第四章 数据流挖掘（上）

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大数据分析与挖掘

主要内容

- 4.1 流数据模型
- 4.2 流数据抽样
- 4.6 窗口内的计数问题
- 4.3 流过滤
- 4.4 流中独立元素的数目统计
- 4.5 矩估计
- 4.7 衰减窗口

New Topic: Infinite Data

High dim. data	Graph data	**Infinite data**	Machine learning	Apps
Locality sensitive hashing	PageRank, SimRank	Filtering data streams	SVM	Recommen der systems
Clustering	Community Detection	Queries on streams	Decision Trees	Association Rules
Dimensional ity reduction	Spam Detection	Web advertising	Perceptron, kNN	Duplicate document detection

So far

- So far we have worked datasets or data bases where all data is available
- In contrast, in **data streams**, data arrives one element at a time often at a **rapid rate** that:
  - If it is not **processed immediately** it is lost forever.
  - It is not feasible to **store** it all

Data Streams

- **In many data mining situations, we do not know the entire data set in advance**
- **Stream Management** is important when the input rate is controlled **externally:**
  - Google queries
  - Twitter or Facebook status updates
- We can think of the **data** as **infinite** and **non-stationary** (无穷无尽，且非平稳的，数据分布会动态变化)

The Stream Model

- Input **elements** enter at a rapid rate,
  at one or more input ports (i.e., **streams**)
  - **We call elements of the stream tuples**

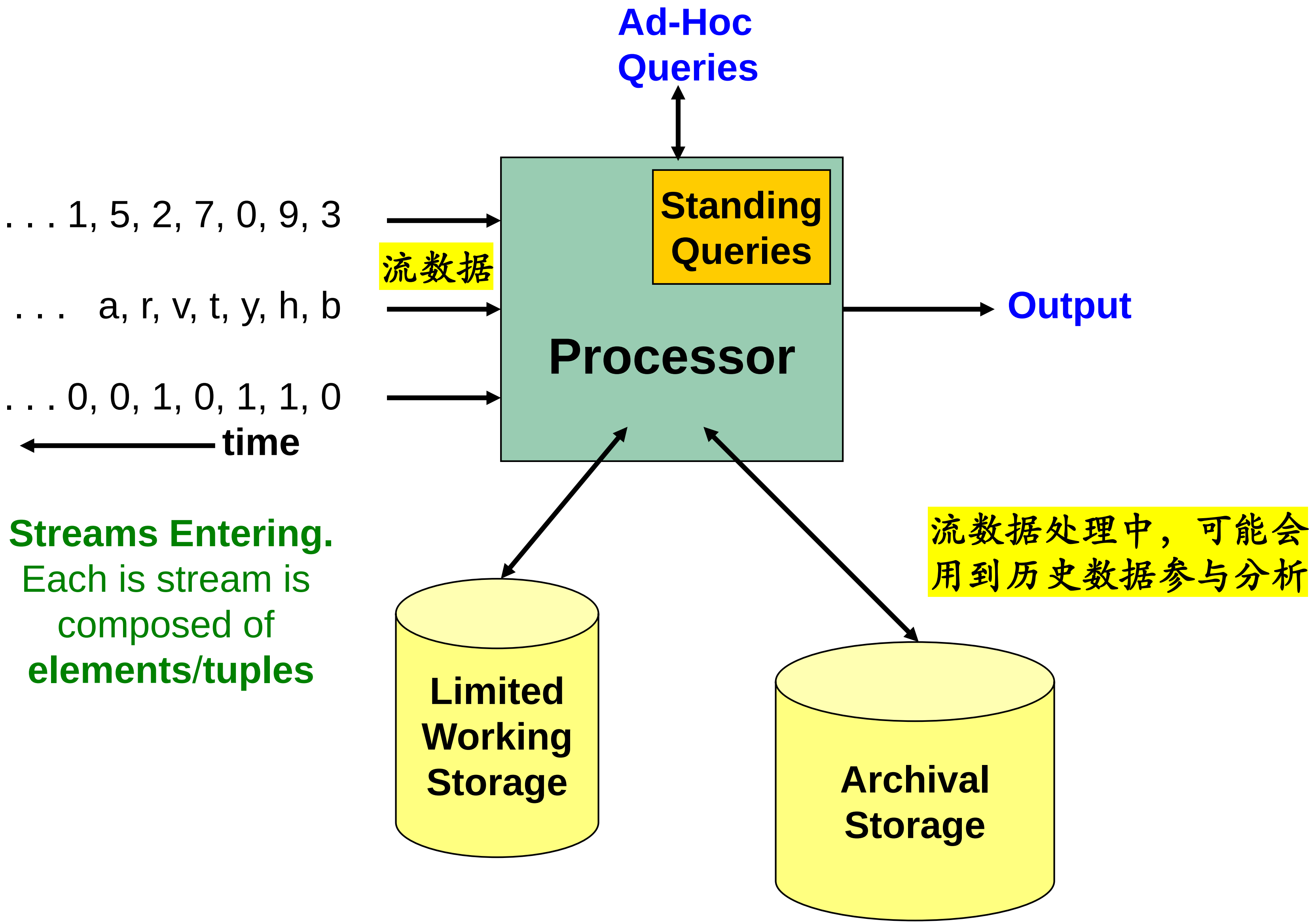
- **The system cannot store the entire stream accessibly**

