Uğur Eren Çanakçı
Candidate Number:277299

Linear Statistical Models Project Report

Summary

An investigation on people's reaction times to a number guessing game was conducted. The results show that there's evidence for font size making a difference on the reaction times of people at 5% significance level. Other than that, some people are affected by the background colour on which the text appears at 5% significance level as well.

Introduction

A sample of 36 tries was collected via a computer game. Three people who played the game are asked to choose if the number shown on the screen is an odd number or an even number. Then the person presses the designated button for the number. Every person plays the game 12 times, with 4 different font sizes and 3 different background colours. The tests are done to see if 1)Font size would make things easier to read and affect the reaction time and 2)Background colour affects the time people react to stimuli.

Method

Font size, people, and background colour is hypothesised to affect the reaction times. All other variables in the game are constant: font family is "Arial", font colour is black and variability is none. The font size is a continuous regressor; 0 denotes font size 8, 2 denotes font size 12, 4 denotes font size 16 and 6 denotes font size 24. Each person is considered to be a category, i.e. qualitative regressor, denoted from 1 to 3. The background colour is a qualitative regressor as well; 1 denotes red, 2 denotes blue and 3 denotes green.

Analysis

We build our model assuming there is interaction between our qualitative regressors:

$$y_{ij} = ß_0 + ß_1 x + \gamma_i + \delta_j + \mu_{ij} + \varepsilon_{ij}$$

$ß_0$ denotes the intercept, or overall mean, $ß_1$ denotes the coefficient of font size x, γ_i denotes the person i's nominal effect for i from 1 to 3, δ_j denotes the background colour j's nominal effect for j from 1 to 3, μ_{ij} denotes the effect of interaction between person i and background colour j, ε_{ij} denotes the error coefficient for the person i and background colour j.

Our null hypothesis is H_0: $ß_0 = ß_1 = \gamma_i = \delta_j = \mu_{ij} = 0$ for each i from 1 to 3, j from 1 to 3.

The summary of the model on table (1.2) shows us that mean reaction time for person 1 and background colour 1 is 0.691383 with error 0.069850 and p-value 2.62e-10. Mean reaction time for person 2 from table (1.2) is 0.427383 with error 0.090176 and p-value 0.00701.Mean reaction time for the interaction between person 2 and background 3 from table (1.2) is 0.267250 with error 0.127528 and p-value 0.04600. Other coefficients from table (1.2) have p-values larger than 0.05, which means there isn't significant evidence to reject the null hypotheses for these coefficients. We also observe on table (1.3) that the F-statistic for the coefficient of person has a p-value 4.073e-05, which is in line with the summary table (1.2) and hence telling us that font size, background colour and the interaction

of people and background colour don't have significant evidence to reject the null hypotheses for these coefficients.

Now we can check if transformations of reaction time can be explained by font or background colour. We apply a Box-Cox transformation (1.4) with lambda=2.494949 to our model for this purpose:

$$(y_{ij})^{2.494949} = ß_0 + ß_1 x + \gamma_i x_{ij} + \delta_j + \mu_{ij} + \varepsilon_{ij}$$

The summary of the dataset on table (1.5) shows us that transformed mean reaction time for person 1 and background colour 1 is 0.380986 with error 0.046779 and p-value 1.26e-08. Mean reaction time for person 2 from table (1.5) is 0.169436 with error 0.060391 and p-value 0.00168. Mean reaction time for the interaction between person 3 and background 2 from table (1.5) is 0.564955 with error 0.085406 and p-value 0.04068. Other coefficients from table (1.5) have p-values larger than 0.05, which means there isn't significant evidence to reject the null hypotheses for these coefficients.Anova table (1.6) for our transformed model tells us that there's significant evidence for only the coefficients of people to be nonzero at 5% level.

Finally, we can apply a transform to the font size:

$$(y_{ij})^{2.494949} = ß_0 + ß_1(x_{ij} + 0.012478)^{0.1} + \gamma_i + \delta_j + \mu_{ij} + \varepsilon_{ij} \qquad (1.7)$$

Above, the number next to x is the nominal effect of the font size to the reaction time after the Box-Cox transformation, which is necessary for a proper transformation of regressors. Compared to the previous case, the new intercept for person 1 and background colour 1 is 0.50515 with error 0.07546 and p-value 4.21e-07. The coefficient for font is -0.15909 with error 0.06242 and p-value 0.06242. Mean reaction time for person 2 from table (1.7) is 0.2936 with error 0.05787 and p-value 0.00114. Mean reaction time for the interaction between person 3 and background 2 from table (1.7) is 0.68912 with error 0.08184 and p-value 0.03329. Other coefficients from table (1.7) have p-values larger than 0.05, which means there isn't significant evidence to reject the null hypotheses for these coefficients.Anova table (1.8) for our final model tells us that there's significant evidence for coefficient of font to be nonzero at 5% level as well as the previously shown coefficients of person.

