Homework 2

2020 年 9 月 30 日

1. Exercise 11

1.

$$\left[\begin{array}{c} A|b \end{array}\right] = \left[\begin{array}{cccc:c} 1 & 2 & 4 & 17 & 1 \\ 3 & 6 & -12 & 3 & 2 \\ 2 & 3 & -3 & 2 & 3 \\ 0 & 2 & -2 & 6 & 4 \end{array}\right] \longrightarrow \left[\begin{array}{cccc:c} 3 & 6 & -12 & 3 & 2 \\ 1 & 2 & 4 & 17 & 1 \\ 2 & 3 & -3 & 2 & 3 \\ 0 & 2 & -2 & 6 & 4 \end{array}\right] \longrightarrow \left[\begin{array}{cccc:c} 3 & 6 & -12 & 3 & 2 \\ \mathbf{1/3} & 0 & 8 & 16 & 1 \\ \mathbf{2/3} & -1 & 5 & 0 & 3 \\ \mathbf{0} & 2 & -2 & 6 & 4 \end{array}\right] \longrightarrow$$

$$\left[\begin{array}{cccc:c} 3 & 6 & -12 & 3 & 2 \\ \mathbf{0} & 2 & -2 & 6 & 4 \\ \mathbf{2/3} & -1 & 5 & 0 & 3 \\ \mathbf{1/3} & 0 & 8 & 16 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{cccc:c} 3 & 6 & -12 & 3 & 2 \\ \mathbf{0} & 2 & -2 & 6 & 4 \\ \mathbf{2/3} & \mathbf{-1/2} & 4 & 3 & 3 \\ \mathbf{1/3} & \mathbf{0} & 8 & 16 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{cccc:c} 3 & 6 & -12 & 3 & 2 \\ \mathbf{0} & 2 & -2 & 6 & 4 \\ \mathbf{1/3} & \mathbf{0} & 8 & 16 & 1 \\ \mathbf{2/3} & \mathbf{-1/2} & 4 & 3 & 3 \end{array}\right] \longrightarrow$$

$$\left[\begin{array}{cccc:c} 3 & 6 & -12 & 3 & 2 \\ \mathbf{0} & 2 & -2 & 6 & 4 \\ \mathbf{1/3} & \mathbf{0} & 8 & 16 & 1 \\ \mathbf{2/3} & \mathbf{-1/2} & \mathbf{1/2} & -5 & 3 \end{array}\right] \tag{1}$$

Then,

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1/3 & 0 & 1 & 0 \\ 2/3 & -1/2 & 1/2 & 1 \end{bmatrix} U = \begin{bmatrix} 3 & 6 & -12 & 3 \\ 0 & 2 & -2 & 6 \\ 0 & 0 & 8 & 16 \\ 0 & 0 & 0 & -5 \end{bmatrix} P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{2}$$

2. We want to solve the system $Ax = b$,we can $PAx = Pb$, then we can use the LU solution techniques discussed earlier to solve this permuted system, can get $LUx = Pb$, $Ly = Pb$, and then solve $Ux = y$ by back substitution.

$$Ly = b \Longrightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1/3 & 0 & 1 & 0 \\ 2/3 & -1/2 & 1/2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 17 \\ 3 \end{bmatrix} \Longrightarrow y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 16 \\ -5 \end{bmatrix} \tag{3}$$

Then solve $Ux = y$ by back substitution.

$$Ux = y \Longrightarrow \begin{bmatrix} 3 & 6 & -12 & 3 \\ 0 & 2 & -2 & 6 \\ 0 & 0 & 8 & 16 \\ 0 & 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 16 \\ -5 \end{bmatrix} \Longrightarrow x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 0 \\ 1 \end{bmatrix} \tag{4}$$