappa_i^0 \rightarrow$ initial curature of curve i

$\dot{\kappa}_i \rightarrow$ curvature rate of curve i

$l_i \rightarrow$ length of curve i

Initial curvature : κ^0_{stitch}

Curvature at Δl_1: $\kappa^0_{\text{stitch}} + \dot{\kappa}_{\Delta l_1}\Delta l_1$

Curvature at $(\Delta l_1 + \Delta l_2)$: $\kappa^0_{\text{stitch}} + \dot{\kappa}_{\Delta l_1}\Delta l_1 + \dot{\kappa}_{\Delta l_2}\Delta l_2$

Curvature at $(\Delta l_1 + \Delta l_2 + \Delta l_3)$: $\kappa^0_{\text{stitch}} + \dot{\kappa}_{\Delta l_1}\Delta l_1 + \dot{\kappa}_{\Delta l_2}\Delta l_2 + \dot{\kappa}_{\Delta l_3}\Delta l_3$

$$\text{Curvature Rate} = \frac{\text{Curvature}_{\text{Final}} - \text{Curvature}_{\text{Initial}}}{\text{Curve Length}}$$

$$\Rightarrow \text{Curvature Rate} = \frac{\kappa^0_{\text{stitch}} + \dot{\kappa}_{\Delta l_1}\Delta l_1 + \dot{\kappa}_{\Delta l_2}\Delta l_2 + \dot{\kappa}_{\Delta l_3}\Delta l_3 - \kappa^0_{\text{stitch}}}{\Delta l_1 + \Delta l_2 + \Delta l_3}$$

$$\Rightarrow \text{Curvature Rate} = \left(\frac{\Delta l_1}{\Delta l_1 + \Delta l_2 + \Delta l_3}\right)\dot{\kappa}_{\Delta l_1} + \left(\frac{\Delta l_2}{\Delta l_1 + \Delta l_2 + \Delta l_3}\right)\dot{\kappa}_{\Delta l_2} + \left(\frac{\Delta l_3}{\Delta l_1 + \Delta l_2 + \Delta l_3}\right)\dot{\kappa}_{\Delta l_3}$$

Since, $\Delta l_1 + \Delta l_2 + \Delta l_3 = l_{\text{stitch}}$

The formulation for the curvature rate is

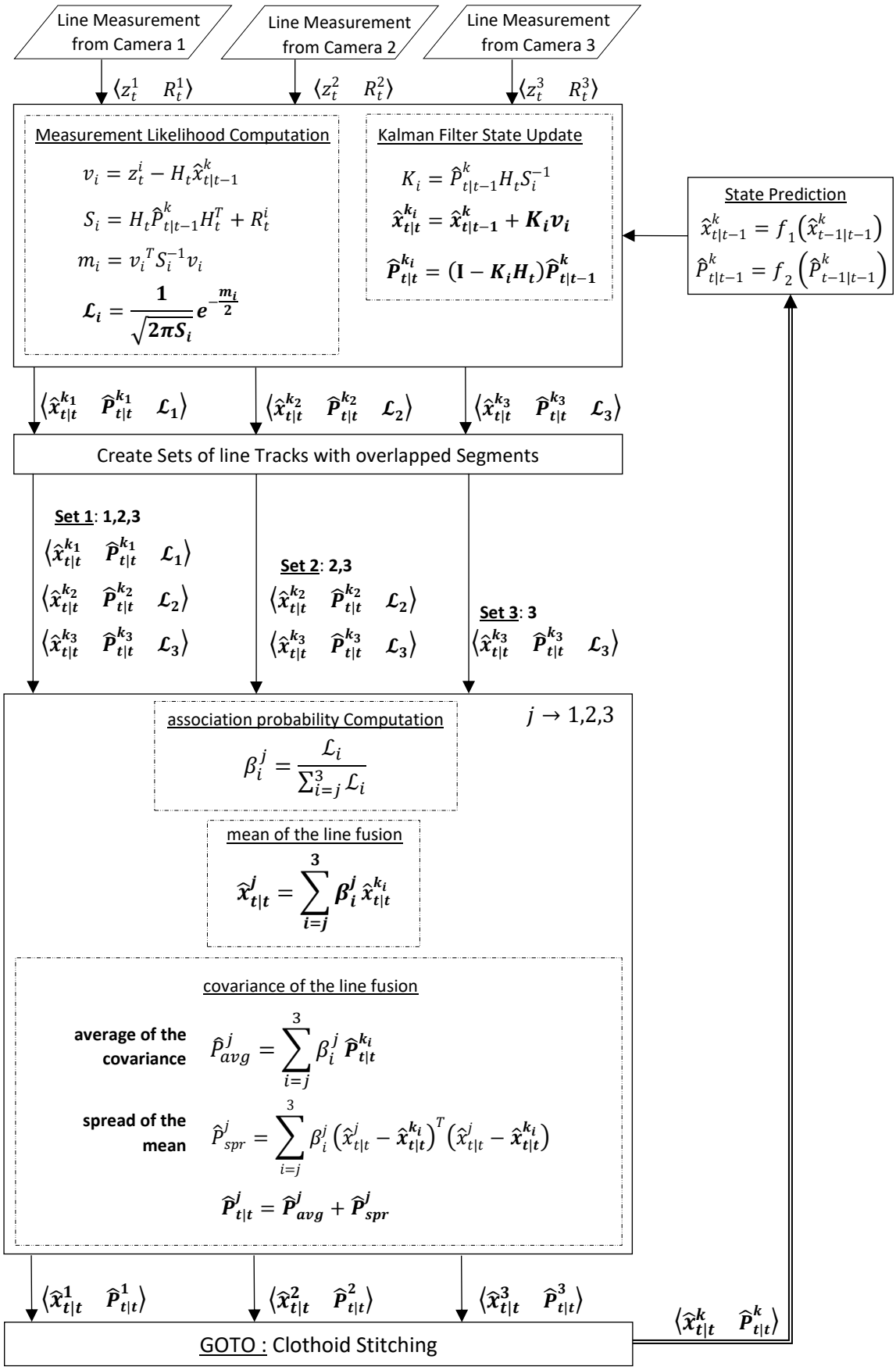
$$\dot{\kappa}_{\text{stitch}} = \left(\frac{l_1}{l_{\text{stitch}}}\right)(w_1\dot{\kappa}_1 + w_2\dot{\kappa}_2 + w_3\dot{\kappa}_3) + \left(\frac{l_2 - l_1}{l_{stitch}}\right)\left(\left(\frac{w_2}{w_2 + w_3}\right)\dot{\kappa}_2 + \left(\frac{w_3}{w_2 + w_3}\right)\dot{\kappa}_3\right) + \left(\frac{l_3 - l_2}{l_{stitch}}\right)\dot{\kappa}_3$$

OUTPUT

$$\begin{Bmatrix} y^0_{\text{stitch}} \\ \alpha^0_{\text{stitch}} \\ \kappa^0_{\text{stitch}} \\ \dot{\kappa}_{\text{stitch}} \\ l_{\text{stitch}} \end{Bmatrix}$$

8.7 Detailed Design

8.7.1 Part 1 : Generation Of Multiple Line Hypothesis and Hypothesis merge



8.7.2 Part 2 : Clothoid Stitch

$$\hat{\boldsymbol{x}}^1_{t|t} = \begin{pmatrix} \boldsymbol{y}^0_1 \\ \boldsymbol{\alpha}^0_1 \\ \boldsymbol{\kappa}^0_1 \\ \dot{\boldsymbol{\kappa}}_1 \\ \boldsymbol{l}_1 \end{pmatrix} \quad \hat{\boldsymbol{P}}^1_{t|t} \qquad \hat{\boldsymbol{x}}^2_{t|t} = \begin{pmatrix} \boldsymbol{y}^0_2 \\ \boldsymbol{\alpha}^0_2 \\ \boldsymbol{\kappa}^0_2 \\ \dot{\boldsymbol{\kappa}}_2 \\ \boldsymbol{l}_2 \end{pmatrix} \quad \hat{\boldsymbol{P}}^2_{t|t} \qquad \hat{\boldsymbol{x}}^3_{t|t} = \begin{pmatrix} \boldsymbol{y}^0_3 \\ \boldsymbol{\alpha}^0_3 \\ \boldsymbol{\kappa}^0_3 \\ \dot{\boldsymbol{\kappa}}_3 \\ \boldsymbol{l}_3 \end{pmatrix} \quad \hat{\boldsymbol{P}}^3_{t|t} \qquad \hat{\boldsymbol{x}}^n_{t|t} = \begin{pmatrix} \boldsymbol{y}^0_n \\ \boldsymbol{\alpha}^0_n \\ \boldsymbol{\kappa}^0_n \\ \dot{\boldsymbol{\kappa}}_n \\ \boldsymbol{l}_n \end{pmatrix} \quad \hat{\boldsymbol{P}}^n_{t|t}$$

Arrange $\{\hat{\boldsymbol{x}}^1_{t|t} \quad \hat{\boldsymbol{x}}^2_{t|t} \quad \hat{\boldsymbol{x}}^3_{t|t} \quad \dots \quad \hat{\boldsymbol{x}}^n_{t|t}\}$ **in the increasing order of** $\{\boldsymbol{l}_1 \quad \boldsymbol{l}_2 \quad \boldsymbol{l}_3 \quad \dots \quad \boldsymbol{l}_n\}$

let $l = \{l_1 \quad l_2 \quad l_3 \quad \dots \quad l_n\}$ $\qquad X = \{\hat{x}^1_{t|t} \quad \hat{x}^2_{t|t} \quad \hat{x}^3_{t|t} \quad \dots \quad \hat{x}^n_{t|t}\}$

$\text{Sort}_{\text{ascending}}(l) \rightarrow l'$ $\qquad$ where $\boldsymbol{l}' = \{\boldsymbol{l}'_1 \quad \boldsymbol{l}'_2 \quad \boldsymbol{l}'_3 \quad \dots \quad \boldsymbol{l}'_n\}$

$\text{Rearrange}(X, l) \rightarrow X'$ $\qquad$ where $\boldsymbol{X}' = \{\hat{\boldsymbol{x}}^{l'_1}_{t|t} \quad \hat{\boldsymbol{x}}^{l'_2}_{t|t} \quad \hat{\boldsymbol{x}}^{l'_3}_{t|t} \quad \dots \quad \hat{\boldsymbol{x}}^{l'_n}_{t|t}\}$

$$\hat{\boldsymbol{x}}^{l'_i}_{t|t} = \begin{pmatrix} \boldsymbol{y}^0_{l'_i} \\ \boldsymbol{\alpha}^0_{l'_i} \\ \boldsymbol{\kappa}^0_{l'_i} \\ \dot{\kappa}'_i \\ \boldsymbol{l}'_i \end{pmatrix}$$

Compute Average initial lateral offset

$$y^0_{stitch} = \frac{1}{n}\sum_{i=1}^{n} \boldsymbol{y}^0_{l'_i}$$

$\boldsymbol{y}^0_{stitch}$ $\qquad \{\boldsymbol{y}^0_{l'_i}\}$

Compute Average initial heading

$$\alpha^0_{stitch} = \frac{1}{n}\sum_{i=1}^{n} \boldsymbol{\alpha}^0_{l'_i}$$

$\boldsymbol{\alpha}^0_{stitch}$ $\qquad \{\boldsymbol{\alpha}^0_{l'_i}\}$

Compute Average initial curvature

$$\kappa^0_{stitch} = \frac{1}{n}\sum_{i=1}^{n} \boldsymbol{\kappa}^0_{l'_i}$$

$\boldsymbol{\kappa}^0_{stitch}$ $\qquad \{\boldsymbol{\kappa}^0_{l'_i}\}$

Compute average Curvature Rates at different overlapped Fused Curve Segments

$$\dot{\kappa}'_1 = \frac{1}{n}\sum_{i=1}^{n} \dot{\kappa}_{l'_i}$$

$$\dot{\kappa}'_2 = \frac{1}{n-1}\sum_{i=2}^{n} \dot{\boldsymbol{\kappa}}_{l'_i}$$

$$\dot{\kappa}'_3 = \frac{1}{n-2}\sum_{i=3}^{n} \dot{\boldsymbol{\kappa}}_{l'_i}$$

$$\vdots$$

$$\dot{\kappa}'_k = \frac{1}{n-k+1}\sum_{i=k}^{n} \dot{\boldsymbol{\kappa}}_{l'_i}$$

$$\vdots$$

$$\dot{\kappa}'_n = \dot{\boldsymbol{\kappa}}_{l'_n}$$

$\{\dot{\boldsymbol{\kappa}}_{l'_i}\}$

Compute the curvature rate for the stitched segments

$$l = \sum_{i=1}^{n} \boldsymbol{\Delta l}'_i$$

$$\dot{\kappa}_{stitch} = \sum_{i=1}^{n} \frac{\boldsymbol{\Delta l}'_i}{l} \dot{\kappa}'_i$$

