2022 High School Mathematical Contest in Modeling

Team Control Number: 13057

Summary Sheet

Declining bee populations have a devastating effects on the environment and economy; there are several reasons for this. In this paper, we examine how different factors affect the total bee population of a hive. These factors include: (1) reproduction rates, (2) starvation, (3) Varroa mite and DWV (deformed wing virus) infestations, (4) how predators kill bees, steal their honey and consume larvae, and (5) destruction from hurricanes. By developing models for each factor and then merging them, we can predict the population of a bee hive over time.

We define an algorithm using R to calculate the number of bee hives required to fully pollinate a 20-acre apple orchard. The algorithm maps out 20 acres and measures the theoretical effect of different concentrations of bee hives on the pollination of the area. Our algorithm relies on two independent variables: the location and population of each bee hive. By running this simulation, we found that the optimal number of bee hives in order to pollinate the 20 acres is 6.

To model reproduction rates and nectar production, we use the flat peak cosine function and integral equations. For lifespan, mites and diseases, we parameterize the time of death and number of deaths. We use the Leslie Matrix—created by mathematician Patrick Holt Leslie—to model the population dynamics of honey bee predators and the impact of predator fluctuation on bees. We use a haversine function to briefly examine the effects of hurricanes on bee population. This gives us a window into how other natural disasters might impact bees. The sensitivity analysis quantifies the magnitude and probability of influence for each factor on the bee population.

BEES ARE BUZZIN

TEAM 13057

ENVIRONMENTAL CONDITIONS

Conditions such as a cold winter or heavy rains hurt bee populations since honey bees cannot leave their hives and therefore cannot produce honey for consumption. During cold winters, it is common for honey bee colonies to throw out male drones when their isn't enough food for all bees. When it rains a lot, bees are at risk of injuring themselves since the raindrops can damage their wings, making it near impossible for them to return to their hives.

PREDATORS

Animals that prey on honey bees, obviously, negatively impact their population, but which animals are these? Bears and striped skunks were the two animals that we looked at for their predation of honey bees. The likelihood of bees being attacked by striped skunks is low, but the number of bees killed in each attack is in the hundreds. Similar to striped skunks, bears—particularly black bears—are unlikely to attack a honey bee hive, but when they do, the bears are impervious to stings on most of their body except their face. This means bears can shake off the honey bees, and eat as much honey and bee larvae as they want.

MITES AND DISEASES

Varroa mites are a common pest that can invade honey bee hives and cause a lot of harm. The mites can spread very quickly because they get into the brood cells of the honey bees before they are sealed, so the mites take nutrients from the bee pupa even before it has matured into an adult bee. The other side of what mites do is that they spread a virus called DWV (deformed wing virus), a congenital abnormality that causes shriveled wings, decreased body size and decoloration of honey bees. Overall, the disease spreads quickly in a hive and it decreases the overall lifespan of the bees. If DWV spreads unchecked in a bee hive, the result is often a complete collapse of the colony often referred to as CCD or colony collapse disorder.

HABITAT DESTRUCTION

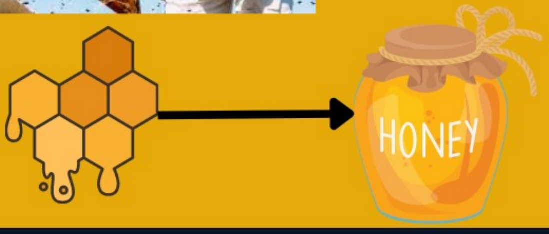
Several factors can lead to habitat destruction, such as bear attacks and natural disasters like tropical storms. The destruction of bee hives can put a lot of stress on bees to rebuild from scratch everything they had built. This can also lead to CCD because bees might abandon their hive if there isn't enough resources to last a long winter. Also, if the environment around the bee hive changes dramatically, worker bees might abandon the hive because conditions are unsuitable for collecting nectar and pollen.

