Formules de trigonométrie

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15-10-2022

I) Addition

Propriétés

$$\begin{aligned}\forall (a,b)\in\mathbb{R}^2 : cos\,(a+b) &= \cos a\times\cos b-\sin a\times\sin b\\ cos\,(a-b) &= \cos a\times\cos b+\sin a\times\sin b\\ sin\,(a+b) &= \sin a\times\cos b+\sin b\times\cos a\\ sin\,(a-b) &= \sin a\times\cos b-\sin b\times\cos a\end{aligned}$$

$$a+b\neq\ \frac{\pi}{2}+k\pi/k\in\mathbb{Z}$$

$$\begin{aligned}tan\,(a+b) &= \frac{sin\,(a+b)}{cos\,(a+b)}=\frac{\sin a\times\cos b+\sin b\times\cos a}{\cos a\times\cos b-\sin a\times\sin b}\\ &= \frac{\cos a\times\cos b\times\left(\frac{\sin a\times\cos b}{\cos a\times\cos b}+\frac{\sin b\times\cos a}{\cos a\times\cos b}\right)}{\cos a\times\cos b\times\left(\frac{\cos a\times\cos b}{\cos a\times\cos b}-\frac{\sin a\times\sin b}{\cos a\times\cos b}\right)}\\ &= \frac{\tan a+\tan b}{1-\tan a\times\tan b}\end{aligned}$$

Duplication

$$\begin{aligned}x &= a=b\\ \cos 2x &= \cos^2 x-\sin^2 x\\ &= 1-2\times\sin^2 x\\ &= 2\times\cos^2 x-1\end{aligned}$$

$$\sin 2x = 2\times\sin x\times\cos x$$

$$\tan 2x=\frac{2\times\tan x}{1-\tan^2 x}$$

II) Produit vers la somme

$$\cos a \times \cos b = \frac{1}{2} \times [cos\,(a+b) + cos\,(a-b)]$$
$$\sin a \times \sin b = \frac{1}{2} \times [cos\,(a+b) - cos\,(a-b)]$$
$$\sin a \times \cos b = \frac{1}{2} \times [sin\,(a+b) + sin\,(a-b)]$$

$$\cos^2 x = \frac{\cos 2x + 1}{2}$$
$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

III) Somme vers le produit

$$On\ pose : \begin{cases} p = a + b \\ q = a - b \end{cases} \Leftrightarrow \begin{cases} a = \frac{p+q}{2} \\ b = \frac{p-q}{2} \end{cases}$$

$$\cos p + \cos q = 2 \times \cos \frac{p+q}{2} \times \cos \frac{p-q}{2}$$
$$\cos p - \cos q = 2 \times \sin \frac{p+q}{2} \times \sin \frac{p-q}{2}$$
$$\sin p + \sin q = 2 \times \sin \frac{p+q}{2} \times \cos \frac{p-q}{2}$$

IV) Résolution d'équation trigonométrique

$$\forall\,(\alpha,\beta,\omega)\in\mathbb{R}^3,(\alpha,\beta)\neq(0,0)$$

$$[E]\Leftrightarrow\alpha\times\cos x+\beta\times\sin x=\omega\Leftrightarrow\frac{\alpha}{\sqrt{\alpha^2+\beta^2}}\times\cos x+\frac{\beta}{\sqrt{\alpha^2+\beta^2}}=\frac{\omega}{\sqrt{\alpha^2+\beta^2}}$$

$$\forall\,(\alpha,\beta)\in\mathbb{R}^2,(\alpha,\beta)\neq(0,0):\frac{\alpha}{\sqrt{\alpha^2+\beta^2}}\in[-1;1]\wedge\frac{\beta}{\sqrt{\alpha^2+\beta^2}}\in[-1;1]$$

$$\exists\mathit{\Phi}\in\mathbb{R}:\begin{cases}\cos\mathit{\Phi}=\frac{\alpha}{\sqrt{\alpha^2+\beta^2}}\\ \sin\mathit{\Phi}=\frac{\beta}{\sqrt{\alpha^2+\beta^2}}\end{cases}$$

$$\begin{aligned}[E]&\Leftrightarrow\cos\mathit{\Phi}\times\cos x+\sin\mathit{\Phi}\times\sin x=\frac{\omega}{\sqrt{\alpha^2+\beta^2}}\\ &\Leftrightarrow cos\,(x-\mathit{\Phi})=\frac{\omega}{\sqrt{\alpha^2+\beta^2}}\\ &\Leftrightarrow\begin{cases}\forall\omega\notin[-1;1]:cos\,(x-\mathit{\Phi})\neq\frac{\omega}{\sqrt{\alpha^2+\beta^2}}\\ \forall\omega\in[-1;1]\,,\exists\theta\in\mathbb{R}:\cos\theta=\frac{\omega}{\sqrt{\alpha^2+\beta^2}}\wedge cos\,(x-\mathit{\Phi})=\cos\theta\end{cases}\end{aligned}$$