INTRODUCTION TO DEEP LEARNING FOR COMPUTER VISION

DAY 3 – CONVOLUTIONAL NEURAL NETWORKS

SEBASTIAN HOUBEN

Schedule

Today

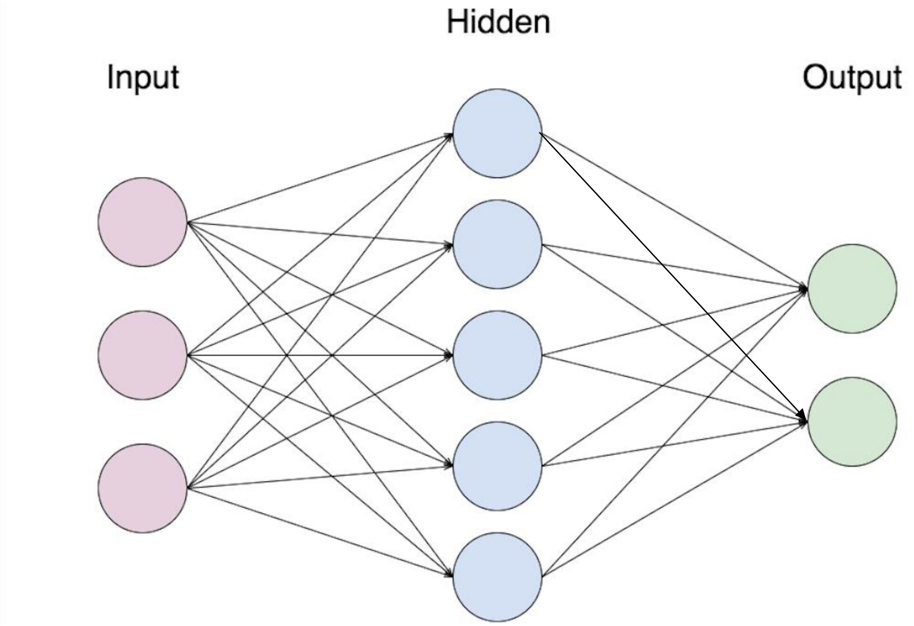
- Neural Nets
- Training of Neural Nets
- Gradient Computation
- Deep Neural Nets
- Bare Necessities for Training Deep Neural Nets
- Tensorflow

