

Chapter 8 Introduction to Evolutionary Computation

Evolution is an optimization process where the aim is to improve the ability of an organism (or system) to survive in dynamically changing and competitive environments.

Chromosomes: One chromosome equals one individual. N genes define this individual.

Population: Sum of individuals

Fitness: Calculated by fitness function. The higher the fitness, the higher the quality of the individual.

Parent: Individual with high fitness who is allowed to reproduce.

Crossover: A certain percentage of genes are randomly taken from a certain number of parents to offspring.

Mutation: Random genetic changes during reproduction. Chance to overcome shortcomings of parents.

Genotype: Chromosomes expressed with genes. (Binary / Value / Permutation)

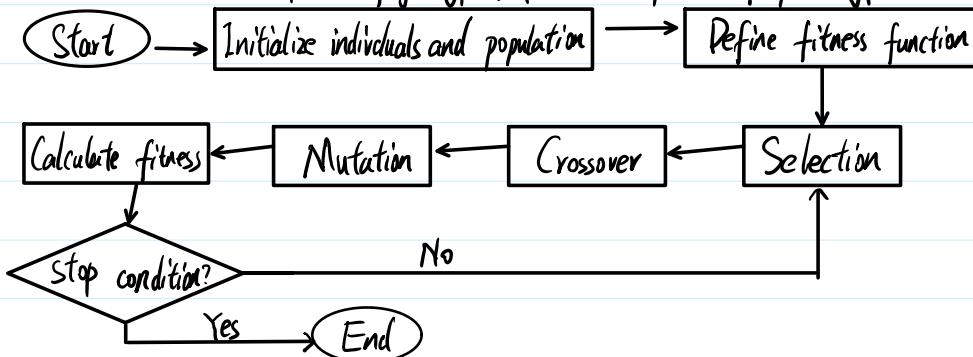
Phenotype: Expression of chromosome

Binary Chromosome

0	1	1	1
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 Each grid is a gene

0111 is the expression of genotype. 7 is the expression of phenotype.



8.2 Representation – The Chromosome

Hamming distance between two strings of equal length is the number of positions at which the corresponding symbols are different

Binary coding : $7 = \begin{smallmatrix} 0 & 1 & 1 & 1 \\ \uparrow & \uparrow & \uparrow & \uparrow \\ 1 & 1 & 1 & 1 \end{smallmatrix}$ $\therefore 7$ and 8 with a Hamming distance of 4
 $8 = \begin{smallmatrix} 1 & 0 & 0 & 0 \end{smallmatrix}$

The numbers are close to each other, but there is a big gap in Hamming distance

\therefore Use Gray coding

Binary \Rightarrow Gray $g_1 = b_1, g_2 = b_1 \bar{b}_2 + \bar{b}_1 b_2$

Binary : $b = 110 \quad g_1 = 1 \quad g_2 = 1\bar{1} + \bar{1}1 = 0 \quad g_3 = 1$

Gray : $b = 101$

8.3 Initial Population

The goal of random initial is to ensure that the initial population is a uniform representation of the entire search space.

If regions of the search space are not covered by the initial population, chances are that those parts will be neglected by the search process.

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8.4 Fitness Function

$$f : T^{n_x} \rightarrow \mathbb{R}$$

$$f : S_c \xrightarrow{\phi} S_x \xrightarrow{\psi} \mathbb{R} \xrightarrow{r} \mathbb{R}_+$$

ϕ : Decoding function, e.g. decode to floating-point

ψ : Objective function r : Scaling function

Different formulation of fitness function for different problems.

Unconstrained optimization problems: No decoding function required, $S_x = S_c$

Constrained op: Add a constraint penalty function

Multi-objective op: Fitness function is a weight sum of all the sub-objectives.

Dynamic and noisy problems: Dynamic fitness functions are time-dependent whereas noisy functions usually have an added Gaussian noisy component.

8.5 Selection

① Selection of the new population ② Reproduction

Selective Pressure: A high selective pressure decreases diversity in the population more rapidly, but limits the exploration abilities of the population.

A low selective pressure may lead to premature convergence to suboptimal solution

Random Selection: Each individual has the same probability of $\frac{1}{n_s}$. No fitness information is used.
The lowest selective pressure.

Proportional Selection: Biases selection toward the most fit individuals

$$\varphi_s(x_i(t)) = \frac{f_r(x_i(t))}{\sum_{i=1}^{n_s} f_r(x_i(t))}$$

$\varphi_s(x_i(t))$ is the probability that x_i will be selected.

$$\text{① } f_r(x_i(t)) = r(x_i(t)) = f_{\max} - \frac{1}{f_{\max} - f_{\min}(t)}$$

$$\text{② } f_r(x_i(t)) = r(x_i(t)) = \frac{1}{1 + f_{\psi}(x_i(t)) - f_{\min}(t)}$$

where $f_{\min}(t)$ is the minimum observed fitness up to time step t .

Roulette Wheel Sampling: Spin a roulette wheel and record which slice ends up at the top; the corresponding individual is then selected.

