Transportation Problems with Multiple Commodities

Transportation Problem and Optimization Techniques

An In-depth Analysis and Solution Modeling

MAT 3991 – Seminar

Group - 05

S/19/SP/871 , S/19/825 , S/19/844 , S/19/815 , S/19/868 , S/19/802 , S/19/867

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CHAPTER 01

INTRODUCTION

The transportation problem is one of the classic linear programming problems concerned with the most efficient way of distributing a product or service. In addition, it has become crucial in the balance between the supply and demand in manufacturing supply chains. Manufacturing firms today face greater competition, and therefore they need an effective logistics system because the markets have become global. From the sourcing of raw materials from suppliers through the transformation process into finished goods, to delivering these products to warehouses and end-users, optimization at each of these steps is key concerning competitive advantage.

Operations researchers have contributed to the innovation, analysis, and implementation of strategies that address these challenges over the years. This area of study mostly encompasses supply chain network design, a critical area of research that investigates the integration of suppliers, manufacturers, warehouses, and retailers. The primary goal is to deliver goods in the right quantities, to the right locations, at the right time—while minimizing total transportation costs and meeting service level expectations.

Facility location problems are a significant subset of supply chain management that has been studied for quite a few decades. The general problem at hand is to find optimal locations for production facilities, warehouses, or distribution centers that achieve overall cost-effectiveness and operational efficiency. No matter how advanced the production processes, transportation systems, or inventory management techniques may be, long-term inefficiencies are inevitable when location choices are poor. Therefore, facility location decisions are important while designing an effective supply chain to consider uncertainties for the future as well.

The multi-commodity distribution network is an extension of the facility location problem, incorporating multiple products with their flow paths. Goods are moved in these systems from manufacturing plants to customers through intermediate distribution centers, besides adding further layers of complication. The design for such networks inherently needs to make a balance between fixed costs related to facility operation, variable costs concerning material flow, and transportation costs across several commodities.

In order to avoid these problems, many manufacturing enterprises apply optimization approaches such as Benders Decomposition. This approach represents an effective application in the solution of big-size, multi-commodity distribution network design problems. It gives a possibility to minimize a combined cost function concerning fixed facility costs and flow costs over the distribution network and transportation. The model provides a contribution towards an efficient, agile supply chain system capable of meeting the demands imposed by contemporary manufacturing and distribution.

1.1 Literature Review

MCTP is an intrinsic optimization problem within the context of logistics and supply chain management, which basically intends to efficiently transfer different types of commodities from diverse sources to varied destinations, considering capacity and cost constraints. It is challenging due to its property of flows of different commodities, which should be managed at the same time across a common network. MILP provides a potent modeling framework for the MCTP, as complex discrete decisions regarding facility location or route selection can be modeled in concert with continuous flow variables. However, this typically also results in computational intractability, especially for instances with a large number of points, and requires the development of specialized solution techniques.

Early research laid the basic foundation needed for modeling and solving MCTP. Geoffrion and Graves (1974) presented a multi-commodity capacitated distribution problem as a MILP and used Benders' decomposition. While not a Branch and Bound or a Cutting Plane method, the principle of problem decomposition is very related to how Branch and Bound works. By decomposing the problem into location and transportation subproblems, similar to branching, it enables the solution of the individual components much more efficiently. This pioneering work threw light on structured approaches to resolve the computational intensiveness of the MCTP.

(Hemmecke, Onn and Weismantel, 2009)proposes a new approach by considering even a much simple problem, viz., several commodities produced in multiple plants and dispersed to several destinations based on several demands. He has developed such a model based on multi-commodity transportation with supply chain involving step-wise cost. In this thesis, the new model is tested on the multi-commodity transportation problem of SAB Miller Europe and compared with the other methods from the literature. Also, the feasibility checking method has been developed for large-scale mixed-integer linear programming problems having binary variables in the objective function. The specialty of the model developed by them is the fixed cost in the objective function can be expressed as a stepwise constant function. The developed model is a mixed integer linear programming model.

Besides above studies, (Afshari et al., 2014) have presented a multi-objective mixed integer linear programming formulation for location within network distribution problem. For the development of this model, some assumptions have been considered, including: there are two potential central warehouses that at least one of them should be located, limited capacities for both central and regional warehouses, the transportation cost per unit is as a coefficient of distance between central and regional warehouses and between regional warehouses and customers. There is a minimum level of customer satisfaction. Two objectives have been considered by them. Minimizing the total cost, including establishment and transportation cost, and maximizing customer satisfaction. The combinatorial nature inherent in MILP problems, especially in the context of MCTP, demands the use of specialized algorithms such as Branch and Bound. It works by systematically exploring the solution space: recursively partitioning the problem into smaller subproblems (branching), and computing bounds on the optimal solution for each such subproblem (bounding). By solving LP relaxations of such subproblems, the algorithm effectively prunes portions of the search tree that cannot contain improved integer solutions. While guaranteed to provide an optimal solution, the computational cost of Branch and Bound can become prohibitive for large MCTP instances, and therefore enhancements, including cutting plane methods, have been developed. The cutting plane method aims at tightening the LP relaxation of MCTP by adding valid inequalities or cuts in such a way that no feasible integer solution will be cut off.

It cuts off a number of fractional solutions that are present in LP relaxation but are not there in the integer feasible region. There are many types of cuts, such as Gomory cuts (Gomory, 1958), Mixed-Integer

Rounding (MIR) cuts (Nemhauser and Wolsey, 1988)**,** and Lift-and-Project cuts (Balas, Sebastián Ceria and Gérard Cornuéjols, 1993), among others, each with different strengths and computational costs. The procedure works by iteratively adding violated cuts and resolving the LP relaxation. This progressively refines the feasible region and speeds up convergence to an integer solution. The most effective solution approach for large-scale MCTP instances generally integrates Branch and Bound with the Cutting Plane method within a Branch-and-Cut framework. It thereby combines the systematic search by Branch and Bound with the strengthening of the LP relaxation provided by the cutting planes.

On each node of the Branch and Bound tree, the cutting plane methods are applied to strengthen the LP relaxation, yielding tighter bounds and, hence, more effective pruning. An integrated approach combines the best of both techniques in an effort toward the solution of challenging real-world instances of MCTP. Future research continues to explore new cutting plane families, improved branching strategies, and specialized algorithms tailored for specific MCTP structures in the pursuit of even better solution efficiency. The MCTP is a basic optimization problem in logistics and supply chain management; it involves the problem of transporting a number of different commodities from various sources to different destinations with a pre-specified structure and cost of the transport network in the most efficient way. This is complex because of the need to handle various commodity flows on one network.

MILP is a powerful framework for modeling MCTP, since it allows the representation of discrete decisions - such as facility location or route selection - together with continuous flow variables. These formulations typically yield computationally intractable problems for large-sized instances and require the development of specific solution techniques. Early research laid the foundation to model and solve MCTP. An example for a multi-commodity capacitated distribution problem using MILP, coupled with Benders' decomposition was given by Geoffrion and Graves (1974).

Although different from Branch-and-Bound or the Cutting Plane approach per se, yet Benders's decomposition shares something critical - decomposing the original problem into other problems, say location and transport sub-problem, is central to how in principle Branch-and-Bound works. Here, the division of the major problem into more 'sub-' problems enables the resolution of the parts separated in this way to be done more effectively. This pioneering work underlined the necessity of structured approaches in order to cope with the computational burden of MCTP. (Lelkes et al., 2005)has developed a new model by considering much simpler problem where several commodities produced in several plants, distributed to several destination sites according to demands. It is developed for multi-commodity transportation and supply chain problems, which include stepwise constant cost.

In this study, it compared the literature methods by one new model, tested in the case of SAB Miller Europe of multi-commodity transportation problems; besides that a method to be used in large-scaled M ILP feasibility problems that contain binary variables in an objective function were also developed. Specialty of the model developed by them is the fix cost in the objective can be expressed as a stepwise constant function. This is a mixed integer linear programming model. Along with the above studies (Afshari et al., 2014)presented a multi-objective mixed integer linear programming formulation for location within network distribution problem. Some assumptions have been considered for the development of this model: there are two potential central warehouses that at least one of them should be located; there is limited capacity for both central and regional warehouses. The transportation cost per unit is a coefficient of distance between central and regional warehouses and between regional warehouses and customers. Besides, there is a minimum level of customer satisfaction. They have considered two objectives:. These could be minimization of total cost, which in turn would amount to summing up the establishment of warehouses along with transportation costs; and on the other hand, customer satisfaction should also be maximized. Intrinsically combinatorial, the approaches, particularly concerning MCTP have to use special algorithms, such as the Branch and Bound approach. It performs a systematic search of the solution space by recursively partitioning the problem into smaller subproblems. The algorithm performs efficient pruning of parts of the search tree that cannot yield any better integer solutions, as determined through the solving of LP relaxations of the subproblems. While guaranteed to find the optimal solution, the computational cost of Branch and Bound may become prohibitive for large-sized MCTP instances, hence motivating enhancements such as cutting plane methods. The idea of the Cutting Plane methods is to strengthen the LP relaxation of the MCTP by adding valid inequalities- cuts-that tighten the feasible region without excluding any feasible integer solution. This cut actually "cuts off" the fractional solutions present in the LP relaxation yet not present within the integer feasible region. Different types of cuts exist, for example, Gomory cuts (Gomory, 1958), MIR cuts, Mixed-Integer Rounding, (Nemhauser and Wolsey, 1988) and Lift-and-Project cuts (Balas, Sebastián Ceria and Gérard Cornuéjols, 1993)**,** with each having different strengths and computational costs.

Iteratively adding violated cuts and resolving the LP relaxation progressively refines the feasible region, yielding faster convergence to an integer solution. In practice, the most effective method to solve large-scale instances of MCTP is often embedding the systematic search of Branch-and-Bound within the strengthening of the LP relaxation provided by the cutting planes in a Branch-and-Cut framework. Methods for the cut plane in every node of the Branch and Bound tree are used to tighten the LP relaxation; hence, tighter bounds are obtained that allow for more effective pruning. It is an integrated approach wherein the power of both techniques is combined to provide an effective tool for solving complex real-world MCTP instances.

Future research continues to explore new cutting plane families, improved branching strategies, and specialized algorithms tailored to specific MCTP structures that will further enhance the efficiency of the solutions.

1.2 Objective of the study

The objective of this work is to determine the minimum total transportation cost of a multi-commodity transportation network. A mathematical model is formulated according to the case study, and a computer code is developed using python.

1.3 Outline of the report

The present thesis contains four chapters in total. Chapter 1 introduces the problem with a literature review, focusing on previous techniques and studies done concerning transportation problems.

Chapter 2 presents the proposed mathematical model of the multi-commodity transportation problem and its solution procedure in a programming language called python.

Chapter 3 contains the result of case study using python (given in Chapter 2) of transportation problem. Case study is discussed in this report is pertained to the real data used in Nesto Confectionery Lanka (Pvt) Ltd located in Kandy, Sri Lanka.

Finally, Chapter 4 summarizes the discussion and conclusion.

CHAPTER 02

MATERIALS AND METHODS

2.1 Network Optimization

Real-world physical networks, such as highways, telephone lines, electric grids, water supply systems, and railway systems, are common examples of networks that facilitate the movement of goods, services, or information. In these networks, the primary objective is often to transport products or resources from one location to another while minimizing total transportation costs or travel distances. This would include route finding with the least length or flow pattern optimization in order to attain minimum cost. Network optimization has become one of the cornerstone problems in operations research and extends its influence into fields such as computer science, applied mathematics, engineering, and management.

A network flow problem is typically represented as a system of nodes connected by arcs. These nodes serve distinct roles based on the type of network:

- **Supply nodes:** Nodes where resources or goods originate.
- **Demand nodes:** Nodes where resources or goods are consumed or required.
- **Transshipment nodes:** Intermediate nodes through which resources or goods pass, facilitating movement between supply and demand nodes.

