



University of Engineering & Management, Kolkata

End Semester Examination, December, 2019

Course: B.Tech (CSE) Semester: 3<sup>rd</sup>

Paper Name: Mathematics - III

Paper Code: M301

Full Marks: 70

Date: 19/12/2019

Time: 9:30am – 12:30pm

Group - A (10 marks)

Answer any 5. Each question is of 2 marks.

1. A) A speaks truth in 75% cases and B speaks truth in 80% cases. What is the probability that they contradict each other in a statement?

B) Find the expectation of the following random variable X

$x_i$	0	1	2	3
$f_i$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{8}$

C) Check whether the following function is a probability density function or not

$$f(x) = \frac{1}{\Gamma(5)} e^{-x} x^4, 0 < x < \infty.$$

D) Check whether the notation \* defined by  $a * b = \frac{a+b}{5}$  is a binary operation on the set of integers.

E) How many generators does a cyclic group of order 7 contain?

F) Find the image of the elements 3 and 4 if

$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 1 & - & - & 3 \end{pmatrix}$  be an odd permutation.

G) Give one example of a ring and a field each.

H) What is the chromatic number of bi-partite graph?

Group - B (15 marks)

Answer any 3. Each question is of 5 marks.

2. If the random variable X has the pdf  $f(x) = \begin{cases} 3x, & 0 < x < 1 \\ 0, & \text{else} \end{cases}$ , then find the pdf of  $y = 4x + 3$ .

3. In a group G, prove that  $(ab)^2 = a^2b^2$  if and only if  $(ab)^{-1} = a^{-1}b^{-1}$  for  $a, b$  in G.

4. If P is a subgroup of S and Q is a normal subgroup of S, then show that  $P \cap Q$  is a normal subgroup of P.

5. Let G be a regular graph, the degree of each of its vertices being 4. Determine the number of vertices of G. if G determines 10 regions.

6. Prove that a complete graph with five vertices is non-planar.

7. Remove one vertex from Kuratowski's 1<sup>st</sup> graph and show it becomes a planar graph.

Group - C (45 marks)

Answer any 3. Each question is of 15 marks.

8. A) On rainy days, Joe is late to college with probability 0.3; on no rainy days, he is late with probability 0.1. With probability 0.7 it will rain tomorrow find the probability that Joe is early tomorrow. Given that Joe was early find the probability that it rained.  
 B) A petrol pump is supplied with petrol once a day. If its daily volume of sales (X) in thousands of litres is distributed by :  $f(x)=5(1-x)^4; 0 \leq x \leq 1$ , what must be the capacity of its tank in order that the probability that its supply will be exhausted in a given day shall be 0.01? 9+6

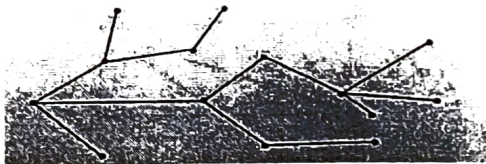
9. A) The probability of a missile hitting a target is  $\frac{1}{4}$ .  
 (i) If 7 such missiles are sent, what is the probability of hitting the target at least twice  
 (ii) How many missiles must be fired so that the probability of hitting the target at least once is more than  $\frac{2}{3}$ .  
 B) If 5% of the electric bulbs manufactured by a company are defective, use Poisson distribution to find the probability that in a sample of 100 bulbs (i) none is defective (ii) 5 bulbs will be defective. 8+7

10. A) Find all cyclic subgroups of the group  $(Z_5, +)$ .  
 B) State Lagrange's Theorem. Use Lagrange's theorem to prove that a group of prime order has no non-trivial subgroup. 8+7

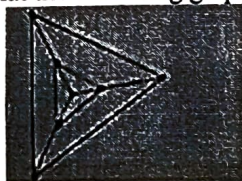
11. A) Prove that a group  $(G, *)$  is abelian if and only if  $(a * b)^{-1} = a^{-1} * b^{-1}$  for all  $a, b \in G$ .

- B) Let S be set of all real matrices  $\left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a^2 + b^2 = 1 \right\}$ . Show that S forms a commutative group under matrix multiplication. 8+7

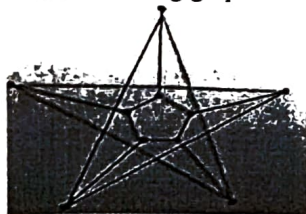
12. A) Find the chromatic polynomial of the following tree



- B) Show that the following graph is self-dual



13. A) How many ways a tree on 5 vertices can be coloured with at most 4 colours.  
 B) Show that the following graph is non planar



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