Neural Net – Multilayer Perceptron

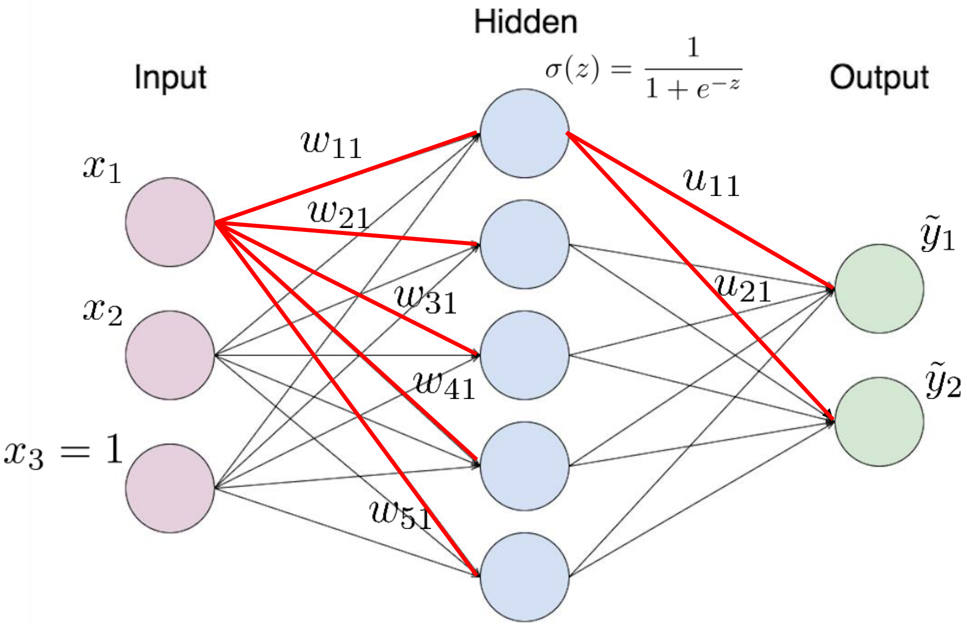
Feature Extraction

$$\begin{bmatrix} 2 \\ 5 \\ 1 \\ 8 \end{bmatrix} \longrightarrow \boxed{\text{Classifier}} \longrightarrow \{cat,dog\}$$

Neural Net – Multilayer Perceptron



Neural Net – Multilayer Perceptron



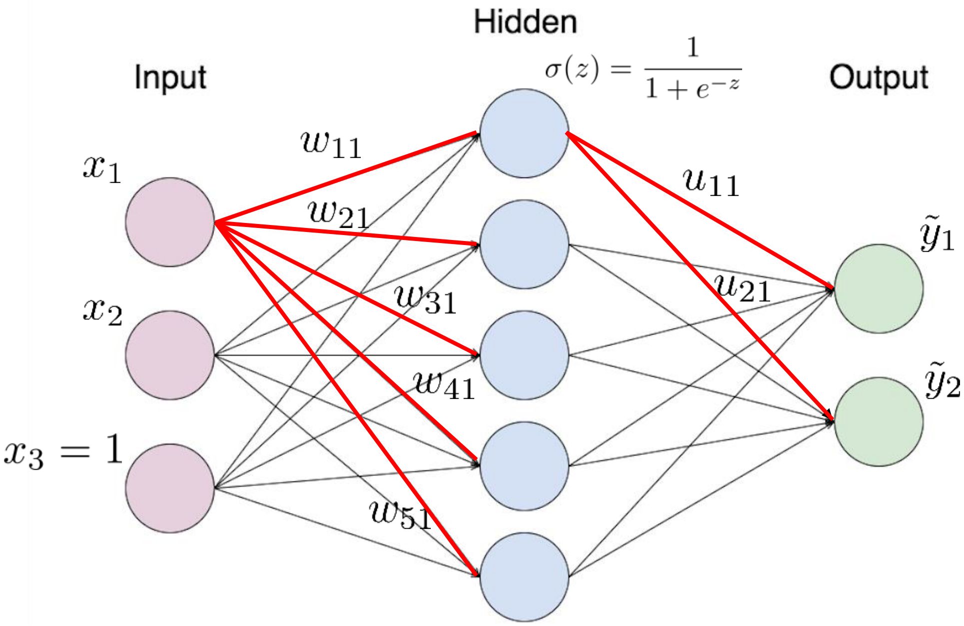
$$W := \begin{pmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \\ w_{41} & w_{42} & w_{43} \\ w_{51} & w_{52} & w_{53} \end{pmatrix}$$

$$U := \begin{pmatrix} u_{11} & u_{12} & u_{13} & u_{14} & u_{15} \\ u_{21} & u_{22} & u_{23} & u_{24} & u_{25} \end{pmatrix}$$

$$x := \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \qquad \tilde{y} := \begin{pmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{pmatrix}$$

