Note:

$\Rightarrow$ EQUIVALENT NOTATIONS: $\Leftrightarrow$ $\emptyset$ $k \Leftrightarrow t$

This Report (Stachniss) $\Leftrightarrow$	Implementation (Barfoot)
1. def EKF(μ_{t-1}, Σ_{t-1}, u_t, z_t):	1. def EKF($\hat{x}_{k-1}$, $\hat{P}_{k-1}$, v_k, y_k):
2. $\bar{\mu}_t = g(u_t, \mu_{t-1})$	2. $\check{x}_k = f(v_k, \hat{x}_{k-1}, 0)$
3. $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$	3. $\hat{P}_k = F_{k-1} \hat{P}_{k-1} F_{k-1}^T + Q'_k$
4. $K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$	4. $K_k = P_k G_k^T (G_k \check{P}_k G_k^T + R_k)^{-1}$
5. $\mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t))$	5. $\hat{x}_k = \check{x}_k + K_k (y_k - g(\check{x}_k, 0))$
6. $\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$	6. $\hat{P}_k = (I - K_k G_k) \check{P}_k$
7. Return μ_t, Σ_t	7. Return $\hat{x}_k$, $\hat{P}_k$

Note: Be careful with notations. See ⊗ if you get confused anywhere..

a1) Localization using Sensor Fusion:

⇒ Task: Estimate 2D robot pose

⇒ EKF Equations:

· Algorithm:

Extended_Kalman_filter$(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$:

$\bar{\mu}_t = g(u_t, \mu_{t-1})$

$\bar{\Sigma}_t = G_t \, \Sigma_{t-1} \, G_t^T + R_t$

$K_t = \bar{\Sigma}_t \, H_t^T (H_t \, \bar{\Sigma}_t \, H_t^T + Q_t)^{-1}$

$\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t))$

$\Sigma_t = (I - K_t \, H_t) \, \bar{\Sigma}_t$

return μ_t, Σ_t

Credits: Cyrill Stachniss

⊗ Given: $\Sigma_0 = \text{diag}\{1, 1, 0.1\}$, $\mu_0 = \begin{bmatrix} 3.019, & 0.071, & -2.910 \end{bmatrix}^T$ ($\mu_{x_0}, \mu_{y_0}, \mu_{\theta_0}$)

→ (Ground Truth for Initialization)

· Prediction: (a) $x_t = g(u_t, x_{t-1}) + \epsilon_t$

Approximation (b)

$$g(u_t, x_{t-1}) \approx g(u_t, \mu_{t-1}) + \underbrace{\frac{\partial g(u_t, \mu_{t-1})}{\partial x_{t-1}}}_{=: G_t} (x_{t-1} - \mu_{t-1})$$

Step 2. $\bar{\mu}_t = g(u_t, \mu_{t-1})$

$$\rightarrow x_k = \begin{bmatrix} x_k \\ y_k \\ \theta_k \end{bmatrix} = g(u_k, x_{k-1}) = \begin{bmatrix} x_{k-1} \\ y_{k-1} \\ \theta_{k-1} \end{bmatrix} + dt \begin{bmatrix} \cos\theta_{k-1} & 0 \\ \sin\theta_{k-1} & 0 \\ 0 & 1 \end{bmatrix} \left(\begin{bmatrix} v_k \\ \omega_k \end{bmatrix} + w_k \right)$$

For first step,

$$\bar{\mu}_1 = g(u_1, \mu_0) = \mu_0 + (0.1) \begin{bmatrix} \cos\theta_0 & 0 \\ \sin\theta_0 & 0 \\ 0 & 1 \end{bmatrix} \left(\begin{bmatrix} v_1 \\ \omega_1 \end{bmatrix} \right)$$

All variables on RHS, μ_0 (or $\begin{bmatrix} \mu_{x_0}, \mu_{y_0}, \mu_{\theta_0} \end{bmatrix}$), v_1, ω_1 known, so $\bar{\mu}_1$ can be found. ⟹ $\boxed{\text{Step 2 - DONE}}$

⊗ $g(u_k, \mu_{k-1})$ is same as $g(u_k, x_{k-1})$ without noise w_k

⊗ $\mu_{k-1} = [\mu_{x_{k-1}}, \mu_{y_{k-1}}, \mu_{\theta_{k-1}}]^T$

$\boxed{\text{Step 3.}}$ $\underset{3\times3}{\bar{\Sigma}_t} = \underset{3\times3}{G_t}\,\underset{3\times3}{\Sigma_{t-1}}\,\underset{3\times3}{G_t^T} + R_t$ $\rightarrow R_t = F_x \Sigma_{u_x} F_x^T$

