Linear Algebra

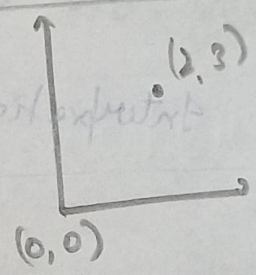
d- Dimensional ~~Hypothesis~~ Hyperplane: $\sum_{i=1}^{d} w_i x_i + w_0 = 0$

$$\underbrace{w^T}_{\text{weight}} \underbrace{x}_{\text{features}} + \underbrace{w_0}_{\text{Bias}} = 0$$

Vectors: A collection of values in d-dimension $[x_1, x_2, x_3, \dots x_d]$ d- dimensional vector

array
list
tuple

L_2 - Norm → Euclidean distance



$d = \sqrt{2^2 + 3^2}$

$\|x\| = 5$

Hyperplane Equation if that passes through origin:

$$w^T x + w_0 = 0$$

Now $x = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{\text{d-dimentional}} \in R^d$

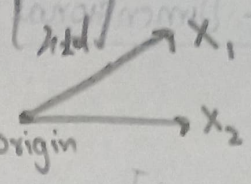
$$[w_0 \ w_1 \ \dots \ w_d] \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + w_0 = 0$$

$\Rightarrow \quad w_0 = 0$

Angle b/w two vectors :

$$x_1 = \begin{bmatrix} x_{11} \\ x_{21} \\ \vdots \\ x_{1d} \end{bmatrix}$$

$$x_1^T \cdot x_2 = \|x_1\| \|x_2\| \cdot \cos\theta$$



$$\theta = \cos^{-1}\left(\frac{x_1 \cdot x_2}{\|x_1\| \cdot \|x_2\|}\right)$$

$$x_2 = \begin{bmatrix} x_{21} \\ x_{22} \\ \vdots \\ x_{2d} \end{bmatrix}$$

* Remember $x_1 \cdot x_2$ is false & not defined because x_1 has dimensions $d \times 1$ and x_2 also have $d \times 1$ dimension so we will have to transpose one of them.

Geometric Interpretion of Hyperlane equation

~~$\rightarrow ax + by + c = 0$~~

~~(a, b) is alway perpendicular to line~~

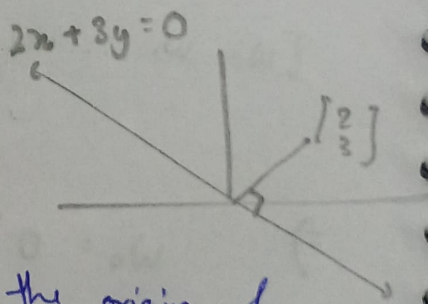
~~(c) is the intercept~~

- a hyperplane passing through origin

2-D (line)

$$\underset{x}{2x_1} + \underset{y}{3x_2} = 0$$

$$w = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$



3-D (plane)

w is a vector that passes through the origin & is perpendicular to the plane

$$w^T x + w_0 = 0$$

- **Hyperplanes:**

2-D $ax + by + c = 0$

$w_1x_1 + w_2x_2 + w_0 = 0$

3-D $ax + by + cz + d = 0$

$w_1x_1 + w_2x_2 + w_3x_3 + w_0 = 0$

$$\boxed{\text{Generalising} \quad \sum_{i=1}^{d} w_i x_i + w_0 = 0}$$

* Equation of Hyperplane

$w^T x + w_0 = 0$

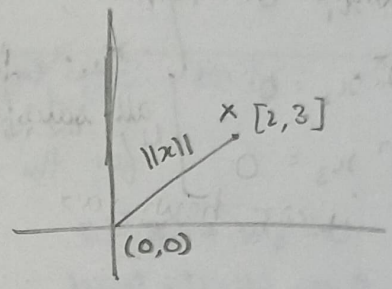
$$\begin{array}{cc} x_1 & x_2 \\ \end{array}$$