$$\tilde{y} = U\sigma\left(Wx\right)$$

Neural Net – Multilayer Perceptron



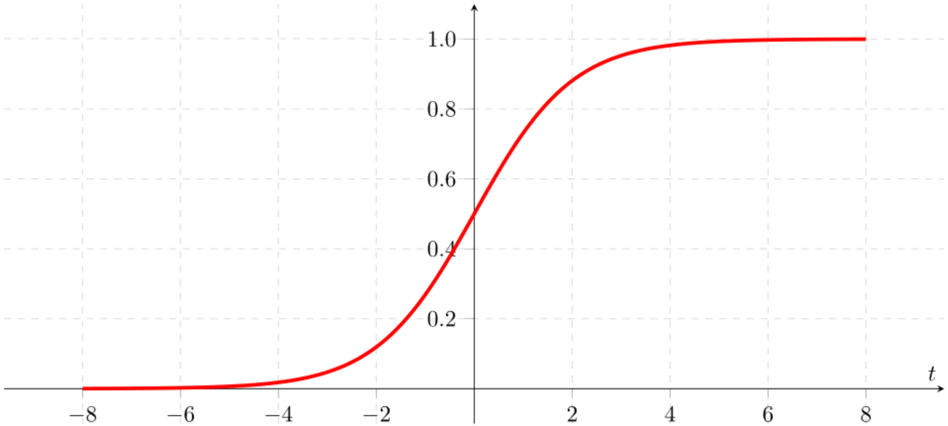
$$W := \begin{pmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \\ w_{41} & w_{42} & w_{43} \\ w_{51} & w_{52} & w_{53} \end{pmatrix}$$

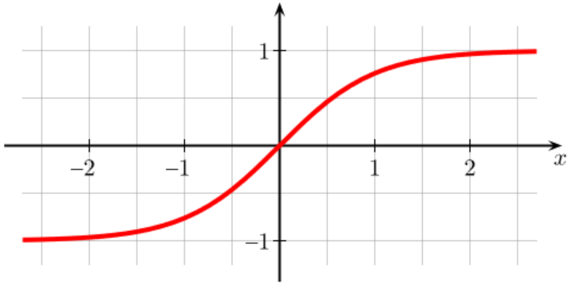
$$U := \begin{pmatrix} u_{11} & u_{12} & u_{13} & u_{14} & u_{15} \\ u_{21} & u_{22} & u_{23} & u_{24} & u_{25} \end{pmatrix}$$

$$x := \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \qquad \tilde{y} := \begin{pmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{pmatrix} \qquad y := \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$\tilde{y} = U\sigma\left(Wx\right)$$