(Note $X \neq x$)
$X = [x\ y\ \theta]^T$

$$\bar{\Sigma}_1 = G_1 \Sigma_0 G_1^T + R_1$$

Σ_0 given (diag{1, 1, 0.1})

$$G_k = \left.\frac{\partial g(u_k, x_{k-1})}{\partial x_{k-1}}\right|_{\mu_{k-1}} = \left[\left(\frac{\partial x_k}{\partial x_{k-1}}\right) \left(\frac{\partial x_k}{\partial y_{k-1}}\right) \left(\frac{\partial x_k}{\partial \theta_{k-1}}\right)\right]$$

$$= \left[\begin{pmatrix}1+0\\0+0\\0+0\end{pmatrix} \begin{pmatrix}0+0\\1+0\\0+0\end{pmatrix} \left(\begin{bmatrix}0\\0\\1\end{bmatrix} + dt\begin{bmatrix}-\sin\theta_{k-1} & 0\\ \cos\theta_{k-1} & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}V_k\\ \omega_k\end{bmatrix}\right)\right]\Bigg|_{\mu_{k-1}}$$

$$\boxed{G_k = \begin{bmatrix}1 & 0 & -(dt)\sin\mu_{\theta_{k-1}}(V_k)\\ 0 & 1 & +(dt)\cos\mu_{\theta_{k-1}}(V_k)\\ 0 & 0 & 1\end{bmatrix}}$$

μ_{k-1}

So, G_1 can be found as μ_{θ_0}, V_1, ω, are known.

Don't get confused: Say a variable x & its mean μ_x are sometimes written interchangeably. For example,

$[\mu_{\theta_0}, V_1, \omega_1]$ is the same as $[\theta_0, \mu_{V_1}, \mu_{\omega_1}]$, ...

Now, $R_k = F_k \, \Sigma_{u_k} \, F_k^T$

$$\boxed{\Sigma_{u_k} = \begin{bmatrix} v_var & 0 \\ 0 & om_var \end{bmatrix}}$$

(Using code notation)

$$F_k = \left.\frac{\partial g(u_k, x_{k-1})}{\partial u_k}\right|_{\mu_{u_k}} = \left.\begin{bmatrix} \left(\frac{\partial x_k}{\partial v_k}\right) & \left(\frac{\partial x_k}{\partial \omega_k}\right) \end{bmatrix}\right|_{\mu_{u_k}}$$

$$F_k = \begin{bmatrix} \begin{pmatrix} 0 + (dt)\cos\theta_{k-1} \\ 0 + (dt)\sin\theta_{k-1} \\ 0 + 0 \end{pmatrix} & \begin{pmatrix} 0 + 0 \\ 0 + 0 \\ 0 + (dt) \end{pmatrix} \end{bmatrix}$$

$$\boxed{F_k = \begin{bmatrix} (dt)\cos\mu_{\theta_{k-1}} & 0 \\ (dt)\sin\mu_{\theta_{k-1}} & 0 \\ 0 & (dt) \end{bmatrix}}$$

So, F_1 can be found as μ_{θ_0} (or θ_0) is known.

R_1 can be found using above Σ_{u_1} & F_1 as follows:

$$R_k = F_k \Sigma_{u_k} F_k^T$$

$$= \begin{bmatrix} (dt)\cos\theta_{k-1} & 0 \\ (dt)\sin\theta_{k-1} & 0 \\ 0 & (dt) \end{bmatrix} \begin{bmatrix} v_var & 0 \\ 0 & om_var \end{bmatrix} \begin{bmatrix} (dt)\cos\theta_{k-1} & (dt)\sin\theta_{k-1} & 0 \\ 0 & 0 & (dt) \end{bmatrix}$$

$$= \left[\begin{array}{c|c|c} \left((dt)\cos\theta_{k-1}\right)^2 v_var & (dt)^2 \sin\theta_{k-1}\cos\theta_{k-1}\, v_var & 0 \\ (dt)^2 \sin\theta_{k-1}\cos\theta_{k-1}\, v_var & \left((dt)\sin\theta_{k-1}\right)^2 v_var & 0 \\ 0 & 0 & om_var \end{array}\right]$$

R_k

... (R_k), $(G_k \,\&\, \Sigma_{k-1})$, $\bar{\Sigma}_k$ .....

$\boxed{\text{Step 3 - DONE}}$ From R_1, $(G_1 \,\&\, \Sigma_0)$, $\bar{\Sigma}_1$ can be found.