- **Q: How do you make critical calculations about the stream using a limited amount of (secondary) memory?**

Side note: SGD is a Streaming Alg.

- **Stochastic Gradient Descent (SGD, 随机梯度下降) is an example of a stream algorithm**
- **In Machine Learning we call this: Online Learning**
  - Allows for modeling problems where we have a continuous stream of data
  - We want an algorithm to learn from it and slowly adapt to the changes in data
- **Idea: Do slow updates to the model**
  - **SGD** (SVM, Perceptron) makes small updates
  - **So:** First train the classifier on training data.
  - **Then:** For every example from the stream, we slightly update the model (using small learning rate)

General Stream Processing Model



Streams Entering.
Each is stream is composed of **elements/tuples**

Problems on Data Streams

- **Types of queries one wants on answer on a data stream:** (we'll do these today)
  - **Sampling data from a stream 采样分析**
    - Construct a random sample
  - **Queries over sliding windows 滑动窗口查询**
    - Number of items of type ***x*** in the last ***k*** elements of the stream

Problems on Data Streams

- **Types of queries one wants on answer on a data stream:** (we'll do these next time)
  - **Filtering a data stream**
    - Select elements with property ***x*** from the stream
  - **Counting distinct elements**
    - Number of distinct elements in the last ***k*** elements of the stream
  - **Estimating moments**
    - Estimate avg./std. dev. of last ***k*** elements
  - **Finding frequent elements**

Applications (1)

- **Mining query streams**
  - Google wants to know what queries are more frequent today than yesterday
- **Mining click streams**
  - Yahoo wants to know which of its pages are getting an unusual number of hits in the past hour
- **Mining social network news feeds**
  - E.g., look for trending topics on Twitter, Facebook

Applications (2)

- **Sensor Networks**
  - Many sensors feeding into a central controller
- **Telephone call records**
  - Data feeds into customer bills as well as settlements between telephone companies
- **IP packets monitored at a switch**
  - Gather information for optimal routing
  - Detect denial-of-service attacks

Sampling from a Data Stream: Sampling a fixed proportion

As the stream grows the sample also gets bigger

Sampling from a Data Stream

- Why is this important?
  - Since **we can not store the entire stream,** a representative **sample** can act like the stream
- **Two different problems:**
  - **(1)** Sample a **fixed proportion** of elements in the stream (say 1 in 10) 固定比例采用，如1/10
  - **(2)** Maintain a **random sample of fixed size** over a potentially infinite stream 固定规模的随机采样
    - At any “time” ***k*** we would like a random sample of ***s*** elements of the stream 1…k
      - **What is the property of the sample we want to maintain?** For all time steps ***k***, each of ***k*** elements seen so far must have **equal probability** of being sampled

Sampling a Fixed Proportion

- **Problem 1: Sampling a fixed proportion**
  - E.g. sample 10% of the stream
  - As stream gets bigger, sample gets bigger
- **Naïve solution:**
  - Generate a random integer in **[0…9]** for each query
  - Store the query if the integer is **0**, otherwise discard
- **Any problem with this approach?**
  - We have to be very careful what query we answer using this sample

需要结合业务需求场景，以体现模型价值和判断合理性

Problem with Naïve Approach

- **Scenario:** Search engine query stream
  - **Stream of tuples:** (user, query, time)
  - **Question:** What fraction of unique queries by an average user are duplicates?
    - Suppose each user issues x queries once and d queries twice in one month
    - total of $x+2d$ query instances
    - then the correct answer to the query is $d/(x+d)$