Neural Net – Non-Linearities



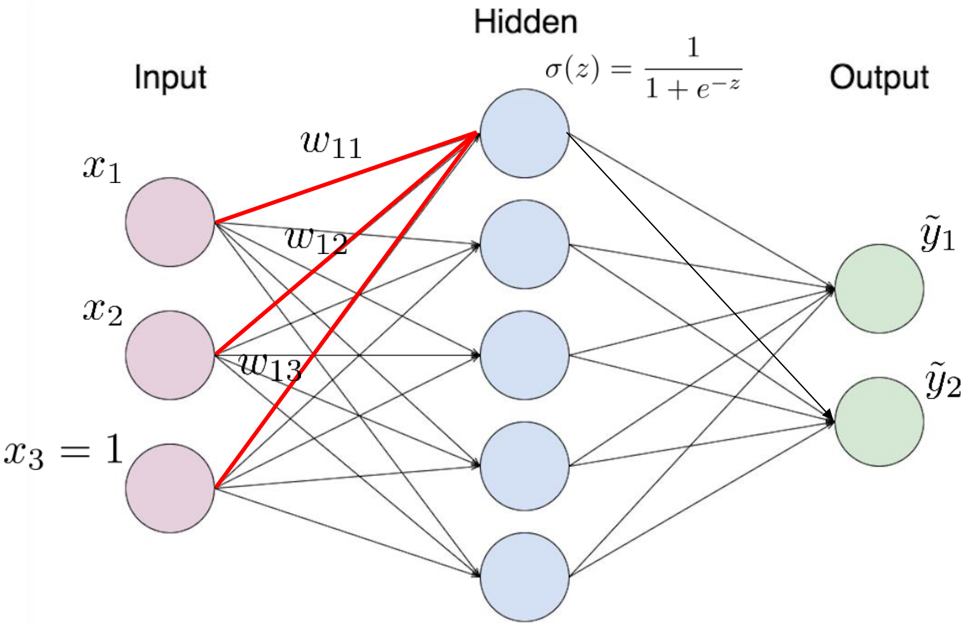


$$\sigma(z) = \frac{1}{1+e^{-z}} = \frac{1}{2}\left(1 + tanh\frac{z}{2}\right)$$

$$\sigma'(z) = \sigma(z)(1-\sigma(z))$$

$$tanh'(z) = 1 - tanh^2(z)$$

Neural Net – Interpretation

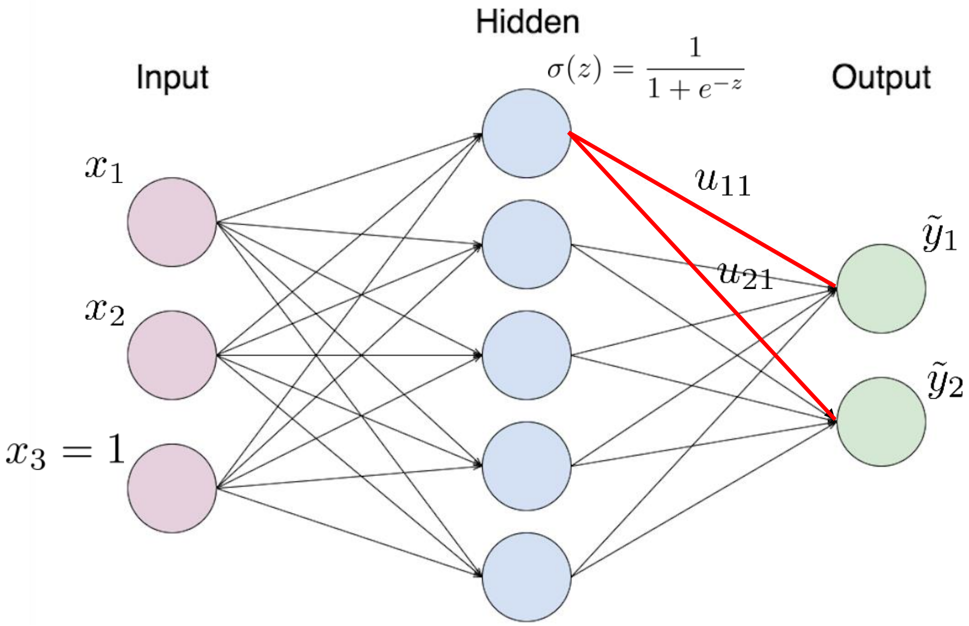


- Input norm should be limited
- Nothing should fire for zero input
- Shift by mean and normalize by standard deviation (over training set)

$$x := \frac{\hat{x} - \text{mean}}{\text{std}}$$

- Hidden neuron reacts if input is similar to weight vector
- Hidden neurons code regions of feature space
- More hidden neurons can devide the feature space in more regions

Neural Net – Interpretation



- Second layer weights control output for each region
- Net can approximate each continuous function
- Polynomials can
- Sine functions can (Fourier series)

Neural Net – Training

- Training data: $(x^{(i)}, y^{(i)}); i = 1, ..., n; y^{(i)} \in \{0;1\}^m; x^{(i)} \in \mathbb{R}^d$
- Predictions (!): $\tilde{y}^{(i)} = \sigma\left(U\sigma\left(Wx^{(i)}\right)\right)$
- Accuracy: $\frac{1}{n}\sum_{i=0}^{n} \mathbf{1}\left[\underset{j}{argmax}\,\tilde{y}_j^{(i)} = \underset{j}{argmax}\,y_j^{(i)}\right]$
- Loss: $L(x,y,W,U) = \sum_{i=1}^{n}\sum_{j=1}^{m}\left(\tilde{y}_j^{(i)} - y_j\right)^2$

$$= \sum_{i=1}^{n}\sum_{j=1}^{m}\left(\sigma\left(U\sigma\left(Wx^{(i)}\right)\right)_j - y_j\right)^2$$