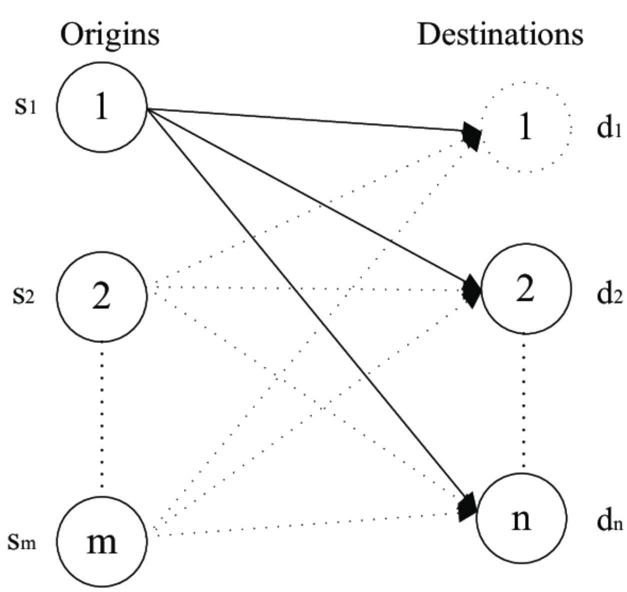


Figure 1 : Network flow

Let G = (N, A) be a directed network defined by a set N of m nodes, and a set A of n directed arcs. Each arc (i, j) in A has an associated cost per unit flow on that arc. Assume that the flow cost varies linearly with the amount of flow. Each arc (i, j) in A also has a maximum and minimum capacity U_{ij} and L_{ij} respectively that can flow on the arc. Supply or demand in each node i can be denoted by integer b(i). If b(i) > 0, node i is a supply node; if b(i) < 0, node i is a demand node; and if b(i) = 0, then node i is a transshipment node.

2.2 Transportation problem

The transportation problem is a significant area within operations research, focusing on optimization. It involves a linear objective function and linear constraints, aiming to determine the optimal quantities to transport from each source to each destination to achieve the minimum total transportation cost. This problem plays a crucial role in supply chain management and logistics, ensuring cost-effective distribution.

The transportation problem has two primary objectives: minimizing shipping costs and maximizing shipping profits between sources and destinations. When the transportation costs for each source-destination pair are known, the problem can be solved to identify the optimal routes that minimize the total transportation cost.

There are two main categories of transportation problems:

- **Balanced Transportation Problem:** This occurs when the total supply equals the total demand, making it easier to allocate resources without adjustments.
- **Unbalanced Transportation Problem:** This arises when total supply does not match total demand. To solve such problems, they are converted into balanced transportation problems by introducing dummy supply nodes or dummy demand nodes to equalize supply and demand.

2.2.1 Mathematical Formulation of Transportation Problem

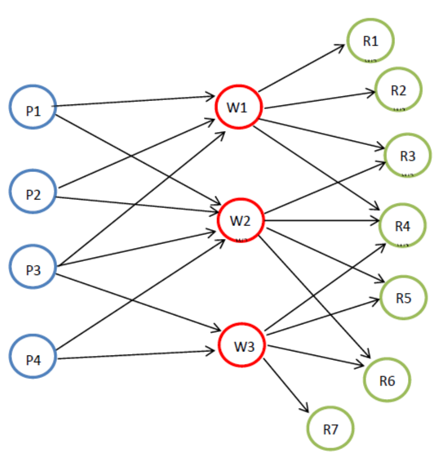


Figure 2: Transportation network with demand and supply

Let, x_{ij}= number of quantity can be transported from i^{th} source to j^{th} destination

c_{ij} = cost of transportation of one unit from i^{th} source to j^{th} destination,

s_i = available amount of i^{th} source

d_j = required amount in j^{th} destination

	1	2	$\cdots$	$\cdots$	n	Supply
1	x_{11} c_{11}	x_{12} c_{12}	$\cdots$ $\cdots$	$\cdots$ $\cdots$	x_{1n} c_{1n}	a_1
2	x_{21} c_{21}	x_{22} c_{22}	$\cdots$ $\cdots$	$\cdots$ $\cdots$	x_{2n} c_{2n}	a_2
3	$\cdots$ $\cdots$	$\cdots$ $\cdots$	$\cdots$ $\cdots$	$\cdots$ $\cdots$	$\cdots$ $\cdots$	a_3
	$\cdots$ $\cdots$	$\cdots$ $\cdots$	$\cdots$ $\cdots$	$\cdots$ $\cdots$	$\cdots$ $\cdots$	
m	x_{m1} c_{m1}	x_{m2} c_{m2}	$\cdots$ $\cdots$	$\cdots$ $\cdots$	x_{mn} c_{mn}	a_m
Demand	b_1	b_2	$\cdots$	$\cdots$	b_n	

Figure 3: Transportation table with demand and supply

Mathematical model can be formulated as follows to determine minimum total transportation cost:

Case 1: (Balanced transportation problem)

$$\text{Minimize } z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \qquad (1)$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i \text{ , } i = 1,2,..m \qquad (2)$$

$$\sum_{i=1}^{m} x_{ij} = b_j \text{ , } j = 1,2,\ldots n \qquad (3)$$

$$x_{ij} \geq 0 \text{ for all } i, j \qquad (4)$$

A necessary and sufficient condition for the existence of a feasible solution for the transportation problem is that the total supply is equal to the total demand.

$$\textbf{i.e.} \sum_i^m a_i = \sum_j^n b_j.$$

This implies that the problem is balanced transportation problem.

Here, equation (1) represents the objective function of the problem which indicates that the total cost of the transportation is to be minimized. The constraints (2) and (3) represent the availability and the requirement of i th source to jth destination respectively. The negative quantity cannot be supplied from sources to destinations. Constraint (4) gives the non-negativity condition.

Case 2: (Unbalanced transportation Problem)

Case (i) Total supply < Total demand $\left(\sum_i^m a_i < \sum_j^n b_j\right)$

$$\text{Minimize } z = \sum_{i=1}^m \sum_{j=1}^n c_{ij}x_{ij} \qquad (5)$$

Subject to,

$$\sum_{j=1}^n x_{ij} = a_i \text{ , } i = 1,2,..m \qquad (6)$$

$$\sum_{i=1}^m x_{ij} \leq b_j \text{ , } j = 1,2,\dots n \qquad (7)$$

$$x_{ij} \geq 0 \text{ for all } i,j \qquad (8)$$

Convert the problem into balanced transportation problem by adding dummy supply

$\left(\sum_j^n b_j - \sum_i^m a_i\right)$ quantity with zero transportation cost.

Case (ii) Total supply > Total demand ($\sum_i^m a_i > \sum_j^n b$ **)**

$$\text{Minimize } z = \sum_{i=1}^m \sum_{j=1}^n c_{ij}x_{ij} \qquad (1)$$

Subject to,

$$\sum_{j=1}^n x_{ij} \leq a_i \text{ , } i = 1,2,..m \qquad (2)$$

$$\sum_{i=1}^m x_{ij} = b_j \text{ , } j = 1,2,\dots n \qquad (3)$$

$$x_{ij} \geq 0 \text{ for all } i,j \qquad (4)$$

Here, quantity of supply is greater than the quantity demanded. This can be converted to a balanced problem by adding dummy column whose quantity will be $\sum_i^m a_i - \sum_j^n b_j$.

2.2.2 Solution procedure of Balanced Transportation Problem

Transportation problem can be solved using two stages; determining initial basic feasible solution and then determine the optimal solution.

There are three methods to find initial basic feasible solution:

- Northwest corner rule
- Minimum cost method
- Vogel's approximation method/Regret method

Optimal solution can be find using Stepping stone method and Modified Distribution Method or U-V Method.

2.3 Multi-commodity transportation problem

The multi-commodity transportation problem involves the movement of multiple commodities from production plants to customers through intermediate distribution centers. This problem is a critical aspect of supply chain and logistics optimization, where the goal is to efficiently allocate resources and minimize costs.

The main objective of the multi-commodity transportation problem is to identify the distribution centers to be utilized while ensuring the following criteria are met:

- Customer Demand Fulfillment: All customer demands for the various commodities are completely satisfied.
- Production Capacity Constraints: The production limits at the plants are adhered to without exceeding their capacities.
- Cost Optimization: The total cost, comprising the fixed costs of operating the distribution centers and the variable transportation costs, is minimized.

2.3.1 Mathematical Model of Multi-commodity transportation problem

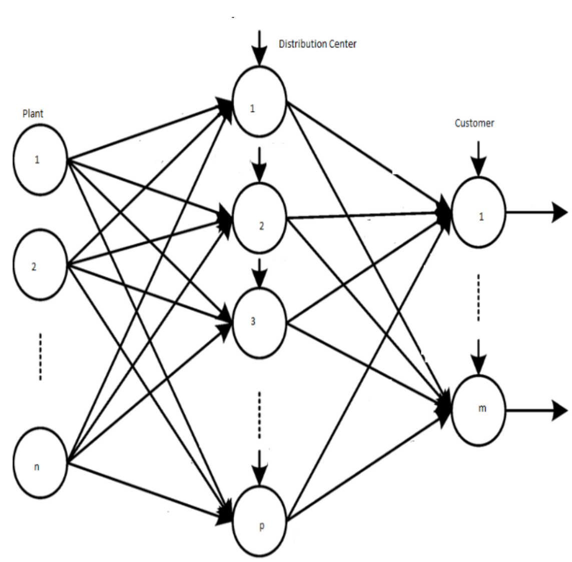


Figure 4: A Multi-commodity Distribution Network

Objective function of multi-commodity transportation problem consists with three parts; minimizing total distribution cost, annual operating cost and the cost of flow through the distribution center.

Total distribution cost is the cost of producing and shipping i th commodity from j th plant to l th customer via k^{th} distribution center.

i.e. $\sum_{i=1}^{q}\sum_{j=1}^{n}\sum_{k=1}^{p}\sum_{l=1}^{m} c_{ijkl}\, x_{ijkl}$

Where,

c_{ijkl} : Unit cost of producing and shipping i^{th} commodity from j^{th} plant to customer l via k^{th} distribution center.

x_{ijkl}: Amount of i^{th} commodity shipped from j^{th} plant to customer l via k^{th} distribution center.

Annual operating cost can be defined as follows:

$$\sum_{k=1}^{p} f_k z_k$$

Where,

f_k : Fixed annual operating cost of the k^{th} distribution center.

$$z_k = \begin{cases} 1, & \text{if } k^{th} \text{ distribution center is opened} \\ 0, & \text{otherwise} \end{cases}$$

Here, operating cost is added only if k th distribution center is open. So z_k is a binary variable.

The cost of flow through the distribution center can be defined as follows:

If k^{th} distribution center serves to the l^{th} customer and if the demand of i^{th} commodity to l^{th} customer is given by D_{il};

$$\textit{i.e}\ \sum_{k=1}^{p}\sum_{i=1}^{q}\sum_{i=1}^{m} v_k\, D_{il} y_{kl}$$

Where,

D_{il} : Demand of i^{th} commodity by l^{th} customer

v_k : Unit cost of flow through the k^{th} distribution center

$$y_{kl} = \begin{cases} 1\,, if\ k^{th} distribution\ center\ surves\ l^{th} customer \\ 0\,, \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad otherwise \end{cases}$$

Objective function can be written as follows

Minimize $\sum_{i=1}^{q}\sum_{j=1}^{n}\sum_{k=1}^{p}\sum_{l=1}^{m} \boldsymbol{c_{ijkl}\, x_{ijkl}} + \sum_{k=1}^{p} \boldsymbol{f_k z_k} + \sum_{k=1}^{p}\sum_{i=1}^{q}\sum_{i=1}^{m} \boldsymbol{v_k\, D_{il} y_{kl}}$

Following requirements should be satisfied to reach to the objective function:

- Number of units of i^{th} commodity that can be transported from j^{th} plant to l^{th} customer through the k^{th} distribution cannot exceed the supply of each commodity.

  i.e. $\sum_{k=1}^{p}\sum_{l=1}^{m} x_{ijkl} \leq S_{ij}$ ***for all i,j***

  Where,

  x_{ijkl}: Amount of i^{th} commodity shipped from j^{th} plant to l^{th} customer via i^{th} k^{th} distribution center.

  S_{ij} : Production capacity of i^{th} commodity at j^{th} plant.

