Matrix Analysis Assignment 4

October 13, 2020

Question 1.

$$\mathbf{A}=\begin{pmatrix} 1 & 3 & 1 & -4 \\ -1 & -3 & 1 & 0 \\ 2 & 6 & 2 & -8 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 3 & 1 & -4 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{A^TA}=\begin{pmatrix} 6 & 18 & 4 & -20 \\ 18 & 54 & 12 & -60 \\ 4 & 12 & 6 & -20 \\ -20 & -60 & -20 & 80 \end{pmatrix} \longrightarrow \begin{pmatrix} 4 & 12 & 6 & -20 \\ 18 & 54 & 12 & -60 \\ 6 & 18 & 4 & -20 \\ -20 & -60 & -20 & 80 \end{pmatrix} \longrightarrow \begin{pmatrix} 4 & 12 & 6 & -20 \\ 0 & 0 & -15 & 30 \\ 0 & 0 & -5 & 10 \\ 0 & 0 & 10 & -20 \end{pmatrix}$$

$$\longrightarrow \begin{pmatrix} 4 & 12 & 6 & -20 \\ 0 & 0 & -15 & 30 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{AA^T}=\begin{pmatrix} 27 & -9 & 54 \\ -9 & 11 & -18 \\ 54 & -18 & 108 \end{pmatrix} \longrightarrow \begin{pmatrix} -9 & 11 & -18 \\ 27 & -9 & 54 \\ 54 & -18 & 108 \end{pmatrix} \longrightarrow \begin{pmatrix} -9 & 11 & -18 \\ 0 & 24 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

由上可知,

$$\operatorname{rank}\left(\mathbf{A}^T\mathbf{A}\right)=\operatorname{rank}(\mathbf{A})=\operatorname{rank}\left(\mathbf{A}\mathbf{A}^T\right)=2$$

Question 2.

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1. 当需要拟合的直线为 $y_1 = \alpha_0 + \alpha_1 x$, 可以假设:

$$A_1 = \begin{pmatrix} 1 & -5 \\ 1 & -4 \\ 1 & -3 \\ 1 & -2 \\ 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \end{pmatrix}, \quad x_1 = \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix}, \quad b = \begin{pmatrix} 2 \\ 7 \\ 9 \\ 12 \\ 13 \\ 14 \\ 14 \\ 13 \\ 10 \\ 8 \\ 4 \end{pmatrix}$$

代入最小二乘法计算公式，得:

$$x_1 = \left(A_1^T A_1\right)^{-1} A_1^T b = \begin{pmatrix} \frac{1}{11} & 0 \\ 0 & \frac{1}{110} \end{pmatrix} \times \begin{pmatrix} 106 \\ 20 \end{pmatrix} = \begin{pmatrix} \frac{106}{11} \\ \frac{2}{11} \end{pmatrix}$$

拟合的直线为：$y_1 = \frac{2}{11} x_1 + \frac{106}{11}$ 此时误差和为:

$$\sum_{i=1}^{m} \varepsilon_i^2 = \varepsilon^T \varepsilon = (\mathbf{A_1 x} - \mathbf{b})^T (\mathbf{A_1 x} - \mathbf{b}) = 162.909$$

2. 当需要拟合的直线为 $y_1 = \alpha_0 + \alpha_1 x + \alpha_2 x^2$, 可以假设:

$$A_2 = \begin{pmatrix} 1 & -5 & 25 \\ 1 & -4 & 16 \\ 1 & -3 & 9 \\ 1 & -2 & 4 \\ 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \\ 1 & 5 & 25 \end{pmatrix}, \quad x_2 = \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{pmatrix}, \quad b = \begin{pmatrix} 2 \\ 7 \\ 9 \\ 12 \\ 13 \\ 14 \\ 14 \\ 13 \\ 10 \\ 8 \\ 4 \end{pmatrix}$$

代入最小二乘法计算公式，得:

$$x_2 = \left(A_2^T A_2\right)^{-1} A_2^T b = \begin{pmatrix} \frac{89}{129} & 0 & -\frac{5}{429} \\ 0 & \frac{1}{110} & 0 \\ -\frac{5}{429} & 0 & \frac{1}{858} \end{pmatrix} \times \begin{pmatrix} 106 \\ 20 \\ 688 \end{pmatrix} = \begin{pmatrix} \frac{198}{143} \\ \frac{2}{11} \\ -\frac{62}{143} \end{pmatrix}$$

拟合的直线为：$y_2 = \frac{198}{143} + \frac{2}{11}x_2 - \frac{62}{143}x_2^2$ 此时误差和为:

$$\sum_{i=1}^{m} \varepsilon_i^2 = \varepsilon^T \varepsilon = (\mathbf{A_2x} - \mathbf{b})^T(\mathbf{A_2x} - \mathbf{b}) = 2.089$$

显然 2.089<162.909, 所以 $y_2 = \frac{198}{143} + \frac{2}{11}x_2 - \frac{62}{143}x_2^2$ 的拟合效果好