Weekly presentation SCM research

2nd presentation: Theory of games

Summary

I/Introduction

II/Two persons zero sum games

1)Description

2)Special methods

3)Linear programming

III/Games against Nature

I/ Introduction

- Assist decision making

- GAME
  - *There are a finite number of participants*
  - *Each participant has a finite number of possible courses of action*
  - *The participant wishing to apply the theory must know all the possible actions*
  - *After all have chosen a course of action their respective gains are finite*
  - *The gain of the participant depends upon the actions of the others as well as himself*
  - *All possible outcomes are calculable*

Two person zero sum games

II/Two person zero sum games

<u>1) Description</u>

- Principle

  «Two interested parties each of which trying to gain as much as possible at the expense of the other »

- Vocabulary
  - Players, A and B
  - Play
  - Strategy(mixed and pure)
  - V, value of the game
  - Gain matrix

A(x,y,z)

B(t,u,v,w)

		B's courses of action			
		T	*U*	*V*	*W*
A's courses of action	*X*	*a*	*b*	*c*	*d*
	Y	*e*	*f*	*g*	*h*
	Z	*k*	*l*	*m*	*n*

Gain Matrix of a 3 × 4 game.

II/Two person zero sum games

<u>1) Description</u>

- What to determine to solve a game

  - *Value of the game (=average amount per play A will win in the long run if A and B each uses his best strategy)*

  - *The strategy to be used by A(r. B) to ensure that his average gain per play is at least equal(r. no more) to V*

II/Two person zero sum games

1) Description

- Saddle points
  - When the solution involves each player using only one course of action throughout the game is said to have a saddle point
  - It is the point of intersection of the two courses of action and the gain at this point is V
  - If there is none the players have to use a mixed strategy

		B 1	2	3	4	min
	1	1	7	3	4	1
A	2	5	6	4	5	(4)
	3	7	2	0	3	0
	max	7	7	(4)	5	

II/Two person zero sum games

1) Description

- Dominance
  - When a strategy is obvisouly less profitable than the others
  - Enables to reduce the game

	B: 1	2	3	4
A: 1	2	2	3	4
2	4	3	2	2

→

A \ B		
	2	3
	3	2

II/Two person zero sum games

2) Special methods

- Two by two games
  - Oddments=frequencies with which the players must use their courses of action in their best strategies
  - We consider first that the sum of the oddments vertically and horizontally are equal **V**

		B: 1	B: 2	
A	1	a_1	a_2	$\lvert b_1 - b_2 \rvert$
	2	b_1	b_2	$\lvert a_1 - a_2 \rvert$
		$\lvert a_2 - b_2 \rvert$	$\lvert a_1 - b_1 \rvert$	

B plays B_1; $$V = \frac{a_1|b_1-b_2|+b_1|a_1-a_2|}{|b_1-b_2|+|a_1-a_2|}$$

B plays B_2; $$V = \frac{a_2|b_1-b_2|+b_2|a_1-a_2|}{|b_1-b_2|+|a_1-a_2|}$$

or using *B*'s oddments.

A plays A_1; $$V = \frac{a_1|a_2-b_2|+a_2|a_1-b_1|}{|a_2-b_2|+|a_1-b_1|}$$

A plays A_2; $$V = \frac{b_1|a_2-b_2|+b_2|a_1-b_1|}{|a_2-b_2|+|a_1-b_1|}.$$

II/Two person zero sum games

2) Special methods

- Example

			B	
			1	**2**
			Attack the smaller store	**Attack the larger store**
A	**1**	**Defend the smaller store**	0 Both survive	-2 The larger one destroyed
	2	**Defend the larger store**	-1 The smaller one destroyed	0 Both survive

II/Two person zero sum games

2) Special methods

- Two by N games
  - The solution involves picking out a 2x2 sub-game which fits the 2xN game (because here the sum of the oddments are supposed equal)

		B		
		1	2	3
	1	-6	4	-1
A	2	7	-5	-2

		B	
		1	2
	1	-6	4
A	2	7	-5

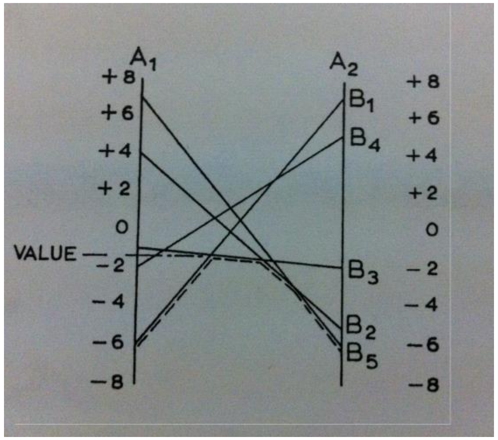
		B	
		1	3
	1	-6	-1
A	2	7	-2

II/Two person zero sum games

2) Special methods

- Graphical treatment
  - Indicates the sub-game which has the same solution as the 2xN game
  - The two lines whixh intersect at the highest point of the bound show the two courses of action B should use in his best strategy
  - This method falls down if there is more than one solution