· Correction : i) $z_t = h(x_t) + \delta_t$ ii) $h(x_t) \approx h(\bar{\mu}_t) + \underbrace{\frac{\partial h(\bar{\mu}_t)}{\partial x_t}}_{=:\, H_t} (x_t - \bar{\mu}_t)$

Remaining Steps

$$
\begin{aligned}
&4: \quad K_t = \overset{3\times3}{\bar{\Sigma}_t}\, \overset{3\times2}{H_t^T} (\overset{2\times3}{H_t}\, \overset{3\times3}{\bar{\Sigma}_t}\, \overset{3\times2}{H_t^T} + \overset{2\times2}{Q_t})^{-1} \\
&5: \quad \mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t)) \\
&6: \quad \Sigma_t = (I - K_t\, H_t)\, \bar{\Sigma}_t \\
&7: \quad \text{return } \mu_t, \Sigma_t
\end{aligned}
$$

Correction

$$
z_t = h(x_t) = \begin{bmatrix} r_k^l \\ \phi_k^l \end{bmatrix} = \begin{bmatrix} \overbrace{\sqrt{(x_l - x_k - d\cos\theta_k)^2 + (y_l - y_k - d\sin\theta_k)^2}}^{\sqrt{\beta^2+\alpha^2}} \\ \operatorname{atan2}(\underbrace{y_l - y_k - d\sin\theta_k}_{\alpha}, \underbrace{x_l - x_k - d\cos\theta_k}_{\beta}) - \theta_k \end{bmatrix} + \mathbf{n}_k
$$

$\boxed{\text{Step 4.}}$ $K_k = \bar{\Sigma}_k H_k^T (H_k \bar{\Sigma}_k H_k^T + Q_k)^{-1}$

$$
H_k = \frac{\partial h(x_k)}{\partial x_k}\Bigg|_{\bar{\mu}_k} = \left[\frac{\partial h(x_k)}{\partial x_k}, \frac{\partial h(x_k)}{\partial y_k}, \frac{\partial h(x_k)}{\partial \theta_k} \right]
$$

$$
H_k^{\cancel{l}} = \begin{bmatrix} \frac{\partial r_k^l}{\partial x_k} & \frac{\partial r_k^l}{\partial y_k} & \frac{\partial r_k^l}{\partial \theta_k} \\ \frac{\partial \phi_k^l}{\partial x_k} & \frac{\partial \phi_k^l}{\partial y_k} & \frac{\partial \phi_k^l}{\partial \theta_k} \end{bmatrix}_{2\times3} \Bigg|_{\bar{\mu}_k}
$$

$\bar{\mu}_k$ from Step 2. Let $\alpha = y_l - y_k - d\sin\theta_k$ & $\beta = x_l - x_k - d\cos\theta_k$

$$
H_k^l = \begin{bmatrix} \frac{-\beta}{\sqrt{\alpha^2+\beta^2}} & \frac{\alpha}{\sqrt{\alpha^2+\beta^2}} & \frac{\beta d\sin\theta_k - \alpha d\cos\theta_k}{\sqrt{\alpha^2+\beta^2}} \\ \frac{\alpha}{\sqrt{\alpha^2+\beta^2}} & \frac{-\beta}{\sqrt{\alpha^2+\beta^2}} & -\left(\frac{\beta d\cos\theta_k + \alpha d\sin\theta_k}{\alpha^2+\beta^2} + 1\right) \end{bmatrix}
$$

$$Q_k = \begin{bmatrix} r\text{-var} & 0 \\ 0 & b\text{-var} \end{bmatrix}$$ (Notation from code)

Therefore, we have H_1, Q_1, $\bar{\Sigma}_1$ (prev-step), we can find K_1

$\boxed{\text{Step 4 - DONE}}$

(3×34) 3×2 (34×1) (3×1) $34 = 17\times 2$

$\boxed{\text{Step 5.}}$ $$\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t))$$

$$z_t = \begin{bmatrix} r \\ b \end{bmatrix}_{2(34)}$$ (Look notation of variables, See code for description)

$$h(x_k = \bar{\mu}_k) \begin{bmatrix} r_k^l \\ \phi_k^l \end{bmatrix} = \begin{bmatrix} \sqrt{(x_l - x_k - d\cos\theta_k)^2 + (y_l - y_k - d\sin\theta_k)^2} \\ \operatorname{atan2}(y_l - y_k - d\sin\theta_k, x_l - x_k - d\cos\theta_k) - \theta_k \end{bmatrix} + \mathbf{n}_k \quad \text{I}$$

We get $\bar{\mu}_1$ from Step 2. (or $\bar{\mu}_k = \bar{\mu}_{x_k}, \bar{\mu}_{y_k}, \bar{\mu}_{\theta_k}$)

↳ we know all 0 in I

⇒ d is given = 0.21901627.