$\dot{\boldsymbol{\kappa}}_{stitch}$ $\qquad \{\dot{\boldsymbol{\kappa}}'_i\}$

$\boldsymbol{l}$ $\qquad \{\boldsymbol{\Delta l}'_i\}$

Compute Lengths of Overlapped segments

$$\boldsymbol{\Delta l}'_1 = \boldsymbol{l}'_1$$

$$\boldsymbol{\Delta l}'_2 = \boldsymbol{l}'_2 - \boldsymbol{l}'_1$$

$$\boldsymbol{\Delta l}'_3 = \boldsymbol{l}'_3 - \boldsymbol{l}'_2$$

$$\vdots$$

$$\boldsymbol{\Delta l}'_n = \boldsymbol{l}'_n - \boldsymbol{l}'_{n-1}$$

$$\boldsymbol{\Delta l}' = \{\boldsymbol{\Delta l}'_1 \quad \boldsymbol{\Delta l}'_2 \quad \boldsymbol{\Delta l}'_3 \quad \dots \quad \boldsymbol{\Delta l}'_n\}$$

$\{\boldsymbol{l}'_i\}$

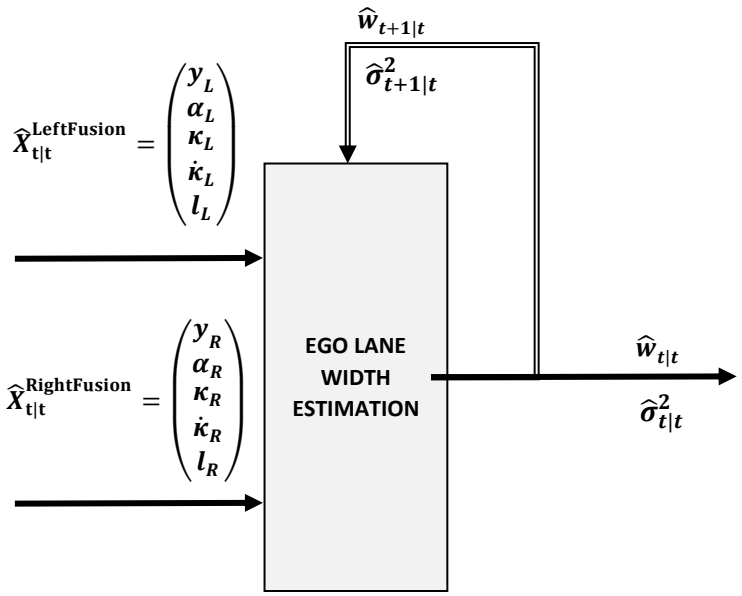
$$\hat{\boldsymbol{x}}^k_{t|t} = \left(\boldsymbol{y}^0_{stitch} \quad \boldsymbol{\alpha}^0_{stitch} \quad \boldsymbol{\kappa}^0_{stitch} \quad \dot{\boldsymbol{\kappa}}_{stitch} \quad \boldsymbol{l}\right)^T$$

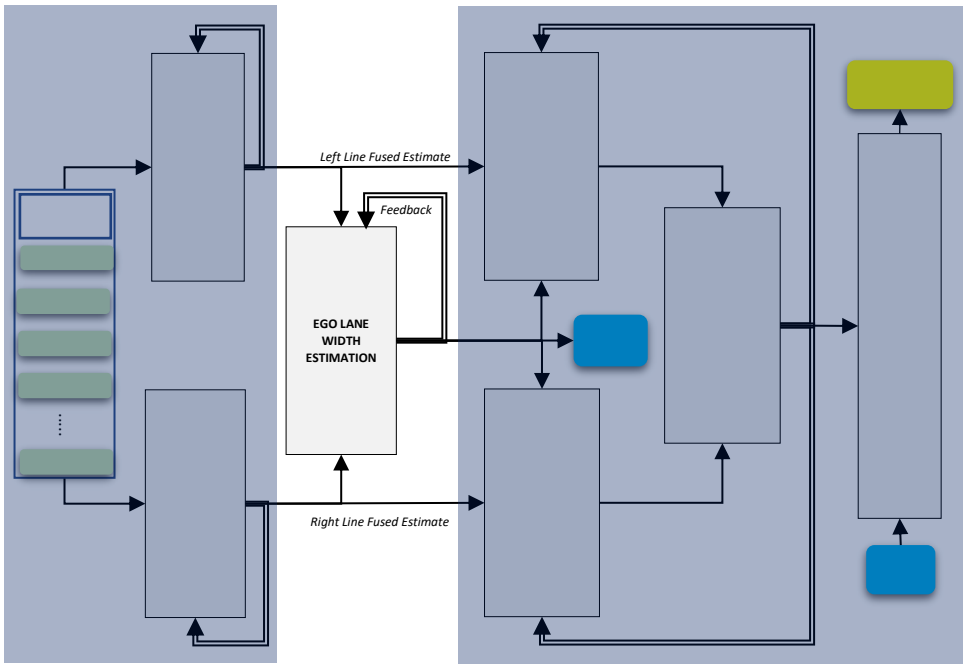
$$\hat{\boldsymbol{P}}^k_{t|t} = \sum_{i=1}^{n} \hat{\boldsymbol{P}}^i_{t|t}$$

$\{\hat{\boldsymbol{P}}^i_{t|t}\}$

9. EGO LANE WIDTH ESTIMATION

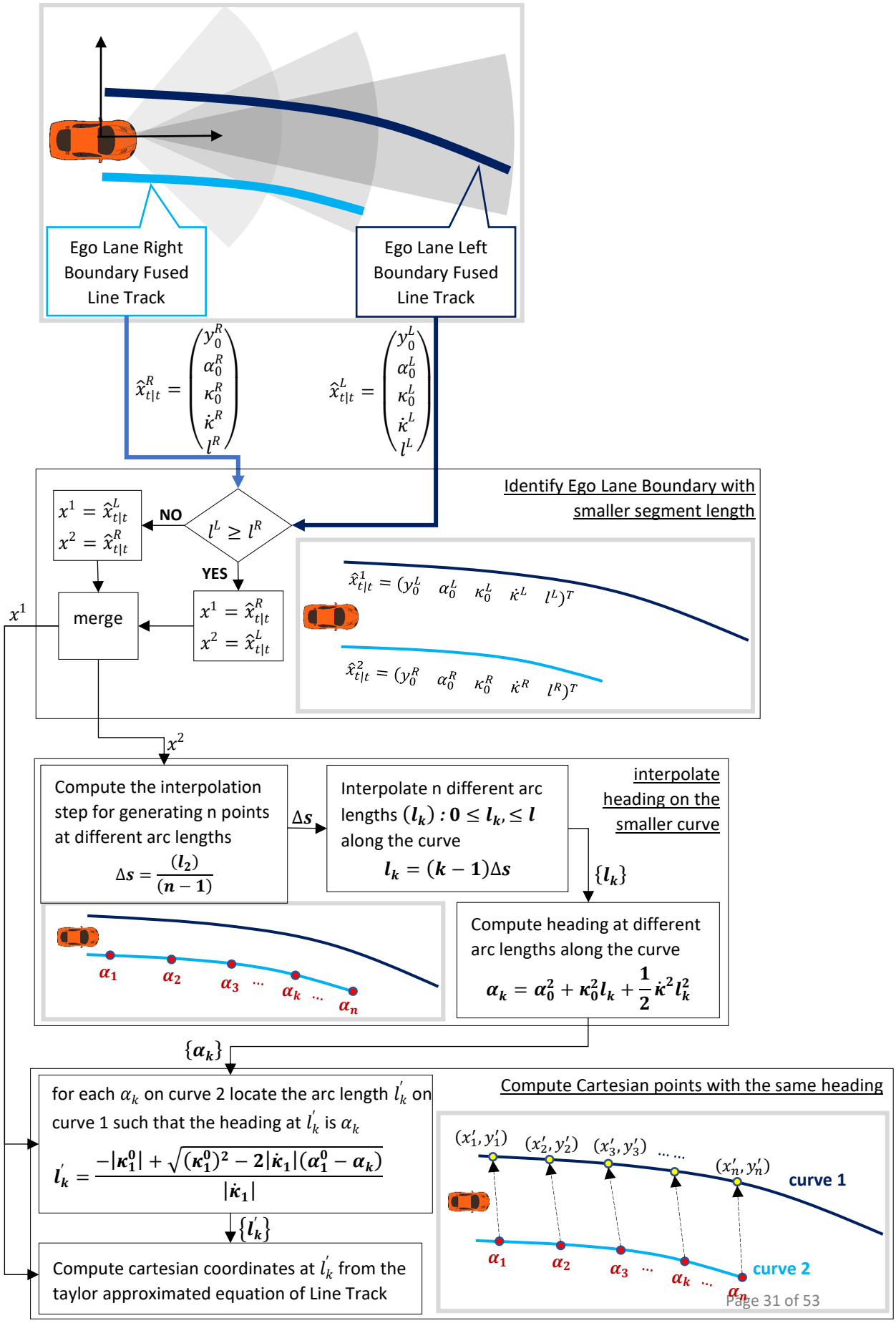
9.1 Data Flow/Block Diagram



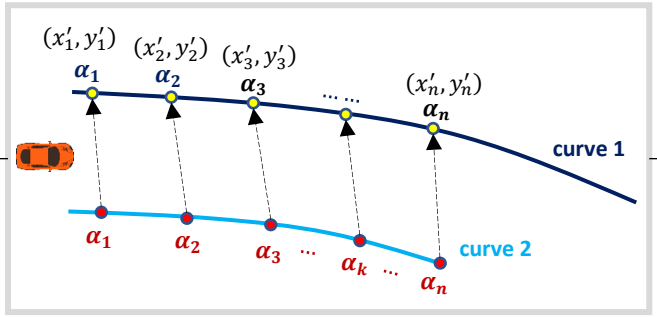


9.2 Detailed Design

9.2.1 Step 1 : Identification of Point Correspondences on Lane Boundary Lines



9.2.2 Step 2 : Width Computation For Each Pair of Point Correspondences



$\{\alpha_k\}$ $\{(x_k', y_k')\}$

$$m_k' = \tan(\alpha_k)$$
$$c_k' = y_k' - m_k' x_k'$$

Equation 1 (tangent at the point (x_k', y_k') on curve 1)

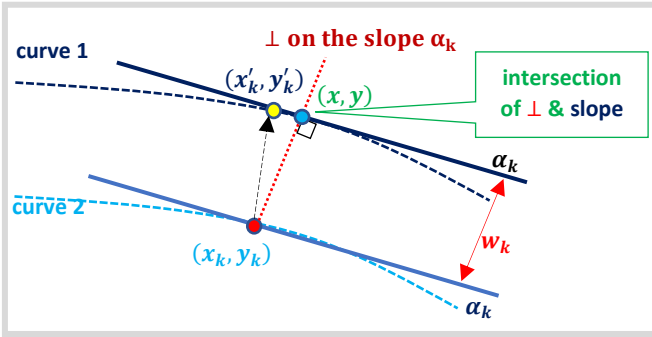
$$m_k^{\perp} = -\frac{1}{\tan(\alpha_k)}$$
$$c_k^{\perp} = y_k - m_k^{\perp} y_k$$

Equation 2 (perpendicular of the tangent at the point (x_k, y_k) on curve 2)