Problem with Naïve Approach

- **Scenario:** Search engine query stream
  - **Proposed solution:** **We keep 10% of the queries**
    - Sample will contain ***(x+2d)*/10** elements of the stream
    - Sample will contain ***d*/100** pairs of duplicates
      - **d/100 = 1/10 · 1/10 · d**
    - There are ***(10x+19d)/100*** unique elements in the sample
      - $(x+2d)/10 - d/100 = (10x+19d)/100$
- **So the sample-based answer is** $\dfrac{\frac{d}{100}}{\frac{x}{10}+\frac{d}{100}+\frac{18d}{100}} = \dfrac{d}{10x+19d}$

$\neq$ ***d/(x+d)***

Solution: Sample Users

Solution:

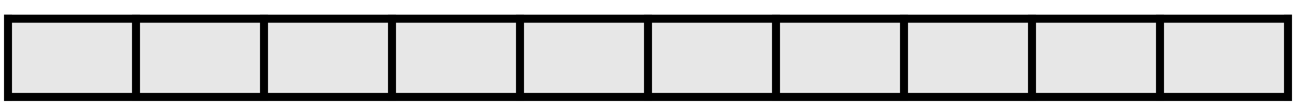
- Pick **1/10th** of **users** and take all their searches in the sample

- Use a hash function that hashes the user name or user id uniformly into 10 buckets

Generalized Solution

- **Stream of tuples with keys:**
  - Key is some subset of each tuple's components
    - e.g., tuple is (user, search, time); key is **user**
  - Choice of key depends on application

- **To get a sample of *a/b* fraction of the stream:**
  - Hash each tuple's key uniformly into ***b*** buckets
  - Pick the tuple if its hash value is at most ***a***



Hash table with **b** buckets, pick the tuple if its hash value is at most **a.**
How to generate a 30% sample?
Hash into b=10 buckets, take the tuple if it hashes to one of the first 3 buckets

Sampling from a Data Stream: Sampling a fixed-size sample

The sample is of fixed size

Maintaining a fixed-size sample

- **Problem 2: Fixed-size sample**
- **Suppose we need to maintain a random sample *S* of size exactly *s* tuples 仅保存s个元组**
  - E.g., main memory size constraint

- 固定大小抽样，希望达到如下效果:
  - **Suppose at time *n* we have seen *n* items**
  - **Each item is in the sample *S* with equal prob. *s/n***

Maintaining a fixed-size sample

例如：

How to think about the problem: say s = 2

Stream: a x c y z k c d e g…

At **n= 5,** each of the first 5 tuples is included in the sample **S** with equal prob.

At **n= 7,** each of the first 7 tuples is included in the sample **S** with equal prob.

看似理想，但不现实的做法是：

存下n个元组（n是不断增大的），然后随机选择s个。

Solution: Fixed Size Sample

- **Algorithm (a.k.a. Reservoir Sampling, 水塘抽样)**
  - Store all the first ***s*** elements of the stream to ***S***
  - Suppose we have seen ***n-1*** elements, and now the ***n***th element arrives (***n*** > ***s***)
    - With probability ***s/n***, keep the ***n***th element, else discard it
    - If we picked the ***n***th element, then it replaces one of the ***s*** elements in the sample ***S***, picked uniformly at random
- **Claim:** This algorithm maintains a sample ***S*** with the desired property:
  - After ***n*** elements, the sample contains each element seen so far with probability ***s/n***

Proof: By Induction

- **We prove this by induction:**
  - Assume that after ***n*** elements, the sample contains each element seen so far with probability ***s/n***
  - We need to show that after seeing element ***n+1*** the sample maintains the property
    - Sample contains each element seen so far with probability ***s/(n+1)*** ***注意n是动态变化的，需要存储n数值***
- **Base case:**
  - After we see **n=s** elements the sample **S** has the desired property
    - Each out of **n=s** elements is in the sample with probability ***s/s = 1***

Proof: By Induction

- **Inductive hypothesis:** After ***n*** elements, the sample ***S*** contains each element seen so far with prob. ***s/n***
- **Now element *n+1* arrives**
- **Inductive step:** For elements already in ***S***, probability that the algorithm keeps it in ***S*** is:

$$\underbrace{\left(1-\frac{s}{n+1}\right)}_{\text{Element } \mathbf{n+1} \text{ discarded}}+\underbrace{\left(\frac{s}{n+1}\right)}_{\text{Element } \mathbf{n+1} \text{ not discarded}}\underbrace{\left(\frac{s-1}{s}\right)}_{\text{Element in the sample not picked}}=\frac{n}{n+1}$$