$x = [2, 3]$

$\|x\| = \sqrt{2^2 + 3^2}$

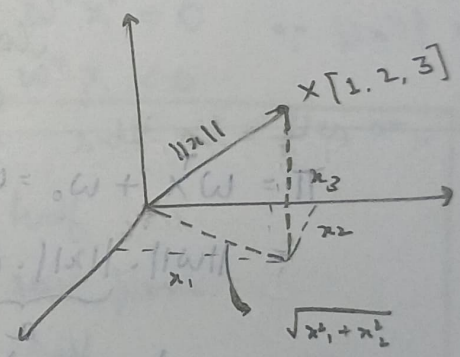
↳ L_2- Norm | Euclidean

Length of the vector



$\|x\| = \sqrt{x_1^2 + x_2^2 + x_3^2}$

$\|x\| = \sqrt{\left(\sqrt{x_1^2 + x_2^2}\right)^2 + x_3^2}$



- **A hyperplane that passes through origin**

$w^T x = 0$

Proof: $x = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \in \mathbb{R}^d$

$w = [w_1 \; w_2 \; w_3 \; \dots \; w_d]$

$$[w_1 \; w_2 \; w_3 \; \dots \; w_d] \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + w_0 = 0$$

$\Rightarrow w_1 \times 0 + w_2 \times 0 \; \dots \; w_d \times 0 + w_0 = 0$

$$\boxed{w_0 = 0}$$

That makes the equation $\boxed{w^T x = 0}$

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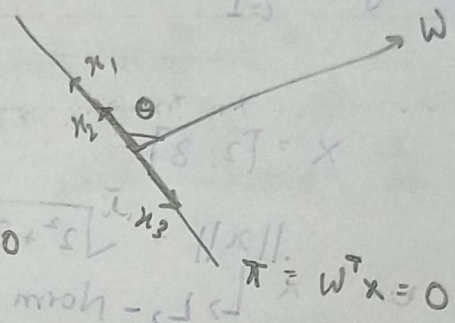
* w^T is always perpendicular to the plane

$$w^T x + w_0 = 0$$

$$\left.\begin{aligned} w^T x_1 &= 0 \\ w^T x_2 &= 0 \\ w^T x_3 &= 0 \end{aligned}\right\} \begin{array}{l} x_1, x_2 \text{ \& } x_3 \\ \text{all satisfy equation} \\ \text{for the plane } \pi = w^T x = 0 \end{array}$$



$$\underset{>0}{\|w\|} \cdot \underset{>0}{\|x_1\|} \cdot \underset{0}{\cos\theta} = 0$$

