Exercise: Random Number Generation and Monte Carlo Method

Using the Minimal Standard Random Number Generator with seed value 1 and trial number of points n=1000, 10000, 100000, 1000000,

1. calculate the following 1D integral using Type 1 Monte Carlo Method

$$\int_0^1 \frac{1}{x^2+1} dx \quad (\text{Exact solution} : \frac{\pi}{4})$$

2. calculate the following 2D area using Type 2 Monte Carlo Method

$13x^2+34xy+25y^2-1<0$ within the region $-1 \leq x, y \leq 1$

(Exact solution: $\frac{\pi}{6}$).

In Arbitrary Range and Dimension

Random Numbers in Arbitrary Range and Dimension

The generators introduced here produces uniform random numbers within the range $[0, 1]$. It is possible to map them in an interval $[a, b]$ using simple linear transformation.

$$v = (b - a)u + a,$$

where u is the random number in the interval $[0, 1]$ and v is the random number mapped in an interval $[a, b]$. Also, random numbers in D-dimensions can be realized by simply calling the random number generator D times.

Exercise: (cont)

3 calculate the following 3D volume using Type 2 Monte Carlo Method

$$(x-1)^2 + (y-0.5)^2 + z^2 - 1 < 0$$

within the region $0 \leq x \leq 2$, $-0.5 \leq y \leq 1.5$, $-1 \leq z \leq 1$.
(Exact solution: $\frac{4\pi}{3}$)

Output of your code should contain for each method:
Number of points; Numerical Result; Error = Numerical Result - Exact.