PESTICIDES

The harm of pesticides on bees is very relevant to their productivity. The chemicals in pesticides are known to adversely affect the memory and learning of honey bees, and at higher levels of exposure to pesticides bees can become infertile. This prevents reproduction and blocks the continued growth of a bee hive. Also, if a bee has direct contact with an insecticide, the bee will die shortly unable to return to its hive with nectar and pollen.

IMPACT ON BEEKEEPERS

The decrease in the number of honey bee colonies has also hurt bee keepers who harvest the honey from their hives to sell on the market. Over the years, beekeepers have recorded a steady decrease in the number of hives they maintain, attributing the loss of hives to mite infestations like with the varroa mites and habitat destruction caused primarily by predators. The decline of domestic honey bee colonies might actually be a good outcome because then wild and endangered bee species face less competition for resources.



DON'T WORRY BEE HAPPY

Contents

1 Introduction

1.1 Background

Honey bees play a crucial role in the function of natural environments as well as human society. According to a Cornell University Study[1], " honey bees pollinated $12.4 billion worth of directly dependent crops and $6.8 billion worth of indirectly dependent crops in 2010." In fact, according the US Department of Agriculture, "Honey bees alone pollinate 80 percent of all flowering plants, including more than 130 types of fruits and vegetables." In biology, a keystone species is crucial to maintain the operation of the ecosystem; the absence of this species would lead to the collapse of the entire ecosystem. Honey bees are a type of keystone species.

However, since the winter of 2006-2007 beekeepers discovered unusually high losses for their bee hives' population. This syndrome of significant decrease in bee population is known as the Colony Collapse Disorder (CCD).

Given how invaluable honey bees are for the environment and their critical position in the entire ecosystem, it is pertinent to evaluate the possible factors that could lead to this loss of honey bees in order to work towards a solution to this problem.

1.2 Problem Restatement

The problem tasks us to find the predicted population of a honey bee hive considering various factors like predators, mites, and natural disasters. We must also calculate an approximate number of hives that a 20 acre area will need to be sufficiently pollinated. We want our model to be used as a resource for beekeepers. We hope it will be useful since it quantifies the risks associated with maintaining a bee colony. It also raises awareness of how detrimental a bear attack or Varroa mite infestation is to the sustainability of a hive. From there, beekeepers can take measures to mitigate the risks associated with beekeeping. Our paper revolves around these three questions:

1. What are the essential factors that could influence the population size of a bee colony?
2. How do each of these factors influence the others and impact the size of a bee colony?
3. How do bees interact with the surrounding environment through pollination?

2 Preliminary Information

2.1 Assumptions

1. We will only look at bee populations in America because it would be cumbersome to model the populations for honeybees in many different environments.

2. Bees do not immigrate to colonies and emigrate from other colonies, honey bees are local and are only born and die. No new bees should be introduced to a hive because the problem explicitly dictates that the model does not have to consider the interactions between hives.

3. There is always going to be a queen for the 5-6 year lifespan of a bee colony, and the queen will always be able to produce eggs. This is justified because when the queen of a hive dies, the worker bees choose a few possible successor larvae and one of them becomes the sexually-reproducing queen bee.

4. Climate change is negligible in the short term; within the life span of a bee colony. It is unlikely that there would be a major climate disaster during the life span of a bee hive.

5. The maximum reproduction rate of a honey bee colony is 2000 eggs per day.

6. We assume that Honey bees would still forage for food during category 1 and 2 hurricanes and sacrifice a fraction of their population, whereas, in larger storms, the honey bees would stay at their hive because the conditions are too dangerous. Most sources agree that light storms rain showers and storms would not stop foraging; however, the risk of a honey bee's wings getting wet and heavy is relatively high during storms, so there is the possibility of the rain injuring and killing a worker bee.