The plots of the final model after Box-Cox transformation of the response variable and exponential transformation of the normalised font size can be seen at (2.2) in Appendix.

Conclusions

Font size, people, and some of the interactions between people and background colour have a nonzero effect on the reaction times within 5% significance. For background colour there was no conclusive evidence to accept its effect being nonzero. Hence, our conclusive model is:

$$(y_{ij})^{2.494949} = 0.50515 - 0.15909(x_{ij} + 0.012478)^{0.1} + \gamma_i + \mu_{ij} + \varepsilon_{ij}$$

where $\gamma_1 = 0$ (already calculated in $ß_0$), $\gamma_2 = -0.211550$, $\mu_{32} = 0.18397$ and 0 for the rest.

It should be noted, though, that the data was collected from a very limited sample. With more test subjects and more background colours used, it is possible to get a more refined model.

Appendix

R Codes and Tables:

(1.1)

Table Name: Mean Reaction Times

MeanReactionTime	Person	FontSizeIndicator	BackgroundColorIndicator	BackgroundColor	FontSize
0,657	1	0	1	Red	8
0,704	1	2	1	Red	12
0,628	1	4	1	Red	16
0,612	1	6	1	Red	24
0,695	1	0	2	Blue	8
0,579	1	2	2	Blue	12
0,569	1	4	2	Blue	16
0,598	1	6	2	Blue	24
0,638	1	0	3	Green	8
0,685	1	2	3	Green	12
0,66	1	4	3	Green	16
0,636	1	6	3	Green	24
0,522	2	0	1	Red	8
0,494	2	2	1	Red	12
0,52	2	4	1	Red	16
0,477	2	6	1	Red	24
0,531	2	0	2	Blue	8
0,537	2	2	2	Blue	12
0,546	2	4	2	Blue	16
0,456	2	6	2	Blue	24
0,558	2	0	3	Green	8
0,505	2	2	3	Green	12
0,483	2	4	3	Green	16
0,492	2	6	3	Green	24
0,749	3	0	1	Red	8
0,687	3	2	1	Red	12
0,739	3	4	1	Red	16
0,715	3	6	1	Red	24
0,852	3	0	2	Blue	8
0,733	3	2	2	Blue	12

0,748	3	4	2	Blue	16
0,871	3	6	2	Blue	24
0,809	3	0	3	Green	8
0,755	3	2	3	Green	12
0,727	3	4	3	Green	16
0,705	3	6	3	Green	24

The initial steps for data analysis are below:

```
> table = Mean_Reaction_Times_Final_Sayfa1
```

Now, our regressors and response variable is prepared for R:

```
> reactiontime = (table$MeanReactionTime/1000)
> person = factor(table$Person)
> font = table$FontSizeIndicator
> backgroundcolor = factor(table$BackgroundColorIndicator)
```

Above, "reactiontime" was divided by 1000 because of an unintended change on the vector while importing the data from a csv file.

(1.2): Initial model

```
> model_interaction = lm(reactiontime ~ font + person*backgroundcolor)
> summary(model_interaction)
Call:
lm(formula = reactiontime ~ font + person * backgroundcolor)

Residuals:
     Min       1Q   Median       3Q      Max
-0.42654 -0.02216  0.00434  0.04058  0.17088

Coefficients:
                           Estimate Std. Error t value Pr(>|t|)
(Intercept)                0.691383   0.069850   9.898 2.62e-10 ***
font                      -0.013711   0.009505  -1.442  0.16111
person2                   -0.264000   0.090176  -2.928  0.00701 **
person3                    0.072250   0.090176   0.801  0.43027
backgroundcolor2          -0.040000   0.090176  -0.444  0.66102
backgroundcolor3          -0.144000   0.090176  -1.597  0.12238
person2:backgroundcolor2   0.171250   0.127528   1.343  0.19093
person3:backgroundcolor2   0.118500   0.127528   0.929  0.36133
person2:backgroundcolor3   0.267250   0.127528   2.096  0.04600 *
person3:backgroundcolor3   0.170500   0.127528   1.337  0.19281
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1275 on 26 degrees of freedom
Multiple R-squared:  0.5989,   Adjusted R-squared:  0.4601
F-statistic: 4.314 on 9 and 26 DF,  p-value: 0.001621
```