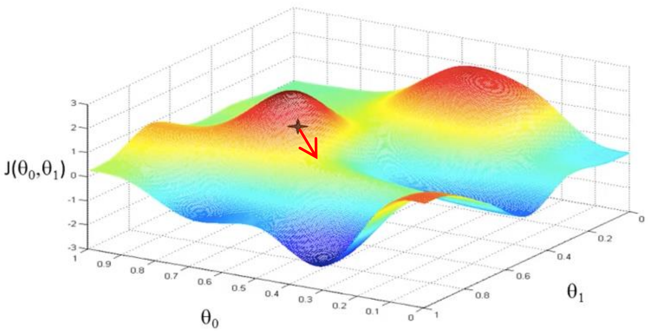
- Training: $W^{(k+1)} = W^{(k)} - \eta\frac{\partial}{\partial W}L(x,y,W^{(k)},U^{(k)})$

$$U^{(k+1)} = U^{(k)} - \eta\frac{\partial}{\partial U}L(x,y,W^{(k)},U^{(k)})$$

$$y^{(i)} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \qquad \tilde{y}^{(i)} = \begin{pmatrix} 0.1 \\ \vdots \\ 0.23 \\ 0.99 \\ 0.61 \\ \vdots \\ 0.44 \end{pmatrix}$$

- *n* samples
- *m* classes
- *d* input size

Neural Net – Training



- Simple case: $d = 1, m = 1$

$$L(x,y,w,u) = \sum_{i=1}^{n} \left(\sigma\left(u\sigma\left(wx^{(i)}\right)\right) - y\right)^2$$

$$\frac{\partial}{\partial w} L(x,y,w,u) = \sum_{i=1}^{n} 2\left(\sigma\left(u\sigma\left(wx^{(i)}\right)\right) - y\right) \cdot \sigma'\left(u\sigma\left(wx^{(i)}\right)\right) \cdot u\sigma'\left(wx^{(i)}\right) \cdot x^{(i)}$$

$$\frac{\partial}{\partial u} L(x,y,w,u) = \sum_{i=1}^{n} 2\left(\sigma\left(u\sigma\left(wx^{(i)}\right)\right) - y\right) \cdot \sigma'\left(u\sigma\left(wx^{(i)}\right)\right) \cdot \sigma\left(wx^{(i)}\right)$$

- Vanishing Gradient

- *n* samples
- *m* classes
- *d* input size

RUHR UNIVERSITÄT BOCHUM

Neural Net – Training

$$W^{(k+1)} = W^{(k)} - \eta \frac{\partial}{\partial W} L(x, y, W^{(k)}, U^{(k)})$$

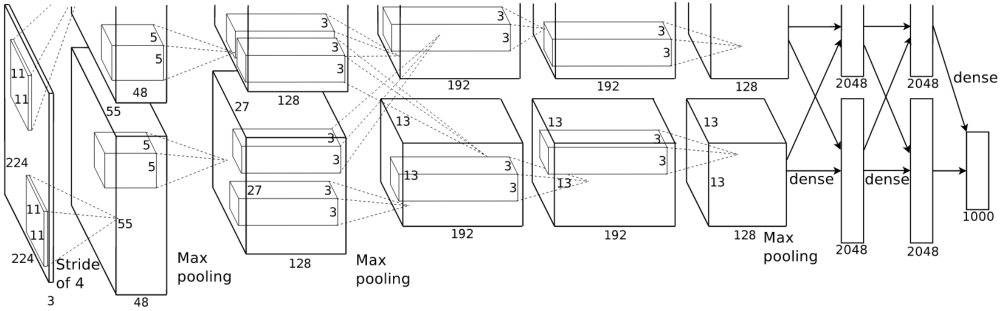
$$U^{(k+1)} = U^{(k)} - \eta \frac{\partial}{\partial U} L(x, y, W^{(k)}, U^{(k)})$$