$$x = \frac{c_k' - c_k^{\perp}}{m_k^{\perp} - m_k'}$$
$$y = m_k' x + c_k'$$

Point of intersection of equation 1 and equation 2

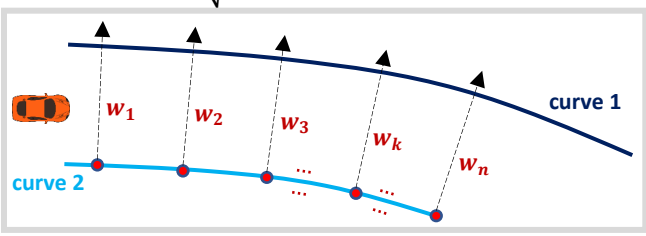
Distance Correction



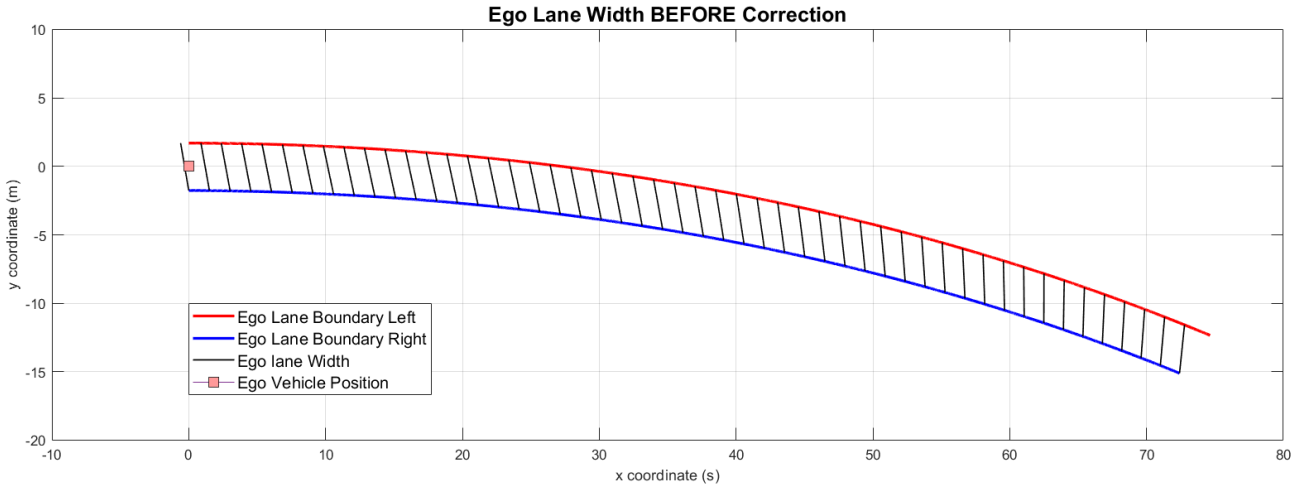
$\{(x, y)\}$

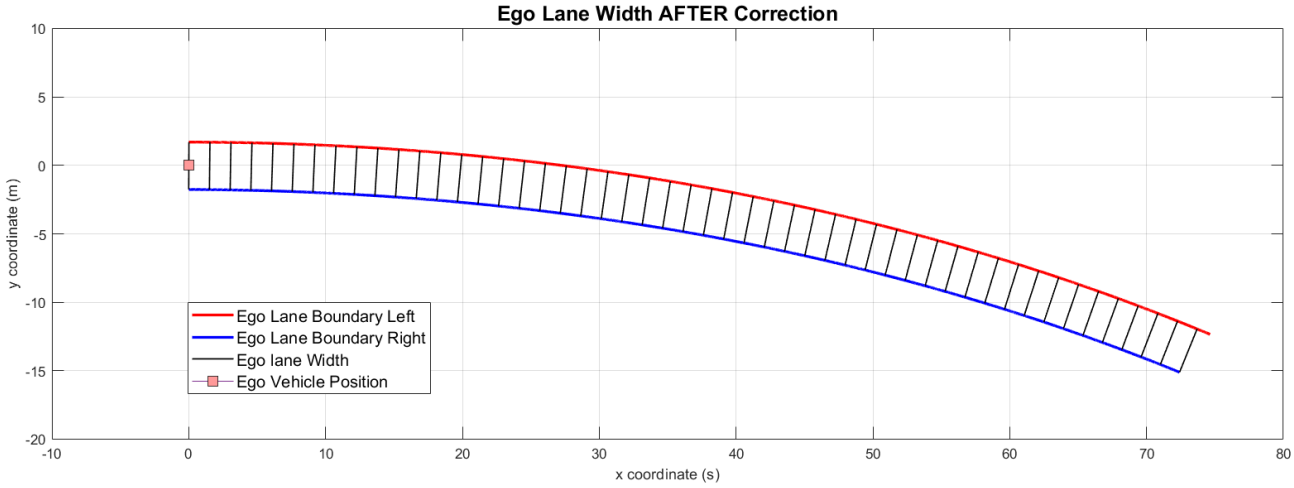
Compute width at k

$$w_k = \sqrt{(x - x_k)^2 + (y - y_k)^2}$$



9.2.3 Plot : Ego Lane Width Before & After Correction



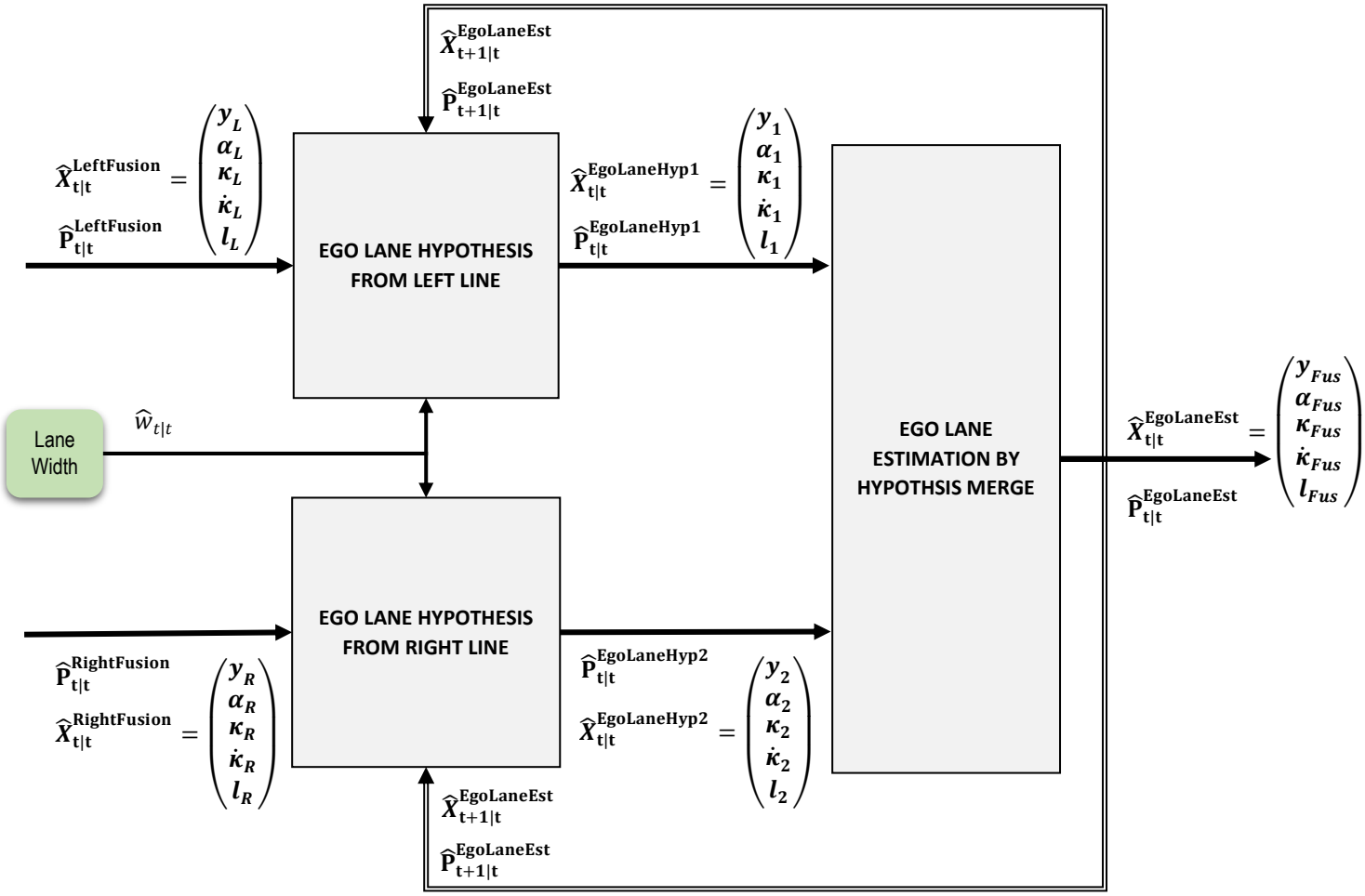


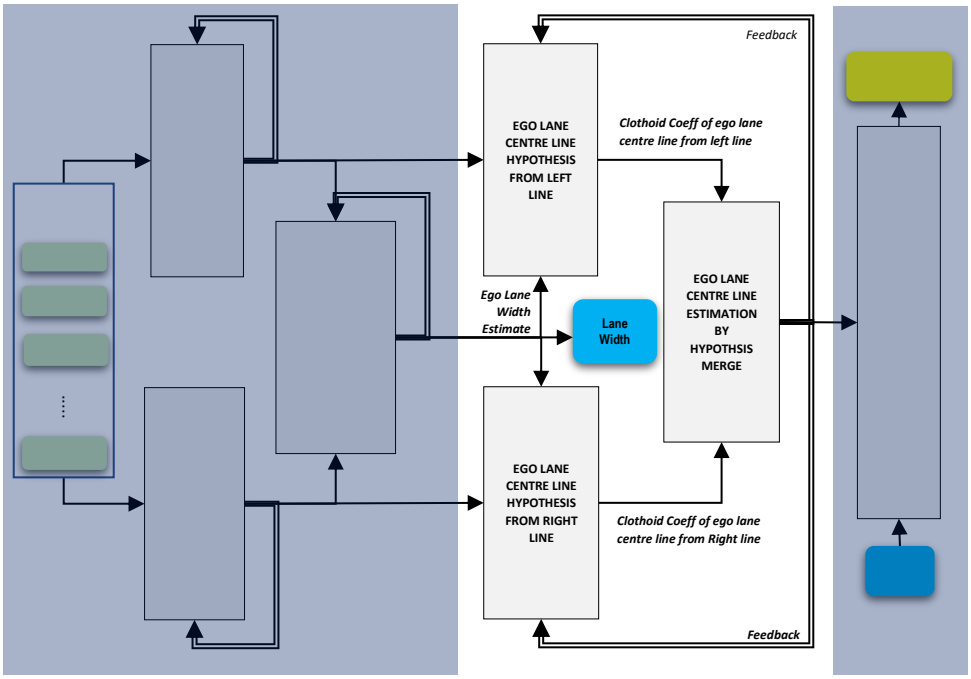
Note :

- Design and implementation of Filtering Recursion for lane width estimation is in progress and shall be released in the next version

