

# Weekly presentation SCM research

*9th presentation: Optimizing the  
Supply Chain Configuration for New  
Products (Part 1)*



# Introduction

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# Introduction

## Papers

- *Supply Chain Configuration and Part Selection in Multigeneration Products*, Massachusetts Institute of Technology (1999)
- *Optimizing the Supply Chain Configuration for New Products*, Management Science (2005)



# Summary

## I/Context

*1) Introduction*

*2) Notation and assumptions*

## II/ Optimization Model

*1) Inventory model*

*2) Mathematical formulation*

## III/ Dynamic Programming



# I/ Context

## 1) Introduction

### •Intent

- Develop support decision tool to use during product development process where the product design has been fixed,
- Vendors, manufacturing technologies, shipments options to be decided

### •Model

- A Supply Chain can be viewed as a network where the nodes represent functionality that must be provided and the arcs the constraints among the functions
- Several options for each functions, characterized by their **direct cost** and **lead time**

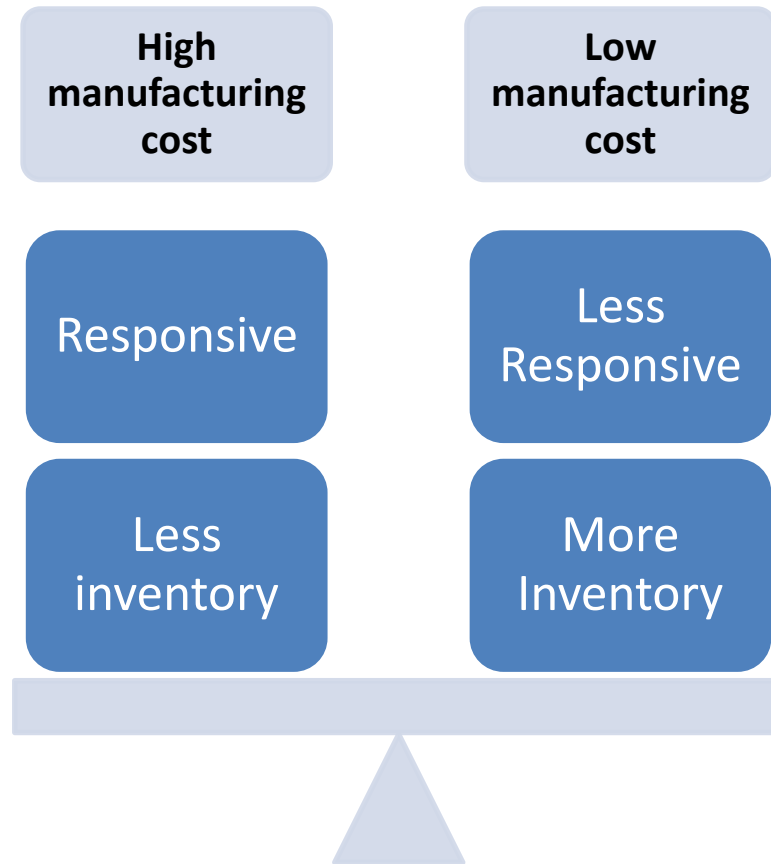
*Objective is to identify the options that can satisfy each function and then select the best one that **minimize the entire Supply Chain cost***



# I/ Context

## *1) Introduction*

- Tradeoff





# I/ Context

## 1) Introduction

*UMC ( Unit Manufacturing Cost) is the dominant criterion in the Supply Chain design*

- Gross margin target
- Easy to calculate directly

• 3 specific costs to minimize

### Safety Stock

- *Expected inventory in order not to fall in shortage*

### Pipeline Stock

- *Work-in-process inventory*

### COGS

- *Cost Of Goods Sold*
- *Total cost of all the units that are delivered to customers during a company-defined period of time*