$$\cos\theta = 0$$

$$\theta = 90^\circ$$

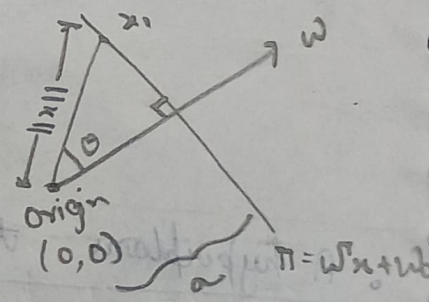
$$\pi = w^T x + w_0 = 0$$

$$= \|w\| \cdot \underbrace{\|x\| \cdot \cos\theta}_{a} + w_0 = 0$$

$$= \|w\| \cdot a + w_0 = 0$$

$$= w_0 = -a\,\|w\|$$

$$= \boxed{a = \frac{-w_0}{\|w\|}}$$



Let's take $\hat{w}$ a unit vector in w direction

$$\hat{w} = \frac{w}{\|w\|}$$

$w_0 = -a$

↳ when $\pi = \hat{w} x + w_0 = 0$

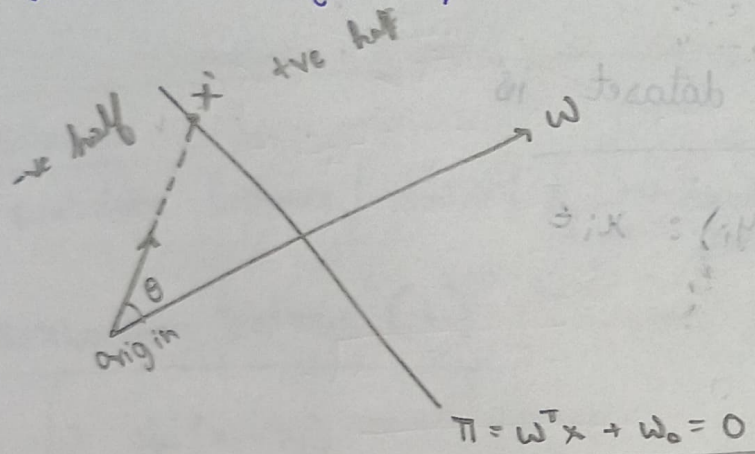
Interpretation of eq of a plane.

$$w^T x + w_0 = 0 \qquad \hat{w}^T x + w_0 = 0$$

w = vector $\perp$ to all vector on the plane passing through origin $= \hat{w}$

$$w_0 = \frac{\text{distance}(0, \pi)}{\|w\|} \qquad w_0 = \frac{\text{distance}(0, \pi)}{1}$$

On which side of the plane does a point lie on?



$$w^T x' + w_0 = 0$$

$$\|w\| \cdot \|x'\| \cos\theta + w_0 = 0$$

$$\|x\| < \|x'\|$$

$$w^T x' = 0 \qquad \because \|x'\| > \|x\|$$

$$w^T x < 0$$

Give a point x, to find which side it lies on,

$$w^T x + w_0 \begin{cases} < 0 \rightarrow \text{-ve half} \\ = 0 \rightarrow \text{on the plane} \\ > 0 \rightarrow \text{+ve half} \end{cases}$$

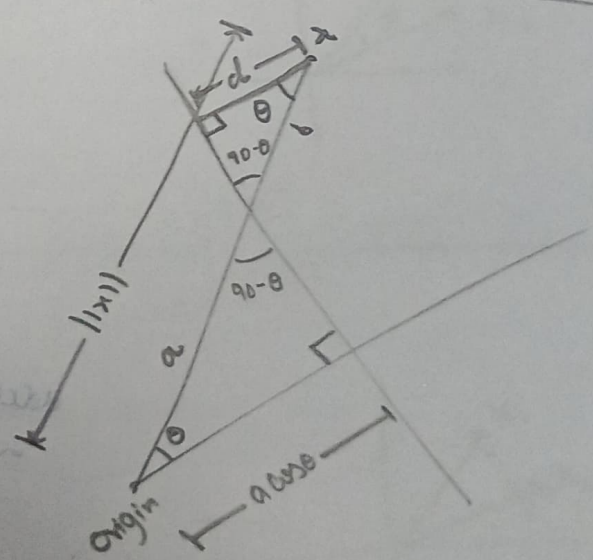
- Distance of point from hyperplane

$$d = b\cos\theta$$

$$a\cos\theta = \frac{-w_0}{\|w\|}$$

$$a = \frac{-w}{\|w\| \cos\theta}$$

$$b = \|x\| - a$$



$$b = \|x\| - \frac{(-w_0)}{\|w\| \cos\theta}$$

$$b = \frac{\|x\| \|w\| \cos\theta + w_0}{\|w\| \cos\theta} = \frac{w^T x + w_0}{\|w\| \cos\theta}$$

$$\boxed{b\cos\theta = d = \frac{w^T x + w_0}{\|w\|}}$$

If we use $\hat{w}'$ distance $= \hat{w}^T x + w_0$

Quantifying how 'good' a dataset is

given $D = \{(x_i, y_i) : x_i \in$

① Count the # of ~~mid~~ misclassification

for (x_i, y_i) in dataset :

prediction $= w^T x_i + w_0$

if (prediction ≤ 0 and $y_i \leq 0$) or (prediction > 0 and $y_i < 0$) :

correct += 1

else

incorrect += 1

accuracy = correct / incorrect

error = incorrect / correct

$$\boxed{\sum_{x_i} \frac{w^T x_i + w_0}{\|w\|} \cdot y_i} \rightarrow \max \qquad \boxed{-\sum_{x_i} \frac{w^T x_i + w_0}{\|w\|} \cdot y_i}$$

Why are we Adding? Why not multiply?