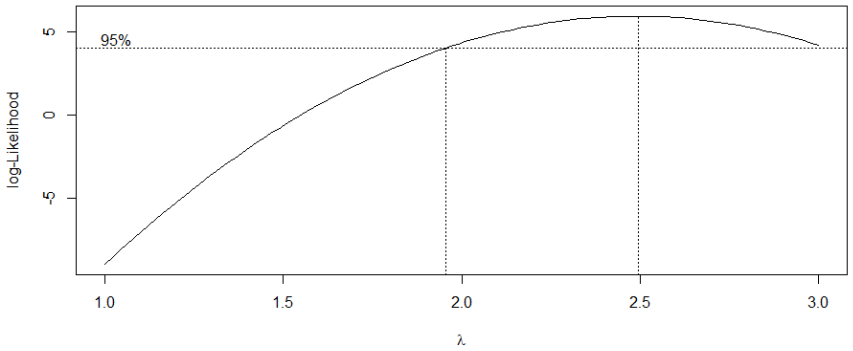
(1.3): ANOVA test for initial model

```
> anova(model_interaction)
Analysis of Variance Table
```

```
Response: reactiontime
                       Df  Sum Sq  Mean Sq F value    Pr(>F)
font                    1 0.03384 0.033839  2.0807    0.1611
person                  2 0.49736 0.248679 15.2908 4.073e-05 ***
backgroundcolor         2 0.02478 0.012388  0.7617    0.4770
person:backgroundcolor  4 0.07549 0.018873  1.1605    0.3509
Residuals              26 0.42284 0.016263
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

(1.4): Box-Cox transformation, lambda calculation

```
bc <- boxcox(model, lambda = seq(1, 3))
```




```
> bc$x[which.max(bc$y)]
[1] 2.494949
```

(1.5): Model after the Box-Cox transformation of the response variable

```
> model_bc = lm(reactiontime_bc ~ font + person*backgroundcolor)
> summary(model_bc)

Call:
lm(formula = reactiontime_bc ~ font + person * backgroundcolor)

Residuals:
      Min        1Q    Median        3Q       Max
-0.246242 -0.027332  0.003973  0.037602  0.164842

Coefficients:
                          Estimate Std. Error t value Pr(>|t|)
(Intercept)               0.380986   0.046779   8.144 1.26e-08 ***
font                     -0.012478   0.006366  -1.960  0.06077 .
person2                  -0.211550   0.060391  -3.503  0.00168 **
person3                   0.101790   0.060391   1.686  0.10386
backgroundcolor2         -0.048204   0.060391  -0.798  0.43199
backgroundcolor3         -0.083697   0.060391  -1.386  0.17754
person2:backgroundcolor2  0.111211   0.085406   1.302  0.20429
person3:backgroundcolor2  0.183969   0.085406   2.154  0.04068 *
person2:backgroundcolor3  0.138760   0.085406   1.625  0.11629
person3:backgroundcolor3  0.127040   0.085406   1.487  0.14892
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

```
Residual standard error: 0.08541 on 26 degrees of freedom
Multiple R-squared:  0.8024,   Adjusted R-squared:  0.734
F-statistic: 11.73 on 9 and 26 DF,  p-value: 4.081e-07
```

(1.6): ANOVA test for the model with Box-Cox transformation of the response variable

```
> anova(model_bc)
Analysis of Variance Table

Response: reactiontime_bc
                       Df  Sum Sq Mean Sq F value    Pr(>F)
font                    1 0.02803 0.02803  3.8423   0.06077 .
person                  2 0.68001 0.34000 46.6130 2.523e-09 ***
backgroundcolor         2 0.01838 0.00919  1.2596   0.30052
person:backgroundcolor  4 0.04363 0.01091  1.4955   0.23248
Residuals              26 0.18965 0.00729
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

(1.7) Final model after Box-Cox transformation of the response variable and exponential transformation of the normalised font size

```
> model_bc_fontnorm = lm(reactiontime_bc ~ I((font + 0.012478)^0.1) + person*backgroundcolor)
> summary(model_bc_fontnorm)

Call:
lm(formula = reactiontime_bc ~ I((font + 0.012478)^0.1) + person *
    backgroundcolor)