- So, at time ***n,*** tuples in ***S*** were there with prob. **s/n**
- Time ***n→n+1,*** tuple stayed in ***S*** with prob. **n/(n+1)**
- So prob. tuple is in ***S*** at time ***n+1*** $= \frac{s}{n} \cdot \frac{n}{n+1} = \frac{s}{n+1}$

窗口内计数问题
Queries over a (long) Sliding Window

Sliding Windows

- A useful model of stream processing is that queries are about a ***window*** of length ***N*** – the ***N*** most recent elements received
- **Interesting case:** ***N*** is so large that the data cannot be stored in memory, or even on disk
  - Or, there are so many streams that windows for all cannot be stored
- **Amazon example:**
  - For every product **X** we keep 0/1 stream of whether that product was sold in the **n**-th transaction
  - We want answer queries, how many times have we sold **X** in the last **k** sales（k小于n，在窗口之内）

Sliding Window: 1 Stream

- **Sliding window on a single stream:**

N = 6

q w e r t y u i o p $\boxed{\text{a s d f g h}}$ j k l z x c v b n m

q w e r t y u i o p a $\boxed{\text{s d f g h j}}$ k l z x c v b n m

q w e r t y u i o p a s $\boxed{\text{d f g h j k}}$ l z x c v b n m

q w e r t y u i o p a s d $\boxed{\text{f g h j k l}}$ z x c v b n m

← Past Future →

Counting Bits (1)

- **Problem:**
  - Given a stream of **0**s and **1**s
  - Be prepared to answer queries of the form
    How many 1s are in the last *k* bits? where $k \leq N$

N是窗口大小，k是任务中设定的一个参数，可变

- **Obvious solution:**
  Store the most recent N bits
  - When new bit comes in, discard the $N+1^{st}$ bit

0 1 0 0 1 1 0 1 1 1 0 1 0 1 0 1 1 0 $\boxed{1\ 1\ 0\ 1\ 1\ 0}$ Suppose N=6

← Past Future →

Counting Bits (2)

- **You can not get an exact answer without storing the entire window**

- **Real Problem:** 真正的问题是无法存下N bits数据
  What if we cannot afford to store *N* bits?
  - **E.g.**, we're processing 1 billion streams and
    ***N* = 1 billion**

0 1 0 0 1 1 0 1 1 1 0 1 0 1 0 1 1 0 ~~1 1 0 1 1 0~~

← Past Future →

不能完整记录N个元组，同时希望得到准确结果

An attempt: Simple solution

- Q: How many 1s are in the last *N* bits?

N

0 1 0 0 1 1 1 0 0 0 1 0 1 0 0 1 0 0 0 1 0 1 1 0 1 1 0 1 1 1 0 0 1 0 1 0 1 1 0 0 1 1 0 1 0

Past Future

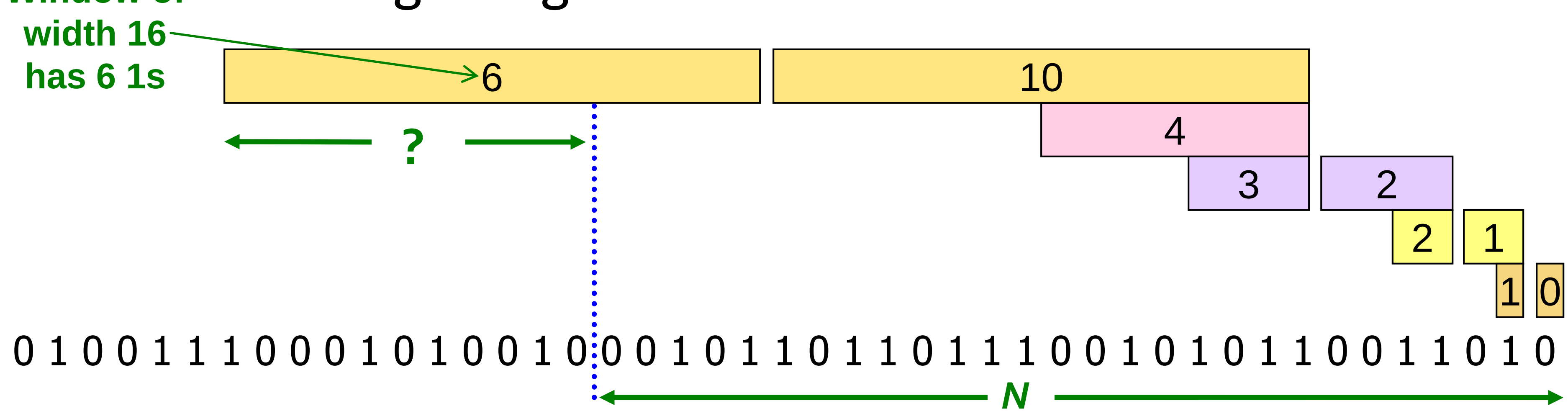
- **Simple solution**
  - **Maintain 2 counters:**
    - ***S***: number of 1s from the beginning of the stream
    - ***Z***: number of 0s from the beginning of the stream
  - **How many 1s are in the last N bits?** $N \cdot \frac{S}{S+Z}$
- A simple solution that does not really solve our problem
- **它用了一致性假设(Uniformity assumption)，实际上数据流可能是非一致性的(non-uniform)**
  - the distribution changes over time

