Noter til IMO

December 9, 2021

Kapitel 3

0.1 Definitioner, teorem, bemærkninger og propositioner

Proposition 3.27 The product AB of two invertible matrices A and B is invertible $(AB)^{-1} = B^{-1}A^{-1}$

Proposition 3.33 Let A be an $m \times r$ matrix and B an $r \times n$ matrix. Then

$$(AB)^T = B^T A^T$$

Definition 3.7.1 Positive definite matrices En positiv definit matrice er givet ved

$$v^T A v > 0$$

3.7.2 Positive semi-definite matrices En positiv semi definit matrice er givet ved

$$v^t A v \leq 0$$

Proposition 3.41 Let A be a symmetric $n \times n$ matrix and B an invertible $n \times n$ matrix. Then A is positive (semi) definite if and only if

$$B^T A B$$

is positive (semi) definite.

0.2 Opgaver

Exercise 3.42 (Overvej kraftigt at bruge 8.26) Beviser at

$$D = \begin{pmatrix} d & 0 \\ 0 & e \end{pmatrix}$$

er positivt definit (ikke semi definit) hvis og kun hvis $e > 0 \wedge d > 0$

Beviser også at

$$A = \begin{pmatrix} a & c \\ c & b \end{pmatrix}$$

Er positivt definit (ikke semi definit) hvis og kun hvis $a > 0 \wedge ab - c^2 > 0$

Kapitel 4

0.3 Definitioner, teorem, bemærkninger og propositioner

Remark 4.6

$$\max_{x \in C} f(x) = \min_{x \in C} -f(x)$$

Definition 4.32 Definitionen på en konveks (ikke strengt) er

$$f((1-t)x + ty) \leq (1-t)f(x) + tf(y)$$

for hvert tal t hvor $0 \leq t \leq 1$

Lemma 4.26 Let $f : \mathbb{R}^d \longrightarrow \mathbb{R}$ be a convex function. Then the subset

$$C = \{x \in \mathbb{R}^d | f(x) \leq a\}$$

is a convex subset of $\mathbb{R}^d$, where $a \in \mathbb{R}^2$

4.5 Fourier-Motzkin elimination

0.4 Opgaver

Kapitel 5

0.5 Definitioner, teorem, bemærkninger og propositioner

5.2.3 Bounded subsets A subset $S \subseteq \mathbb{R}^d$ is called bounded if there exists $M \in \mathbb{R}$, such that

$$u \cdot u \leq M$$

for every $u \in S$

Teorem 5.19 If $x_0 \in \mathbb{R}^n$ is a solution to the system

$$(A^T A)x = A^T b$$

of n linear equations with n unknows, then x_0 is an optimal solution to

$$\min_{x \in \mathbb{R}^n} |b - Ax|^2$$

Cauchy-Schwarz inequality For two vectors $u, v \in \mathbb{R}^d$,

$$|u \cdot v| \leq |u||v|$$

og

$$-1 \leq \frac{u \cdot v}{|u||v|} \leq 1$$

og

$$\cos(\theta) = \frac{u \cdot v}{|u||v|}$$

Teorem 5.30 For two vectors $u, v \in \mathbb{R}^d$,

$$|u + v| \leq |u| + |v|$$

og

$$|u - v| = |(u - w) + (w - v)| \leq |u - w| + |w - v|$$

0.6 Opgaver