- Basic form of gradient: $\sum_{i=1}^{n} d\left(x^{(i)}, y^{(i)}, W, U\right)$
- Gradient descent: $W^{(k+1)} = W^{(k)} - \eta \sum_{i=1}^{n} d(x^{(i)}, y^{(i)}, W^{(k)}, U^{(k)})$
- Stochastic gradient descent: $W^{(k+1)} = W^{(k)} - \eta d(x^{(i)}, y^{(i)}, W^{(k)}, U^{(k)})$
- Batch gradient descent $W^{(k+1)} = W^{(k)} - \eta \sum_{i \in \text{BATCH}} d(x^{(i)}, y^{(i)}, W^{(k)}, U^{(k)})$

Deep Neural Nets

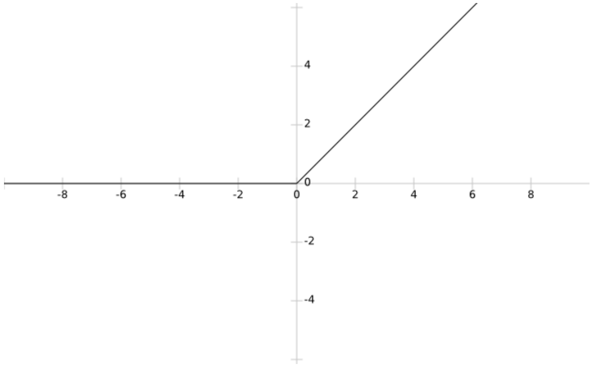
- 3-Layer network can approximate any continuous function
- More layers tend to work better
  - Not quite clear why
  - Handwavy: Natural phenonemons are hierarchically structered
  - Hopefully layers will adapt to those different phenomenons
- Vanishing Gradient Problem
- Many, many parameters

Alex Krizhevsky et al.



RUHR UNIVERSITÄT BOCHUM

Deep Neural Nets - Adaptations



- Non-Linearity: $\sigma(z) \to \mathrm{ReLU}(z) = \max\{0, z\}$

  Output: $\mathrm{softmax}(z) = \dfrac{exp\,(z_i)}{\sum_k exp\,(z_k)}$

- Loss function: $(f(x) - y)^2 \to -\sum_i y_i \log \mathrm{softmax}(z)_i$

  Cross-entropy

- Weight initialization: $W \sim \mathcal{N}(0, 0.1)$

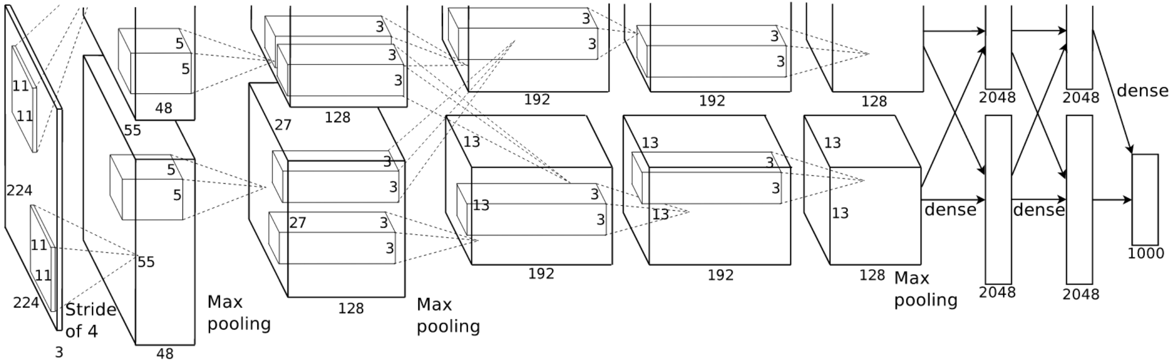
- Data preparation: $x^{(k)} := \dfrac{x^{(k)} - \mathrm{mean}(x^{(1)}, \ldots, x^{(n)})}{\mathrm{std}(x^{(1)}, \ldots, x^{(n)})}$