10. EGO LANE CENTER LINE HYPOTHESIS

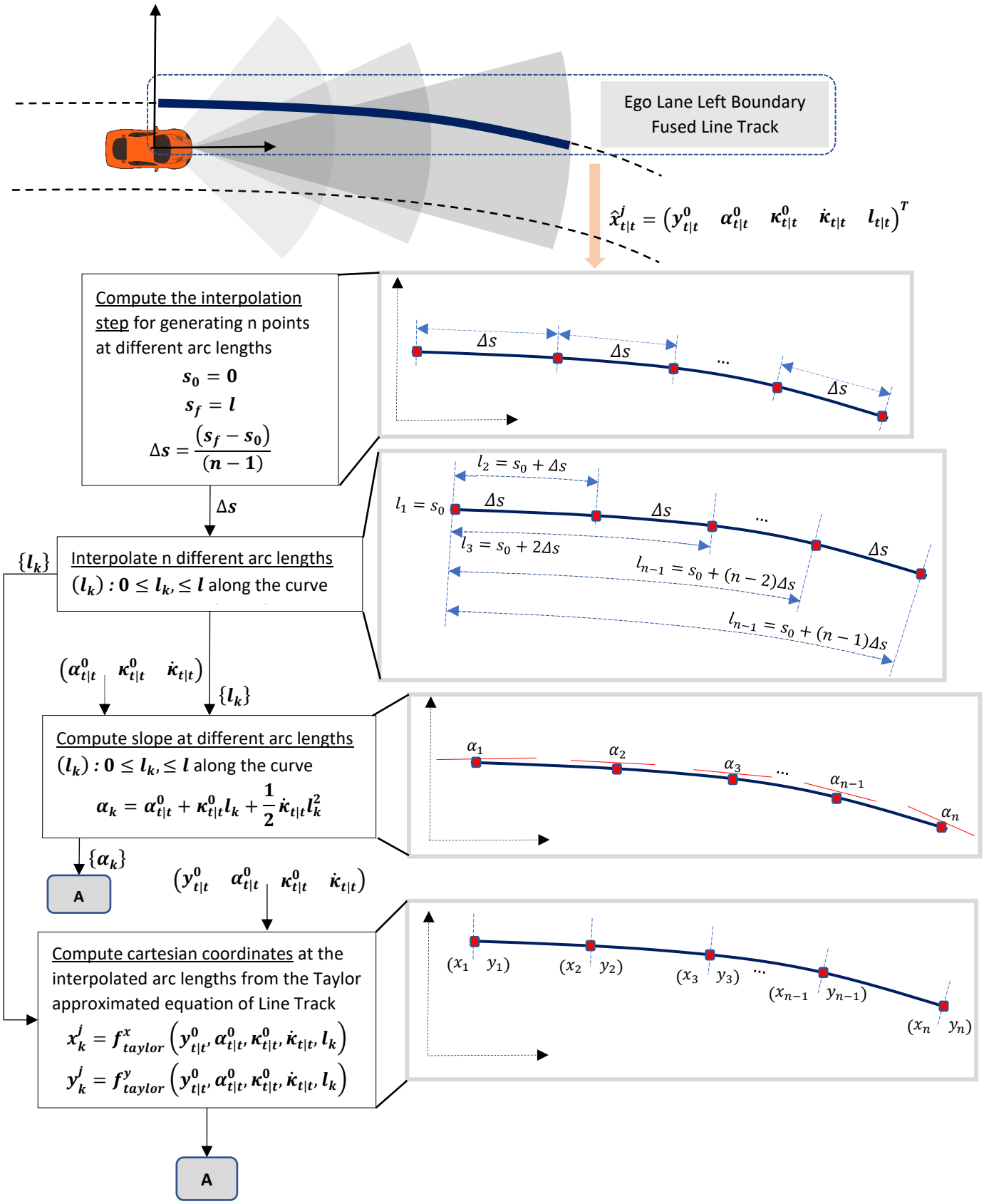
10.1 Data Flow/Block Diagram



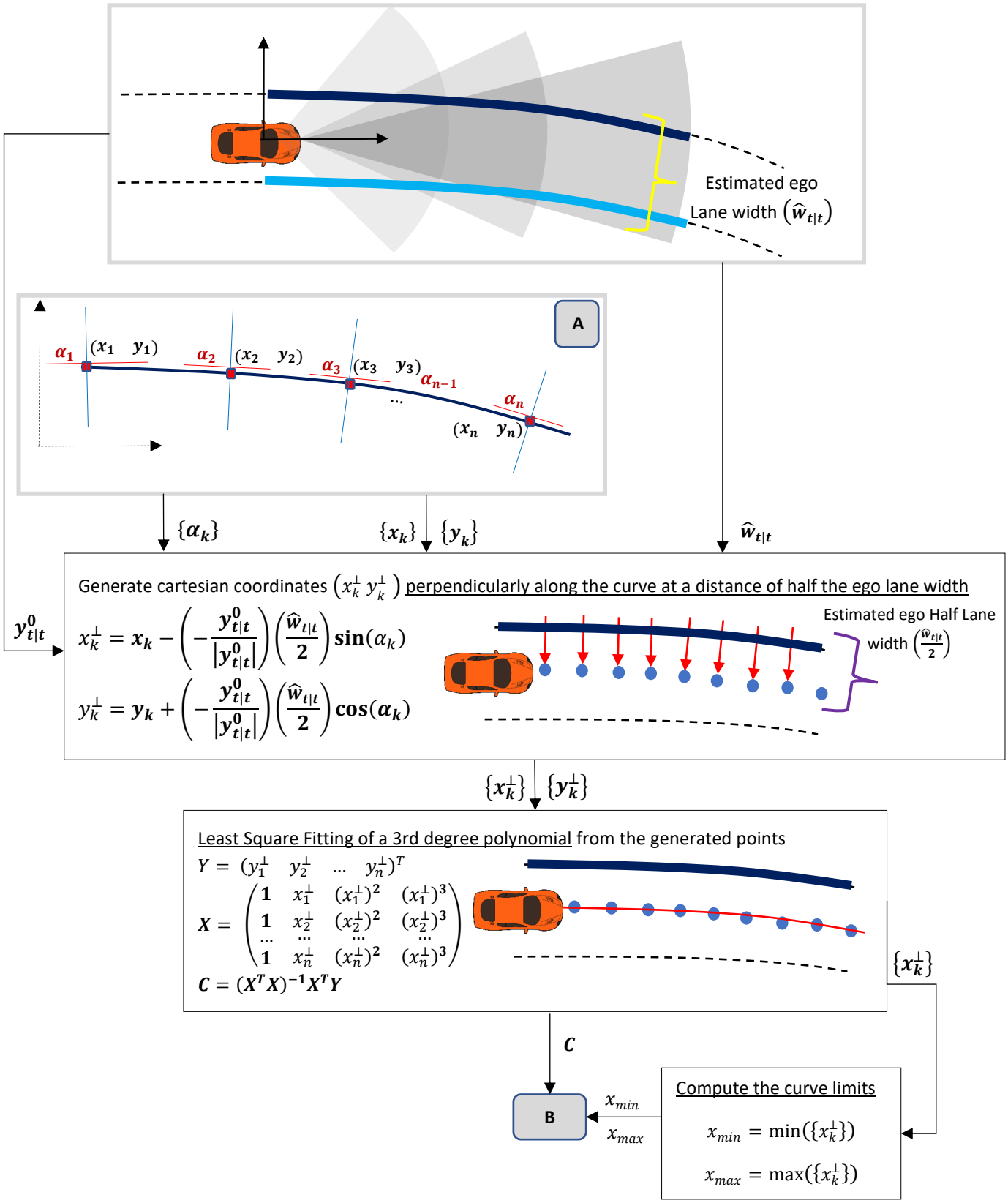


10.2 Detailed Design

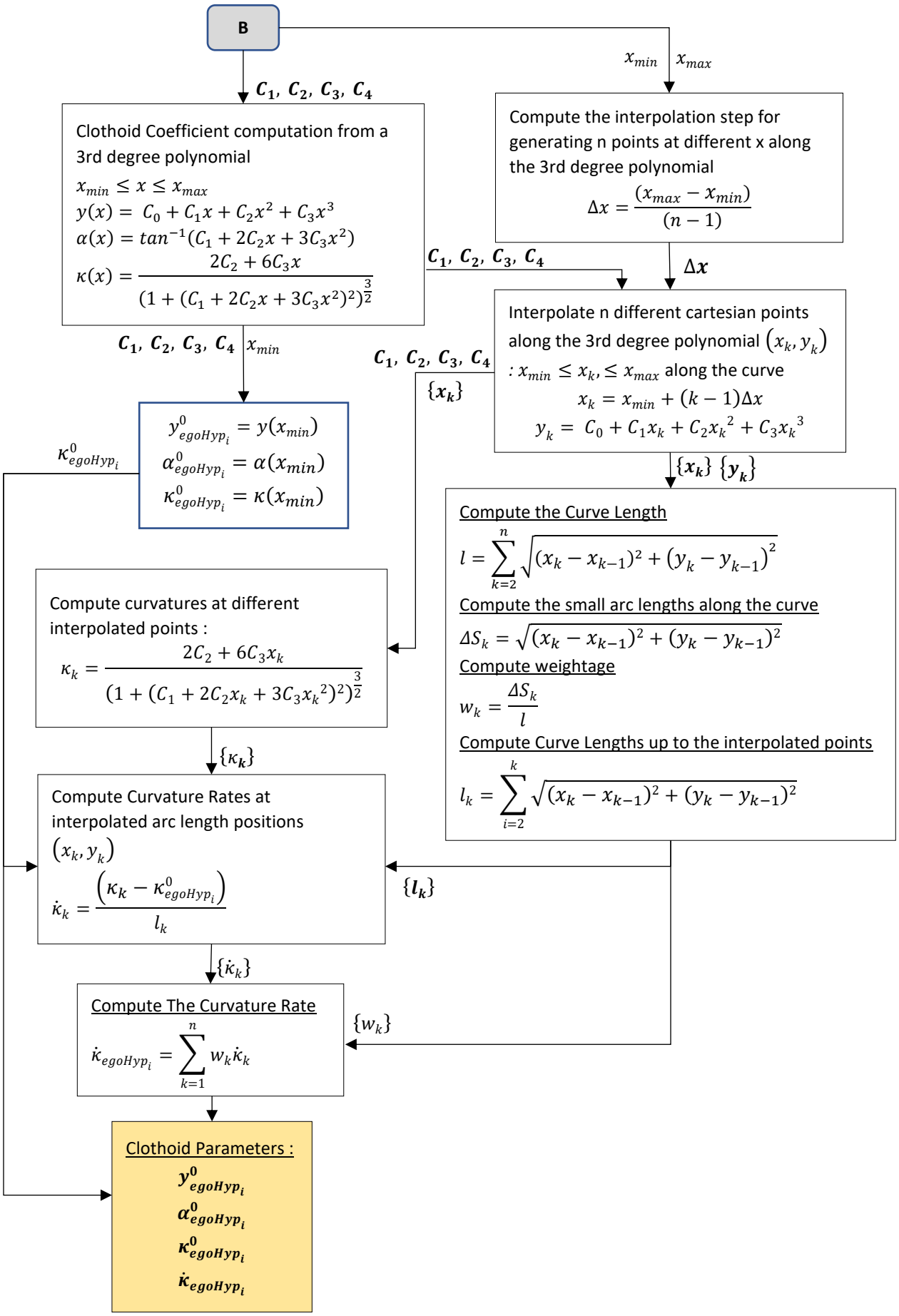
10.2.1 Step 1 : Generate Points in Cartesian Coordinates along the Line



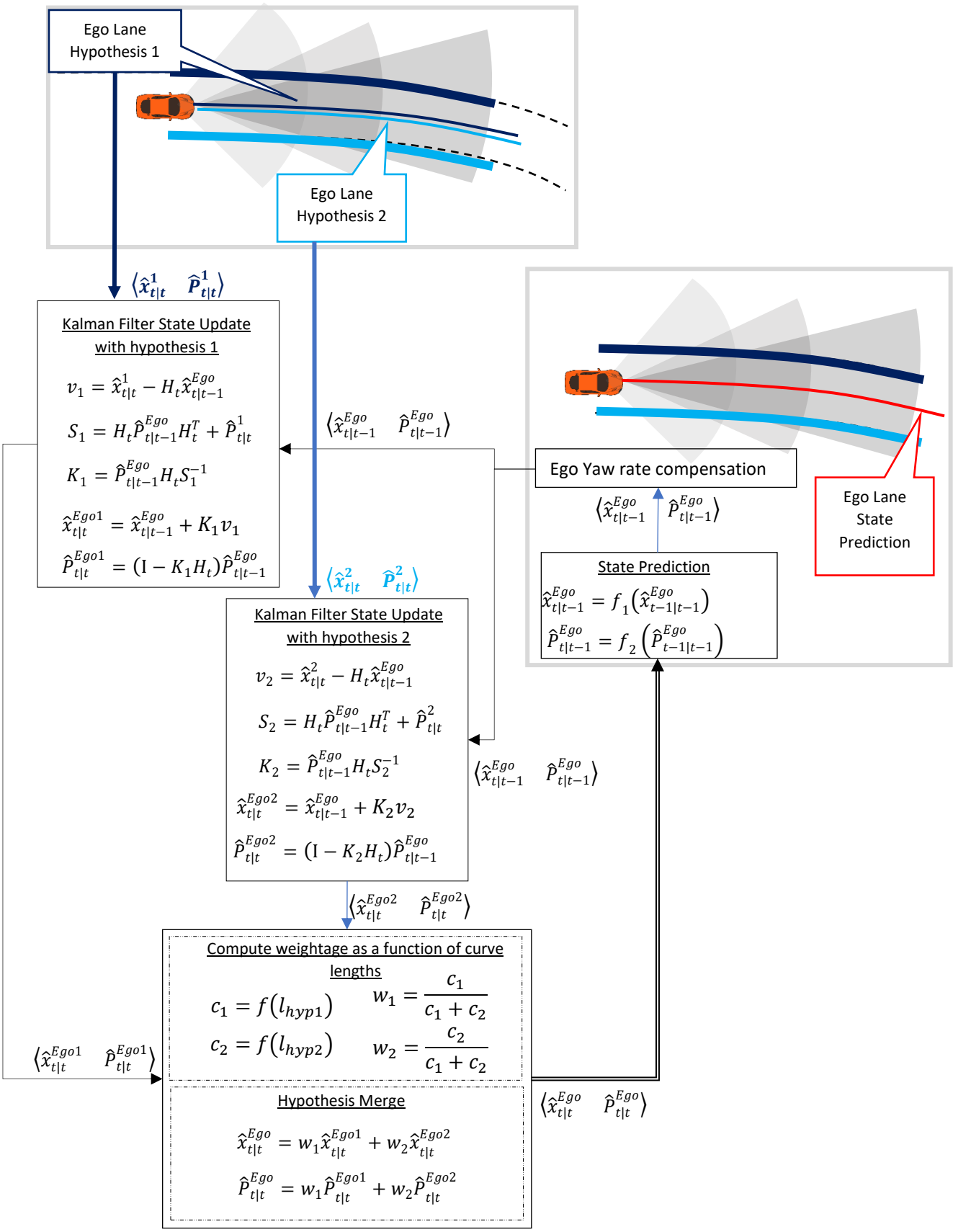
10.2.2 Step 2 : Ego Lane Centre Line Polynomial Fitting by Least Squares Estimation