- Number of amounts of i^{th} commodity shipped from j^{th} plant to l^{th} customer via k^{th} distribution center must be equal to the amount of that i^{th} commodity produced at all plants that is destined for that customer and shipped via that distribution center. This constraint forces x_{ijkl} to take the value 0,if

  y_{kl} =0. That is whenever y_{kl} =0, l^{th} customer is not served by k^{th} distribution center.

  i.e $\sum_{j=1}^{n} \boldsymbol{x_{ijkl}} = \boldsymbol{D_{il} y_{kl}}$ ***(all i,k,l)***

  Where,

  D_{il} : Demand for i^{th} commodity by l^{th} customer.

  $$y_{kl} = \begin{cases} 1\,, if\ k^{th} distribution\ center\ surves\ l^{th} customer \\ 0\,, \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad otherwise \end{cases}$$

- Each customer is served by only one distribution center.

  i.e $\sum_{k=1}^{p} \boldsymbol{y_{kl}} = \mathbf{1}$ ***(l = 1,…,m)***

Where,

$$y_{kl} = \begin{cases} 1, if\ k^{th} distribution\ center\ surves\ l^{th} customer \\ 0, \quad otherwise \end{cases}$$

- Flow through k^{th} distribution center, if open (that is $z_k = 1$) is between the lower bound L_k and the upper bound U_k. That ensure the logical relationship between the y and z variables; that is $z_k = 1$ if and only if y_{kl} =1 for some *l*.

  i. e $\boldsymbol{L_k z_K \leq \sum_{i=1}^{q} \sum_{l=1}^{m} D_{il}\, y_{kl} \leq U_k z_k}$ ***(k=1,..,p)***

  Where,

  L_k, U_k :Minimum and maximum allowable annual flow through the k^{th} distribution center.

  D_{il} : Demand for i^{th} commodity by l^{h} customer.

  $$y_{kl} = \begin{cases} 1, if\ k^{th} distribution\ center\ surves\ l^{th} customer \\ 0, \quad otherwise \end{cases}$$

  $$z_k = \begin{cases} 1, if\ k^{th} distribution\ center\ is\ opened \\ 0, \quad otherwise \end{cases}$$

- Amount of commodity shipped to customer via distribution center cannot be negative.

  i.e $\boldsymbol{x_{ijkl} \geq 0}$ ***(all i,j,k,l)***

  Where,

  x_{ijkl} : Number of amount of i^{th} commodity shipped from i^{th} plant to l^{th} customer via k^{th} distribution center.

- y_{kl} is a binary variable. y_{kl} =1 if customer is served by the distribution center k.

  i.e $\boldsymbol{y_{kl} \geq 0\ or\ 1}$ ***(all k,l).***

  Where,

  $$y_{kl} = \begin{cases} 1, if\ k^{th} distribution\ center\ surves\ l^{th} customer \\ 0, \quad otherwise \end{cases}$$

- z_k is also a binary variable. If $z_k = 1$, k^{th} distribution center is opened on that site and if $z_k = 0$, there is no a distribution center.

  i.e $\boldsymbol{z_k \geq 0}$ ***or 1 (k=1,…,p)***

  Where,

  $$z_k = \begin{cases} 1 \ , if\ k^{th} distribution\ center\ is\ opened \\ 0 \ , \qquad\qquad\qquad\qquad\qquad\qquad otherwise \end{cases}$$

2.3.2 Formulated mathematical model for multi-commodity transportation problem

Minimize $\sum_{i=1}^{q}\sum_{j=1}^{n}\sum_{k=1}^{p}\sum_{l=1}^{m} c_{ijkl}\, x_{ijkl} + \sum_{k=1}^{p} f_k z_k + \sum_{k=1}^{p}\sum_{i=1}^{q}\sum_{i=1}^{m} v_k\, D_{il} y_{kl}$

Subject to,

$$\sum_{k=1}^{p}\sum_{l=1}^{m} x_{ijkl} \leq S_{ij} \ \text{(alli,j)}$$

$$\sum_{j=1}^{n} x_{ijkl} = D_{il} y_{kl} \qquad \text{(all i,k,l)}$$

$$\sum_{k=1}^{p} y_{kl} = 1 \qquad (l = 1,\ldots,m)$$

$$L_k Z_K \leq \sum_{i=1}^{q}\sum_{l=1}^{m} D_{il}\, y_{kl} \leq U_k z_k \qquad (k=1,..,p)$$

$$x_{ijkl} \geq 0 \qquad \text{(all i,j,k,l)}$$

$$y_{kl} = \begin{cases} 1 \, , if\ k^{th} distribution\ center\ surves\ l^{th} customer \\ 0 \, , \qquad\qquad\qquad\qquad\qquad\qquad otherwise \end{cases}$$

$$z_k = \begin{cases} 1 \ , if\ k^{th} distribution\ center\ is\ opened \\ 0 \ , \qquad\qquad\qquad\qquad\qquad\qquad otherwise \end{cases}$$

i : Index for commodities, *i*=1,2,…,q

j : Index for plants, *j*=1,2,…,n

k : Index for distribution centers, *k*=1,2,..,p

l : Index for customers, l =1,2 ,….m

S_{ij} : Production capacity for commodity i at plant j

D_{il} : Demand for i^{th} commodity by l^{th} customer

L_k, U_k : Minimum and maximum allowable annual flow through the k^{th} distribution center

f_k : Fixed annual operating cost of the k^{th} distribution center at site k

v_k : Per unit cost of flow through the k^{th} distribution center

c_{ijkl} : Per unit cost of producing and shipping i^{th} commodity to j^{th} plant to l^{th} customer via k^{th} distribution center

$$y_{kl} = \begin{cases} 1\ , if\ k^{th} distribution\ center\ surves\ l^{th} customer \\ 0\ , \qquad otherwise \end{cases}$$

$$z_k = \begin{cases} 1\ , if\ k^{th} distribution\ center\ is\ opened \\ 0\ , \qquad otherwise \end{cases}$$

CHAPTER 03
Case Study

3.1 Case Study

Nesto Confectionery Lanka (Pvt) Ltd is a private company categorized under Candy and Confectionery-Manufacturers Equipment and located in Kandy, Sri Lanka. It was established in and incorporated in 1985. Company has only one plant which is located at Kundasale, Kandy. Chocolate toffees, wafers biscuit, extruded snacks and many more products are produce in this plant. Case study was conducted using the data collected from the above company.

Figure 5: Image of a store house

This problem involves the transportation of a truckload of their products from a plant to seven distribution centers, and to 15 customers at a minimal transportation cost. Production capacity of the plant is adequate to satisfy their customer demands.

Table 1: Unit cost (in Rupees) of producing and shipping toffees to each customer through each distribution center

	Toffee						
Distribution center	**Kandy**	**Mannar**	**Badulla**	**Galle**	**Colombo**	**Rathnapura**	**Anuradhapura**
Customer							
1	1.27	1.32	1.30	1.30	1.27	1.30	1.30
2	1.27	1.33	1.30	1.30	1.27	1.30	1.30
3	1.29	1.35	1.31	1.32	1.28	1.31	1.31
4	1.27	1.31	1.29	1.30	1.26	1.29	1.30
5	1.60	1.55	1.68	1.69	1.54	1.69	1.57
6	1.34	1.41	1.32	1.34	1.31	1.33	1.35
7	1.35	1.42	1.33	1.35	1.32	1.34	1.36
8	1.41	1.49	1.40	1.38	1.37	1.41	1.49
9	1.40	1.47	1.39	1.37	1.36	1.40	1.48
10	1.36	1.40	1.37	1.36	1.33	1.36	1.37
11	1.37	1.41	1.38	1.37	1.33	1.37	1.37
12	1.33	1.39	1.35	1.34	1.31	1.32	1.39
13	1.35	1.42	1.37	1.36	1.32	1.34	1.42
14	1.35	1.35	1.43	1.37	1.32	1.39	1.33
15	1.35	1.36	1.43	1.38	1.32	1.41	1.34

Table 2: Unit cost (in Rupees) or producing and shipping Chocolates to each customer through each distribution center

	Chocolate						
Distribution center	**Kandy**	**Mannar**	**Badulla**	**Galle**	**Colombo**	**Rathnapura**	**Anuradhapura**
customer							
1	30.60	33.22	31.77	32.13	30.31	31.77	31.77
2	30.71	33.40	31.79	32.20	30.28	31.79	31.89
3	30.86	33.45	31.98	32.43	30.37	32.06	32.03
4	31.11	33.36	32.21	32.72	30.43	32.26	32.31
5	44.15	42.19	47.62	48.31	41.85	48.31	43.23
6	33.89	37.13	33.02	33.93	32.56	33.48	34.11
7	34.78	38.70	33.87	34.91	33.22	34.39	35.30
8	37.33	41.33	36.73	35.83	35.27	37.33	41.33
9	38.17	42.35	37.52	36.61	35.83	38.25	42.87
10	34.93	36.67	35.18	34.76	33.33	34.76	35.18
11	34.56	36.65	34.99	34.63	33.09	34.56	34.59
12	34.07	36.99	34.72	34.42	32.79	33.21	36.87
13	34.91	38.17	35.70	35.43	33.35	34.08	38.20
14	34.36	34.63	38.36	35.35	32.82	36.67	33.25
15	34.35	34.86	38.16	35.76	32.85	37.21	33.58

Table 3 : Unit cost (in Rupees) of producing and shipping Biscuit to each customer through each distribution center

	Biscuit packet						
Distribution center	**Kandy**	**Mannar**	**Badulla**	**Galle**	**Colombo**	**Rathnapura**	**Anuradhapura**
customer							
1	29.73	31.48	30.51	30.75	29.54	30.51	30.51
2	29.83	31.63	30.55	30.83	29.54	30.55	30.62
3	29.76	31.34	30.44	30.72	29.45	30.49	30.47
4	29.83	31.15	30.48	30.78	29.43	30.51	30.54
5	39.42	38.03	41.86	42.35	37.79	42.35	38.77
6	32.23	34.56	31.61	32.26	31.28	31.93	32.39
7	32.16	34.56	31.60	32.24	31.20	31.92	32.48
8	33.85	36.36	33.47	32.91	32.56	33.85	36.36
9	34.36	36.97	33.95	33.38	32.89	34.41	37.30
10	32.59	33.74	32.75	32.47	31.52	32.47	32.75
11	32.23	33.58	32.51	32.28	31.29	32.23	32.25
12	32.24	34.28	32.70	32.49	31.34	31.64	34.20
13	32.54	34.68	33.05	32.88	31.51	31.99	34.70
14	32.37	32.55	35.12	33.05	31.31	33.96	31.61
15	32.41	32.76	35.05	33.39	31.37	34.40	31.87

Table 4: Demand for each commodity by the customer

	Demand		
Customer	**Toffee**	**Chocolate**	**Biscuit**
1	2460000	49400	74190
2	2475000	50000	74286
3	2063000	48200	78571
4	2690000	47000	80000
5	2235000	52000	73571
6	2570000	52600	73207
7	2470000	46000	75000
8	2466800	49500	78929
9	2625000	46000	73571
10	2390000	48450	73214
11	2280500	50650	78500
12	2750000	51400	73571
13	2400000	46000	70064
14	2470000	49800	72500
15	2360000	49500	71286

Table 5 : Capacity of each commodity at each distribution center

Distribution Center	Commodity		
	Toffee	Chocolate	Biscuit
Kandy	10000000	200000	310000
Mannar	2500000	58000	75000
Badulla	5500000	102000	166000
Galle	5500000	100000	160428
Colombo	5200000	110000	160000
Rathnapura	5300000	100000	150000
Anuradhapura	5000000	110000	150000

Table 6 : Fixed cost (in Rupees) at each distribution center and per unit cost of flow through the distribution center

Distribution Center	Toffee	Chocolate	Biscuit	Fixed cost
Kandy	0.58	7.56	7.56	3470869.80
Mannar	0.84	10.20	10.20	2654948.57
Badulla	0.60	7.80	7.80	3921082.71
Galle	0.84	10.20	10.20	5772084.00
Colombo	0.64	8.16	8.16	4952240.57
Rathnapura	0.61	7.92	7.92	4043157.86
Anuradhapura	0.60	7.80	7.80	3963476.57

Table 7 : Capacity of each plant for each commodity

Commodity	Plant
1	39000000
2	780000
3	1120461

With respect to the data gathered from company and to satisfy the company requirements, mathematical model can be formulated as follows:

Total distribution cost is the cost of producing and shipping i th commodity from the plant to l th customer via k th distribution center.

i.e. $\sum_{i=1}^{3}\sum_{k=1}^{7}\sum_{l=1}^{15} c_{ikl}\, x_{ikl}$

Where,

c_{ikl} : Unit cost of producing and shipping i th commodity from the plant to l th customer via k th distribution center.

x_{ijkl}: Amount of i^{th} commodity shipped from j^{th} plant to customer l via k^{th} distribution center.

Annual operating cost can be defined as follows:

$\sum_{k=1}^{7} f_k z_k$

Where,

f_k : Fixed annual operating cost of the k^{th} distribution center.

$$z_k = \begin{cases} 1 \ , if\ k^{th} distribution\ center\ is\ opened \\ 0 \ , \quad otherwise \end{cases}$$

Here, operating cost is added only if k th distribution center is open.