$$\mathrm{softmax}\begin{pmatrix} 21 \\ 34 \\ \vdots \\ 10,000 \\ \vdots \\ 680 \end{pmatrix} \to \underbrace{\begin{pmatrix} 0.002 \\ 0.004 \\ \vdots \\ 0.927 \\ \vdots \\ 0.03 \end{pmatrix}}_{\sum=1}$$

RUHR UNIVERSITÄT BOCHUM RUB

Convolutional Neural Nets

- Neural net with parameter reuse
- Each layer gets an image with *c* channels as input
- This is convoluted with d filters of size $n \times n \times c$
- resulting in an image with d channels
- Idea: Find certain local image patches / patterns

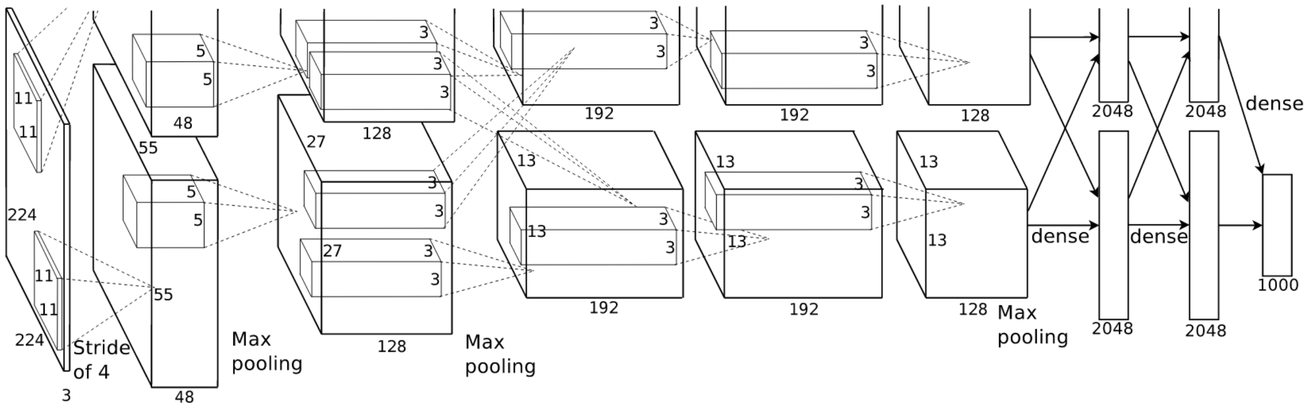


Alex Krizhevsky et al.

RUHR UNIVERSITÄT BOCHUM RUB

Convolutional Neural Nets

- Idea: Exact location of image patch is not so important
- Compress information
- Maxpool-Layers: Take small window (e.g., 2x2) and only propagate maximum value to next layer