		B 1	B 2	B 3
	1	-6	4	-1
A	2	7	-5	-2



II/Two person zero sum games

2) Special methods

- Three by three games
  - Solving it the same way as previously would be tedious
  - Method that only works if both players use all their plays in their best strategy

		B 1	B 2	B 3
A	1	6	0	6
A	2	8	-2	0
A	3	4	6	5

II/Two person zero sum games

2) Special methods

- Summary
  - Look for saddle points. If one is found the game is solved
  - Look for dominance, to reduce the size of the game
  - Substract each row from the one above it and write the result below the matrix (B's oddments)
  - If the A's and B's oddments sum is equal, the reduced game is solved
  - If not, all the matrices of order one less than the one must be considered

  → apply the process again

II/Two person zero sum games

3) Linear Programming

- More general method
- Conditions in which it cannot be used
  - If the gain matrix is 4x4 or higher order and the game needs to be reduced
  - When player B is « Nature »
- Example

		B		
		1	2	3
	1	6	0	3
A	2	8	-2	3
	3	4	6	5

II/Two person zero sum games

<u>3) Linear Programming</u>

		B					
		1	**2**	**3**			
	1	**6**	**0**	**3**	6	-3	0
A	**2**	**8**	**-2**	**3**	10	-5	0
	3	**4**	**6**	**5**	-2	1	0
		-2	2	0			
		4	-8	-2			
		4	4	8			
							16

- Linear solving
  - *Consider the game from B's point of view*
  - *B is trying to minimise V*
  - *B(y1,y2,y3) is his optmimum strategy*
  - *p,q,r slack variables*
  - *V=V1-V2*
  - *s, artificial variable with very high value M (see allocation chapter)*

II/Two person zero sum games
3) Linear Programming

- Minimise V1-V2+Ms

	y1	y2	y3	p	q	r	s	V1	V2	P
p	6	0	3	1				-1	1	0
q	8	-2	3		1			-1	1	0
r	4	6	5			1		-1	1	0
s	1	1	1				1			1
w	0	0	0				-M	-1	1	

- Same iterations that in chapter 2

- Two solutions

A(0,1,5) and B(1,0,2)
A(0,1,5) and B(2,1,0)
V=14/3

II/Games against Nature

- Nature cannot be considered as an interested party, Nature is not trying to do her best
- Applying some restrictions
- Calculate the best strategy for A against the worst Nature is able to do
- Solved by linear programming

II/Games against Nature

- Example

N(x1,x2,x3,x4)

		Nature 1	2	3	4
Firm	1	-8	7	7	-8
	2	-9	6	-4	-4
	3	-5	-5	-5	-5

- Constraints:
  - $x2+x3 \geq 0{,}1$
  - $\dfrac{x_4}{x_1 + x_4} \geq 0{,}7$
  - $\dfrac{x_2}{x_2 + x_3} \geq 0{,}8$

II/Games against Nature

	x1	x2	x3	x4	p	q	r	s	t	u	y	V1	V2	P
p	-8	7	7	-8	1							-1	1	0
q	-9	6	-4	-4		1						-1	1	0
r	-5	-5	-5	-5			1					-1	1	0
s	1	1	1	1				1						1
t	1			1					1					0,9
u	0,7			-0,3						1				0
y		-0,2	0,8								1			0
w									-M			-1	1	0

II/Games against Nature

	x1	x2	x3	x4	p	q	r	s	t	u	y	V1	V2	P
x1	1								0,03	0,1				0,27
x2		1						0,8	-0,08		-0,2			0,08
x3			1					0,2	-0,02		0,2			0,02
x4				1					0,07	-0,1				0,63
p					1	-1		-3	0,55	-0,5	-0,2			1,95
r						-1	1	9	-0,95	-0,5	-2			0,45
V2						1		-4	0,95	0,5	2	-1	1	4,55
w						-1		4	-0,95	-0,5	-2			-4,55

- N(0,27;0,08;0,02;0,63)
- Firm (0,1,0)
- V=-4,55

4)Next steps

Thank you for your attention!