The cost of flow through the distribution center can be defined as follows:

If k^{th} distribution center serves to the l th customer and if the demand of i th commodity to l th customer is given by D_{il} ;

i.e $\sum_{k=1}^{7} \sum_{i=1}^{3} \sum_{l=1}^{15} v_k D_{il} y_{kl}$

Where,

D_{il} : Demand of i^{th} commodity by l^{th} customer

v_k : Unit cost of flow through the k^{th} distribution center

$$y_{kl} = \begin{cases} 1, if\ k^{th} distribution\ center\ surves\ l^{th} customer \\ 0, \quad otherwise \end{cases}$$

Objective function can be written as follows

Minimize$\sum_{i=1}^{3} \sum_{k=1}^{7} \sum_{l=1}^{15} c_{ikl}\, x_{ikl} + \sum_{k=1}^{7} f_k z_k + \sum_{k=1}^{7} \sum_{i=1}^{3} \sum_{l=1}^{15} v_k D_{il} y_{kl}$

According to the company requirements these conditions should be satisfied to reach to the objective function.

- Number of units of i th commodity that can be transported from j th plant to l th customer through the k th distribution cannot exceed the supply of each commodity

  i.e $\sum_{k=1}^{7} \sum_{l=1}^{15} x_{ijkl} \leq s_i$ (all *i,j*)

- Amount of commodity shipped to customer via distribution center must equal the amount of that commodity produced at the plant that is destined for that customer and shipped via that distribution center. This constraint forces x_{ikl} to take the value 0,if y_{kl} =0. That is whenever y_{kl} =0 customer l is not served by distribution center k.

  i.e $x_{ikl} = D_{il} y_{kl}$ (all *i,k,l*)

Where,

D_{il} : Demand for i^{th} commodity by l^{th} customer

$$y_{kl} = \begin{cases} 1, if\ k^{th} distribution\ center\ surves\ l^{th} customer \\ 0, \qquad\qquad\qquad\qquad\qquad\qquad otherwise \end{cases}$$

- Each customer *is* served by only one distribution center.

  i.e $\sum_{k=1}^{7} y_{kl} = 1$ (l = 1,…,*15*)

Where,

$$y_{kl} = \begin{cases} 1, if\ k^{th} distribution\ center\ surves\ l^{th} customer \\ 0, \qquad\qquad\qquad\qquad\qquad\qquad otherwise \end{cases}$$

- Flow through k^{th} distribution center, if open (that is $z_k = 1$) is less than the upper boundU_k. That ensure the logical relationship between the y and z variables; that is $z_k = 1$ if and only if y_{kl} =1 for some l

  i. e $\sum_{i=1}^{3} \sum_{l=1}^{15} D_{il}\, y_{kl} \leq U_k z_k$ (k=1,..,*7*)

Where,

U_k : Maximum allowable annual flow through the k^{th} distribution center.

D_{il} : Demand for i^{th} commodity by l^{h} customer. $y_{kl} = \begin{cases} 1, if\ k^{th} distribution\ center\ surves\ l^{th} customer \\ 0, \qquad\qquad\qquad\qquad\qquad\qquad otherwise \end{cases}$

$$z_k = \begin{cases} 1 \ , if\ k^{th} distribution\ center\ is\ opened \\ 0 \ , \qquad\qquad\qquad\qquad\qquad otherwise \end{cases}$$

- Amount of commodity shipped to customer via distribution center cannot be negative.

  i.e $\boldsymbol{x_{ijkl}} \geq \mathbf{0}$ **(all *i,j,k,l*)**

Where,

x_{ijkl} : Number of amount of i^{th} commodity shipped from i^{th} plant to l^{th} customer via k^{th} distribution center.

- y_{kl} is a binary variable. y_{kl} =1 if customer is served by the distribution center *k*.

  i.e $\boldsymbol{y_{kl}} \geq \mathbf{0} \; \boldsymbol{or} \; \mathbf{1}$ **(all *k,l*)**.

Where,

$$y_{kl} = \begin{cases} 1, if\ k^{th} distribution\ center\ surves\ l^{th} customer \\ 0, \qquad\qquad\qquad\qquad\qquad\qquad\qquad otherwise \end{cases}$$

- z_k is also a binary variable. If $z_k = 1$, k^{th} distribution center is opened on that site and if $z_k = 0$, there is no a distribution center.

  i.e $\boldsymbol{z_k} \geq \mathbf{0} \; \boldsymbol{or} \; \mathbf{1}$ (*k*=1,…,*p*)

Where,

$$z_k = \begin{cases} 1, if\ k^{th} distribution\ center\ is\ opened \\ 0, \qquad\qquad\qquad\qquad\qquad otherwise \end{cases}$$

Then the mathematical model developed to satisfy the conditions of the company transportation problem can be presented as follows,

Minimize$\sum_{i=1}^{3}\sum_{k=1}^{7}\sum_{l=1}^{15} c_{ikl}\, x_{ikl} + \sum_{k=1}^{7} f_k z_k + \sum_{k=1}^{7}\sum_{i=1}^{3}\sum_{l=1}^{15} v_k\, D_{il} y_{kl}$

Subject to,

$\sum_{k=1}^{7}\sum_{l=1}^{15} x_{ijkl} \le s_i$ (for all i)

$x_{ikl} = D_{il} y_{kl}$ ($all i,k,l$)

$\sum_{k=1}^{15} y_{kl} = 1$ (l=1..15)

$\sum_{i=1}^{3}\sum_{l=1}^{15} D_{il}\, y_{kl} \le U_k$ (k=1,..,7)

$x_{ikl} \ge 0$ (all i,k,l)

$$y_{kl} = \begin{cases} 1\,, if\ k^{th} distribution\ center\ surves\ l^{th} customer \\ 0\,, \quad otherwise \end{cases}$$

$$z_k = \begin{cases} 1\ , if\ k^{th} distribution\ center\ is\ opened \\ 0\ , \quad otherwise \end{cases}$$

i : Index for commodities, i=1,2,…,q

j : Index for plants, j=1,2,…,n

k : Index for distribution centers, k=1,2,..,p

l : Index for customers, l =1,2 ,….m

S_{ij} : Production capacity for i^{th} commodity at j^{th} plant

D_{il} : Demand for i^{th} commodity by l^{th} customer

L_k, U_k : Minimum and maximum allowable annual flow through the k^{th} distribution center

f_k : Fixed annual operating cost of the k^{th} distribution center at site k

v_k : Per unit cost of flow through the k^{th} distribution center

c_{ijkl} : Per unit cost of producing and shipping i^{th} commodity to j^{th} plant to l^{th} customer via k^{th} distribution center

$$y_{kl} = \begin{cases} 1, & if\ k^{th} distribution\ center\ surves\ l^{th} customer \\ 0, & otherwise \end{cases}$$

$$z_k = \begin{cases} 1, & if\ k^{th} distribution\ center\ is\ opened \\ 0, & otherwise \end{cases}$$

Formulated mathematical model for multi-commodity transportation problem has been solved using Python.

Chapter 4
RESULTS AND DISCUSSION

The chapter deals with the solution of a multi-commodity transportation problem through two techniques. The optimization of Branch and Bound method and Cutting Plane method. Cost minimization, utilization of each distribution center, shipment allocations, and comparison against each other are discussed. Graphs and tables have been embedded for better clarity and visualization of results.

4.1 Results of the Branch and Bound Method

Using the Branch and Bound method, an overall cost of 138,152,172.79 was achieved while using all seven distribution centers (Anuradhapura, Badulla, Colombo, Galle, Kandy, Mannar, and Rathnapura). The method shows an effective utilization of resources at distribution centers for meeting the demand of commodities. Central roles of selected distribution centers at specific levels, in the light of the supply network, could be underlined within the allocation pattern.

Python Code for Branch and Bound method

```
from pulp import *
import pandas as pd
import numpy as np
import pandas as pd
from scipy.optimize import linprog

# File paths
toffee_costs_path = '/Users/senadhinipun/Documents/University/Project/EXcel data files/Toffee_Distribution_Costs.xlsx'
chocolate_costs_path =
'/Users/senadhinipun/Documents/University/Project/EXcel data files/Chocolate_Distribution_Costs.xlsx'
biscuit_costs_path = '/Users/senadhinipun/Documents/University/Project/EXcel data files/Biscuit_Distribution_Costs.xlsx'
plant_capacity_path = '/Users/senadhinipun/Documents/University/Project/EXcel data files/Plant_Capacity_for_Commodities.xlsx'
demand_path = '/Users/senadhinipun/Documents/University/Project/EXcel data files/Commodity_Demand.xlsx'
fixed_cost_path = '/Users/senadhinipun/Documents/University/Project/EXcel data files/Fixed_Cost_Distribution.xlsx'
distribution_capacity_path =
'/Users/senadhinipun/Documents/University/Project/EXcel data files/Capacity_Commodity_Distribution.xlsx'

# Load data
```

```python
toffee_costs = pd.read_excel(toffee_costs_path, index_col='Customer')
chocolate_costs = pd.read_excel(chocolate_costs_path, index_col='Customer')
biscuit_costs = pd.read_excel(biscuit_costs_path, index_col='Customer')
plant_capacity = pd.read_excel(plant_capacity_path)
demand = pd.read_excel(demand_path)
fixed_costs = pd.read_excel(fixed_cost_path)
distribution_capacity = pd.read_excel(distribution_capacity_path)

# Data preprocessing
customers = list(demand['Customer'])
distribution_centers = list(fixed_costs['Distribution Center'])

commodities = ['Toffee', 'Chocolate', 'Biscuit']
cost_data = {'Toffee': toffee_costs, 'Chocolate': chocolate_costs, 'Biscuit':
biscuit_costs}

# Extract relevant capacity data for each commodity at distribution centers
capacity_data = {
    'Toffee': distribution_capacity[['Distribution Center', 'Toffee']],
    'Chocolate': distribution_capacity[['Distribution Center', 'Chocolate']],
    'Biscuit': distribution_capacity[['Distribution Center', 'Biscuit']]
}

# Extract demand for each customer and commodity
demand_data = {
    'Toffee': demand[['Customer', 'Toffee']],
    'Chocolate': demand[['Customer', 'Chocolate']],
    'Biscuit': demand[['Customer', 'Biscuit']]
}

# Create optimization problem
model = LpProblem("Multi-Commodity_Transportation_Problem", LpMinimize)

# Decision variables
x = LpVariable.dicts("shipment", ((i, j, k) for i in commodities for j in
distribution_centers for k in customers), lowBound=0, cat='Continuous')
z = LpVariable.dicts("open", (j for j in distribution_centers), cat='Binary')

# Objective function: Minimize transportation and fixed costs
model += (
    lpSum(
        cost_data[i].loc[k, j] * x[i, j, k]
        for i in commodities
        for j in distribution_centers
        for k in customers
    ) +
    lpSum(fixed_costs.loc[fixed_costs['Distribution Center'] == j, 'Fixed
cost'].values[0] * z[j]
    for j in distribution_centers)
)

# Constraints
# 1. Customer demand must be met for each commodity
```

```
for i in commodities:
    for k in customers:
        model += lpSum(x[i, j, k] for j in distribution_centers) ==
demand_data[i].loc[demand_data[i]['Customer'] == k, i].values[0],
f"Demand_{i}_{k}"

# 2. Distribution center capacity constraints
for i in commodities:
    for j in distribution_centers:
        model += lpSum(x[i, j, k] for k in customers) <=
capacity_data[i].loc[capacity_data[i]['Distribution Center'] == j,
i].values[0], f"Capacity_{i}_{j}"

# 3. Only open distribution centers can have shipments
M = 2750000  # Updated realistic large number based on maximum demand
for i in commodities:
    for j in distribution_centers:
        for k in customers:
            model += x[i, j, k] <= M * z[j], f"Open_center_{i}_{j}_{k}"

# Solve the problem using Branch and Bound
model.solve()

# Output the results
for v in model.variables():
    print(f"{v.name} = {v.varValue}")

print(f"Total cost: {model.objective.value()}")
```

4.1.1Key Observations

1. **Commodity Distribution**

Toffee- Considering the proportion of toffees, a large proportion was routed through Kandy, Rathnapura, and Anuradhapura because they have the capacity and are strategically located to handle bulk quantities.

Chocolate- Most exports were concentrated in Kandy and Anuradhapura, demonstrating its aptitude for this commodity distribution.

Biscuit- Heavy routing of biscuits through Kandy and Badulla is an indication of efficiency in handling the logistics of this commodity.