10.2.3 Step 3 : Polynomial Coefficients to Clothoid Parameters Transformation

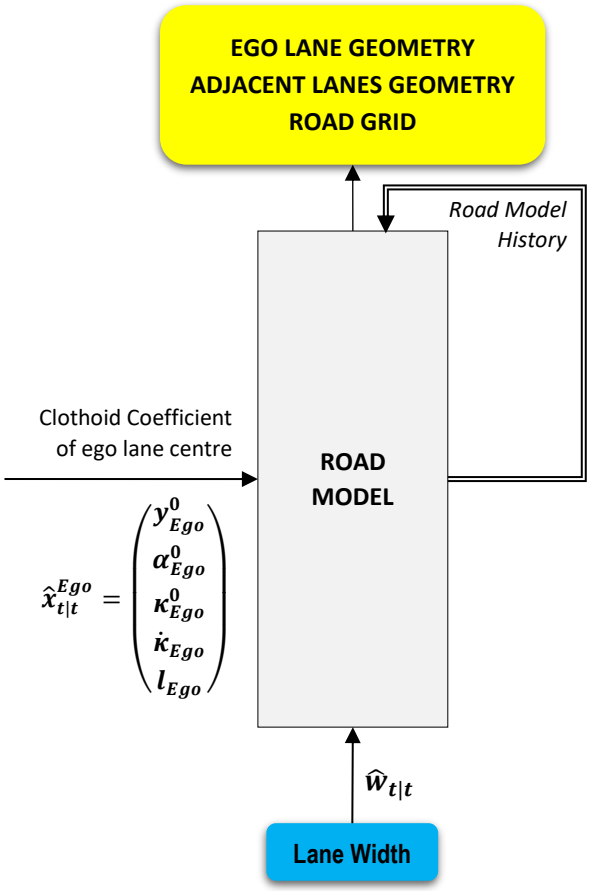


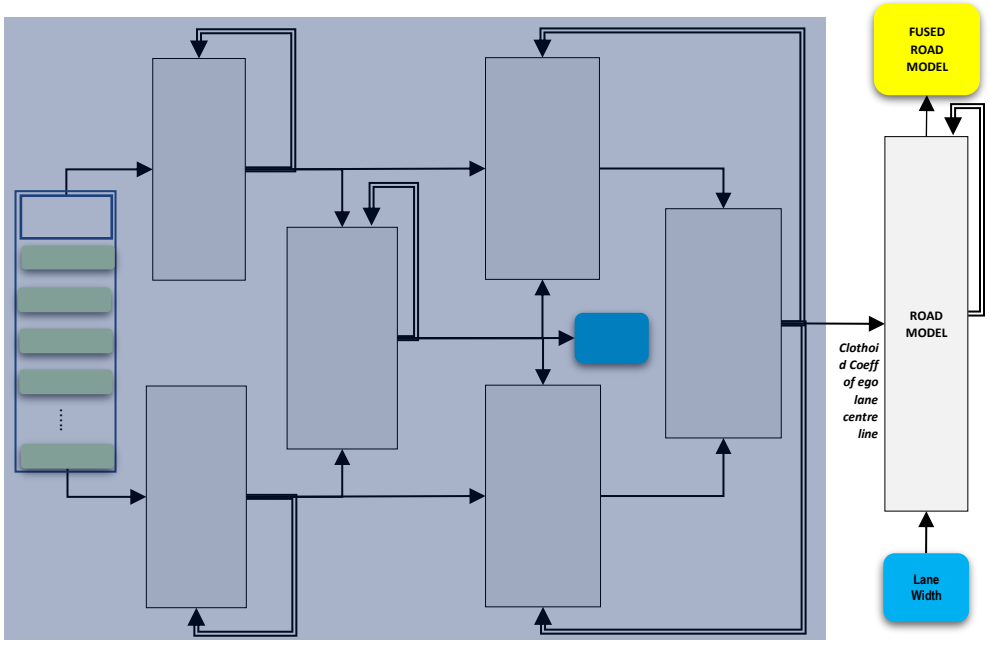
10.2.4 Step 4 : Ego Lane Centre Line Estimation by Multiple Hypothesis Merge



11. ROAD MODEL COMPUTATION

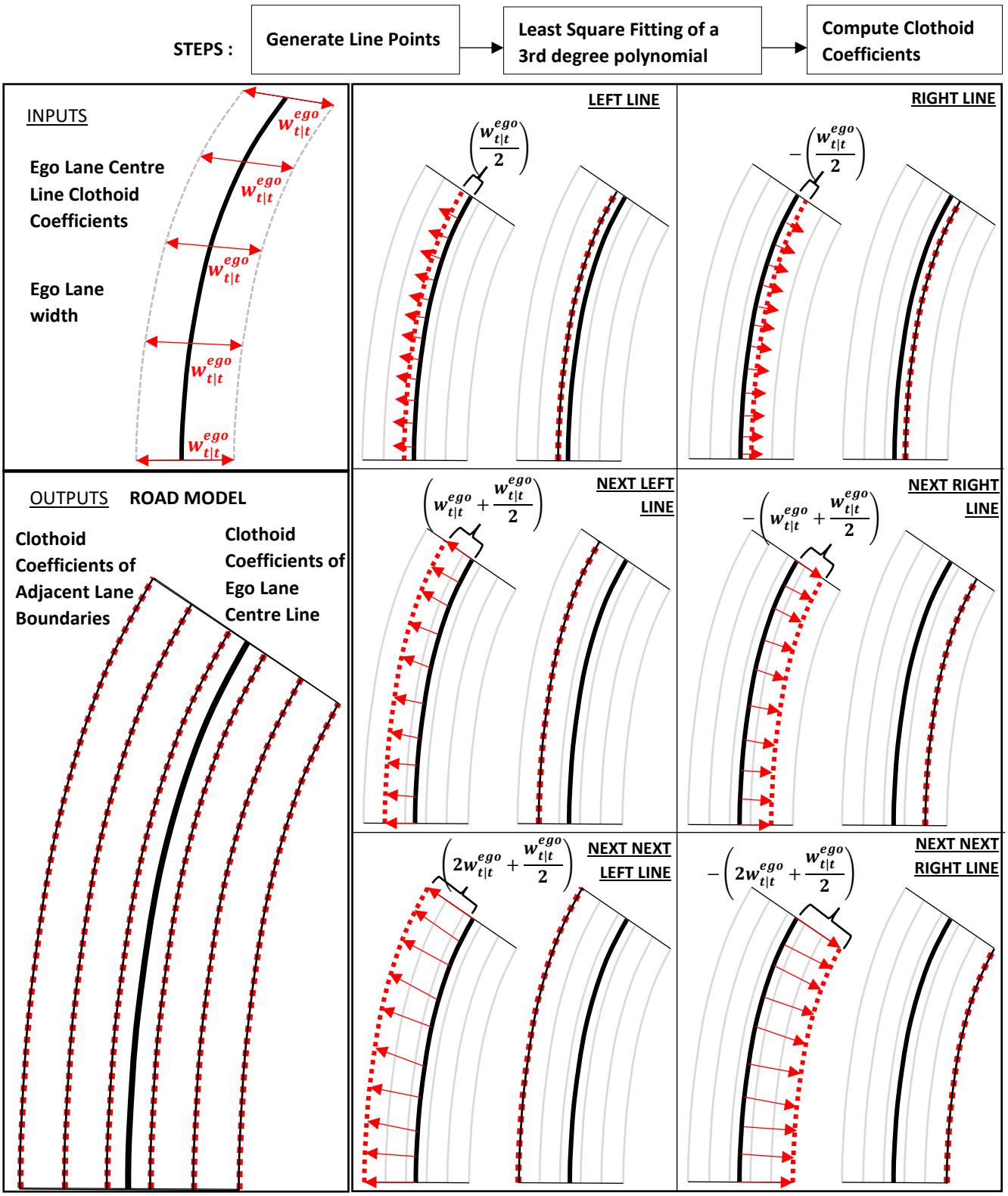
11.1 Data Flow/Block Diagram



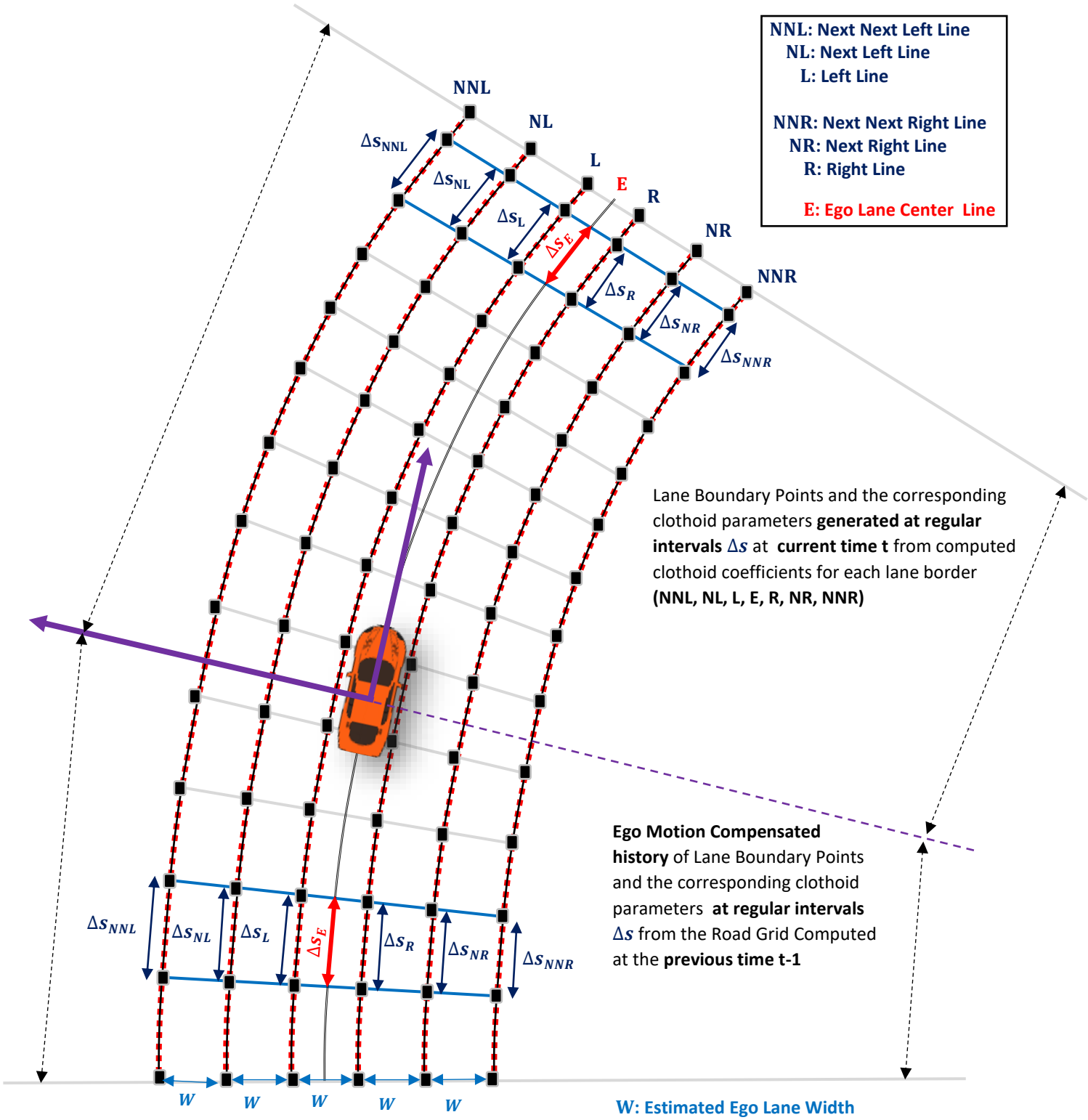


11.2 High Level Design

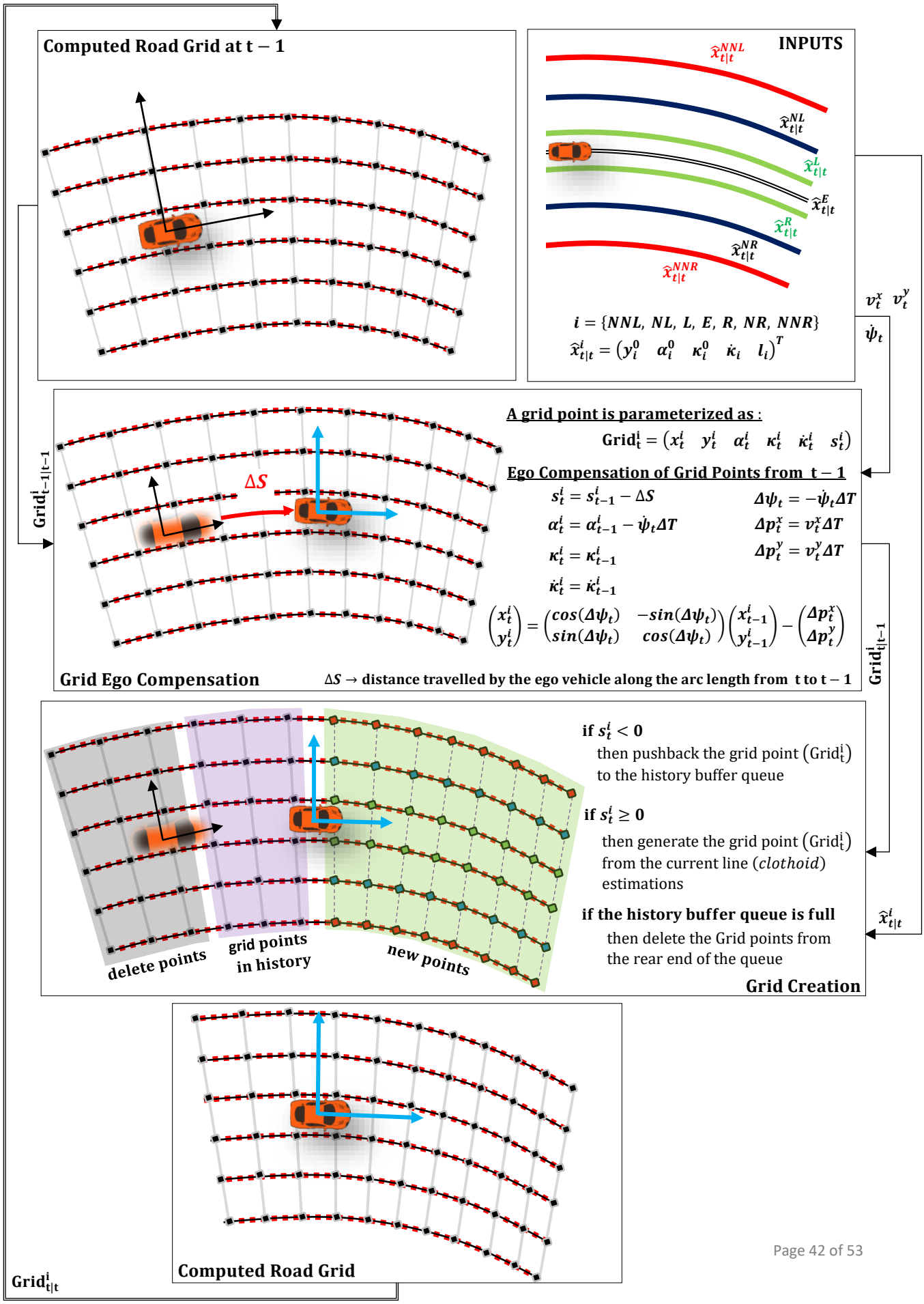
11.2.1 Step 1 : Adjacent Lane Boundary Line Geometry Computation



