

# Formules de trigonométrie

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## I) Addition

### Propriétés

$$\begin{aligned}\forall (a, b) \in \mathbb{R}^2 : \cos(a + b) &= \cos a \times \cos b - \sin a \times \sin b \\ \cos(a - b) &= \cos a \times \cos b + \sin a \times \sin b \\ \sin(a + b) &= \sin a \times \cos b + \sin b \times \cos a \\ \sin(a - b) &= \sin a \times \cos b - \sin b \times \cos a\end{aligned}$$

$$a + b \neq \frac{\pi}{2} + k\pi / k \in \mathbb{Z}$$

$$\begin{aligned}\tan(a + b) &= \frac{\sin(a + b)}{\cos(a + b)} = \frac{\sin a \times \cos b + \sin b \times \cos a}{\cos a \times \cos b - \sin a \times \sin b} \\ &= \frac{\cos a \times \cos b \times \left( \frac{\sin a \times \cos b}{\cos a \times \cos b} + \frac{\sin b \times \cos a}{\cos a \times \cos b} \right)}{\cos a \times \cos b \times \left( \frac{\cos a \times \cos b}{\cos a \times \cos b} - \frac{\sin a \times \sin b}{\cos a \times \cos b} \right)} \\ &= \frac{\tan a + \tan b}{1 - \tan a \times \tan b}\end{aligned}$$

### Duplication

$$\begin{aligned}x &= a = b \\ \cos 2x &= \cos^2 x - \sin^2 x \\ &= 1 - 2 \times \sin^2 x \\ &= 2 \times \cos^2 x - 1\end{aligned}$$

$$\sin 2x = 2 \times \sin x \times \cos x$$

$$\tan 2x = \frac{2 \times \tan x}{1 - \tan^2 x}$$

## II) Produit vers la somme

$$\cos a \times \cos b = \frac{1}{2} \times [\cos(a+b) + \cos(a-b)]$$

$$\sin a \times \sin b = \frac{1}{2} \times [\cos(a+b) - \cos(a-b)]$$

$$\sin a \times \cos b = \frac{1}{2} \times [\sin(a+b) + \sin(a-b)]$$

$$\cos^2 x = \frac{\cos 2x + 1}{2}$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

## III) Somme vers le produit

$$\text{On pose : } \begin{cases} p = a + b \\ q = a - b \end{cases} \Leftrightarrow \begin{cases} a = \frac{p+q}{2} \\ b = \frac{p-q}{2} \end{cases}$$

$$\cos p + \cos q = 2 \times \cos \frac{p+q}{2} \times \cos \frac{p-q}{2}$$

$$\cos p - \cos q = 2 \times \sin \frac{p+q}{2} \times \sin \frac{p-q}{2}$$

$$\sin p + \sin q = 2 \times \sin \frac{p+q}{2} \times \cos \frac{p-q}{2}$$

## IV) Résolution d'équations trigonométriques

$$\forall (\alpha, \beta, \omega) \in \mathbb{R}^3, (\alpha, \beta) \neq (0, 0)$$

$$[E] \Leftrightarrow \alpha \times \cos x + \beta \times \sin x = \omega \Leftrightarrow \frac{\alpha}{\sqrt{\alpha^2 + \beta^2}} \times \cos x + \frac{\beta}{\sqrt{\alpha^2 + \beta^2}} = \frac{\omega}{\sqrt{\alpha^2 + \beta^2}}$$

$$\forall (\alpha, \beta) \in \mathbb{R}^2, (\alpha, \beta) \neq (0, 0) : \frac{\alpha}{\sqrt{\alpha^2 + \beta^2}} \in [-1; 1] \wedge \frac{\beta}{\sqrt{\alpha^2 + \beta^2}} \in [-1; 1]$$

$$\exists \Phi \in \mathbb{R} : \begin{cases} \cos \Phi = \frac{\alpha}{\sqrt{\alpha^2 + \beta^2}} \\ \sin \Phi = \frac{\beta}{\sqrt{\alpha^2 + \beta^2}} \end{cases}$$

$$[E] \Leftrightarrow \cos \Phi \times \cos x + \sin \Phi \times \sin x = \frac{\omega}{\sqrt{\alpha^2 + \beta^2}}$$

$$\Leftrightarrow \cos (x - \Phi) = \frac{\omega}{\sqrt{\alpha^2 + \beta^2}}$$

$$\Leftrightarrow \begin{cases} \forall \omega \notin [-1; 1] : \cos (x - \Phi) \neq \frac{\omega}{\sqrt{\alpha^2 + \beta^2}} \\ \forall \omega \in [-1; 1], \exists \theta \in \mathbb{R} : \cos \theta = \frac{\omega}{\sqrt{\alpha^2 + \beta^2}} \wedge \cos (x - \Phi) = \cos \theta \end{cases}$$