Lyon 1

Mesh and Computational Geometry

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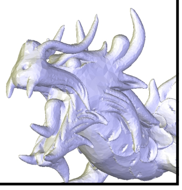


1

Differential operators

- Fait au tableau
  - Gradient de la function qui donne l'aire d'un triangle quand son sommet s0 varie

61

Differential operators

- Functions defined on a surface
  - Let u be a scalar function defined on a surface
  - We would like to express local variations of u
- Let consider the discrete case of a surface being approximated by a simplicial mesh
  - Function u discretized on vertices
  - Gradient of u discretized on triangles
  - Divergence of a vector (defined on triangles) discretized on vertices
- Bibliography : Keenan Crane

62

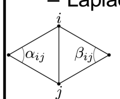
Differential operators

- Gradient d'une fonction définie sur les sommets d'un triangle
  - Gradient de l'interpolation linéaire de la fonction sur le triangle
- Laplacien en un sommet d'une fonction définie sur les sommets
  - Différence entre la moyenne des valeurs sur les voisins et la valeur au sommet
  - Quels coefficients choisir pour faire la moyenne?
  - Choisir des coefficients cohérents avec le fait que le laplacien de la fonction de position des points sur une surface doit être liée à la courbure moyenne et à la normale à la surface (et correspond au gradient de l'aire locale de la surface quand la position du point varie).

63

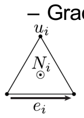
Differential operators

- Simplicial meshes
  - Laplacian Δ of u at vertex i = sum over neighbor vertices j



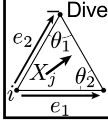
$$(Lu)_i = \frac{1}{2A_i}\sum_j (\cot\alpha_{ij} + \cot\beta_{ij})(u_j - u_i)$$

  - Gradient ∇ of u inside a triangle = sum over the 3 vertices I



$$\nabla u = \frac{1}{2A_f}\sum_i u_i(N \times e_i)$$

  - Divergence at vertex i of a vector X defined on faces = sum over incident faces j

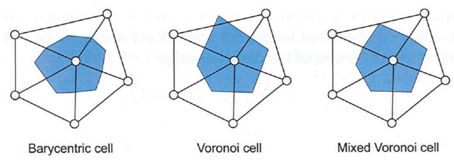


$$2A_f \nabla \cdot X = \frac{1}{2}\sum_j \cot\theta_1(e_1 \cdot X_j) + \cot\theta_2(e_2 \cdot X_j)$$

64

Area A_i

- Computed by duality to a vertex



65

65

Differential geometry

- How to generalize normal, curvatures?
- Consider u corresponding to each coordinate fonction in turn
  - $u = {}^t(x,y,z)$

$$\triangle_{\mathbf{x}} \mathbf{u} = -2H\mathbf{n}$$

H : mean curvature

n : normal vector

66

66

Signals being studied in Computer Graphics

- Signals defined on $\mathbb{R}^2$
  - Scalar signals :
    - height value (terrain)
    - density of some fluid flowing in the plane
  - Vector fields
    - Surface parameterization $\Omega \subset \mathbb{R}^2 \rightarrow \mathbb{R}^3$
    - Displacement field of some fluid flowing in the plane
- Signals defined in $\mathbb{R}^3$
    - Density of a volume material (scalar)
    - Displacement field of some fluid (vector)

67

67

Signals being studied in Computer Graphics

- Signals defined on a surface $\mathbb{S}$
  - Scalar values :
    - Temperature, grey color …
    - Position coordinates (x, y or z)
  - Vector fields
    - Normal vector
    - Maximal/minimal curvature direction
    - Displacement field
    - …

68

68