7. We assume that all bee hives will be planted in an habitable environment (E.g. a tree or the corner of a door frame). Otherwise, hives would fall victim to poor weather and they would get destroyed. However, this does not rule out the possibility of bee hives being destroyed by predators such as bears and skunks.

8. We assume that all the types of flowers are the same and have the same nectar/pollen production and provides the same nutrition value to all honey bees. Otherwise, it would be a great undertaking just to model the different production rates of nectar and pollen from each species of flower.

9. We assume that all the nectar brought back to the hive will stay in the hive—No loss, all is usable for food. It would be unreliable to predict how much nectar is lost and not used for food production.

10. We assume that drone bees don't die during reproduction. After a drone bee does a mating flight with the queen bee, the drone bee continues working on improving the hive.

11. Queen bees have the same diet as a worker bee. Having a separate diet of royal jelly is very insignificant for starvation considering the number of bees in a hive.

2.2 Definitions

1. Bee colony means a natural group of honey bees containing seven thousand or more workers and one or more queens, housed in a man-made hive with movable frames, and operated as a beekeeping unit.

2. CCD(Colony Collapse Disorder) is a phenomenon where pests/diseases, a genetically poor queen bee, exposure to chemical toxins, varroa mites, and malnutrition/starvation can cause the collapse and abandonment of a colony.

3. Forage: Honey bee foraging consists of several behavioral components that include the search for food, identification and memorization of locations where to find nectar, carrying and storing of food, and interactions/communication between honey bees about the location of flowers.

3 Mathematical Model

3.1 Reproduction Rate

The reproductive rate of the queen is a critical piece for a colony's survival. The following variables are used to model that rate.

Table 1: Nutrition supply and starvation Variables

Variables	Definition	Units
$R_{rate}(t)$	The egg laying rate with respect to t	$eggs/day$
$W_{rate}(t)$	The egg laying rate of Worker bees	$eggs/day$
$D_{rate}(t)$	The egg laying rate of Drones	$eggs/day$
W_{ratio}	The Worker to Drone egg ratio parameter	Unitless

Depending on the season the queen can vary its egg laying rate: peaking in the summer with 2000 $eggs/day$ and in the winter plunging down to 0 $eggs/day$ (or close enough). We also assumed (assumption 5) that there would always be a queen, therefore our model should be continuous. To achieve the seasons and periodicity we used the flat peak cosine function [2].

$$R_{rate}(t) = 1000\sqrt{\frac{1+3.5^2}{1+3.5^2\cdot\cos^2\left(\frac{\pi}{182.5}\left(t-\frac{1095}{8}\right)\right)}}\cos\left(\frac{\pi}{182.5}\left(t-\frac{1095}{8}\right)\right)+1000 \tag{1.1}$$

Where $R_{rate}(t)$ is the reproduction rate (egg/day) of the queen bee and t is the number of days from January 1st ($t=0$). $3.5, 1000, \pi/182.5$, and $1095/8$ determined the flatness, maximum and minimum production rate, period (365 days), and origin value (January 1st) respectively.

$$W_{rate} = R_{rate}\cdot W_{ratio} \tag{1.2}$$

$$D_{rate} = R_{rate}\cdot(1-W_{ratio}) \tag{1.3}$$

Using the predefined parameter W_{ratio} we can determine the egg laying rates of Worker Bees (equation 1.2) and Drones (equation 1.3 shown respectively in the equations above.

3.2 Nectar supply and starvation

In this section we are determining the hive nectar supply a hive has starvation. The following are the significant variables to model Nutrition Supply and Starvation.