Residuals:
      Min        1Q    Median        3Q       Max
-0.237515 -0.029554  0.005176  0.042747  0.156157

Coefficients:
                          Estimate Std. Error t value Pr(>|t|)
(Intercept)                0.50515    0.07546   6.694 4.21e-07 ***
I((font + 0.012478)^0.1) -0.15909    0.06242  -2.549  0.01706 *
person2                  -0.21155    0.05787  -3.655  0.00114 **
person3                   0.10179    0.05787   1.759  0.09037 .
backgroundcolor2         -0.04820    0.05787  -0.833  0.41247
backgroundcolor3         -0.08370    0.05787  -1.446  0.16006
person2:backgroundcolor2  0.11121    0.08184   1.359  0.18588
person3:backgroundcolor2  0.18397    0.08184   2.248  0.03329 *
person2:backgroundcolor3  0.13876    0.08184   1.695  0.10194
person3:backgroundcolor3  0.12704    0.08184   1.552  0.13270
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.08184 on 26 degrees of freedom
Multiple R-squared:  0.8185,   Adjusted R-squared:  0.7557
F-statistic: 13.03 on 9 and 26 DF,  p-value: 1.436e-07
```

(1.8): Final model after Box-Cox transformation of the response variable and exponential transformation of the normalised font size

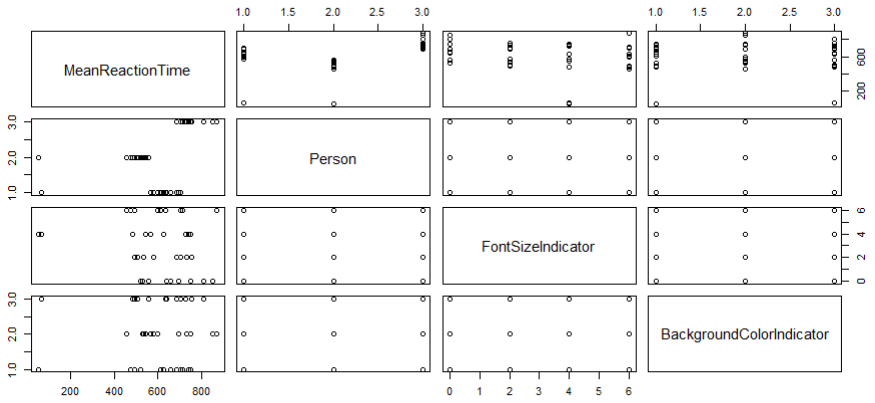
```
> anova(model_bc_fontnorm)
Analysis of Variance Table

Response: reactiontime_bc
                            Df  Sum Sq Mean Sq F value    Pr(>F)
I((font + 0.012478)^0.1)     1 0.04352 0.04352  6.4963   0.01706 *
person                       2 0.68001 0.34000 50.7584 1.053e-09 ***
backgroundcolor              2 0.01838 0.00919  1.3716   0.27145
person:backgroundcolor       4 0.04363 0.01091  1.6285   0.19706
Residuals                   26 0.17416 0.00670
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Plots:

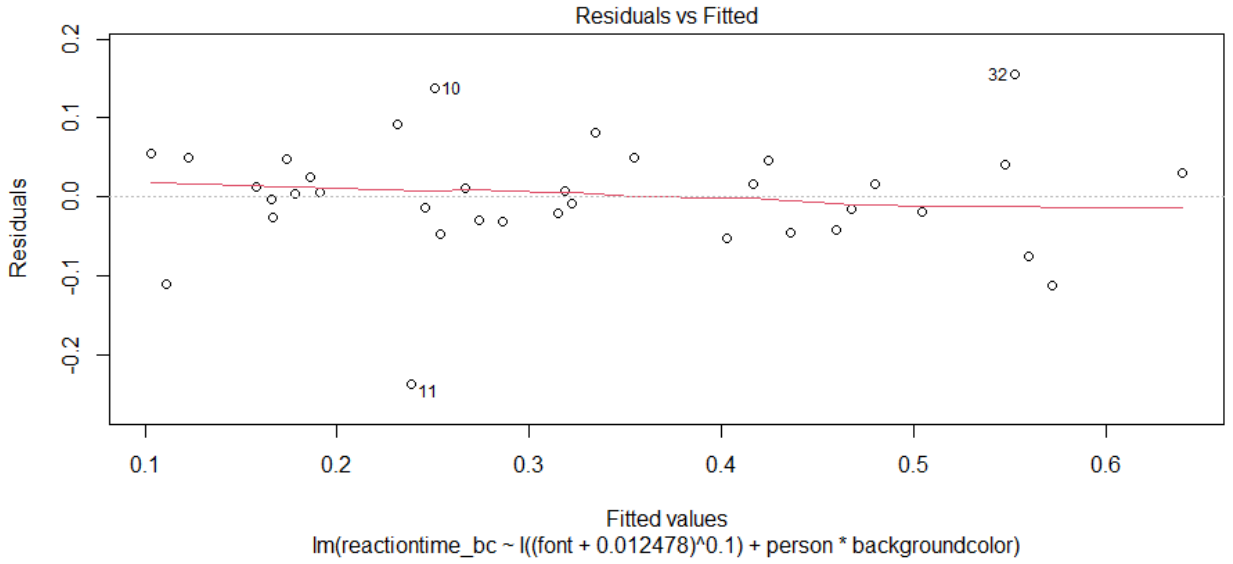
(2.1): Initial plot of the data set

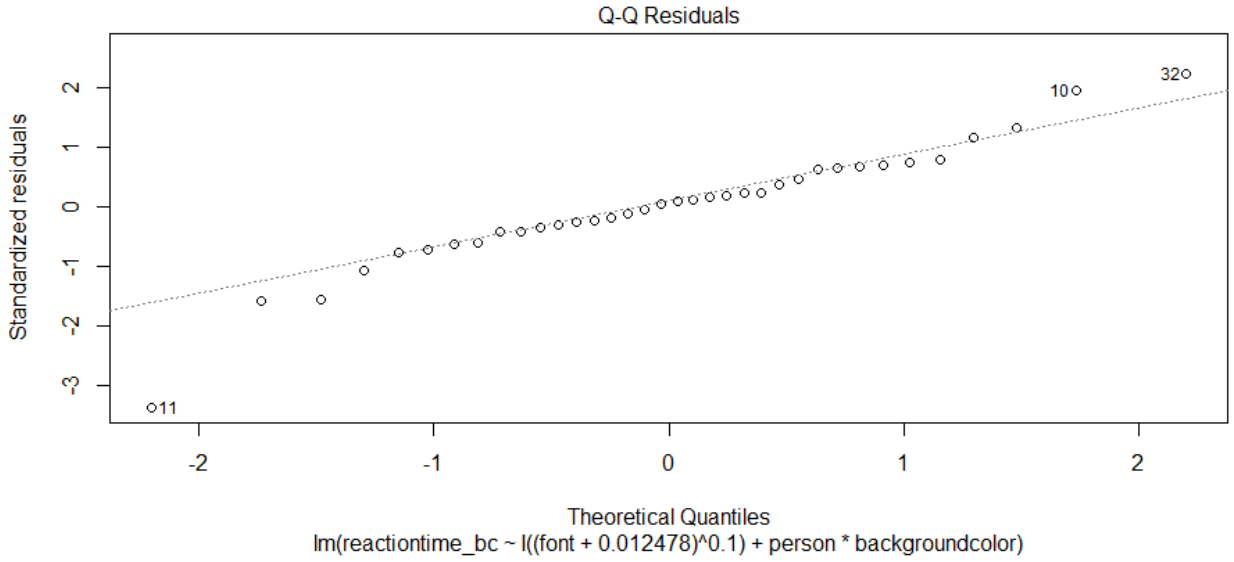
```
> plot(Mean_Reaction_Times_Final_Sayfa1)
```