# I/ Context

## *2) Notation and Assumptions*

- Supply Chain seen as a network of stages, with several options to choose
- Lead Time is the time to perform the function at the stage, provided all the inputs are available
- Direct Cost represents the direct material and direct labor costs associated with the option
- Periodic review policy
- Base-Stock replenishment policy
- No time delay in ordering



# I/ Context

## 2) *Notation and Assumptions*

- Demand process

- External demand occurs only at nodes with no successors, stationary process with average demand per period  $\mu_j$

- Internal stage

$$d_i(t) = \sum_{(i,j) \in A} d_j(t) \qquad \mu_i(t) = \sum_{(i,j) \in A} \mu_j(t)$$

- Demand is bounded by the function  $D_j$ , which is increasing and concave



# I/ Context

## 2) *Notation and Assumptions*

- Guaranteed service time
- Outbound service time  $s_i^{out}$  by which the stage will satisfy its demand with 100% service
- Maximum service time  $S_j$
- Inbound service time service  $s_i^{in}$  time to receive all the required inputs from its suppliers



# II/ Optimization Model

## 1) Inventory model

- $I_i(t)$ , the finished inventory at stage  $i$  at the end of period  $t$

$$I_i(t) = B_i - d_i(t - s_i^{in} - t_i, t - s_i^{out})$$

- $B_i = I_i(0)$
- $d_i(t - s_i^{in} - t_i, t - s_i^{out})$  represents the inventory shortfall between the cumulative replenishment and the cumulative shipment
- In order to guarantee 100% service,  $B_i = D_i(\tau)$   
with  $\tau = \max \{0; s_i^{in} + t_i - s_i^{out}\}$
- the demand over the net replenishment time is demand that has been filled but has not yet been replenished
- If the replenishment time is negative then there is no need for inventory



# II/ Optimization Model

## 1) *Inventory model*

- Safety Stock

$$SS_i = D_i(s_i^{in} + t_i - s_i^{out}) - (s_i^{in} + t_i - s_i^{out})\mu_i$$

- Pipeline inventory

$$PI_i(t) = t_i\mu_i$$



# II/ Optimization Model

## 2) Mathematical formulation

$$\begin{aligned} \text{P min } \sum_{i=1}^N & \alpha c_i [D_i(s_i^{\text{in}} + t_i - s_i^{\text{out}}) - (s_i^{\text{in}} + t_i - s_i^{\text{out}})\mu_i] \\ & + \alpha(c_i - \frac{x_i}{2})t_i\mu_i + \beta x_i\mu_i \\ & \text{Pipeline inventory} \quad \text{COGS} \end{aligned}$$

•Where

- $D_i()$  = maximum demand function for stage  $i$
- $\alpha$  = scalar representing the holding cost rate
- $\beta$  = scalar converting the model's underlying time unit into the company's time interval of interest ( typically 1 year)
- $\mu_i$  = mean demand rate at stage  $i$
- $c_i$  = cumulative cost at stage  $i$
- $t_i$  = selected option's lead time at stage  $i$
- $x_i$  = selected option's cost at stage  $i$



# II/ Optimization Model

## 2) Mathematical formulation

- Constraints

- Stage cost and lead time

$$\sum_{i=1}^{o_i} T_{ik} y_{ik} - t_i \quad \text{for } i = 1, 2, \dots, N \quad \sum_{i=1}^{o_i} C_{ik} y_{ik} - x_i \quad \text{for } i = 1, 2, \dots, N$$