2. **Customer Distribution**

Colombo and Kandy have emerged as the two major distribution centers, serving the largest number of customers. This indicates their strategic positions in the network, as they were able to serve varied customer demands for a variety of commodities.

4.1.2 Descriptions of Visuals

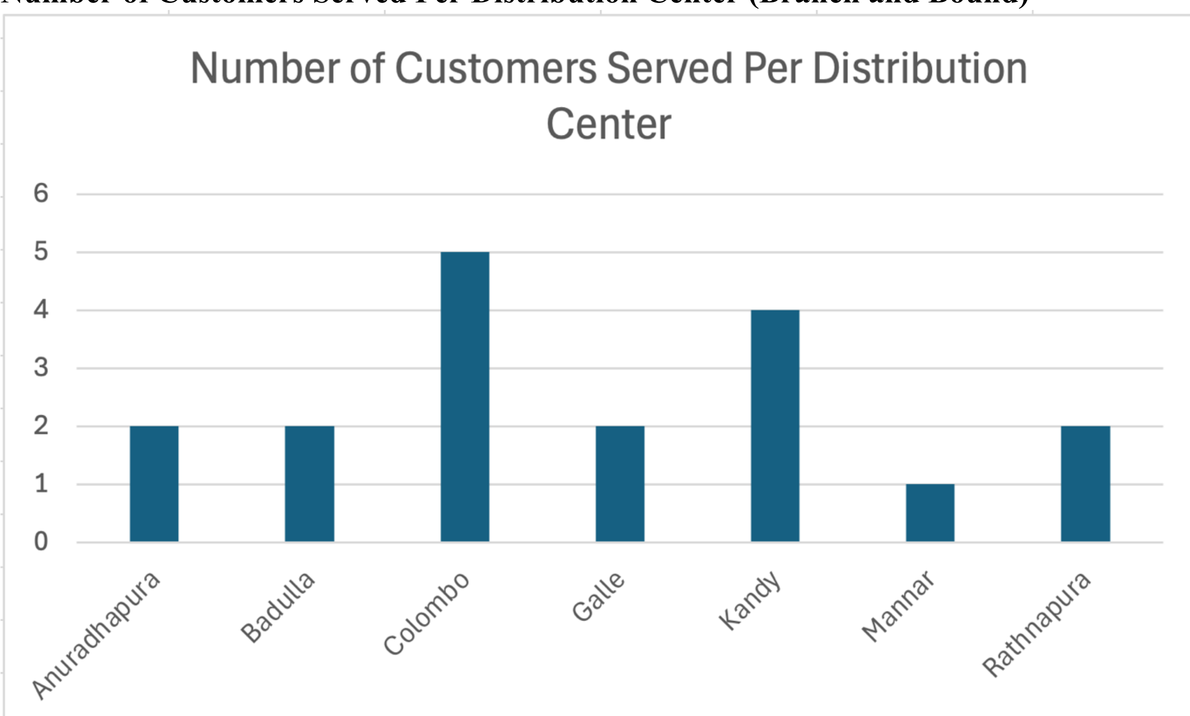
Shipment Allocations from the Branch and Bound Method

	Customer	Commodity	Distribution Center	Quantity
0	1	Biscuit	Kandy	74190
1	1	Chocolate	Kandy	49400
2	1	Toffee	Kandy	2460000
3	2	Biscuit	Kandy	74286
4	2	Chocolate	Kandy	50000
5	2	Toffee	Kandy	2475000
6	3	Biscuit	Kandy	78571
7	3	Chocolate	Kandy	48200
8	3	Toffee	Kandy	2063000
9	4	Biscuit	Kandy	80000
10	4	Chocolate	Kandy	47000
11	4	Toffee	Kandy	2690000
12	5	Biscuit	Mannar	73571
13	5	Chocolate	Mannar	52000
14	5	Toffee	Mannar	2235000
15	6	Biscuit	Badulla	73207
16	6	Chocolate	Badulla	52600
17	6	Toffee	Badulla	2570000
18	7	Biscuit	Badulla	75000
19	7	Chocolate	Badulla	46000
20	7	Toffee	Badulla	2470000
21	8	Biscuit	Galle	78929
22	8	Chocolate	Galle	49500
23	8	Toffee	Galle	2466800
24	9	Biscuit	Galle	73571
25	9	Chocolate	Colombo, Galle	46000
26	9	Toffee	Galle	2625000
27	10	Biscuit	Colombo	73214
28	10	Chocolate	Colombo	48450
29	10	Toffee	Colombo	2390000

30	11	Biscuit	Colombo	78500
31	11	Chocolate	Colombo	50650
32	11	Toffee	Colombo	2280500
33	12	Biscuit	Rathnapura	73571
34	12	Chocolate	Rathnapura	51400
35	12	Toffee	Rathnapura	2750000
36	13	Biscuit	Rathnapura	70064
37	13	Chocolate	Rathnapura	46000
38	13	Toffee	Colombo, Rathnapura	2400000
39	14	Biscuit	Anuradhapura	72500
40	14	Chocolate	Anuradhapura	49800
41	14	Toffee	Anuradhapura	2470000
42	15	Biscuit	Anuradhapura, Colombo	71286
43	15	Chocolate	Anuradhapura	49500
44	15	Toffee	Anuradhapura	2360000

The table is detailed, providing for the customers' commodity and distribution center allocation in a specific format. A separate row for a given customer highlights what commodity is being routed to the proper distribution center along with the quantity being shipped. Emphasize in key distribution centers managing big shipment quantities.

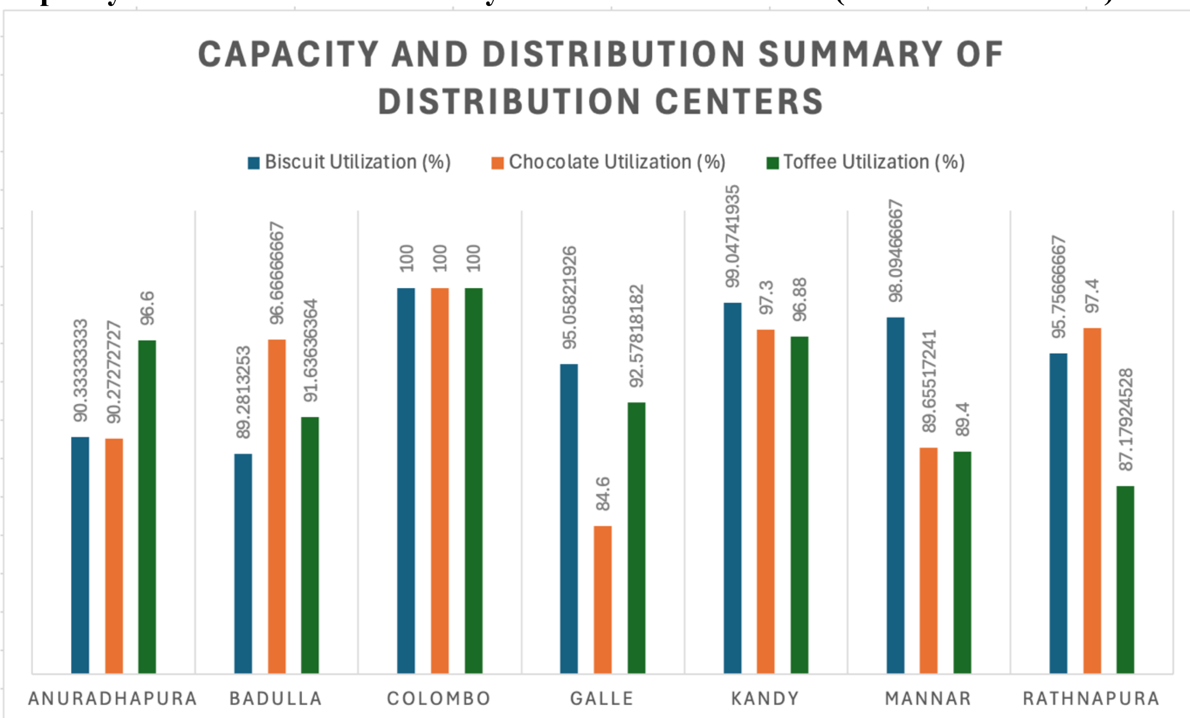
Number of Customers Served Per Distribution Center (Branch and Bound)



The graph above shows the number of customers served by each distribution center. From the above graph on customer coverage, the dominance of Colombo and Kandy can be seen, hence their importance in the whole distribution network.

The bar chart shows that a maximum number of customers were served by Colombo, closely followed by Kandy. Other centers like Badulla and Anuradhapura fall in the moderately covered category, while other centers, like Mannar, have a smaller but critical share. This clearly shows the strategic importance of highly utilized centers through this visualization.

Capacity and Distribution Summary of Distribution Centers (Branch and Bound)



Above graph gives utilization% of the commodities , that is, Biscuit, Chocolate, and Toffee, in different distribution centers, and also gives the amount of operational efficiency or how much capacity used from available capacity.

Colombo has emerged as a fully utilized center, achieving 100% capacity utilization for all three commodities(Biscuit, Chocolate, and Toffee). It shows the importance of Colombo as a key hub for distribution, with its demand met well without leaving any slack in its capacity. High utilization is also found in Kandy, where for all commodities, utilization percentages have crossed 96%, with Biscuit at the highest utilization of about 98.04%. This is indicative of the strategic importance of Kandy in meeting commodity demand efficiently. On the other hand, Rathnapura also posted a good performance, especially for Toffee distribution with 97.4% utilization, while maintaining a balanced utilization of Biscuit and Chocolate.

Badulla, the moderately utilized centers, ranged from 89% to 96.6%, signaling very efficient but slightly underutilized capacity compared to the highly utilized centers like Colombo and Kandy. Similarly, Anuradhapura also depicted a situation of moderate utilization of slightly over 90% for Biscuit and Chocolate, while relatively higher utilization was recorded for Toffee at 96.6%. Underutilized centers like Galle and Mannar showed scope for improvement. The lowest recorded utilisation for Chocolate stood at 84.6% in Galle, while those for Biscuit and Toffee were a little higher, with percentages of 92.58% and 95.06%, respectively. Mannar utilization was thus relatively better yet underutilized, its range falling from 88.4% to 89.65% for all commodities, indicating the possibilities of better integration and matching of demand.

Colombo and Kandy

These two centers are the backbone for the distribution network, which operates at full capacity on a regular basis. Further efficiency could be achieved by investment in expanding infrastructure or taking off the operational load by redistribution.

Improvement Opportunities at Galle and Mannar

Utilization for Galle was strikingly low in Chocolate, insinuating there could be operational inefficiencies or lower demand. Mannar shows consistent underutilization in all commodities and thus might help absorb spillover from fully used centers like Colombo.

Commodity Trends

Toffee had the highest utilization rates across most centers, showing its strong position in the distribution network.

Chocolate was more volatile, with utilization falling below 85% in Galle, which may indicate lower demand or inefficiencies in logistics.

Biscuit had a balanced usage across all centers but peaked in Colombo and Kandy.

1. **Capacity Management**

As some centers are coming closer to full utilization, like Colombo and Kandy, close monitoring with possible capacity expansion in the near future will be needed.