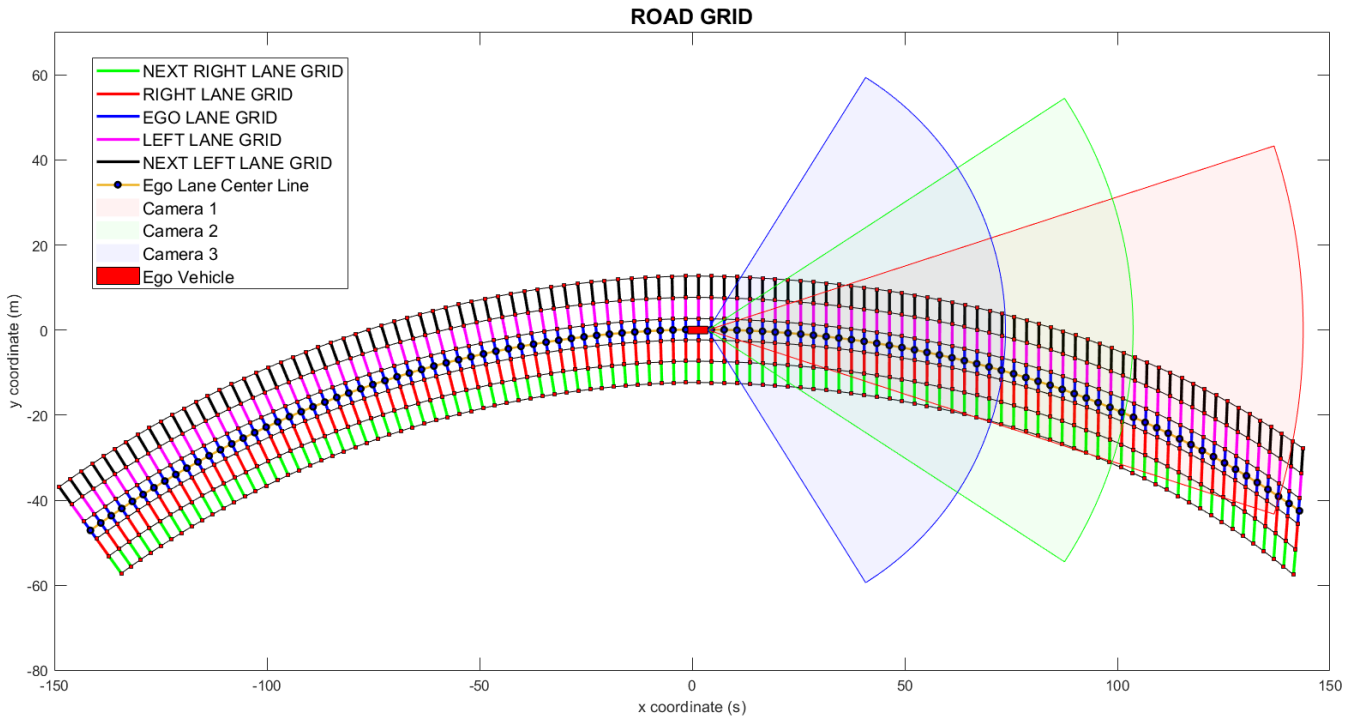
11.2.2 Step 2 : Spatial Parameterization of Road Grid



11.2.3 Step 3 : Road Grid Computation and History Generation



11.3 Grid Plot in MATLAB



The following are the Grid Parameters as depicted in the above plot

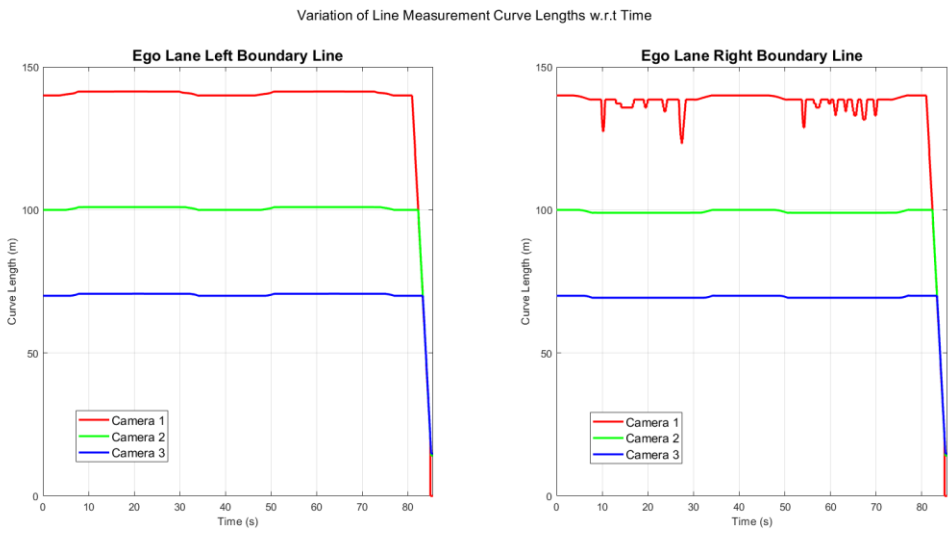
- Line Grid **Length Front** Of Ego Vehicle : *150 meters*
- Line Grid **Length Rear** Of Ego Vehicle : *150 meters*
- Line Grid **Resolution** : *3 meters*
- **Width** Of a Grid Cell : *Estimated Lane Width*

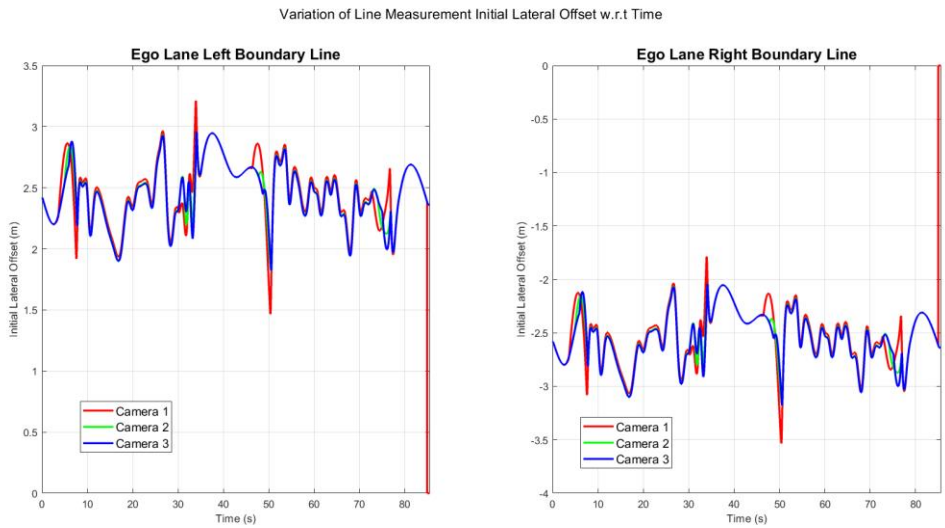
<u>Typical uses of a Road Grid are</u> :

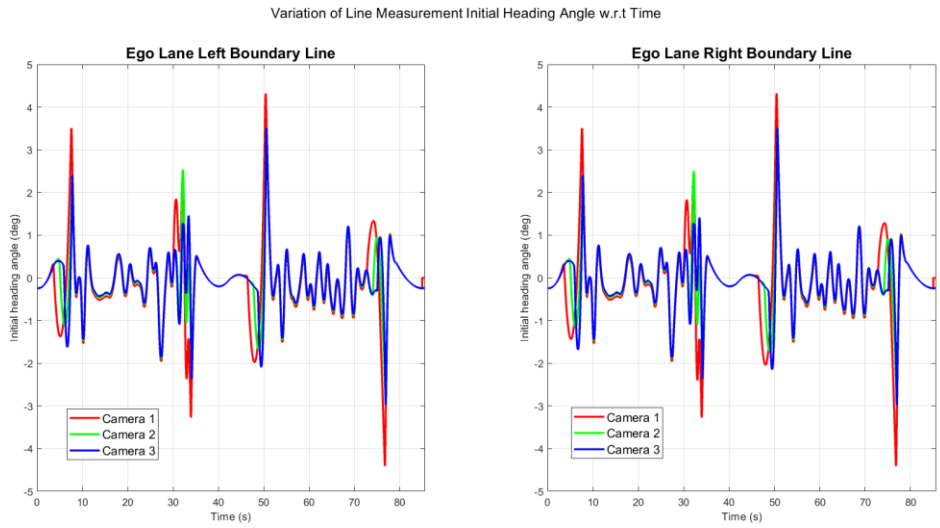
- Situation Assessment for Lane Change event
- Trajectory Planning
- Autonomous Guidance

12. PLOTS

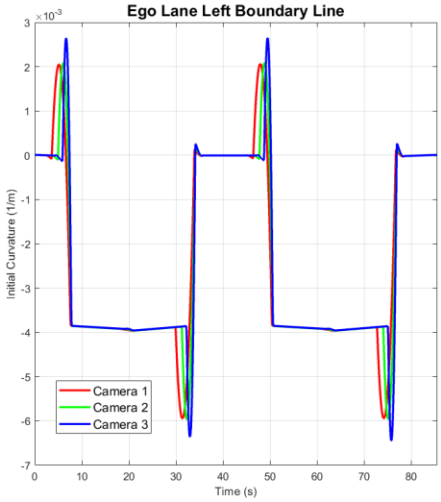
12.1 Clothoid Parameters as Line Measurement from Cameras

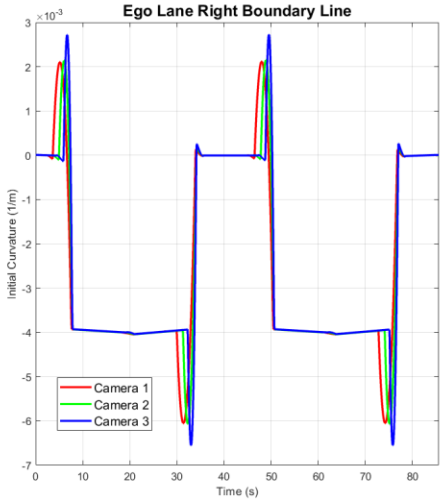




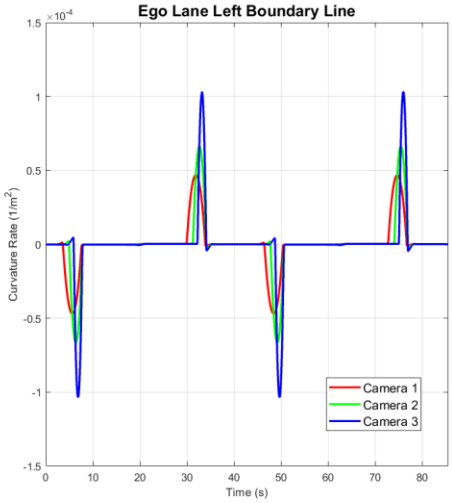


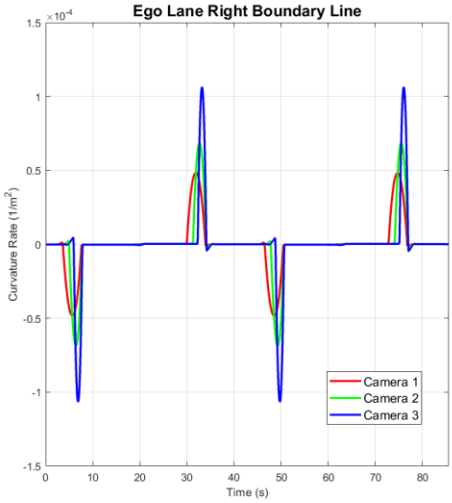
Variation of Line Measurement Initial Curvature w.r.t Time