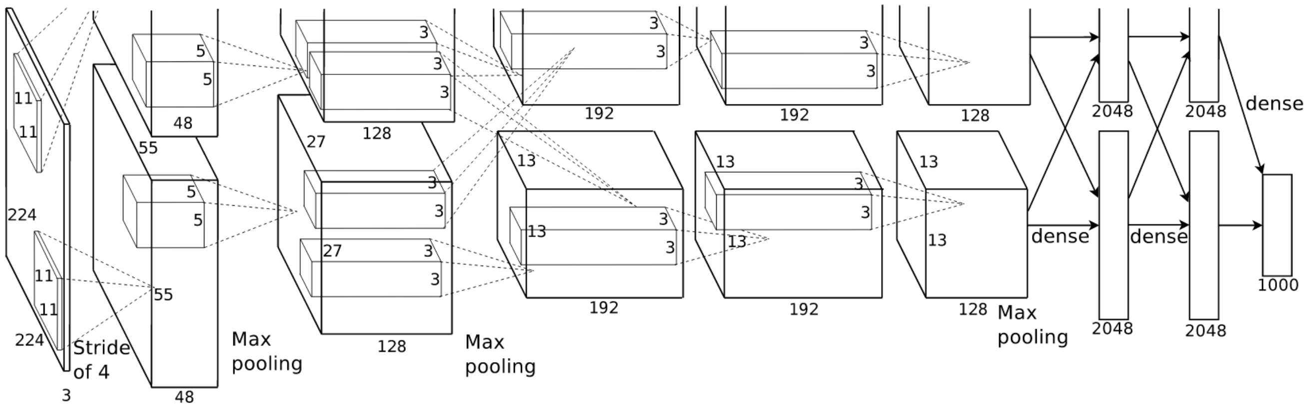


Alex Krizhevsky et al.

RUHR UNIVERSITÄT BOCHUM RUB

Convolutional Neural Nets

- Idea: at the end only relevant information is propagated
- Use classical neural net (fully-connected) to classify results



Alex Krizhevsky et al.

Tensorflow

- Python library for Deep Learning
  - Gradient computation
  - Backpropagation
  - 2000+ operations (e.g., convolution, maxpooling)
- Symbolic computation
  - Write a program that writes (and executes) a program
  - Similar to Numpy

Tensorflow

- GPU paralellization (via CUDA kernels)
- Caveats:
  - Slightly hard to learn
  - Hard to debug

- Alternatives:
  - PyTorch (Torch)
  - Theano (basically the same)
  - Caffe (C++)
  - Keras (Simplification of Theano / Tensorflow)

Tensorflow: Layout

```
import tensorflow as tf
import numpy as np

tf.reset_default_graph()                                              # tensorflow internal reset

x = tf.Variable( np.array( [2, 1] ), dtype=tf.float32, name= “x“ )    # a variable in the program our program writes
y = tf.constant( np.array( [3, 5] ) , dtype=tf.float32, name= “y“ )   # a constant in the program our program writes

z = tf.placeholder( shape=[None, 2], dtype=tf.float32, name= “z“ )    # an input in the program our program writes

loss = tf.reduce_sum((x - y + z)**2)                                  # many other numpy operations are implemented

train_step = tf.train.GradientDescentOptimizer(0.1).minimize(loss)# a subroutine that takes one gradient descent step on loss

z_ = np.array([[2,0]])

with tf.Session() as sess:
  sess.run(tf.global_variables_initializer())                         # initialize variables in program
  print( loss.eval( feed_dict={z:z_}), x.eval()  )                     # 17.0, [ 2.  1.]
  for k in range(100):
    train_step.run( feed_dict={z:z_})                                 # compute loss, compute backpass (derivative), one step downwards

  print( loss.eval( feed_dict={z:z_}), x.eval() )                     # 9.66338e-13,  [1.00000024  4.99999905]
```

Tensorflow: Checkpointing

```
import tensorflow as tf
import numpy as np

tf.reset_default_graph()                                   # tensorflow internal reset

# fancy model implemented here ...

train_step = tf.train.GradientDescentOptimizer(0.1).minimize(loss)

saver = tf.train.Saver()

with tf.Session() as sess:
  sess.run(tf.global_variables_initializer())              # initialize variables in program
  saver.restore(sess, tf.train.latest_checkpoint(os.path.dirname(os.path.realpath(__file__)))) # restore last checkpoint
  for # ...
    # optimization going on here ...
    saver.save(sess, os.path.dirname(os.path.realpath(__file__)) + '/tsd_model', global_step=epoch_cnt, write_meta_graph=False)
    # save current state of variables (but not the model)
```

Tensorflow

- NWHC order
    - stacking of images
    - number, width, height, channel
    - try to adapt to this order

QUESTIONS?

EXERCISES.