- Stage cumulative cost

$$c_i - \sum_{j: (i,j) \in A} c_j - x_i \quad \text{for } i = 1, 2, \dots, N$$

- Service times

$$s_i^{in} \geq s_j^{out} \quad \text{for } i = 1, 2, \dots, N; j: (i,j) \in A$$

$$s_i^{in} + t_i - s_i^{out} \geq 0 \quad \text{for } i = 1, 2, \dots, N$$

$$s_j^{out} \leq S_j \quad \text{for all demand nodes } j$$

- Options sourcing

$$\sum_{k=1}^{o_i} y_{ik} = 1 \quad \text{for } i = 1, 2, \dots, N$$



# III/ Dynamic programming

## *1) Special SC cases*

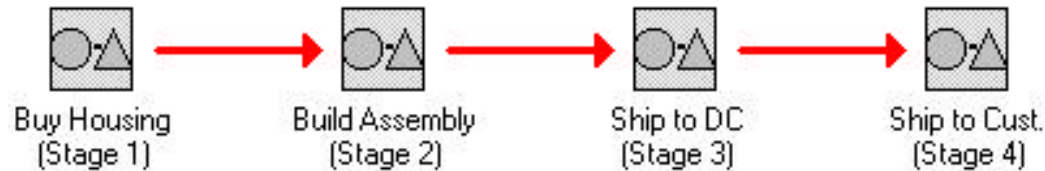
- Programming pattern
  - *Identify the appropriate network model for the Supply Chain*
  - *Define the different subgraphes to solve the problem step by step*
  - *Define the supply chain cost for the subnetwork*
  - *Deduct the functional equation (minimum supply chain cost) according to the constraints*
  - *Solve using forward recursion*



# III/ Dynamic programming

## 1) *Special SC cases*

- Serial line



- Only upstream cost included

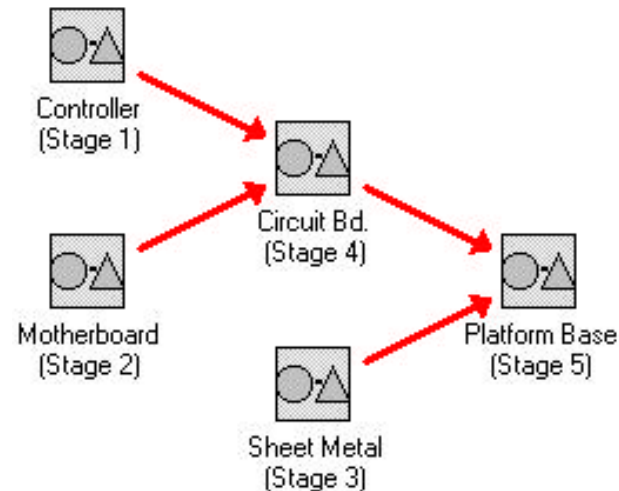
- First evaluate  $f_N(c, SN)$  then solve backtrack



# III/ Dynamic programming

## 1) *Special SC cases*

- Assembly Network



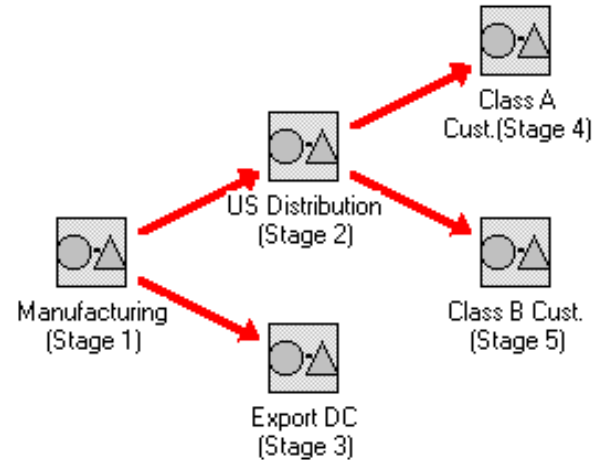
- *Severals suppliers but only one downstream stage*
- *Topologically ordered*
- *One cumulative cost can represent different configurations*
- *CI = incoming cumulative cost*
- *Evaluate all combinations for each value of CI*



# III/ Dynamic programming

## 1) *Special SC cases*

- Distribution Network



- Several customers but only one upstream stage
- Same service time for all customers
- Start solving at  $Fi(0,0)$  then forward



