作业 1

Exercise6

（a） If use 3-digit arithmetic with no pivoting to solve this system，then the result is

$$\left(\begin{array}{cc|c} 10^{-3} & -1 & 1 \\ 1 & 1 & 0 \end{array}\right) \xrightarrow{-10^3 R_1 + R_2} \left(\begin{array}{cc|c} 10^{-3} & -1 & 1 \\ 0 & 10^3 & -10^3 \end{array}\right)$$

Because $fl(1+10^3) = fl(.1001\times10^4) = 10^3$

Then $x = 0, y = -1$

（b） If use partial pivoting and 3-digit arithmetic to solve the original system, then the result is

$$\left(\begin{array}{cc|c} 10^{-3} & -1 & 1 \\ 1 & 1 & 0 \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 10^{-3} & -1 & 1 \end{array}\right) \xrightarrow{-10^{-3} R_1 + R_2} \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & -1 & 1 \end{array}\right)$$

Because $fl(\text{-}1\text{-}10^{-3}) = fl(\text{-}.1001\times10^1) = \text{-}1$

Then $x = 1, y = -1$

Exercise4(a)

Reducing the augmented matrix $\left[A|b\right]$ to $E_{\left[A|b\right]}$ yields

$$A = \left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 3 \\ 2 & 4 & 1 & 3 & 4 \\ 3 & 6 & 1 & 4 & 5 \end{array}\right) \longrightarrow \left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 3 \\ 0 & 0 & -1 & -1 & -2 \\ 0 & 0 & -2 & -2 & -4 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 3 \\ 0 & 0 & -1 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \longrightarrow \left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 3 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) = E\left[A|b\right]$$

Then
$$\begin{aligned} & x_1 = 1 - 2x_2 - x_4 \\ & x_2 \longrightarrow free \\ & x_3 = 2 - x_4 \\ & x_4 \longrightarrow free \end{aligned}$$

Determine the general solution of nonhomogeneous systems is

$$x=\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix}=\begin{pmatrix}1-2x_2-x_4\\x_2\\2-x_4\\x_4\end{pmatrix}=\begin{pmatrix}1\\0\\2\\0\end{pmatrix}+x_2\begin{pmatrix}-2\\1\\0\\0\end{pmatrix}+x_4\begin{pmatrix}-1\\0\\-1\\1\end{pmatrix}$$