Table 2: Nutrition supply and starvation Variables

Variables/Constants	Definition	Units
$N_{rate}(t)$	Nectar Foraging Rate	mg/day
$S_{state}(t)$	Sunlight hours of a US State	hours
$W(t)$	Number of worker bees	bees
$D(t)$	Number of drones	bees
$A(t)$	Boolean (0 or 1) Active function	Unitless
$N_{demand}(t)$	Nectar demand of a bee hive population	mg/day
$N_{supply}(t)$	Physical beehive nectar supply	mg
N_0	Initial amount of nectar	mg
75	Number of flowers a worker bee visits per hour	$flowers/hours$
11	Amount of nectar a bee consumes per day	mg/day
40/100	Amount of Nectar per 100 flowers	$mg/flowers$

Firstly, we determine the hive Nectar forage rate $N_{rate}(t)$ (how many flowers a hive travels to per day). The amount of flowers a hive forages to depends on the hours that bees are active. The bees are active when there is day light (sunrise to sunset)[3]. This means that the Nectar forage rate is proportional to the hours of sunlight.

$$N_{rate}(t) \propto S_{state}(t)$$

Where $N_{rate}(t)$ is the Nectar forage rate in mg/day and $S_{state}(t)$ is the sunlight hours in a particular state (eg. $S_{FL}(t)$ is the sunlight hours model for Florida) as a function of t (in days). We used data from the suncalc[1] and sinusoidal regression to model the $S_{state}(t)$ of each of of the 50 state (see appendix).

$$A(t) = \text{round}\left(\left|\cos\left(\frac{\pi}{\frac{4015}{12}}\left(x - \frac{4015}{24}\right)\right)\sqrt{\frac{1+1.7^2}{1+1.7^2\cos^2\left(\frac{\pi}{\frac{4015}{12}}\left(x-\frac{4015}{24}\right)\right)}}\right|\right) \tag{2.1}$$

We combined the flat peak cosine function, the round operator and absolute values to get a Boolean value (True = 1, False = 0) to create the active function (equation 2.1. If $A(t) = 1$ that means that the worker bees are actively foraging for nectar, which is during the spring, summer and fall days. In contrast, $A(t) = 0$ means that the bees are inactive, which is during the winter days.

$$N_{rate}(t) = \left(\frac{40}{100}\right) W(t) \cdot S_{state}(t) \cdot A(t) \tag{2.2}$$

Equation 2.2 models the amount of Nectar in mg/day a honey bee colony gets per day. Where $\frac{40\ \ mg}{100 flowers}$ is the amount of Nectar a bee gets per 100 flowers[5], 75 is the constant value for the number of flowers a single bee can visit per hour, and $W(t)$ is the population of worker bees as a function of t in days. Using $A(t)$, the worker bees don't forage any nectar ($N_{rate}(t) = 0$) during the winter and forages else where.

$$N_{demand}(t) = \left(\frac{11mg}{day}\right)(W(t)+1) + 3\left(\frac{11mg}{day}\right)D(t) \tag{2.3}$$

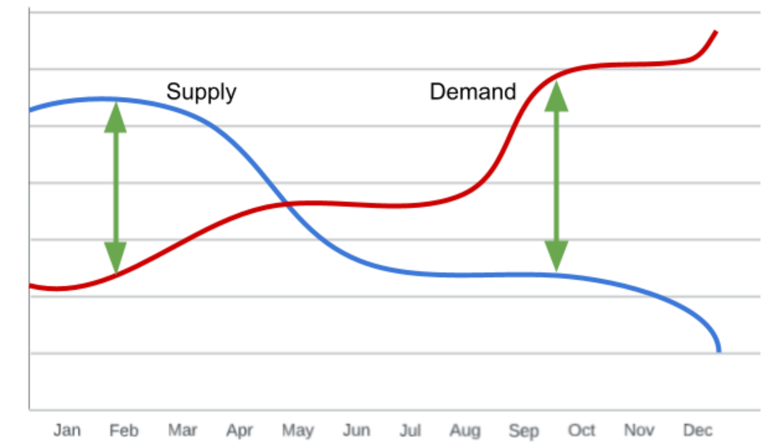
[1]Suncalc[4] is an R package created by Benoit Thieurmel and Achraf Elmarhraoui. The R code used to generate data (see appendix)