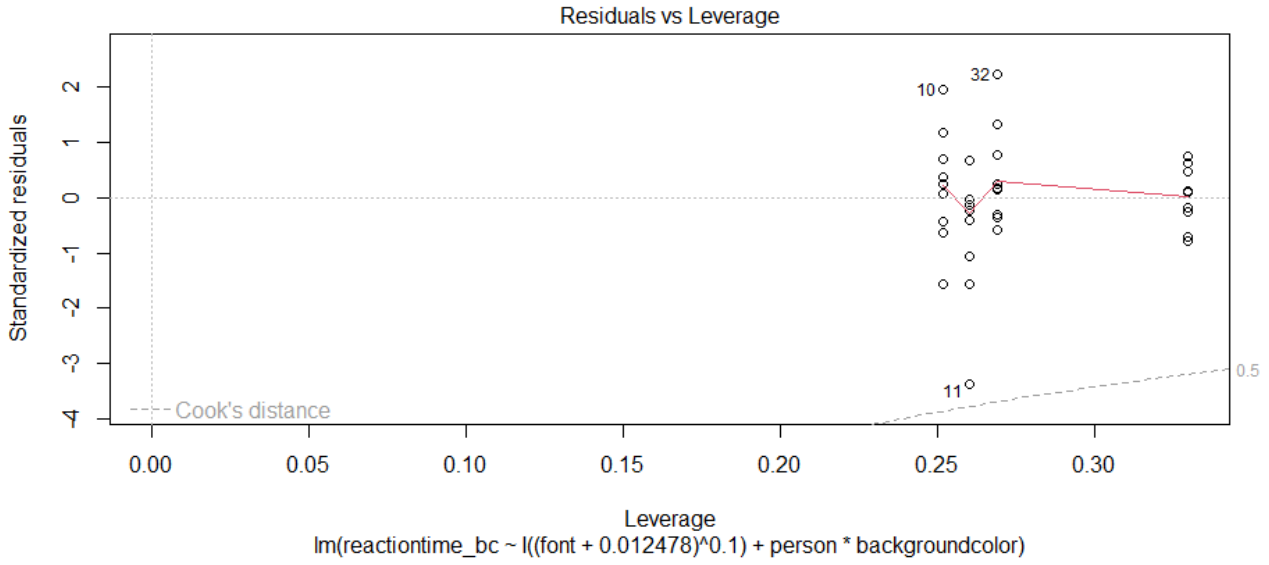


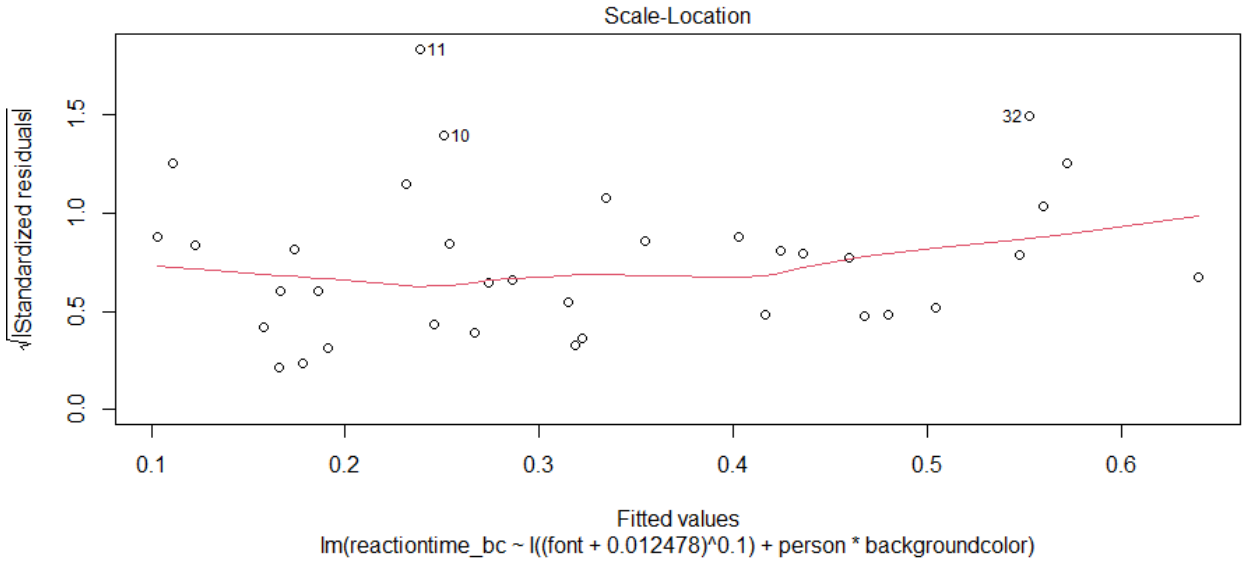
(2.2): Plots after the Box-Cox transformation on the response variable and exponential transformation of the normalised font size

```
> plot(model_bc_fontnorm)
```






Residuals vs Leverage
Standardized residuals
Leverage
Cook's distance
0.5
10
32
11
lm(reactiontime_bc ~ I((font + 0.012478)^0.1) + person * backgroundcolor)


Scale-Location
√|Standardized residuals|
Fitted values
11
10
32
lm(reactiontime_bc ~ I((font + 0.012478)^0.1) + person * backgroundcolor)