Let $i=1$, where i denote the chromosome index;

Calculate $\varphi_s(x_i)$;

Sum = $\varphi_s(x_i)$;

Choose $r \sim U(0, 1)$;

while sum < r do

$i = i+1$, i.e. advance to next chromosome;

sum = sum + $\varphi_s(x_i)$;



end

Return X_i as the selected individual

The probability of being selected is correlated with the fitness by a cumulative function

The best individual may not be selected during a given generation

∴ Stochastic Universal Sampling : λ_i denotes the number of offspring per individual

for $i=1, \dots, n_s$ do

$$\lambda_i(t) = 0;$$

end

$$r \sim U(0, \frac{1}{n}) , \lambda = \sum_{i=1}^{n_s} \lambda_i;$$

$$\text{sum} = 0.0;$$

for $i=1, \dots, n_s$ do

$$\text{sum} = \text{sum} + \varphi_s(X_i(t));$$

while $r < \text{sum}$ do

$$\lambda_i++;$$

$$r = r + \frac{1}{n};$$

end

end

$$\text{return } \lambda = (\lambda_1, \lambda_2, \dots, \lambda_{n_s})$$

Tournament Selection : ① Select a group of n_{ts} individuals randomly from the population

② The performance of n_{ts} individuals is compared and the best individual from this group is select

Rank-Based Selection : Use the rank ordering of fitness values to determine the probability of selection, and not the absolute fitness values.

① Non-deterministic linear sampling : Select individual X_i , $i \sim U(0, V(0, n_s - 1))$

The rank of worst individual is 0, and that of the best individual is $n_s - 1$ #加上不太对吧

② Linear ranking : Assume the best individual create $\hat{\lambda}$ offsprings, $1 \leq \hat{\lambda} \leq 2$, and the worst individual $\tilde{\lambda}$, $\tilde{\lambda} = 2 - \hat{\lambda}$

$$\varphi_s(X_i(t)) = \frac{\hat{\lambda} + (-f_r(X_i(t)) / (n_s - 1))(\hat{\lambda} - \tilde{\lambda})}{n_s} \quad \text{i.e. divide the equidistance between } \hat{\lambda}, \tilde{\lambda}$$

where $f_r(X_i(t))$ is the rank of $X_i(t)$

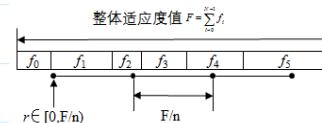
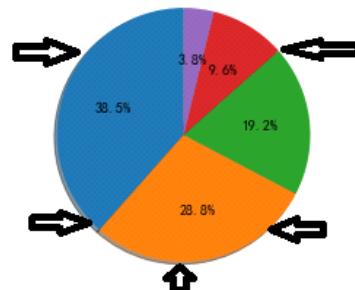
$$\text{③ Nonlinear ranking : } \varphi_s(X_i(t)) = \frac{1 - e^{-\beta f_r(X_i(t))}}{\beta} \quad \text{or} \quad \varphi_s(X_i) = V(1 - V)^{1/p - 1 - f_r(X_i)}$$

where $f_r(X_i)$ is the rank of X_i , β is a normalization constant, and V indicates the probability of selecting the next individual.

$$\text{Boltzmann Selection : } \varphi(X_i(t)) = \frac{1}{1 + e^{f_r(X_i(t))/T(t)}}$$

where $T(t)$ is the temperature parameter. $T(t)$ is reduced from its initial large value

图例：A (Blue), B (Orange), C (Green), D (Red), E (Purple)



to a small value. As $T(\epsilon)$ become smaller, selection focuses more and more on the good individuals.

($\mu + \lambda$) - Selection : μ indicates the number of parents (which is the size of population), and λ is the number of offspring .

After production of the λ offspring :

① (μ, λ) - selection selects the best μ offspring for the next population

② ($\mu + \lambda$) - selection selects the best μ individuals from both the parents and the offspring.

Elitism : The best individuals are copied to the new population without being mutated

Hall of Fame : Contain an archive of the best individuals from the first generation. It can be used as a parent pool for the crossover operator, or, at the last generation, the best individual is selected as the best one in the hall of fame.

8.6 Reproduction Operators

Reproduction : The process of producing from selected parents parents by applying crossover and/or mutation operators.

Crossover : The process of creating one or more new individuals through the combination of genetic material randomly selected from two or more parents.

Mutation : The process of randomly changing the values of genes in a chromosome.

Object : Introduce new genetic material into the population, thereby increasing genetic diversity.

The less fit the individual, the more it is mutated.

8.7 Stopping Conditions

- ① Terminate when no improvement is observed over a number of consecutive generations.
- ② Terminate when there is no change in the population
- ③ Terminate when an acceptable solution has been found.
- ④ Terminate when the objective function slope is approximately zero.

8.8 Evolutionary Computation versus Classical Optimization

The search process : CO : deterministic rules, probabilistic transition rules.

EC : parallel search, sequential search

Search surface information : CO : derivative information

EC : fitness values