DGIM Method

- **Datar-Gionis-Indyk-Motwani Algorithm**
  - **DGIM solution that does <u>not</u> assume uniformity**
- 用 $\boldsymbol{O}(\log^2 \boldsymbol{N})$位，表示大小为N的窗口
- **窗口内1的数量的估计误差，不超过50%**
- **后续的改进算法，可以不断降低错误率**

Idea: Exponential Windows

- **Solution that doesn't (quite) work:**
  - Summarize **exponentially increasing** regions of the stream, looking backward
  - Drop small regions if they begin at the same point as a larger region



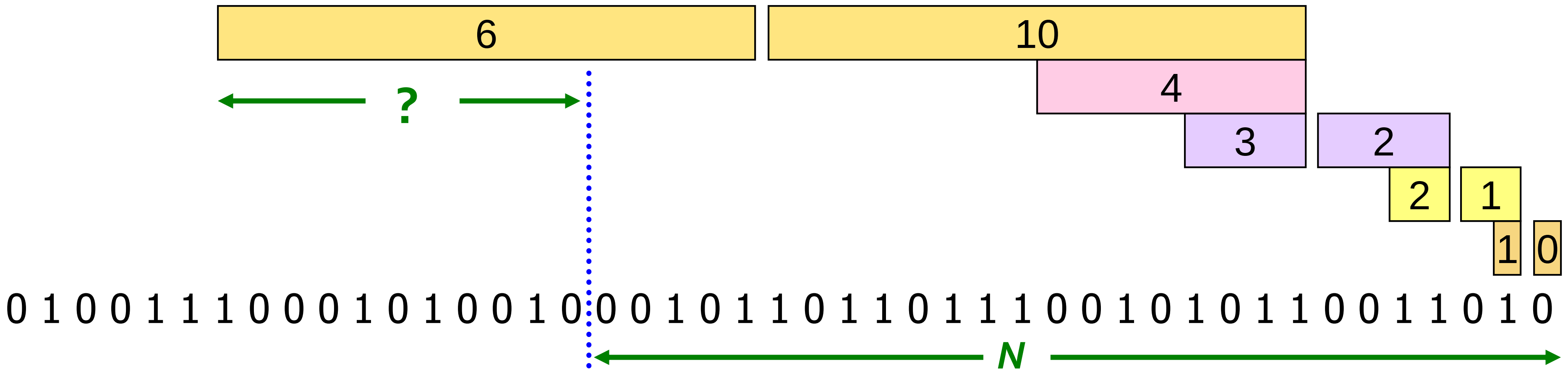
We can reconstruct the count of the last ***N*** bits, except we are not sure how many of the last **6 1s** are included in the ***N***

What's Good?

- **Stores only O($\log^2 N$) bits**
  - $\boldsymbol{O}(\log \boldsymbol{N})$ counts of $\log_2 \boldsymbol{N}$ bits each
- **Easy update as more bits enter**
- Error in count no greater than the number of **1s** in the "**unknown**" area

What's Not So Good?