1) datatype will not have enough precision to hold the ans.
2) if any value is zero then it will make everything 0
3) Multiplication is slower than additions!
   2-8 ns 0.1 ns

Distances

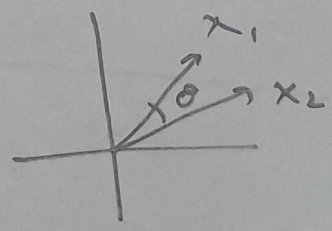
Euclidean Distance / Norm / L_2 → shortest length b/w two points.

Manhattan Distance (L_1)

- abs $(x_1 - x_2)$
- Used to define metric space often in Rational nums.

Cosine Similarity:

cosine similarity $(x_1, x_2) = \cos\theta$

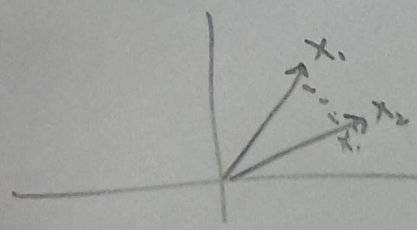


- 0, iff vectors are ⊥ to each other
- 1, iff vectors are ∥ to each other

Projection of Vectors → PCA

~~x_1' is the projection of x_1 onto x_2~~

$$\cancel{\boxed{\|x_1\| \cdot \cos\theta = \frac{x_1 \cdot x_2}{|x_2|}}}$$



* $\|x_1'\| = \|x_1\| \cos\theta$

And direction is a unit vector along x_2

$$\boxed{\text{proj}_{x_2} x_1 = \frac{x_1 \cdot x_2}{\|x_2\|} \hat{x}_2 \quad \Big| \quad \hat{x}_2 = \frac{x_2}{\|x_2\|}}$$

- ~~Cosine Difference~~ : Cosine distance : The term is commonly used for the complement of cosine similarity is positive space.

$$d(\vec{a}, \vec{b}) = 1 - \cos\theta$$

* Used to calculate how different two vectors are
* $[0, 2]$ range

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* Note : Why do we take cos of the angle in cosine similarity?

- $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ Intersecting

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ Coincident

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ Parrallel

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Vector: Computer -> List, tuple, Array
Math -> Anything for which the following operation are defined:
(i) Addition b/w two vectors
-> If you can define addition then
(ii) Multiplication by a scalar $\in R$

Vector Space: ① a set
They follow the following things
(i) Added together
(ii) Multiplied together by scalar

- $S = \{ \vec{v} \mid \ldots \}$

S is closed under vector addition iff $\forall\, v_1, v_2$ then $v_1 + v_2 \in S$

Vector Space should staify
(i) Closed under Addition
(ii) Closed under scalar multiplication
(iii) Vector addition (the way we define) should be commutative
$\vec{u} + \vec{v} = \vec{v} + \vec{u}$

(iv) $\vec{u} + (\vec{v} + \vec{w}) = (\vec{v} + \vec{w}) + \vec{u}$

(v) Addivitive Identity
↳ example
↳ $x + 0 = x$
↳ $x \cdot 1 = x$
↳ $\min(\infty, x) = x$

- It means that we should be able to define a zero vector

$$\vec{u} + \vec{0} = \vec{u}$$

(vi) <u>Additive inverse</u>

For every vector $\vec{v} \in S$ we should have $a - \vec{v} \in S$

$$\boxed{\vec{v} \oplus -\vec{v} = \vec{0}}$$

$\oplus$ ↓ operation

$\vec{0}$ ↓ Additive identity of the operation

★ <u>Some Crazy examples</u>

★ For polynomials we we zero vectors as additive ~~[illegible]~~ identity

(vii) $a(b\vec{v}) = (ab)\vec{v}$

(viii) $1\vec{v} = \vec{v}$ [identity for scalar multiplication]

(ix) $a(\vec{v} + \vec{w}) = a\vec{v} + a\vec{w}$

(x) $(a+b)\vec{v} = a\vec{v} + b\vec{v}$

S = set of all $= \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| \; a,b,c,d \in \mathbb{R} \right\}$

is a vector space.