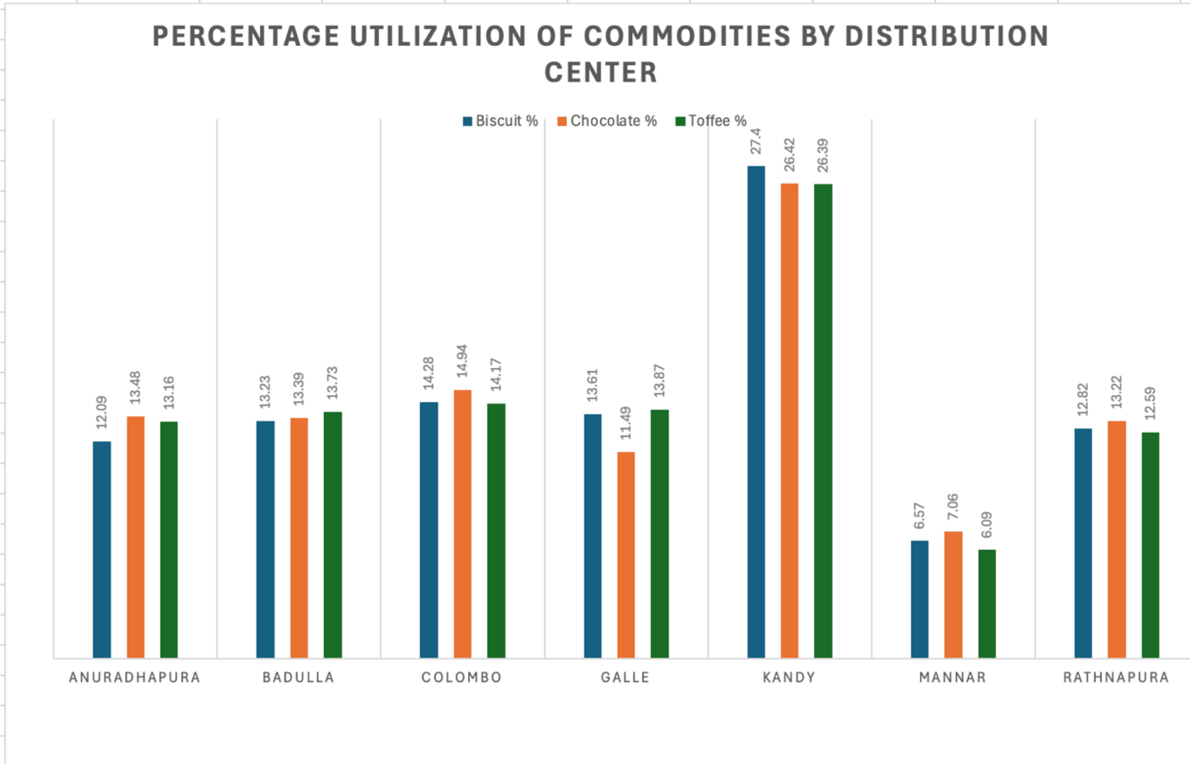
This could be optimized by redistributing shipments from overutilized centers to underutilized ones, such as Galle and Mannar.

2. **Commodity-Specific Adjustments**

The demand pattern analysis for Chocolate and Toffee will definitely help in redistributing the resources appropriately, especially to underutilized centers like Galle and Mannar.

Targeted improvements in commodity handling and logistical strategies may improve the balance of utilization across the network.

Percentage Utilization of Commodities by Distribution Center (Branch and Bound)



This graph represents the percentage share of each commodity-Biscuit, Chocolate, and Toffee-delivered by each distribution center to the overall distribution network. This chart shows the contribution of each center in fulfilling the demand for these commodities with respect to the total allocations across all distribution centers.

Key Observations of the graph

Kandy is the dominant contributor in the logistics network, as it is the highest among all commodities: 27.42% for Biscuit, 26.42% for Chocolate, and 26.39% for Toffee. This goes to point to the pivotal role of Kandy as a big hub, probably due to a high demand that has been satisfied with strategic capacity utilization. Colombo has almost equal contributions in Biscuit (14.28%), Chocolate (14.94%), and Toffee (14.17%), which reflect its strategic placement and ability to efficiently handle a diverse mix of commodities.

Both Rathnapura and Badulla are of utmost importance to the distribution network. Rathnapura provides a quite consistent level of around 12.82% for Biscuit, 13.22% for Chocolate, and 12.59% for Toffee, hence a relatively stable and moderately important contributor. In the case of Badulla, it provides around 13.23% for Biscuit, 13.39% for Chocolate, and 13.73% for Toffee, highlighting its critical support role toward meeting demand.

In contrast, Galle and Mannar are underperforming. The contributions of Galle are relatively low, at 13.61% for Biscuit, 11.49% for Chocolate, and 13.87% for Toffee, indicating a lower importance for certain commodities, especially Chocolate. Mannar is the lowest contributor among all centers, with only 6.57% for Biscuit, 7.06% for Chocolate, and 6.09% for Toffee. These figures highlight Mannar's auxiliary role in the network, signaling potential opportunities for improvement.

From a commodity perspective, Toffee is well spread across most centers but has a high concentration in Kandy, Colombo, and Badulla. Biscuit has a slightly higher dependence on Kandy and Colombo, with lower contributions from Galle and Mannar. Similarly, Chocolate is also well spread across most centers but has lower involvement from Galle and Mannar, which indicates the need to remove inefficiencies from these areas.

Insights

Some key insights into the strategic roles of various distribution centers in the logistics network that this analysis underlines are discussed below. Kandy and Colombo are the clear leading hubs, with the highest volumes on all commodities and underlining their pivotal position as the nodal points within the distribution structure. These centers have a crucial role in managing such high demand with effective resource utilization.

Supplementing the main hubs are Badulla and Rathnapura, both of which act as complementary centers. Their contributions are important in balancing the overall network so that demand is met without overburdening the main hub. This effective support delineates their integrality to the maintenance of an efficient distribution system.

While Galle and Mannar are relatively underutilized, especially for the distribution of Chocolate and Biscuit, their performance indicates some potential inefficiency or misalignment in the allocation of demand. Strategy development should therefore be considered for enhancing their respective logistical importance either by increasing the volume of demand allocation or improving capacity to achieve a better network efficiency and reduction of reliance on the primary hubs.

Implications

These demand patterns bring into focus the key nodes of the chain: the high percentage utilization of Kandy underlines its central position in the network, no doubt driven by the demands of the regions which Kandy is best placed to serve. By the same token, the good, generally consistent contributions from Colombo in all commodities underline the versatility and strategic importance of that centre in managing a mixed portfolio of logistical requirements.

In contrast, the centers of Galle and Mannar may require operational adjustments. Their underutilization indicates possible inefficiencies that could be addressed by redistributing shipments or better leveraging their available capacities. Improvement in their roles within the network could contribute to a more balanced and efficient system, reducing the strain on highly utilized hubs such as Kandy and Colombo.

This graph shows, in quite detail, the commodity-wise distribution responsibilities across the network, underlining the strengths and imbalances of the optimization results coming from the Branch and Bound method.

4.2Results of the Cutting Plane Method

The Cutting Plane method resulted in a slightly lower total cost of 136,365,752.13, proving its ability for iterative refinement of solutions. Like the Branch and Bound method, all seven distribution centers were utilized, but allocation patterns showed a greater degree of diversification.

Python Code for Cutting Plane method

```
import numpy as np
import pandas as pd
from scipy.optimize import linprog


# File paths (replace with your actual file paths)
toffee_costs_path = '/Users/senadhinipun/Documents/University/Project/EXcel data files/Toffee_Distribution_Costs.xlsx'
chocolate_costs_path = '/Users/senadhinipun/Documents/University/Project/EXcel data files/Chocolate_Distribution_Costs.xlsx'
biscuit_costs_path = '/Users/senadhinipun/Documents/University/Project/EXcel data files/Biscuit_Distribution_Costs.xlsx'
plant_capacity_path = '/Users/senadhinipun/Documents/University/Project/EXcel data files/Plant_Capacity_for_Commodities.xlsx'
demand_path = '/Users/senadhinipun/Documents/University/Project/EXcel data files/Commodity_Demand.xlsx'
fixed_cost_path = '/Users/senadhinipun/Documents/University/Project/EXcel data files/Fixed_Cost_Distribution.xlsx'
distribution_capacity_path = '/Users/senadhinipun/Documents/University/Project/EXcel data files/Capacity_Commodity_Distribution.xlsx'

# Load data
toffee_costs = pd.read_excel(toffee_costs_path, index_col='Customer')
chocolate_costs = pd.read_excel(chocolate_costs_path, index_col='Customer')
biscuit_costs = pd.read_excel(biscuit_costs_path, index_col='Customer')
plant_capacity = pd.read_excel(plant_capacity_path)
demand = pd.read_excel(demand_path)
fixed_costs = pd.read_excel(fixed_cost_path)
distribution_capacity = pd.read_excel(distribution_capacity_path)
```

```python
# Data preprocessing
customers = list(demand['Customer'])
distribution_centers = list(fixed_costs['Distribution Center'])
commodities = ['Toffee', 'Chocolate', 'Biscuit']
cost_data = {'Toffee': toffee_costs, 'Chocolate': chocolate_costs, 'Biscuit':
biscuit_costs}

capacity_data = {
    'Toffee': distribution_capacity[['Distribution Center',
'Toffee']].set_index('Distribution Center'),
    'Chocolate': distribution_capacity[['Distribution Center',
'Chocolate']].set_index('Distribution Center'),
    'Biscuit': distribution_capacity[['Distribution Center',
'Biscuit']].set_index('Distribution Center')
}

demand_data = {
    'Toffee': demand[['Customer', 'Toffee']].set_index('Customer'),
    'Chocolate': demand[['Customer', 'Chocolate']].set_index('Customer'),
    'Biscuit': demand[['Customer', 'Biscuit']].set_index('Customer')
}

# Cutting Plane Method implementation

def solve_cutting_plane():
    # Problem initialization
    num_customers = len(customers)
    num_centers = len(distribution_centers)
    num_commodities = len(commodities)

    # Variables

    shipment = np.zeros((num_commodities, num_centers, num_customers),
dtype=int)
    open_center = np.zeros(num_centers)

    # Objective coefficients
    c = []
    for i in commodities:
        for j in distribution_centers:
            for k in customers:
                c.append(cost_data[i].loc[k, j])
    c += list(fixed_costs['Fixed cost'])

    # Constraints
    A_eq = []
    b_eq = []

    # Demand constraints
    for i, commodity in enumerate(commodities):
        for k, customer in enumerate(customers):
```

```
            row = [0] * (num_commodities * num_centers * num_customers +
num_centers)
            for j, center in enumerate(distribution_centers):
                row[i * num_centers * num_customers + j * num_customers + k]
= 1
            A_eq.append(row)
            b_eq.append(demand_data[commodity].loc[customer, commodity])

    # Capacity constraints
    A_ub = []
    b_ub = []
    for i, commodity in enumerate(commodities):
        for j, center in enumerate(distribution_centers):
            row = [0] * (num_commodities * num_centers * num_customers +
num_centers)
            for k in range(num_customers):
                row[i * num_centers * num_customers + j * num_customers + k]
= 1
            row[num_commodities * num_centers * num_customers + j] = -
capacity_data[commodity].loc[center, commodity]
            A_ub.append(row)
            b_ub.append(0)

    # Link shipment to open centers (Big-M constraints)
    M = 2750000
    for i, commodity in enumerate(commodities):
        for j, center in enumerate(distribution_centers):
            for k in range(num_customers):
                row = [0] * (num_commodities * num_centers * num_customers +
num_centers)
                row[i * num_centers * num_customers + j * num_customers + k]
= 1
                row[num_commodities * num_centers * num_customers + j] = -M
                A_ub.append(row)
                b_ub.append(0)

    # Bounds
    bounds = [(0, None)] * (num_commodities * num_centers * num_customers) +
[(0, 1)] * num_centers

    # Solve using linprog
    res = linprog(c, A_ub=np.array(A_ub), b_ub=np.array(b_ub),
A_eq=np.array(A_eq), b_eq=np.array(b_eq), bounds=bounds, method='highs')

    # Extract results
    if res.success:
        solution = res.x
        shipment = solution[:num_commodities * num_centers *
num_customers].reshape((num_commodities, num_centers, num_customers))
        open_center = solution[num_commodities * num_centers *
num_customers:]

        # Force binary results for open centers
```

```
        open_center = np.round(open_center)

        # Output results
        for j, center in enumerate(distribution_centers):
            print(f"open_{center} = {int(open_center[j])}")

        for i, commodity in enumerate(commodities):
            for j, center in enumerate(distribution_centers):
                for k, customer in enumerate(customers):
                    print(f"shipment_('{commodity}',_'{center}',_{customer})
= {shipment[i, j, k]:.2f}")

        print(f"Total cost: {res.fun:.2f}")
    else:
        print("Optimization failed.")

# Run the solver
solve_cutting_plane()
```

4.2.1 Key Findings

Some key facts arose from the Cutting Plane method of analysis that proved efficiency and effectiveness in the optimization of the multi-commodity transportation problem. Distribution Center Utilization Kandy, Rathnapura, and Colombo emerged as vital hubs that can serve diverse allocations across all commodities with a distribution center utilization that showed a more even approach to how this problem will be dealt with using the Branch and Bound method in efficiently allocating resources across a network.

In Commodity Allocation, Toffee shipments were mainly concentrated in Kandy, Rathnapura, and Anuradhapura, indicating that they are strategic. Chocolate was routed mainly through Kandy, Mannar, and Anuradhapura, indicating a focused approach for this commodity. Biscuit allocations were significant in Colombo, Kandy, and Badulla, reflecting their capacity and ability to handle such shipments.

Lastly, Customer Distribution is where the distribution by the Cutting Plane method gives better balanced customer coverage, for instance, Colombo and Kandy being some of the major distribution centers serving key customers effectively. The balance increases supply chain reliability and points to better optimization by the method.