- As long as the **1s** are fairly evenly distributed, the error due to the unknown region is small – **no more than 50%**
- But it could be that all the **1s** are in the unknown area at the end
- In that case, **the error is unbounded!**



[Datar, Gionis, Indyk, Motwani]

Fixup: DGIM method

- **Idea:** Instead of summarizing fixed-length blocks, summarize blocks with specific number of **1s**:
  - Let the block ***sizes*** (number of **1s**) increase exponentially

- **When there are few 1s in the window, block sizes stay small, so errors are small**

1001010110001011 0101010101010101 1 0 10101010101011 1 0 1010101 110101 000 101 1 00 1 0

← N →

DGIM: Timestamps

- Each bit in the stream has a ***timestamp***, starting **1**, **2,** …

- Record timestamps modulo ***N*** (**the window size**), so we can represent any **relevant** timestamp in $\boldsymbol{O(log_2 N)}$ bits

DGIM: Buckets

- A ***bucket*** in the DGIM method is a record consisting of:
  - **(A) The timestamp of its end** [O(log *N*) bits] 最近的时间
  - **(B) The number of 1s between its beginning and end** [O(log log *N*) bits]

- **Constraint on buckets:**
  Number of **1s** must be a power of **2** 只用记录指数
  - That explains the **O(log log *N*)** in **(B) above**

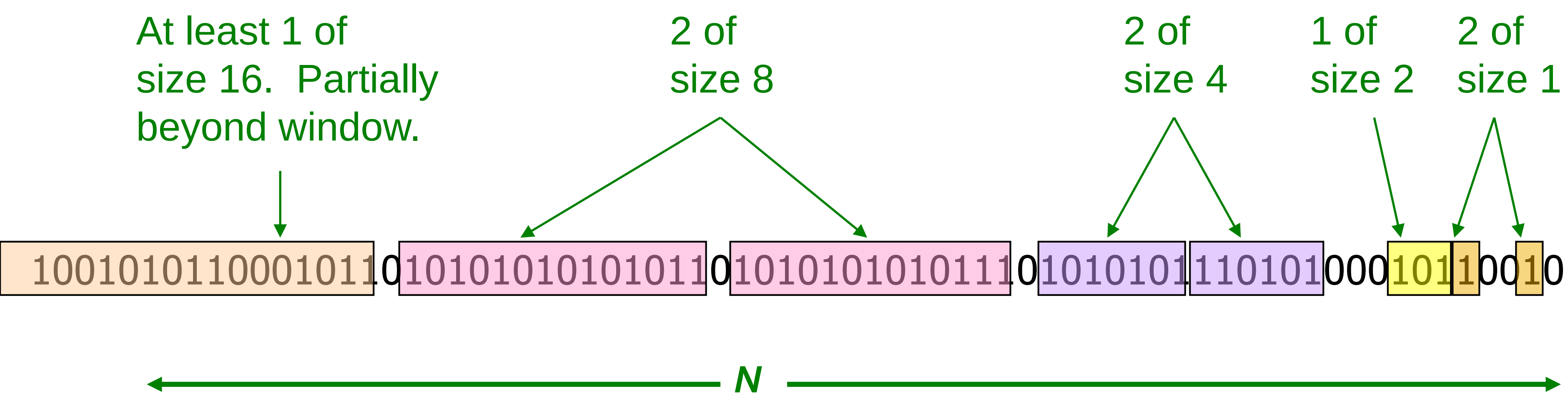
10010101100010110101010101010110101010101011101010101110101000101100010

N

Representing a Stream by Buckets

- Either **one** or **two** buckets with the same **power-of-2 number** of **1s**
- **Buckets do not overlap in timestamps**
- **Buckets are sorted by size**
  - Earlier buckets are not smaller than later buckets
  - 早期bucket逐渐变大
- Buckets disappear when their end-time is **>** ***N*** time units in the past

Example: Bucketized Stream



Three properties of buckets that are maintained:

- Either **one** or **two** buckets with the same **power-of-2** number of **1s**
- Buckets do not overlap in timestamps
- Buckets are sorted by size

Updating Buckets (1)

- When a new bit comes in, drop the last (oldest) bucket if its end-time is prior to ***N*** time units before the current time

2 cases: Current bit is **0** or **1**

- **If the current bit is 0:**
  no other changes are needed

Updating Buckets (2)

If the current bit is 1:

- **(1)** Create a new bucket of size **1**, for just this bit
  - **End timestamp = current time**
- **(2)** If there are now **three buckets of size 1**, **combine the oldest two into a bucket of size 2**
- **(3)** If there are now **three buckets of size 2**, **combine the oldest two into a bucket of size 4**
- **(4) And so on …**

Example: Updating Buckets

Current state of the stream:

1001010110001011 0 1010101010101011 0 101010101010111 0 1010101 110101 000 101 1 00 1 0

Bit of value 1 arrives

001010110001011 0 1010101010101011 0 101010101010111 0 1010101 110101 000 101 1 00 1 0 **1**