Where $11mg/day$ is the amount of Nectar a worker bee (and queen, who has same diet by assumption 13). That number is being multiplied by $W(t) + 1$ which is the number of worker bees plus 1 queen bee (who has the same diet). $D(t)$ has triple the diet of a worker bee[6].

$$N_{supply}(t) = F_2\Big(N_0 + \int_0^t N_{rate}(x) - N_{demand}(x)\, dx\Big) \tag{2.4}$$

In equation 2.4 we take the supply rate minus the demand rate to get the net nectar rate. Then we summed the initial nectar amount (N_0) and integral of net nectar rate to get the $N_{supply}(t)$. Because the supply mathematically can be negative but can't be realistically, we used the "Zero or One" equation ($F_2(x)$) to make any negative numbers 0 (Refer to appendix).

$$N_{disparity}(t) = \int_{t-1}^{t} N_{supply}(x) - N_{demand}(x) dx \tag{2.5}$$



In equation 2.5 and 3.2 we defined the variable $N_{disparity}(t)$, it shows how much extra (if $N_{disparity}(t) > 0$) or insufficient ($N_{disparity}(t) < 0$) Nectar amount there is.

$$B_{disparity}(t) = \left\lceil \frac{N_{disparity}(t)}{11} \right\rceil \tag{2.6}$$

To simply the model further, we defined $B_{disparity}(t)$ (In units of number of bees) that the $N_{disparity}(t)$ can provide. We used the ceiling function to account for any small differences in the bee consumption, for example some eating a little less because of lack of food.

For bees to die by starvation they must have a lack of food for more than a day because a bee starves to death after 1 day[7], in other words if they starve for close to a day, they will live but more it will die. To analyze the starvation system we set up random mock numbers shown in the starvation table.

Using $t = 1$ as an example: When $B_{disparity}(1) = -7$ that means that there is not enough food for 7 bees (Negative of $B_{disparity}$). We then connected the pattern between $B_{starve}(t)$, $B_{disparity}(t)$ and $B_{starve}(t-1)$.

$$B_{starve}(t) = F_1\left(-\left(B_{disparity}(t) + B_{starve}(t-1)\right)\right) \tag{2.7}$$

$$B_{starve}(t) = \frac{1}{2}\left(\frac{\left|B_{disparity}(t) + B_{starve}(t-1)\right|}{B_{disparity}(t) + B_{starve}(t-1)} - 1\right)\left(B_{disparity}(t) + B_{starve}(t-1)\right) \tag{2.8}$$

Table 3: Starvation

Day / Value	0	1	2	3	4	5	6
$N_{disparity}$	0	-83	-24	5	91	-99	-41
$B_{disparity}$	0	-7	-2	1	9	-9	-3
		$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
B_{starve}	0	$\rightarrow$ 7	$\rightarrow$ 0	$\rightarrow$ 0	$\rightarrow$ 0	$\rightarrow$ 9	$\rightarrow$ 0
		$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$B_{starvedie}$	0	0	2	0	0	0	3

In equation 2.7 we use the Zero-Original Equation (Refer to appendix). We found that if $-(B_{disparity}(t)+B_{starve}(t-1) < 0$, which means that if the number of currently starving bees (they started starving the day before) received enough food the next day, they would all stop starving, therefore $B_{starve} = 0$. On the other hand if $-(B_{disparity}(t) + B_{starve}(t-1) > 0$ that means that the ones that started starving the day before either died or recovered by getting enough food.

Using day 2 as an example. $B_{disparity}(2) = -2$, which means that there is not enough food for 2 bees, and $B_{starve}(2-1) = 7$, which means that 7 bees began starving yesterday, two bees would pass away because there were starving bees and 2 continued to starve on day 2. The rest however, recovered because they got enough food, $-(-2+7) < 0- > B_{starve} = 0$.