4.2.2 Visual Aids for Cutting Plane Method

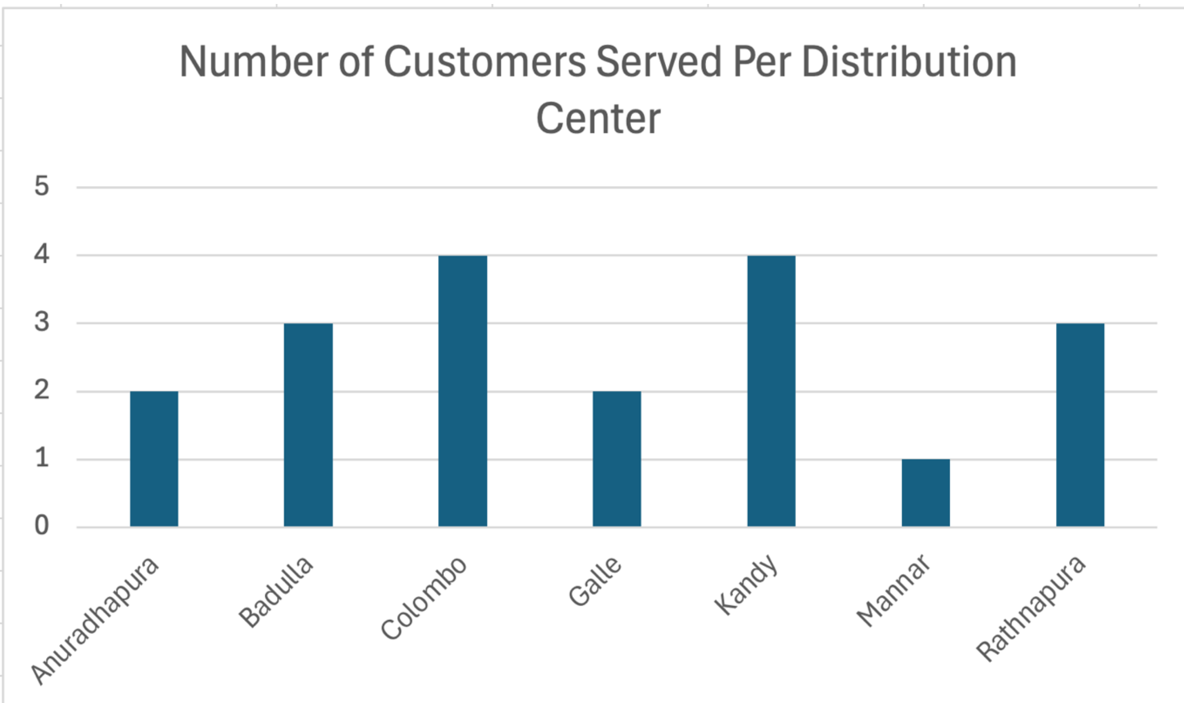
Shipment Allocations from the Cutting Plane Method

Customer	Commodity	Distribution Center	Quantity
1	Biscuit	Kandy	74190
1	Chocolate	Kandy	49400
1	Toffee	Kandy	2460000
2	Biscuit	Kandy	74286
2	Chocolate	Kandy	50000
2	Toffee	Kandy	2475000
3	Biscuit	Kandy	78571
3	Chocolate	Kandy	48200
3	Toffee	Kandy	2063000
4	Biscuit	Kandy	80000
4	Chocolate	Kandy	47000
4	Toffee	Kandy	2690000
5	Biscuit	Colombo, Mannar	73570
5	Chocolate	Mannar	52000
5	Toffee	Mannar	2235000
6	Biscuit	Badulla	73207
6	Chocolate	Badulla	52600
6	Toffee	Badulla	2570000
7	Biscuit	Badulla	75000
7	Chocolate	Badulla	46000
7	Toffee	Badulla	2470000
8	Biscuit	Badulla,Galle	78929
8	Chocolate	Galle	49500
8	Toffee	Colombo,Galle	2466799
9	Biscuit	Colombo,Galle	73571
9	Chocolate	Colombo,Galle	46000
9	Toffee	Colombo,Galle,Badulla	2624999
10	Biscuit	Colombo	73214
10	Chocolate	Colombo,Rathnapura	48450
10	Toffee	Colombo,Rathnapura	2389999
11	Biscuit	Colombo,Kandy,Anuradhapura	78500
11	Chocolate	Colombo,Kandy,Anuradhapura	50650
11	Toffee	Colombo	2280500
12	Biscuit	Rathnapura	73571
12	Chocolate	Rathnapura	51400
12	Toffee	Rathnapura	2750000
13	Biscuit	Rathnapura	70064

13	Chocolate	Rathnapura	46000
13	Toffee	Rathnapura	2400000
14	Biscuit	Anuradhapura	72500
14	Chocolate	Anuradhapura	49800
14	Toffee	Anuradhapura	2470000
15	Biscuit	Anuradhapura	71286
15	Chocolate	Anuradhapura	49500
15	Toffee	Anuradhapura	2360000

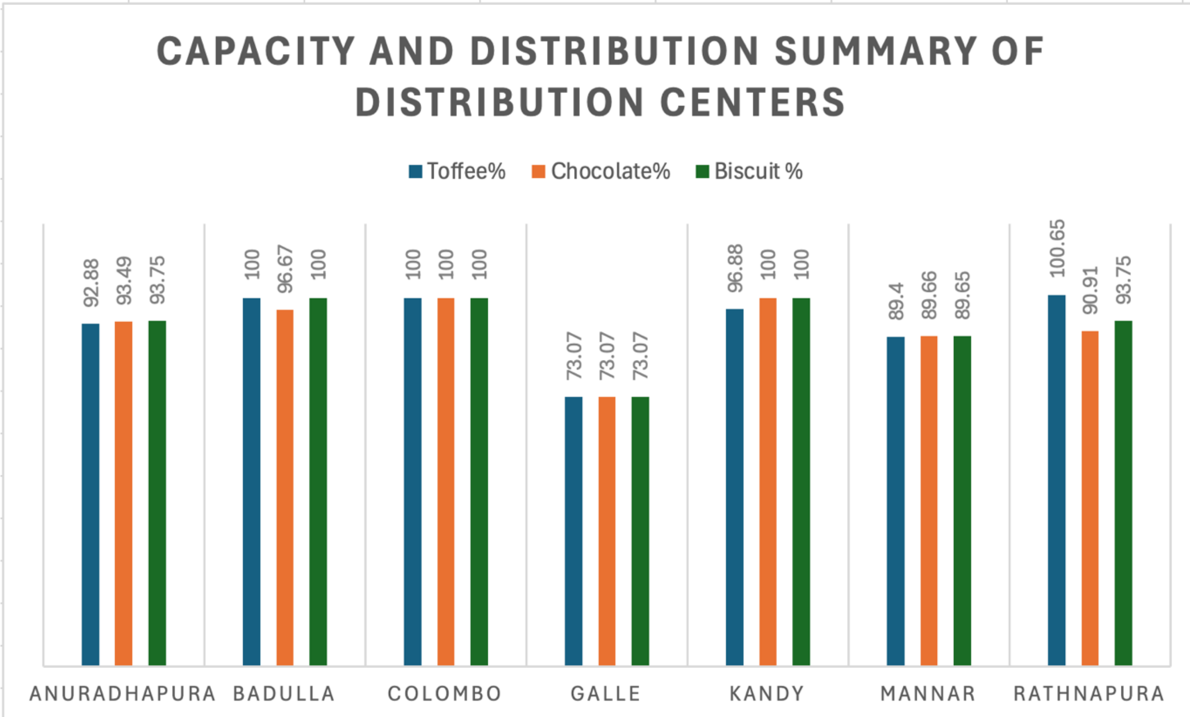
This table presents a detailed breakdown of shipment quantities for each commodity routed to customers across the distribution centers.

Number of Customers Served Per Distribution Center (Cutting Plane)



A bar chart illustrating the customer-serving capability of each distribution center, with prominent centers like Colombo and Kandy.

Capacity and Distribution Summary of Distribution Centers (Cutting Plane)



This graph is plotted to show the percentage utilization of each commodity-that is, Toffee, Chocolate, and Biscuit-across all distribution centers by using the Cutting Plane Method.

It shows the effective utilization of the available capacities for each commodity, reflecting operational efficiency and balance in the distribution network.

Key Observations of the graph

Some of the key observations were noted in performing the Cutting Plane method for this problem on distribution center utilization is fully utilized. Colombo also turned out with 100% utilization across all commodities, which reiterated its strategic importance within the distribution network; full utilization signals optimal capacity utilization, with resources being deployed out without slack. Similarly, for Kandy, the utilization turns out to be 100% across all commodities, thus justifying its value in managing demand for Biscuit, Chocolate, and Toffee. Rathnapura also attained over 90% utilization for all the commodities. Toffee, on adjusted utilization, was 100.65%, thus very important in meeting the logistical demand.

In the category of highly utilized centers, Badulla had effective utilization at 96.67% for Toffee and 93.75% for Biscuit, indicating slightly underutilized capacity. Anuradhapura also showed a balanced utilization rate above 92% for all commodities, though for Toffee, it was a bit underutilized at 92.88%. These plants present good operational performance with capacity utilization scope.

On the other hand, underutilized centers included Galle, which showed lower utilization rates at about 73% for Biscuit, Chocolate, and Toffee, indicating either reduced demand or inefficiency in its role as a key hub. Similarly, Mannar maintained consistent utilization percentages of about 89%, suggesting underuse compared to highly active centers like Colombo and Kandy. These observations point to opportunities for redistributing demand or enhancing efficiency to better utilize the network's overall capacity.

Insights

The insights obtained with the Cutting Plane method show some critical issues of the logistics network. Colombo and Kandy were the two most important distribution centers, utilized at full capacity for all commodities, which underlines their strategic role. This may imply potential investments in view of future demand increases or alternative strategies for distributing the operational load more effectively. On the other hand, Galle has low usages across commodities, which pinpoints possible inefficiencies or misalignment with demand and offers an avenue for further analyses to optimize resources distribution and better utilize Galle's capabilities.

Toffee remained the most in-demand commodity, reaching high utilization across almost all centers, with over-capacity utilization in Rathnapura. Chocolate utilization was more balanced across the network, though a little low in Galle, similar to the observations under the Branch and Bound method. Biscuit has generally shown a balanced utilization profile, but utilization dips in Galle and Mannar show potential areas for capacity improvements or demand redistributions. These all point to a need for focused investments and strategic changes to improve overall network efficiency.

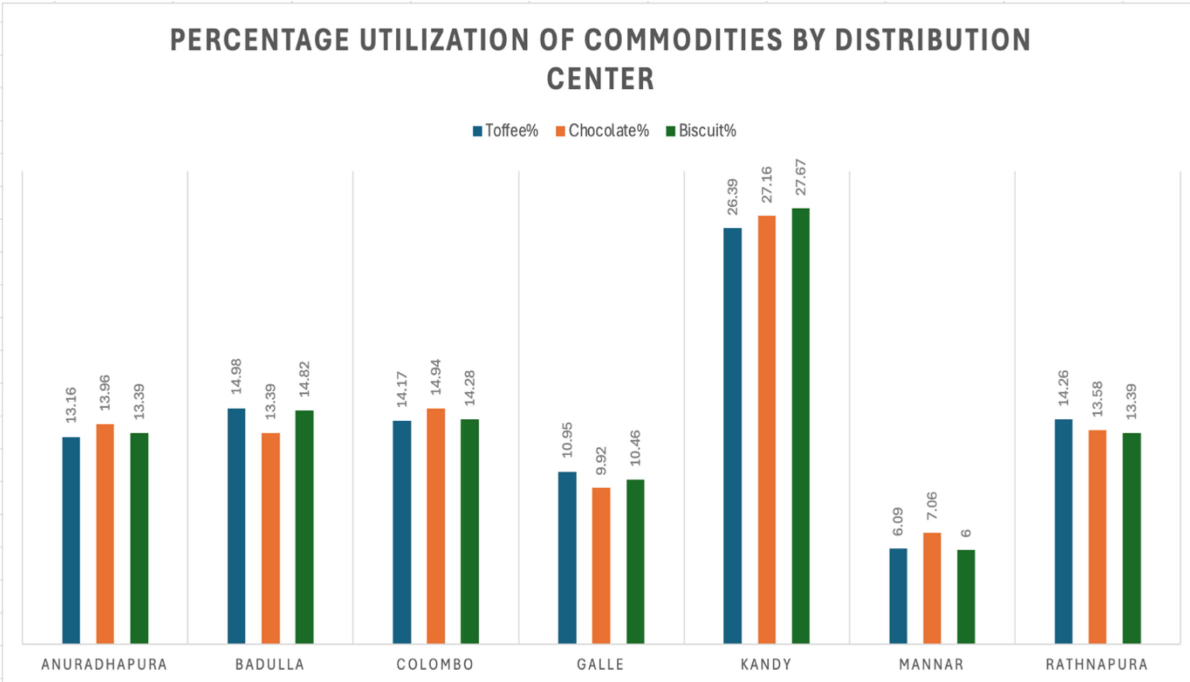
Implications

The implications derived from the analysis give critical areas for improvement and strategic adjustments in the logistics network. The full utilization at major hubs such as Colombo and Kandy suggest the need for capacity expansion or redistribution strategies to avoid potential bottlenecks. These centers are vital to the network, and any inefficiencies may affect overall performance. On the contrary, facilities such as Galle and Mannar are less utilized, and improving these two to a higher extent may facilitate the network at an optimal level. Thus, rearrangement of shipment will decrease burdens on the two hubs with extremely high utilizations.