Two orange buckets get merged into a yellow bucket

001010110001011 0 1010101010101011 0 101010101010111 0 1010101 110101 000 101 1001 0 1

Next bit 1 arrives, new orange bucket is created, then 0 comes, then 1:

010110001011 0 1010101010101011 0 101010101010111 0 1010101 110101 000 101 1001 0 1 **1** **0** **1**

Buckets get merged…

010110001011 0 1010101010101011 0 101010101010111 0 1010101 110101 000 101 1001 0 1 **1** **0** **1**

State of the buckets after merging

010110001011 0 10101010101010110101010101010111 0 1010101110101 000 1011001 0 11 0 1

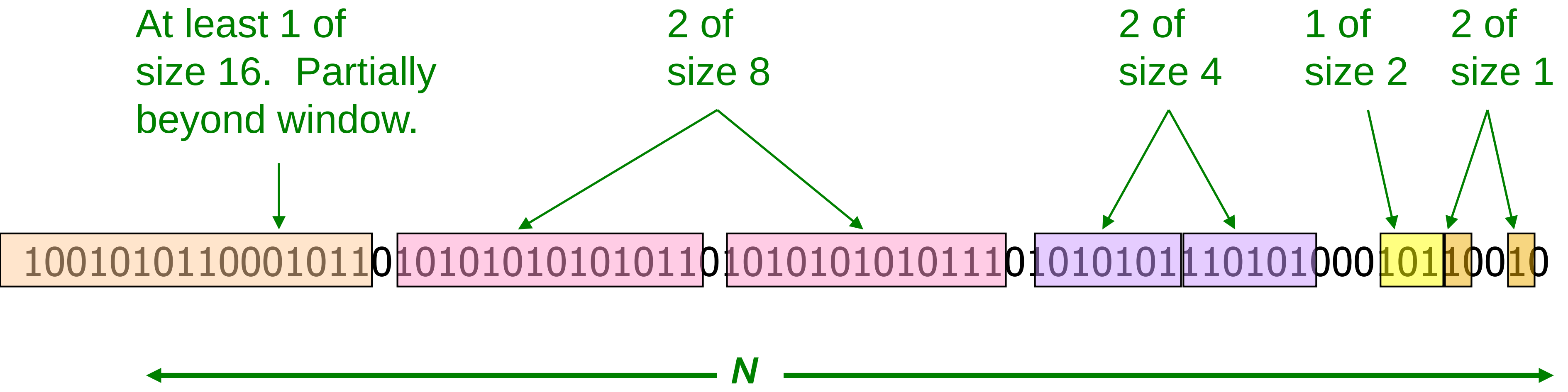
How to Query?

- **To estimate the number of 1s in the most recent *N* bits:**
  1. **Sum the sizes of all buckets but the last**

     (note "size" means the number of 1s in the bucket)
  2. **Add half the size of the last bucket**

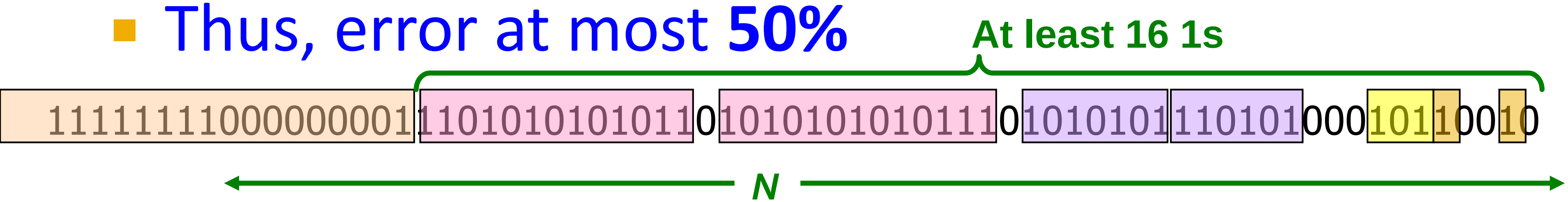
- **Remember:** We do not know how many **1s** of the last bucket are still within the wanted window

Example: Bucketized Stream



Error Bound: Proof

- **Why is error 50%? Let's prove it!**
- Suppose the last bucket has size $\mathbf{2^{r}}$
- Then by assuming $\mathbf{2^{r-1}}$ (i.e., half) of its **1s** are still within the window, we make an error of at most $\mathbf{2^{r-1}}$
- Since there is at least one bucket of each of the sizes less than $\mathbf{2^{r}}$, the true sum is at least $\mathbf{1 + 2 + 4 + .. + 2^{r-1} = 2^{r} - 1}$
- Thus, error at most **50%**