They satisfy all axioms

▲ <u>Identity</u>: performing some particular operation with the value doesn't change the other value

Examples

① $\mathbb{R}^n = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \middle| x_i \in \mathbb{R} \right\}$

$\downarrow$

vector space

② $P^{(d)}(x) = \left\{ \sum_{i=0}^{d} a_i \cdot x^i \;\middle|\; \begin{matrix} a_i \in \mathbb{R} \\ a_i \neq 0 \end{matrix} \right\}$

S = 2d plane $\rightarrow$ $\vec{0} = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$ is S_1 a vector space?

↳ Yes it is a vector space.

- The set of all the vectors we can reach | get from a set of given vectors it is called the span of the vectors.

Basis $\rightarrow$ set of vectors
$\rightarrow$ set of starting | fundamental vectors

$\underbrace{\hat{i} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \hat{j} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{\text{Basis}}$ then $\left\{ \begin{bmatrix} a \\ b \end{bmatrix} \middle| a, b \in \mathbb{R} \right\}$

- A set of vectors are linearly dependent iff $c_1 v_1 + + c_2 v_2 \ldots$ $(v_1, v_2 \ldots v_n)$ has only 1 solution which is $c_1 = c_2 + c_2 = c_3 \ldots \quad c_n = 0$

↳ The <u>basis of a vector</u> space is a set of independent vector that span the whole space.

- **Basis** a set of vectors such that they are:
  (i) linearly independant
  (ii) span the entire space

- **Span:** span of basis $B = \{v_1, v_2 \dots\}$ is the the set of all vector which are linear combination of vectors.

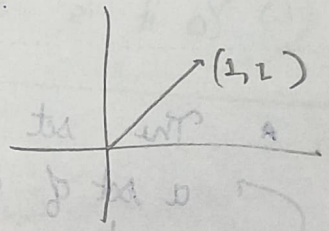
- A basis with d independant vectors will span d-dimensional plane in some space.

example:

$$\text{Space} = \mathbb{R}^2$$

$$B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\text{span } B = \left\{ \begin{bmatrix} c \\ c \end{bmatrix} \;\middle|\; c \in \mathbb{R} \right\}$$



- **Linear Transformation (T)**

T is a transformation from a vector space (V) to a vector space (W)

f: Domain $\rightarrow$ Codomain

- T is a rule/function that assign a value $\vec{w}$ to every value $\vec{v} \in V$

T is linear iff it preserves the linear combinations that is:

$$T(u) + T(v) = T(u + v)$$

$$c \cdot T(v) = T(c \cdot v)$$

(i) All lines must remain lines

(ii) Origin must remain fixed.

- To find the transformation we only need the output of the basis vector.

Examples

1) $T(i) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}_{0}$, $T = [0_1 \ 0_2]$ or $T = \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix}$

$T(j) = \begin{bmatrix} 5 \\ 1 \end{bmatrix}_{0}$, $T = \left[\begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \end{bmatrix} \right]$

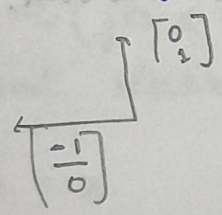
- Matrix Multiplication is the way to apply linear transformatio

where does $\hat{i}$ land

$$\begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

where does $\hat{i}$ land?

Question: Find the ~~vector~~ transformation that rotates the vector space ~~by~~ ~~con~~ counter clockwise by $90°$.