⇒ x_l y_l are the locations of landmarks which can be obtained from "l" array (17×2).

Therefore, we know all values on RHS of (I), so $h(\bar{\mu}_t)$ can be found out.

So, we now know, $\bar{\mu}_t$, K_t, z_t, & $h(\bar{\mu}_t)$ which means μ_t can be found out (or μ_1)

$\boxed{\text{Step 5 - DONE}}$

$\boxed{\text{Step 6 - DONE}}$: $\Sigma_t = (I - K_t H_t)\bar{\Sigma}_t$

K_t, H_t & $\bar{\Sigma}_t$ have already been obtained in steps 4 & 3.

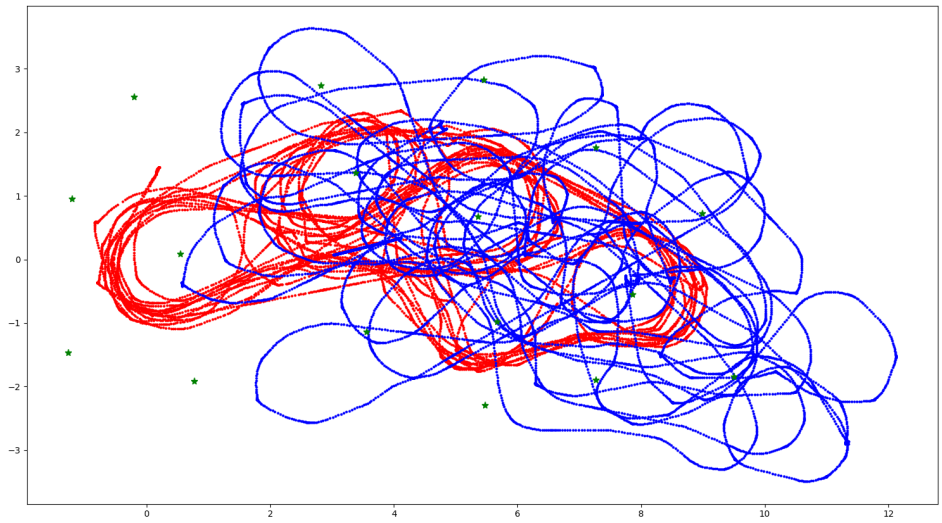
Therefore, Σ_1 can be found out.

$\boxed{\text{Step 7.}}$

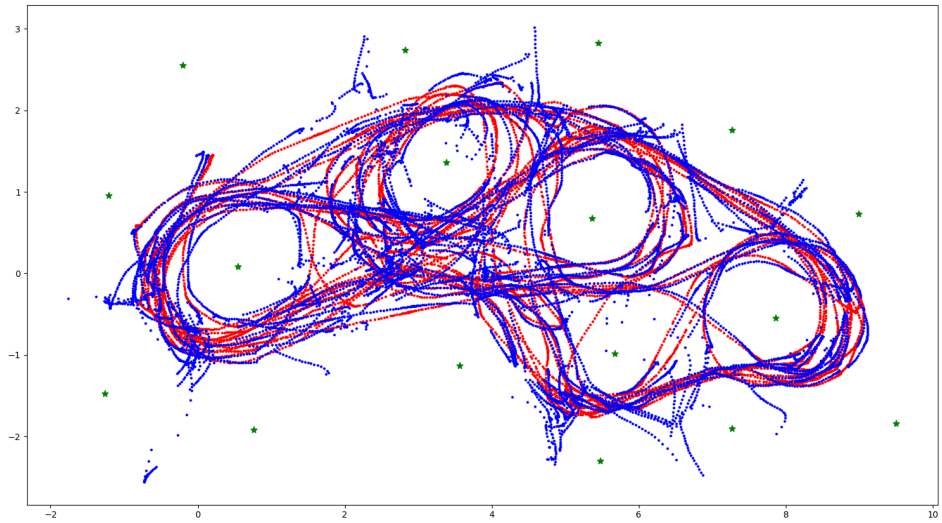
Alas! We now return μ_1 & Σ_1 & repeat all the steps 2 to 7. to find μ_2 & Σ_2, μ_3 & Σ_3 and so on. EKF thus recursively estimates & updates our state.

⇒ RESULTS:

1. GroundTruth versus Noisy Trajectory. Groundtruth is shown in red trajectory while blue is the noisy trajectory. Green stars are landmarks location.



2. GroundTruth versus EKF Trajectory. Groundtruth is shown in red trajectory while blue is the EKF trajectory. Green stars are landmarks location.

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