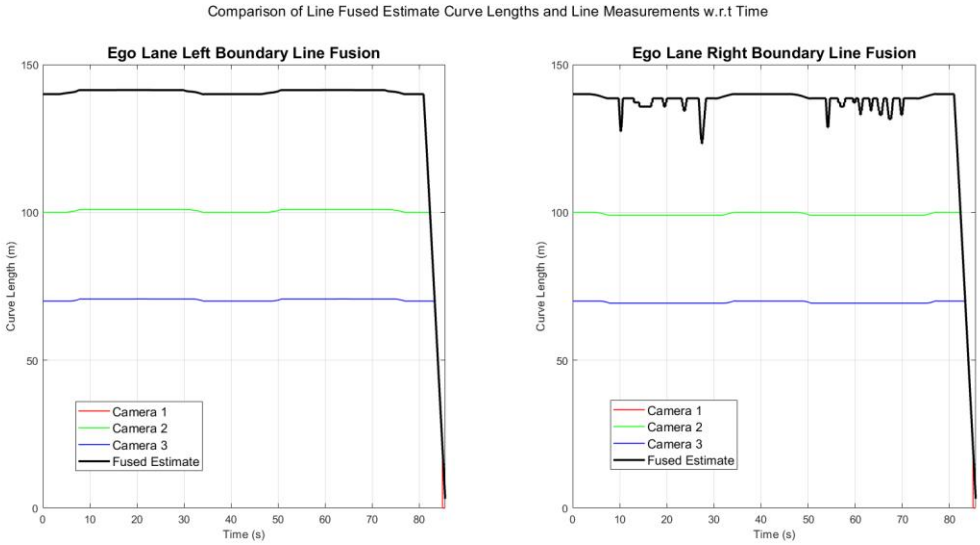


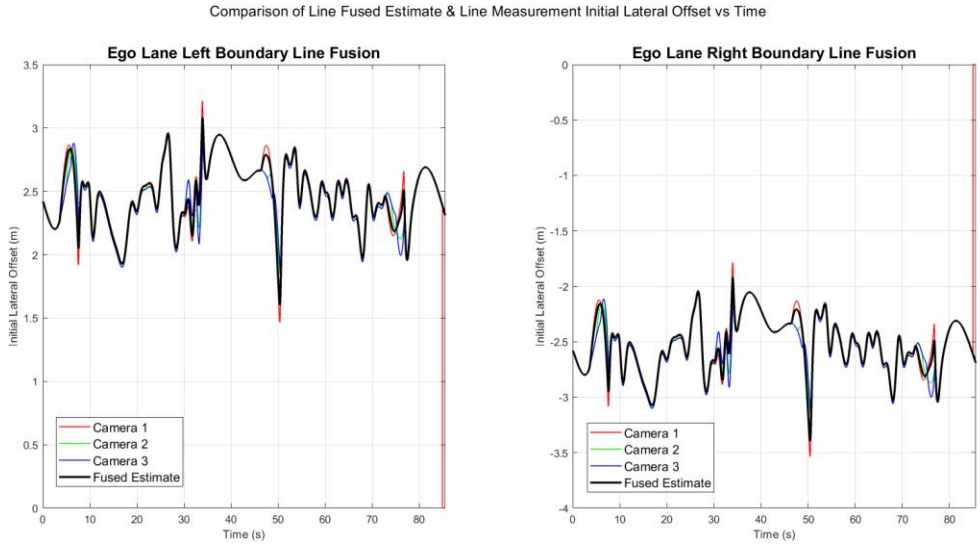
Variation of Line Measurement Curvature Rate w.r.t Time

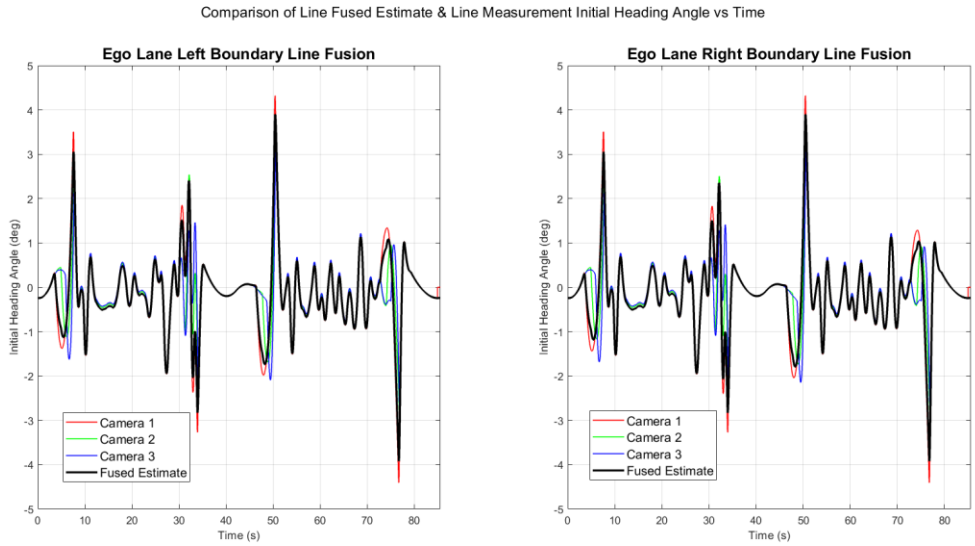




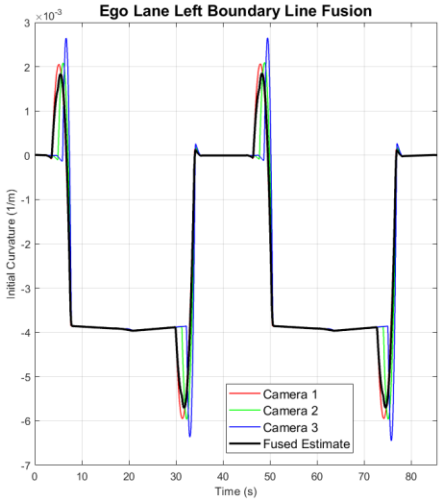
12.2 Ego Lane Boundary Measurement to Line Track Fusion

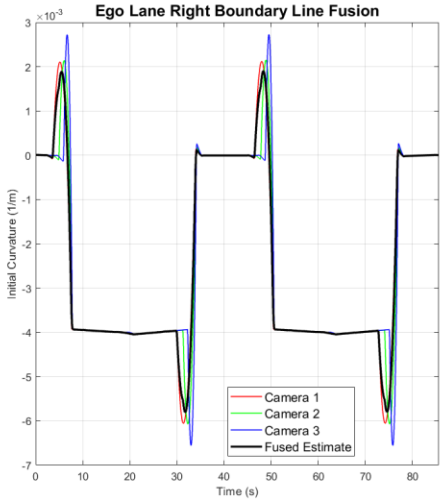




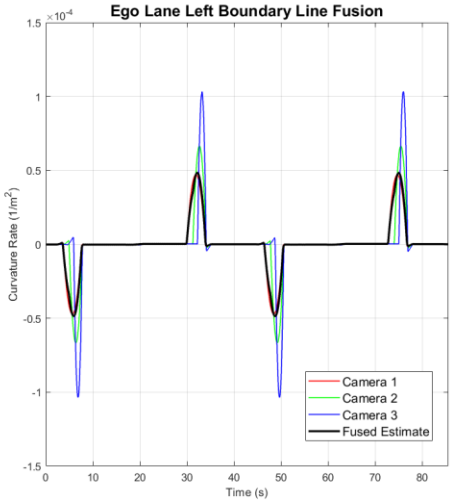


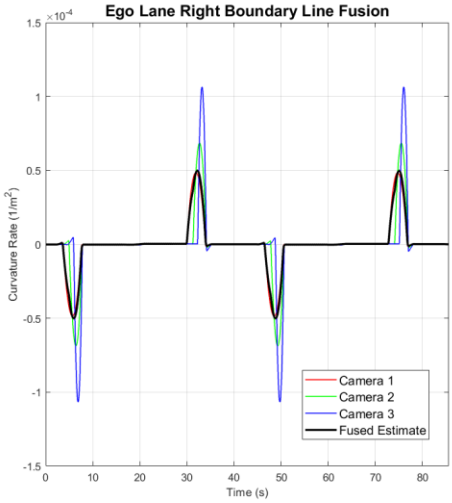
Comparison of Line Fused Estimate & Line Measurement Initial Curvature vs Time



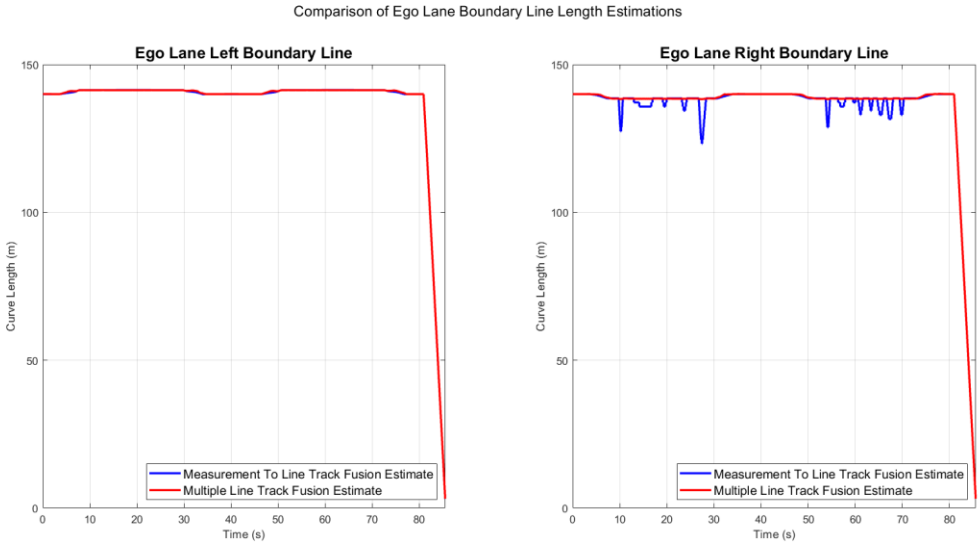


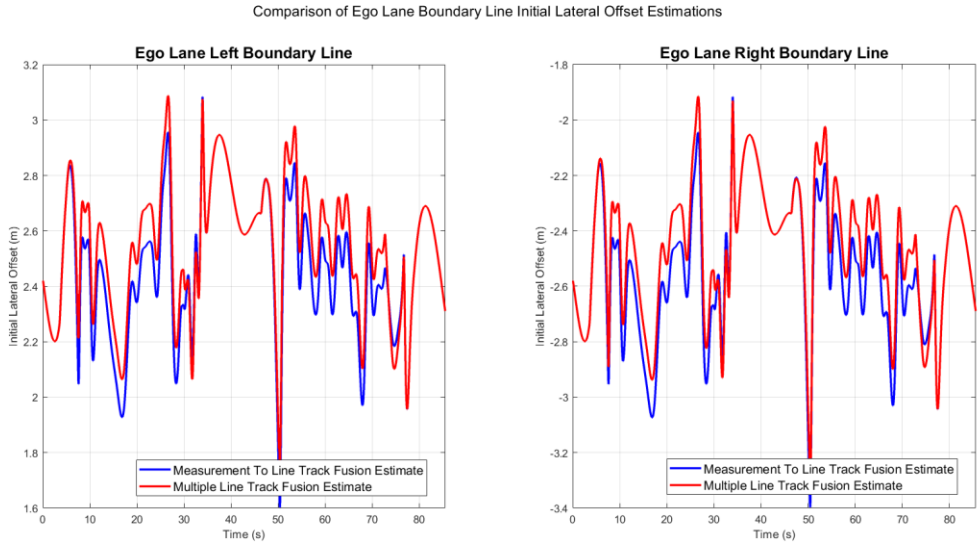
Comparison of Line Fused Estimate & Line Measurement Curvature Rate vs Time

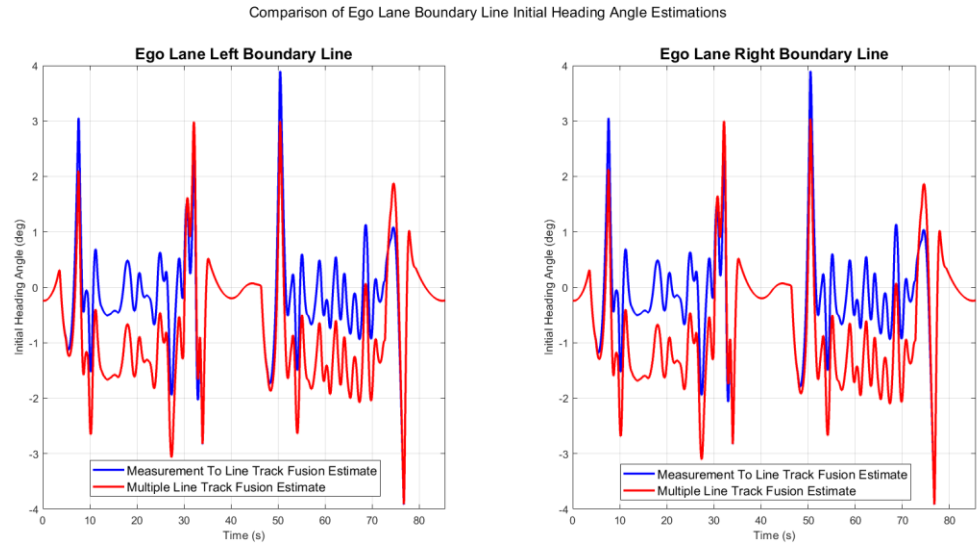




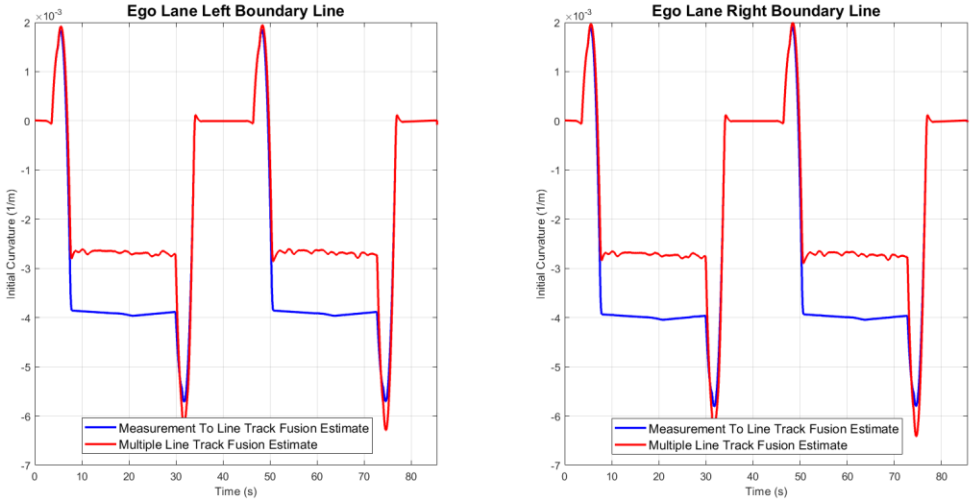
12.3 Ego Lane Boundary Line Estimate after Multi-hypothesis Fusion