$T(\ \ ) =$ 

$i \rightarrow \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$j \rightarrow \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -b \\ a \end{bmatrix}$$

Question: Find the transformation that "scales" the space $2x$.

$i \rightarrow \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ $j \rightarrow \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2a \\ b \end{bmatrix}$$

- Shear

$$\underbrace{\begin{bmatrix} & \\ & \end{bmatrix}}_{\text{rotation}} \underbrace{\begin{bmatrix} & \\ & \end{bmatrix}}_{\text{scaling}} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\rightarrow \underbrace{\begin{bmatrix} & \\ & \end{bmatrix}}_{\text{Scalling}} \left(\underbrace{\begin{bmatrix} & \\ & \end{bmatrix}}_{\text{Rotation}} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \right)$$

$$f(g(x))$$

* Be carful withe the oder.

Subspaces: Given a space S a subspace H is a set subset of S such that H itself is also a space.

H must satisfy the following:

1) Closed under vector addition
2) Closed under scalar multiplication
3) $\vec{0}$ of S should be in H.
4) There can only be one $\vec{0}$ in S.

Prove: Why we can't have we have 2 $\vec{0}$ in one vector space for one operation?

Proof by contradiction: Assume (fts of C) that there are two distinct identies for $\odot$

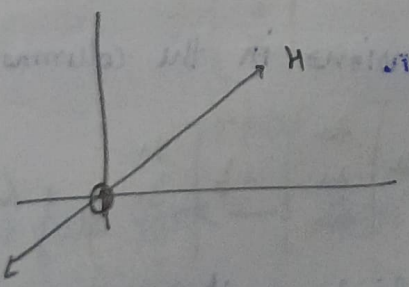
$$id_1 \odot x = x \qquad \forall x$$
$$id_2 \odot x = x$$

$$id_2 \odot id_2 = id_2 = id_1$$

Hence proved

Example: $v \in H,\ c \in R \rightarrow c \cdot v \in H$

S: entire 2-D Plane



→ S is a subspace of itself

→ any line that passes through origin is a subspace

Q $R^5 = \left\{ \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} \middle| \; a, b, c, d, e \in R \right\}$

is R^4 subspace of S? $\longrightarrow$ $\boxed{\text{NO}}$

Dimensions are diff

Q Is $\left\{ \begin{bmatrix} a \\ b \\ c \\ d \\ 0 \end{bmatrix} \middle| \; a, b, c, d \in R \right\}$ a subspace of R^5?

$\rightarrow$ Yes

Q is $\left\{ \begin{bmatrix} a+2 \\ b \\ c \\ d \\ 5 \end{bmatrix} \middle| \; a, b, c, d \in R \right\}$ a subspace by R^5?

$\rightarrow$ No

- $A^{-1} = \dfrac{\text{adj}(A)}{\det(A)}$

<u>Rank</u>: The number of dimensions in the output of the transformation.
↳ Number of linearly independant columns ↱ That gives how many dimensions our output will have

<u>Column Space</u>: The set of all possible linear combination of the columns. $\Rightarrow$ What the output space looks like
↳ Number of dimensions in the columns

$$\begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = a \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + b \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} + c \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \vec{v}$$

- When the rank is <u>as high as it can be</u> it is called full rank.
  ↳ equal to the number of column
- $\vec{0}$ is always there in the column space

- **Row Space**: linear combination of all rows

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}_{n \times m}$$

↳ Colum space = n
Row Space = m

- **Null Space**: set of all possible solutions to equations

$$AX = [0]$$

↳ Basically, when we squished the plane after transformation then the values for which we will get 0.

- **Projections**:

We have a three dimensional object and we want a projection of it. That will be on a 2-d plane That means we have to squish this 3-D object to a 2-D object.

$$\underset{2\times3}{\begin{bmatrix} & & \\ & & \end{bmatrix}} \underset{\substack{3\text{-dim} \\ 3\times1}}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{\substack{2\text{-dim} \\ 2\times1}}{\begin{bmatrix} \\ \end{bmatrix}}$$

- iff A is full rank → then we have one solution of $AX = 0$
  ↳ trivial solution → $\vec{X} = 0$

- iff A is not full rank → ~~for~~ infinite solutions of $AX = 0$

- The column space of an matrix $m \times n$ is a linear subspace of the space. The dimensions of the ~~matrix~~ column space is called the rank of the matrix and is at most $\min(m,n)$

- Rank -> max number of linearly independent rows or columns of A.