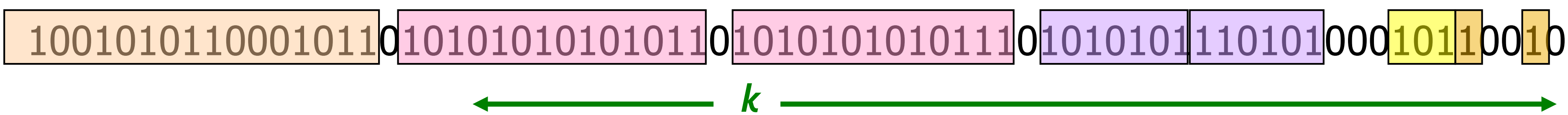


Further Reducing the Error

- Instead of maintaining **1** or **2** of each size bucket, we allow either ***r*-1** or ***r*** buckets (***r* > 2**)
  - Except for the largest size buckets; we can have any number between **1** and ***r*** of those
- **Error is at most *O*(1/*r*)**
- By picking ***r*** appropriately, we can tradeoff between number of bits we store and the error

Extensions

- Can we use the same trick to answer queries **How many 1's in the last *k*?** where ***k*** **<** ***N***?
  - **A:** Find earliest bucket **B** that at overlaps with ***k***. Number of **1s** is the **sum of sizes of more recent buckets + ½ size of B**



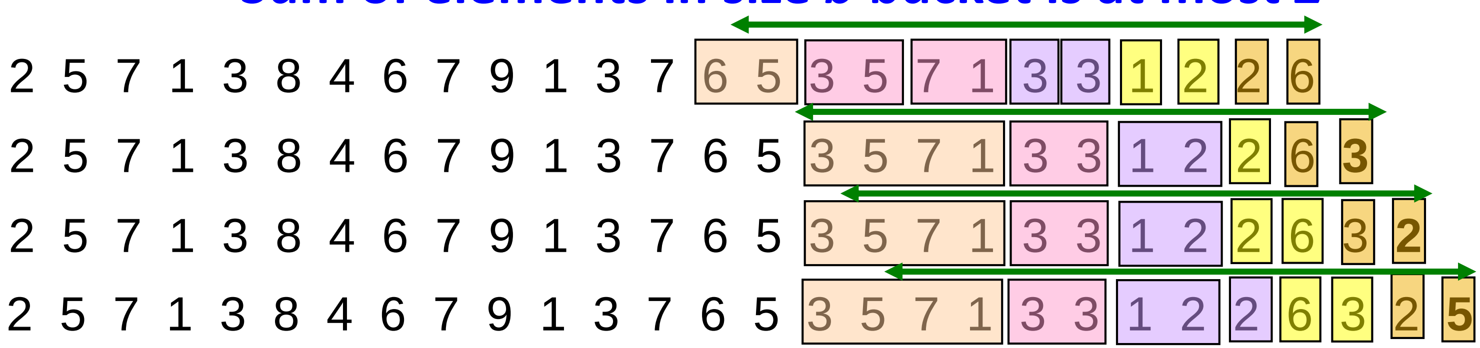
- **Can we handle the case where the stream is not bits, but integers, and we want the sum of the last *k* elements?**

Extensions

- **Stream of positive integers**
- **We want the sum of the last *k* elements**
  - **Amazon:** Avg. price of last **k** sales
- **Solution:**
  - **(1) If you know all have at most *m* bits**
    - Treat ***m*** bits of each integer as a separate stream
    - Use DGIM to count **1s** in each integer
    - The sum is $= \sum_{i=0}^{m-1} c_i 2^i$

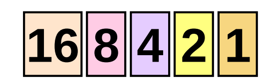
c_i …estimated count for **i-th** bit

  - **(2) Use buckets to keep partial sums**
    - **Sum of elements in size *b* bucket is at most 2^b**



Idea: Sum in each bucket is at most 2^b (unless bucket has only **1** integer)

Bucket sizes:



Summary

- **Sampling a fixed proportion of a stream**
  - Sample size grows as the stream grows
- **Sampling a fixed-size sample**
  - Reservoir sampling
- **Counting the number of 1s in the last N elements**
  - Exponentially increasing windows
  - Extensions:
    - Number of 1s in any last k (k < N) elements
    - Sums of integers in the last N elements