# Design and Analysis of Algorithms (18 Fall)

## Assignment 1

## Due: Oct. 9, 2018

1. Arrange the following functions in ascending asymptotic order of growth rate:

(a) $f_1(n) = n^{2.5} + 5^{100}n^2$, $f_2(n) = 2^{\log n + \log\log n}$, $f_3(n) = \sqrt{n^{3.5}}$, $f_4(n) = 2^{2n}$, $f_5(n) = 3^n$;

(b) $f_1(n) = n^{\log^2 n}$, $f_2(n) = 2^{\log n + \log\log n}$, $f_3(n) = \log^n \log^2 n$, $f_4(n) = n^{\sqrt{n}\log n}$.

2. Both divide-and-conquer and dynamic programming consider recursive relation of a problem. Which type of problems makes dynamic programming more suitable compared with divide-and-conquer? What is the major difference between divide-and-conquer and dynamic programming in terms of the technique?

3. Give an asymptotic tight bound for f(n) in the following recurrence relations:

1、 $f(n) = 6f(\frac{n}{4}) + O(n\log n)$.

2、 $f(n) = 3f(n-3) + O(n)$.

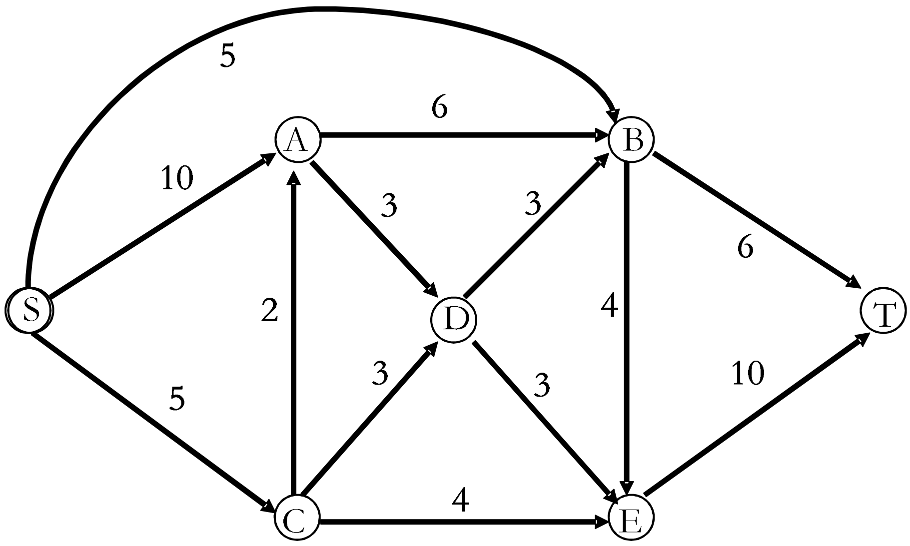
3、 $f(n) = 16f(\frac{n}{4}) + O(n^2)$.

4. You are required to use the dynamic programming method to design an effective algorithm for finding a longest monotonically increasing subsequence in a sequence $S = s_1, s_2, \cdots, s_n$ of n numbers. Let $L_i$ be the length of the longest monotonically increasing subsequence in $S_i = s_1, s_2, \cdots, s_i$.

(a) Construct the recursive relation of $L_i$.

(b) Given the following sequence $S = 3,11,2,5,7,6,14,8,12,15,4,10,13,9,18$, find the longest monotonically increasing subsequence in S by showing $L_i$ for all $i$.

5. Compute a maximum flow from S to T in the following graph.




6. Given an edge-weighted graph (where each edge is associated with a weight). How to detect whether the graph has a *negative cycle* or not? A cycle is called a negative cycle if the sum of all the weights on the edges in the cycle is negative.