Homework 1 - Image filtering

1. (2 pts) Provided a **continuous** 1D signal $x(t)$, **describe and prove** the effect of shifting in time on the spectrum of the signal (i.e. how does the spectrum of $x(t-\delta)$ look like wrt the one of $x(t)$?).

   Hint: Fourier Shift Theorem

2. (3 pts) As we observed in class, convolution in space correspond to an element-wise multiplication in the spectral domain:
   a. Implement a MATLAB script realizing 2D **Cooley-Tukey** FFT for images (**radix-2 decimation-in-time)** and ensure it works by computing the forward and inverse transform on *lena.png* (print the MSE between the original image and the reconstructed one after inverse DFT).
   b. Low pass filter the provided *lena.png* both in space and frequencies with a 5 x 5 gaussian filter and std = 2 (to avoid problems with size of the image use MATLAB's fft2 and ifft2 routines) and compare the two results (print MSE between the two filtered images).
   c. How much zero-padding do we have to introduce to produce consistent results between filtering in space and in the frequency domain?
   d. What happens if we just zero-pad the filter to have the image size but we don't increase the size of the image itself?

   Hint: convert the RGB image in grey-scale with *rgb2gray* and in range [0,1] with *im2double*. Additionally, ensure the size of the image is a power of 2 on both dimensions for (a).

3. (3 pts) Differently from classic low-pass filters, Bilateral Filters (BFs) avoid diffusion of information across borders:
   a. How does an image filtered with BF look like wrt one filtered with a classic low-pass filter?
   b. Describe the efficient implementation of bilateral filter proposed by Durand & Dorsey.
   c. Implement a MATLAB script realizing Bilateral Filter (not necessarily D&D version) and filter the provided *lena.png* with $\sigma_d = 2$ and $\sigma_r = 25$. Visualize the image in the end.
   d. How does the final filtered image look like? What happens if we impose extremely high values of geometric spread σ_d and photometric spread σ_r (i.e. $\sigma_d = 1000,\ \sigma_r = 50$)? How does the final image look like wrt one filtered with a lower σ_d (i.e. $\sigma_d = 2$, $\sigma_r = 50$)? Why? Visualize the images produced with BF for the different mentioned std.

Hint: convert the RGB image in grey-scale with *rgb2gray*, **above std are provided wrt 8-bits grey-scale values (i.e. max value = 255).**

4. (2 pts) Describe non-local means (NLM), implement a MATLAB script realizing it and apply NLM on the provided *lena.png.* Visualize the result in the end.

   Compare one pixel only with the ones in its 11 x 11 neighborhood, use S = 5 for describing the behavior of one pixel and h = 0.01 as std. Avoid any smoothing with gaussian kernel for simplicity.

   Hint: convert the RGB image in grey-scale with *rgb2gray* and in range [0,1] with *im2double*.

Solutions must be submitted by **April 11**
on iCorsi or to federico.monti@usi.ch