第六次作业

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习题5.1.1

Exercise 5.1.1: Compute the PageRank of each page in Fig. 5.7, assuming no taxation.

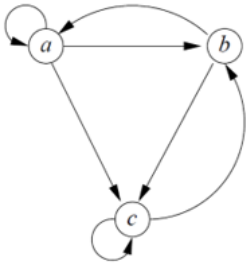

Figure 5.7: An example graph for exercises

图5.7的转移矩阵为：

$$\begin{bmatrix} 1/3 & 1/2 & 0 \\ 1/3 & 0 & 1/2 \\ 1/3 & 1/2 & 1/2 \end{bmatrix}$$

不采用"抽税"法，我们从一个所有分量均为1/3的初始向量开始迭代计算，每次迭代都左乘转移矩阵。最终得到的向量序列为：

$$\begin{bmatrix} 0.2778 \\ 0.2778 \\ 0.4444 \end{bmatrix}, \begin{bmatrix} 0.2315 \\ 0.3148 \\ 0.4537 \end{bmatrix}, \begin{bmatrix} 0.2346 \\ 0.3040 \\ 0.4614 \end{bmatrix}, \begin{bmatrix} 0.2302 \\ 0.3089 \\ 0.4609 \end{bmatrix}, \begin{bmatrix} 0.2302 \\ 0.3089 \\ 0.4609 \end{bmatrix}, \begin{bmatrix} 0.2312 \\ 0.3072 \\ 0.4616 \end{bmatrix}, \dots, \begin{bmatrix} 0.2308 \\ 0.3077 \\ 0.4615 \end{bmatrix}$$

习题5.1.2

Exercise 5.1.2: Compute the PageRank of each page in Fig. 5.7, assuming $\beta = 0.8$.

假定β=0.8，向量v'的迭代公式为：

$$v' = \beta M v + \frac{(1-\beta)e}{n} = \begin{bmatrix} 4/15 & 2/5 & 0 \\ 4/15 & 0 & 2/5 \\ 4/15 & 2/5 & 2/5 \end{bmatrix} v + \begin{bmatrix} 1/15 \\ 1/15 \\ 1/15 \end{bmatrix}$$

我们从一个所有分量均为1/3的初始向量开始迭代计算，最终得到的向量序列为：

$$\begin{bmatrix} 0.2889 \\ 0.2889 \\ 0.4222 \end{bmatrix}, \begin{bmatrix} 0.2593 \\ 0.3126 \\ 0.4281 \end{bmatrix}, \begin{bmatrix} 0.2608 \\ 0.3071 \\ 0.4321 \end{bmatrix}, \begin{bmatrix} 0.2590 \\ 0.3091 \\ 0.4319 \end{bmatrix}, \begin{bmatrix} 0.2594 \\ 0.3085 \\ 0.4321 \end{bmatrix}, \begin{bmatrix} 0.2592 \\ 0.3087 \\ 0.4321 \end{bmatrix}, \dots, \begin{bmatrix} 0.2593 \\ 0.3086 \\ 0.4321 \end{bmatrix}$$

习题5.1.6

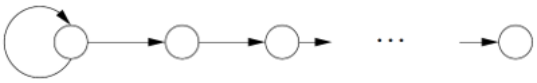

Figure 5.9: A chain of dead ends

Exercise 5.1.6: Suppose we recursively eliminate dead ends from the graph, solve the remaining graph, and estimate the PageRank for the dead-end pages as described in Section 5.1.4. Suppose the graph is a chain of dead ends, headed by a node with a self-loop, as suggested in Fig. 5.9. What would be the PageRank assigned to each of the nodes?

在图5.9中，最左边有自环的节点的PageRank为1，其余所有节点的PageRank为1/2。

习题5.1.7

Exercise 5.1.7: Repeat Exercise 5.1.6 for the tree of dead ends suggested by Fig. 5.10. That is, there is a single node with a self-loop, which is also the root of a complete binary tree of n levels.

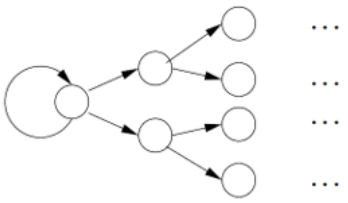

Figure 5.10: A tree of dead ends

在图5.10中，最左边有自环的节点的PageRank为1，第i(i>=2)层的节点的PageRank为$1/(3*2^{i-2})$。

习题5.3.1

5.3.5 Exercises for Section 5.3

Exercise 5.3.1: Compute the topic-sensitive PageRank for the graph of Fig. 5.15, assuming the teleport set is:

(a) A only.

(b) A and C.

图5.15的转移矩阵为：

$$\begin{bmatrix} 0 & 1/2 & 1 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{bmatrix}$$

假设β=0.8。

(a) 仅包含A

则$e_S=[1,0,0,0]^T$，$|S|=1$。

PageRank向量的迭代公式为：

$$v' = \beta M v + \frac{(1-\beta)e_S}{|S|} = \begin{bmatrix} 0 & 2/5 & 4/5 & 0 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 1/2 & 0 & 0 \end{bmatrix} v + \begin{bmatrix} 1/5 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

我们从初始向量$e_S/|S|$开始迭代计算，最终得到的向量序列为：

$$\begin{bmatrix} 1.000 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0.2000 \\ 0.2667 \\ 0.2667 \\ 0.2667 \end{bmatrix}, \begin{bmatrix} 0.5200 \\ 0.1600 \\ 0.1600 \\ 0.1600 \end{bmatrix}, \begin{bmatrix} 0.3920 \\ 0.2027 \\ 0.2027 \\ 0.2027 \end{bmatrix}, \begin{bmatrix} 0.4432 \\ 0.1856 \\ 0.1856 \\ 0.1856 \end{bmatrix}, \begin{bmatrix} 0.4227 \\ 0.1924 \\ 0.1924 \\ 0.1924 \end{bmatrix}, \cdots, \begin{bmatrix} 0.4285 \\ 0.1904 \\ 0.1904 \\ 0.1904 \end{bmatrix}$$

(b) 包含A和C

则$e_S=[1,0,1,0]^T$，$|S|=2$。

PageRank向量的迭代公式为：

$$v' = \beta M v + \frac{(1-\beta)e_S}{|S|} = \begin{bmatrix} 0 & 2/5 & 4/5 & 0 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 1/2 & 0 & 0 \end{bmatrix} v + \begin{bmatrix} 1/10 \\ 0 \\ 1/10 \\ 0 \end{bmatrix}$$

我们从初始向量$e_S/|S|$开始迭代计算，最终得到的向量序列为：

$$\begin{bmatrix} 0.500 \\ 0 \\ 0.500 \\ 0 \end{bmatrix}, \begin{bmatrix} 0.500 \\ 0.1333 \\ 0.2333 \\ 0.1333 \end{bmatrix}, \begin{bmatrix} 0.3400 \\ 0.1867 \\ 0.2967 \\ 0.1867 \end{bmatrix}, \begin{bmatrix} 0.4040 \\ 0.1653 \\ 0.2653 \\ 0.1653 \end{bmatrix}, \begin{bmatrix} 0.3784 \\ 0.1739 \\ 0.2739 \\ 0.1739 \end{bmatrix}, \begin{bmatrix} 0.3886 \\ 0.1705 \\ 0.2705 \\ 0.1705 \end{bmatrix}, \cdots, \begin{bmatrix} 0.3857 \\ 0.1714 \\ 0.2714 \\ 0.1714 \end{bmatrix}$$