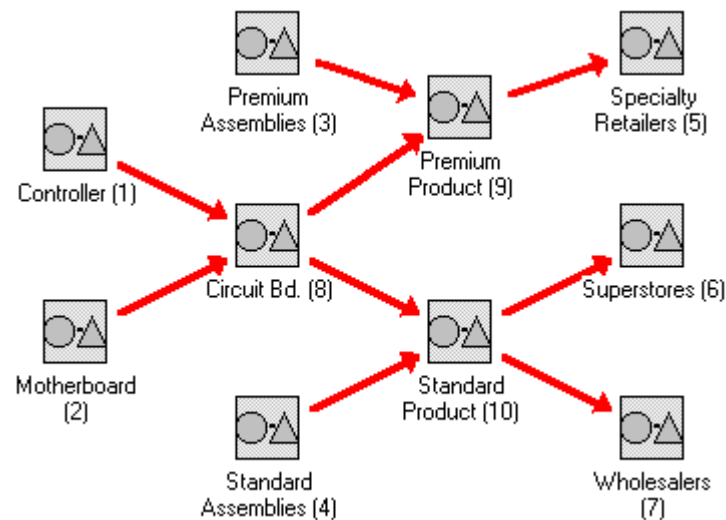
# III/ Dynamic programming

## 2) Spanning tree

- Spanning tree = connected graph that contains  $N$  nodes and  $N-1$  arcs

- Represent numerous kinds of real world supply chains

*Common component that goes into different final assemblies that have each different distribution channels*





# III/ Dynamic programming

## 2) *Spanning tree*

- Nodes are labeled such that for each node  $i$  there is at most one adjacent node with a higher number, called  $i$ 's parent,  $p(i)$
- $N_i$ , subset of nodes  $\{1, 2, \dots, i\}$  that are connected to  $i$

$$N_i = \{i\} + \bigcup_{h < i, (h,i) \in A} N_h + \bigcup_{j < i, (i,j) \in A} N_j$$

- 2 forms of the functional equation
- Minimum cost for the supply chain configuration in a subnetwork with node set  $N_i$

- When  $p(i)$  is downstream,  $f_i(c^T, s^{out})$
- When  $p(i)$  is upstream,  $g_i(c^1, s^{in})$



# III/ Dynamic programming

## 2) Spanning tree

- Supply chain cost for the subnetwork with node set  $N_i$  when option  $k$  is selected for stage  $i$

$$\begin{aligned}
 Z_{ik}(s^{in}, c^1, c^2, s^{out}) = & \\
 & \alpha c^T [D_i(s_i^{in} + T_{ik} - s_i^{out}) - (s_i^{in} + T_{ik} - s_i^{out})\mu_i] \\
 & \text{Safety stock} \\
 & + \alpha \left( c^T - \frac{C_{ik}}{2} \right) T_{ik} \mu_i \quad + \beta C_{ik} \mu_i + \sum_{\{j:(i,j) \in A, j < i\}} g_j(c^T, s^{out}) + \\
 & \text{Pipeline inventory} \quad \text{COGS} \quad \text{downstream} \\
 & \min_{\sum_{\{h:(h,i) \in A, h < i\}} c_h = c^2} \left\{ \sum_{\{h:(h,i) \in A, h < i\}} f_h(c_h, s^{in}) \right. \\
 & \text{upstream}
 \end{aligned}$$



# III/ Dynamic programming

## 2) *Spanning tree*

- Dynamic Program

For  $i=1$  to  $N-1$

- If  $p(i)$  is downstream of  $i$ , for all feasible values of the variables evaluate

$$f_i(c^T, s^{out}) = \min_{k, s^{in}} \{Z_{ik}(s^{in}, 0, c^2, s^{out})\}$$

- If  $p(i)$  is upstream of  $i$ , evaluate

$$g_i(c^1, s^{in}) = \min_{k, c^2, s^{out}} \{Z_{ik}(s^{in}, c^1, c^2, s^{out})\}$$

- Minimize  $g_N(0, s^{in})$  for all  $s^{in}$  feasible to obtain the optimal objective function value

- Computational complexity is of order  $k^N N M^2$



***Thank you for your  
attention!***

