

UEM

University of Engineering & Management, Kolkata

1st Term Examination, September, 2019

Course: B.Tech (CSE)

Semester: 3rd

Paper Name: Mathematics - III

Paper Code: BSC302

Full Marks: 70

Time: 3 hours

Group - A (10 marks)

Answer any 5. Each question is of 2 marks.

1. A) What is the probability that either an odd number or a number greater than 4 will turn up when a fair die is thrown?
B) A speaks truth in 75% cases and B speaks truth in 80% cases. What is the probability that they contradict each other in a statement?
C) If A and B be events with $P(A \cap B) = \frac{1}{2}$, $P(A^c \cap B^c) = \frac{1}{3}$, $P(A) = P(B) = p$, then find p .
D) If the exponential distribution is given by the probability density function $f(x) = e^{-x}$, $0 < x < \infty$, then find the mean.
E) A random variable X has uniform distribution on $(-5, 5)$. Find $P(X > 3)$.
F) Define abelian group with an example.
G) Give an example of a cyclic group with 2000 number of elements.
H) Let us define a binary operation $*$ on the set of real numbers \mathbb{R} as follows
$$a * b = ab + 1, \forall a, b \in \mathbb{R}.$$

Prove that this binary operation is commutative but not associative.

Group - B (15 marks)

Answer any 3. Each question is of 5 marks.

2. Define Binomial distribution. Find its mean and variance.
3. Show that the probability of occurrence of only one of the events A and B is $P(A) + P(B) - 2P(A \cap B)$.
4. A continuous random variable has a p.d.f. $f(x) = 3x^2$ if $0 \leq x \leq 1$ and $f(x) = 0$ otherwise. Find a and b such that
 - a) $P(X \leq a) = P(X > a)$
 - b) $P(X > b) = 0.05$.
5. If X is uniformly distributed in $[-\alpha, \alpha]$ with $\alpha > 0$, then determine α such that

$$P(X > 1) = \frac{1}{3}.$$

6. Show that the set of all real matrices of the form $\begin{bmatrix} x & y \\ 0 & x \end{bmatrix}$ where $x \neq 0$, forms a group w.r.t. matrix multiplication.
7. Show that the identity element and the inverse of an element in a group (G, \circ) are unique.

Group - C (45 marks)

Answer any 3. Each question is of 15 marks.

8. A) In a bolt factory, machines A, B and C manufacture respectively 25%, 35% and 40% of the total output of which 5%, 4% and 2% are defective bolts. A bolt is drawn at random and is found to be defective. What are the probabilities that it has been manufactured by machine A, B and C?

B) A random variable X has the density function

$$f(x) = \frac{a}{x^2 + 1}, -\infty < x < \infty.$$

Find (i) a (ii) the probability that X^2 lies between $\frac{1}{3}$ and 1 (iii) the distribution function of X.

7+8

9. A) A student has to answer a multiple choice question with 5 alternatives. What is the probability that the student knows the answer, given that he answered it correctly?

B) If the random variable X has the pdf $f(x) = \frac{1}{4}, -2 \leq x \leq 2$, and $f(x) = 0$ otherwise,

find (i) $P(X < 1)$ (ii) $P(|X - 1| \geq \frac{1}{2})$.

7+8

10. If the weekly wage of 10,000 workers in a factory follows normal distribution with mean and s.d Rs. 70 and Rs 5 respectively, find the expected number of workers whose weekly wages are (i) between Rs. 66 and 72 (ii) less than Rs. 66 (iii) more than Rs. 72

[Given that $\frac{1}{\sqrt{2\pi}} \int_0^z e^{-\frac{t^2}{2}} dt = 0.1554$ and 0.2881 according as $z = 0.4$ and $z = 0.8$]

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11. A) The probability of a missile hitting a target is $\frac{1}{4}$.

(i) If 7 such missiles are sent, what is the probability of hitting the target at least twice.

(ii) How many missiles must be fired so that the probability of hitting the target at least once is more than $\frac{2}{3}$.

B) If 5% of the electric bulbs manufactured by a company are defective, use Poisson distribution to find the probability that in a sample of 100 bulbs (i) none is defective (ii) 5 bulbs will be defective.

7+8

12. A) Prove that every cyclic group is abelian group but converse is not true.

B) Show that the 4th root of unity form a cyclic group under ordinary multiplication.

7+8

13. A) Define centre of a group. Prove that centre of a group is a subgroup of the group.

B) Prove that a group $(G, *)$ is abelian if and only if $(a * b)^{-1} = a^{-1} * b^{-1}$ for all $a, b \in G$.

7+8