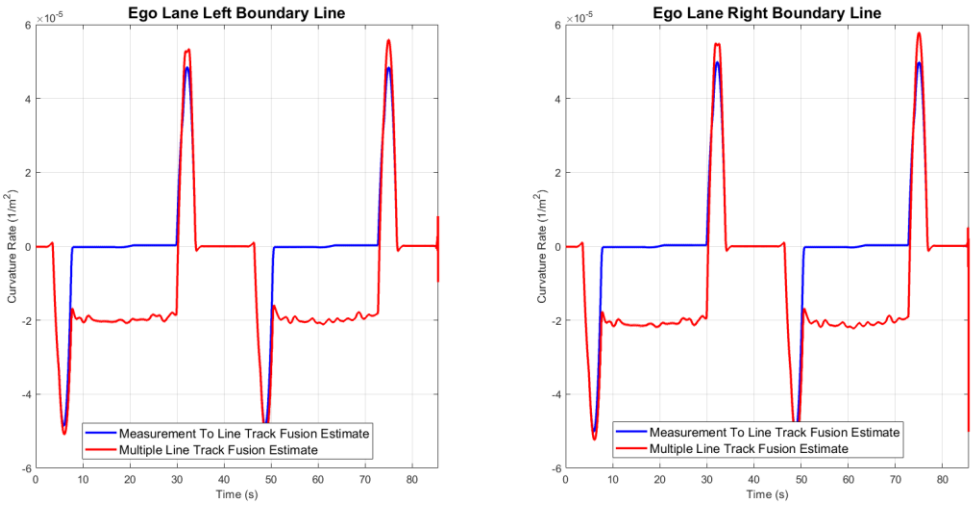




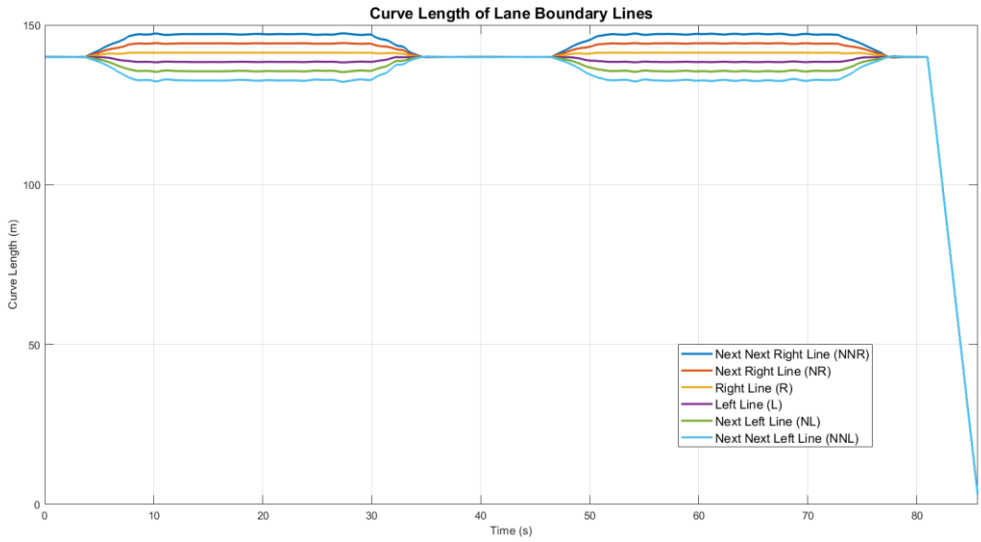
Comparison of Ego Lane Boundary Line Initial Curvature Estimations

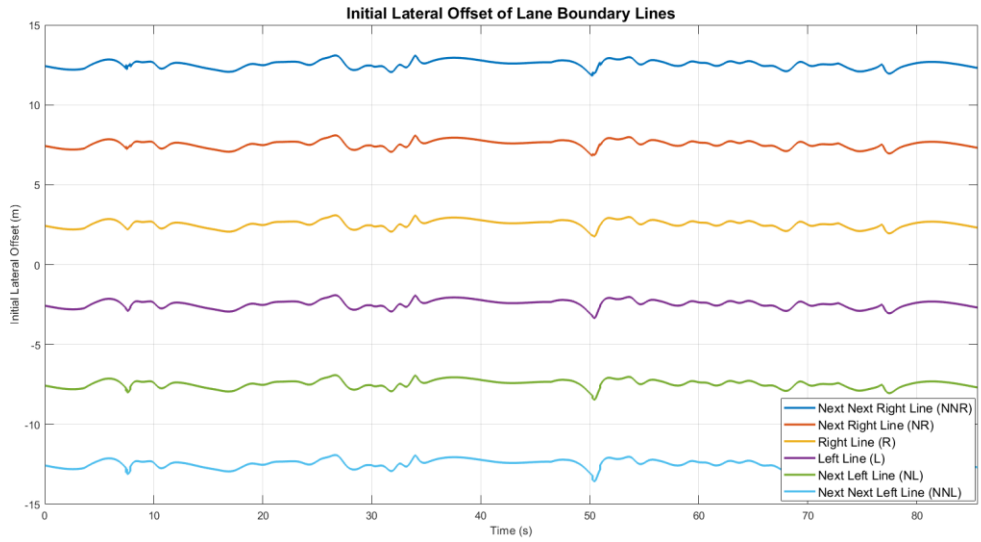


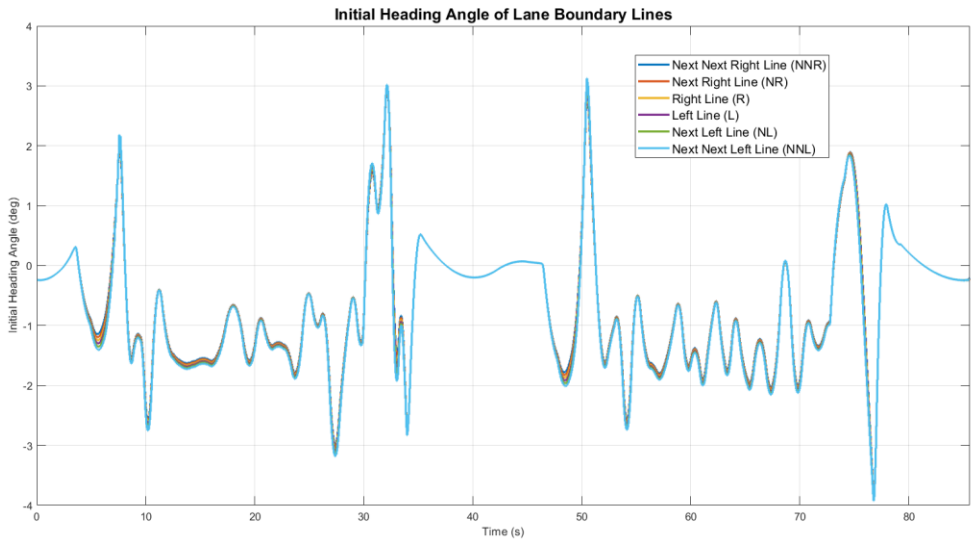
Comparison of Ego Lane Boundary Line Curvature Rate Estimations

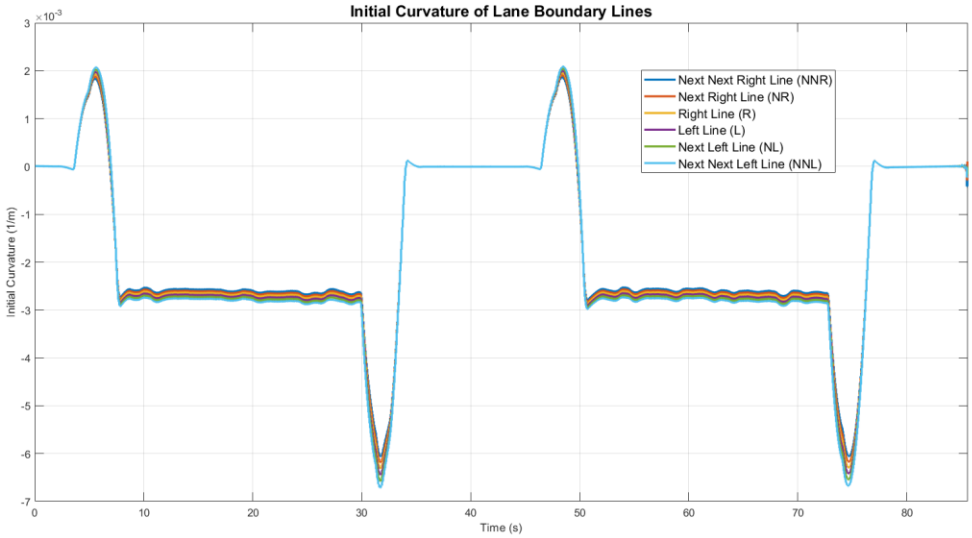


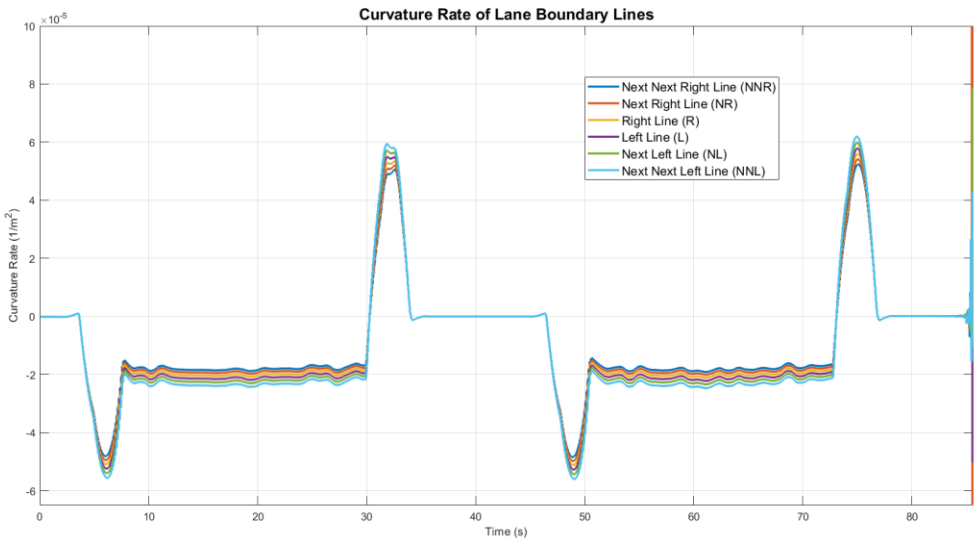
12.4 Road Geometry








Initial Curvature of Lane Boundary Lines
Initial Curvature (1/m)
Time (s)
Next Next Right Line (NNR)
Next Right Line (NR)
Right Line (R)
Left Line (L)
Next Left Line (NL)
Next Next Left Line (NNL)


Curvature Rate of Lane Boundary Lines
Curvature Rate (1/m²)
Time (s)
Next Next Right Line (NNR)
Next Right Line (NR)
Right Line (R)
Left Line (L)
Next Left Line (NL)
Next Next Left Line (NNL)

13. CONCLUSION

This project in its current version is aimed at designing a robust and scalable fusion architecture with multiple cameras. The main highlight is the utilization of clothoid line parameters in fusion framework for system run time efficiency. The technique presented here can support the road model fusion as the number of cameras scales up. Techniques involving particle filter estimation based on more accurate approximation of clothoid equation is not considered in this version . In the next version piece-wise polynomial fitting shall be performed for improving the accuracy of line estimation. In subsequent versions more state of the art techniques shall be explored and integrated as a fusion library into the proposed fusion framework. The Road Grid shall be utilized in the next version for Target To Lane association. Hope the reader finds this document useful, informative and engaging.

Project repository :

- *https://github.com/UditBhaskar19/ROAD_MODEL_FUSION.git*

14. References

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- Yaakov bar-shalom and Fred Daum, "The probabilistic data association filter", IEEE control systems, 2010.