Secondly, demand specific adjustments might help realize efficiencies, particularly for items such as Chocolate in Galle, since a relatively lower utilization rate would also tend to indicate very low demand or operational inefficiency. Ways through which this facility can increase output or adjust the flow of the products will enhance efficiency and effectiveness toward better contributions in the logistical network. The findings bring into view that these call for dynamic and adaptive capacity and demand management.

This graph emphasizes the operational strengths and weaknesses of the network under the Cutting Plane Method, hence providing the necessary insights into the enhancement of efficiency and equity in the distribution center utilization.

Percentage Utilization of Commodities by Distribution Center (Cutting Plane)



This graph depicts the proportional utilization of distribution centers for each commodity (Toffee, Chocolate, and Biscuits) based on their allocated shipment quantities as a percentage of the total shipments across all centers.

Key Observations of the graph

Some of the key observations derived from the above analysis on the distribution network and its utilization include Kandy stands as the most important distribution center as it accounted for 26.39% of Toffee shipments, 27.16% of Chocolate shipments, and 27.67% of Biscuit shipments. Its strategic importance could be due to its location or the huge capacity that it possesses, which would enable it to handle large portions of all commodities efficiently.

Other distribution centers, such as Anuradhapura, Badulla, Colombo, and Rathnapura, have a balanced utilization for all commodities between 13% and 15%. This even dispersion indicates a well-optimized network design where resource utilization is spread efficiently to avoid overloading particular centers.

On the other hand, Mannar represents the minimum percentage utilization for all commodities 6.09% for Toffee, 7.06% for Chocolate, and 6% for Biscuits. This minimum contribution may indicate a low capacity or limited demand coverage in areas served by Mannar, hence it could be underutilized. Meanwhile, Galle, though less important than Kandy, supports a contribution of 10.95% for Toffee, 9.92% for Chocolate, and 10.46% for Biscuits. This underlines the role of Galle in balancing the distribution network, hence making operations smoother despite its smaller share.

Implications

The strategic importance of Kandy is underlined by its dominance in the logistics network, particularly for high-demand commodities such as Toffee and Biscuits. Being a pivotal hub in Kandy, this indicates that it plays a very crucial role in meeting regional demand efficiently and that it has superior capacity or advantageous location.

Along with Kandy, centers such as Colombo, Rathnapura, Anuradhapura, and Badulla exhibit a balancing strategy. The use of these centers is effectively done to prevent the network from overloading any one center while maintaining a consistent and fair distribution of resources.

However, the small contribution of Mannar to the logistics network indicates potential for further improvement. Its underutilization could indicate a need for infrastructure improvements or more effective integration methods to maximize its value and distribute the demands on the network more evenly.

4.3 Comparative Analysis of Methods

Cost Minimization

The Cutting Plane method had a minimum result, with a reduction of about 1.3 million rupees compared to the Branch and Bound method. This is indicative of how the Cutting Plane method precisely works out cost optimizations iteratively.

Distribution Center Utilization

Both methods effectively utilized all seven distribution centers, highlighting their strategic importance. The Cutting Plane method, however, demonstrated slightly better diversification of utilization, particularly in centers like Colombo.

Commodity Allocations

Branch and Bound relied heavily on key centers like Kandy for heavy routing of biscuits and chocolates.

Cutting Plane showed more scattered allocations, utilizing many centers such as Colombo, Rathnapura, and Mannar.

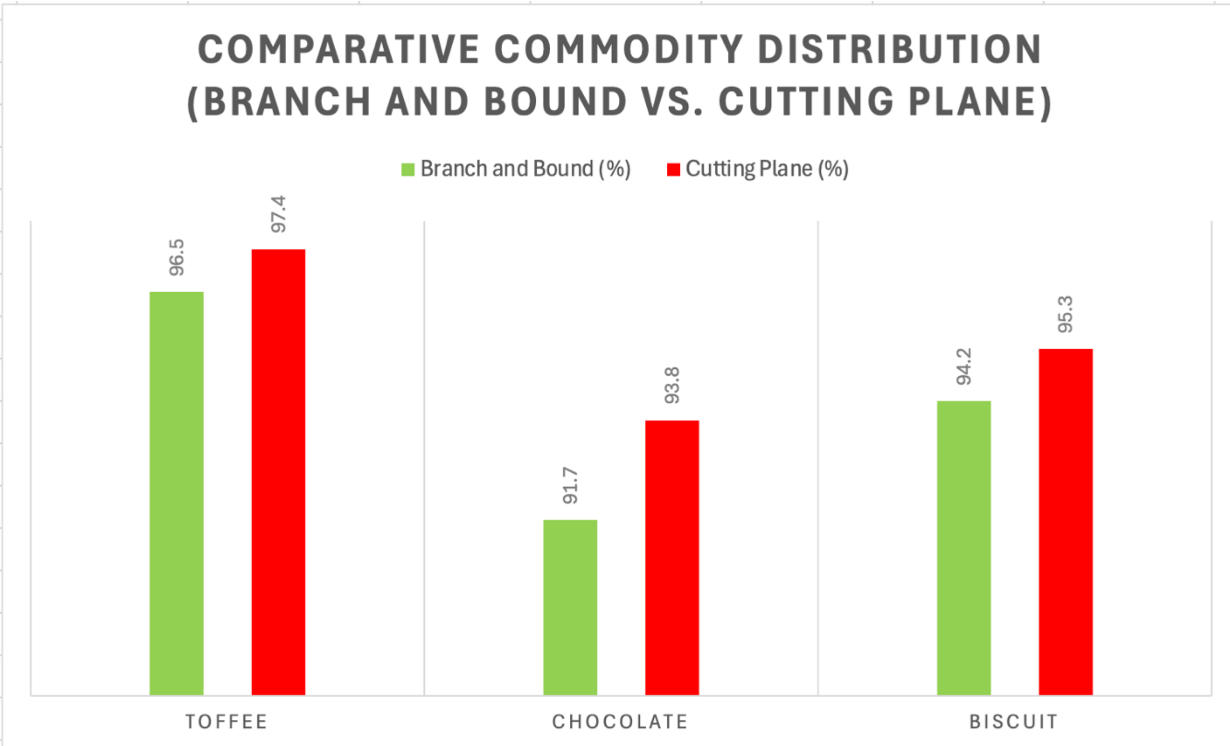
Computational Efficiency

Branch and Bound: Simpler and faster for smaller datasets but limited scalability for large-scale problems.

Cutting Plane: Computationally intensive but highly effective for complex, large-scale datasets, offering refined optimization.

4.3.1 Visual Aids for Comparison

Comparative Commodity Distribution (Branch and Bound vs. Cutting Plane



The graph titled "Comparative Commodity Distribution (Branch and Bound vs. Cutting Plane)" provides an in-depth analysis of how commodities Toffee, Chocolate, and Biscuit are distributed under the Branch and Bound and Cutting Plane methods. It highlights the percentage utilization of each commodity across all distribution centers. For Toffee, the Cutting Plane method demonstrates a slight optimization in distribution, with a marginally higher percentage of utilization compared to the Branch and Bound method. This indicates that the Cutting Plane method effectively diversifies or concentrates shipments to achieve better cost efficiency.

Similarly, for Chocolate, the Cutting Plane method shows a more balanced distribution, with percentages slightly higher than the Branch and Bound method. This suggests improved resource allocation and capacity utilization for this commodity. Biscuit shipments, however, show minimal differences between the two methods, reflecting consistent and effective distribution patterns regardless of the optimization technique used.

The graph underscores the overall efficiency of the Cutting Plane method, which achieves finer adjustments in commodity distribution while maintaining operational consistency. These insights provide valuable implications for the choice of optimization methods, particularly for logistics networks handling multi-commodity scenarios. This comparative visualization highlights the nuanced improvements brought about by iterative refinements in the Cutting Plane method, emphasizing its suitability for complex, large-scale problems.

4.4 Implications and Recommendations

The results of the comparative analysis of the Branch and Bound and Cutting Plane methods have a number of critical implications for the optimization of multi-commodity transportation networks. These can form a basis for strategic decisions and logistics and supply chain management improvements.

4.4.1 Implications

The implications of the findings are three-way, providing rich insights into optimizing multi-commodity transportation systems. First, the Cutting Plane method emerged as more cost-effective, yielding a significant reduction in cost when compared to the Branch and Bound method. This also reflects the strength of iterative refinement methods, especially when dealing with complex and large-sized transportation problems. Organizations that look for incremental benefits in cost efficiency should adopt the Cutting Plane technique over traditional methods of optimization.

Both models yielded the strategic positions of Kandy, Colombo, and Rathnapura as distribution centers. The facilities had a strategic position in the network, further solidified by the higher utilization rates of these facilities. Efficient allocation to these centers will make all the difference in the overall cost and operational performance. Investment in capacity expansion and infrastructure development of critical hubs may bring substantial improvement in the efficiency of the system.

Furthermore, the patterns of allocation for commodities like Toffee, Chocolate, and Biscuit were very interesting. The Cutting Plane method gave a more balanced distribution for commodities like Chocolate and Toffee, which indicates that tailored approaches to commodity routing may enhance capacity utilization and reduce potential bottlenecks. Lastly, the Cutting Plane method also achieved a more balanced distribution of customer coverage across centers, underlining its capability to optimize allocation effectively. This balanced approach will directly enhance customer service reliability and strengthen supply chain resilience, especially in responding to fluctuating demand patterns.

4.4.2 Future Recommendations

To optimize transportation systems effectively, several strategic recommendations emerge from the analysis. First, adopting hybrid optimization techniques that combine the strengths of both the Branch and Bound and Cutting Plane methods is advised. The computational simplicity of the Branch and Bound method, coupled with the iterative refinement capabilities of the Cutting Plane method, can facilitate faster convergence to optimal solutions while maintaining cost efficiency. Additionally, strategic investments in critical distribution centers, such as Kandy, Colombo, and Rathnapura, are essential. Enhancing these hubs by increasing storage capacities, upgrading transportation infrastructure, and integrating advanced technologies like predictive analytics will bolster the overall performance of the logistics network.

A commodity-specific strategy should also be developed to align with the unique demands and characteristics of each product. High-demand commodities like Toffee should be routed through high-capacity centers, while balanced allocations for lower-volume commodities such as Biscuit can reduce the risk of underutilization. Furthermore, the computational intensity associated with the Cutting Plane method underscores the importance of investing in robust computational resources. High-performance computing systems or cloud-based optimization tools can facilitate the application of advanced techniques to large-scale problems, improving scalability and efficiency.

To maintain effectiveness over time, continuous monitoring and periodic sensitivity analyses are necessary. Such analyses will help assess the impacts of changes in demand patterns, transportation costs, and capacity constraints, enabling dynamic adjustments to the optimization models. Lastly, exploring real-time optimization systems can significantly enhance decision-making processes. By integrating these methods into dynamic optimization platforms, organizations can generate near-instantaneous solutions for routing and allocation. These systems can adapt to real-time data inputs, such as fluctuating demand or transportation delays, thereby enhancing supply chain agility and responsiveness.

4.5 Conclusion

The Branch and Bound and Cutting Plane methods provide a comparison that, once again, evokes method selection for cost-effective and efficient transportation solutions. This may refer to investing in infrastructure to reduce transportation costs or even computing routing strategies for commodities that apply especially well to specific commodities. These recommendations provide actionable pathways toward sustained improvement and long-term competitiveness of operations concerning multi-commodity supply chains.

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