- Vector Space is a set whose elements, often called vectors, may be added together and multiplied by numbers called scalars.
  - Scalars are elements of the field
  - The operations of vector addition and scalar multiplications must satisfy the following axioms.

• System of Linear Equation :

↳ Where do these plans intersect?

→ exactly one point
→ ∞ points (line & plane)
→ no intersection (0)

} won't have 2, 13 or solutions like that as it's linear

$$\underset{A}{\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}} \underset{\vec{x}}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{\vec{b}}{\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}}$$

↳ What is the vector $\vec{x}$ that lands on $\vec{b}$ after applying transformation A.

↳ If col(A)(i) is the same as original input space
↳ unique solution

(ii) is not the same then
(a) no solution
(b) infinite solution

Gauss Elimination Algorithm :

• Steps to solve equation
1) Create an argmented matrix
2) Apply row operations to convert to reduced echelon form.

Equivalent Systems : If two system of linear equation

$$[A_1 | C_1] \quad [A_2 | C_2]$$

have same set of solution then they are equivalent.

Row operations: certain operations that maintain equivalence

↳ (i) Swap two rows

(ii) Multiply one equation with some ~~scalar~~ non-zero constant.

(iii) Add a scalar multiple of a row

- $R \to \frac{R_1}{a_1}$
- $R_2 \to R_2 - a_2 \cdot R_1$
- $R_3 \to R_3 - a_3 \cdot R_1$
- $R_2 \to \frac{1}{b_2} \cdot R_2$

Row echelon form:

- All rows having only zero entries are at the bottom.
- The leading entry of every non-zero row is to the right of leading entry of the every row above it.

Reduced row echelon form: $\to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

- Consistant → ∞ or 1 solution

Inconsistant → 0 solutions.

- **Determinant** → How much change in area/volume after transformation it will cause to any other.

$$|A| = \frac{adj(A)}{A^{-1}}$$

- $D \neq 0$ ⇒ Unique solution

$D = 0$ ⇒ Infinite solutions or No solution

• Properties of determinant :

① det (Non-square) → Not defined

② $|A| = |A^T|$

③ swap two rows that toggles the sign of determinant

④ If two rows are same $|A| = 0$
↳ Even if they are proportional

⑤ $\det(A \cdot B) = \det(A) \cdot \det(B)$

⑥ Multiplying with a scalar

$$k\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} ka_1 & kb_1 & kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ ka_2 & kb_2 & kc_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ kc_3 & kb_3 & kc_3 \end{vmatrix}$$

• $$\begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix} = -\begin{bmatrix} p & q & r \\ a & b & c \\ x & y & z \end{bmatrix}$$

or

$$\begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix} = -\begin{bmatrix} b & a & c \\ q & p & r \\ y & x & z \end{bmatrix}$$

• $$\begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix} = \begin{bmatrix} x & y & z \\ a & b & c \\ p & q & r \end{bmatrix}$$

• $|kA| = k^n |A|$ → dimension

• Elementary Row transformations :

$R_1 \longrightarrow R_1 + kR_2 + mR_3$

$C_1 \longrightarrow C_1 + kC_2 + mC_3$

• Addition in determinant :

$$\begin{bmatrix} x_1 & x_2 & x_3 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} + \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} = \begin{bmatrix} x_1 + a_1 & x_2 + b_1 & x_3 + c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

To calculate the determinant by elimination method :

- Transform one row into $[1\ 0\ 0]$ this form
- Keep track of all the scalars that you multiplied the determinant while transforming and in the end divide the determinant by that.