We can then calculate $B_{starvedie}(t)$, the number of bees that die by starvation at a certain t value, using another pattern. We found that $B_{starvedie}(t) + B_{starve}(t) = -B_{disparity}(t)$ for when $B_{disparity}(t) < 0$. On the contrary, if $B_{disparity}(t) > 0$, then $B_{starvedie}(t) + B_{starve}(t) = 0$. This relationship can be described with a variant of "Zero or One" (Refer to appendix) in 2.9

$$B_{starvedie}(t) = F_1\left(B_{disparity}\right) \cdot (-left(B_{disparity}(t) - B_{starve}(t)) \tag{2.9}$$

$$B_{starvedie}(t) = -\frac{1}{2}\left(\frac{\left|B_{disparity}(t)\right|}{B_{disparity}} - 1\right)(= B_{disparity}(t) - B_{starve}(t)) \tag{2.10}$$

Equation 2.10 is the final equation for $B_{starvedie}(t)$. We can test this by explaining an example of $t = 2$. There was seven bees that started starving on $t = 1$ and two bees continued to starve on $t = 2$ because there was not enough food for two ($B_{disparity}(2) = -2$), therefore two bees dies, which matches with $B_{starvedie}(2) = 2$.

3.3 Lifespan of bees

The life span of bees (from birth to death) are not always the same for all bees. This section will model life span of bees. The following are the significant variables.

Table 4: Nutrition supply and starvation Variables

Variables	Definition	Units
$W_{lifespan}$	Seasonal lifespan of Worker Bees	*days*
$M(t)$	Varroa mite population	Mites
$M_p(t)$	Varroa mite to bee population percentage	
$B_{mitedie}(t)$	Number of bees killed by Varroa mite (and DMV)	*bees*
$W_{lifespanmite}(t)$	Worker bee expected lifespan including Varroa (and DMV)	*days*

3.3.1 Seasonal Lifespan

3.3.1.1 Worker bees

To begin, we model the lifespan of worker bees over different seasons. The reason being is that the the lifespan of worker bees will be shorter in the spring and summer due to the high workload (foraging) and longer in the fall and winter.

$$W_{lifespan}(t) = 69\sqrt{\frac{1+1.858^2}{1+1.858^2\cdot\cos^2\left(\frac{\pi}{182.5}\left(t+11.5+21\right)\right)}}\cos\left(\frac{\pi}{182.5}\left(t+11.5+21\right)\right)+111 \tag{3.1}$$

We are again using the flat peak cosine (see appendix) to make a graph. $W_{lifespan}(t)$ is the ideal lifespan of worker bees(days) born on t day, and January 1st when $t = 0$. In spring and summer, the lifespan will go down to 42 - 49 days, and in fall and winter, the lifespan will go up to 180 days[8]. The amplitude will be 69, the difference of 180 and 42, and we need to move the graph 111 up so that the maximum and minimum value will be 180 and 42. We used 1.858 and added 11.5 to t to make the lifespan be 49 days in the mid spring and mid summer. The period is 365, which is decided by 182.5. The bees need approximately 21 days to be an adult[9], and because only an adult bee works, we shifted the graph 21 left so that only the lifespan of adult bees are affected by the workload.

3.3.1.2 Drone bees

$$D_{lifespan}(t) = 42.5\sqrt{\frac{1+b^2}{1+b^2\cos^2\left(\frac{\pi}{182.5}\left(x+152\right)\right)}}\cos\left(\frac{\pi}{182.5}\left(t+152\right)\right)+47.5 \tag{3.2}$$

Similar to the seasonal lifespan of Worker bees, Drones are similar but with different values. The maximum lifespan of a drone is $90 days$ [10] and the minimum life span is $5 days$ because they get thrown out due to having no purpose.

3.3.2 Mites and Deformed Wing Virus

Varroa Destructor mites are a pests and a prominent factor that leads to a colony collapse. Varroa mites are also a vector for many different viruses (around 20) that affect bees. We are only considering the most frequent appearing (>80% of the time) virus[11], the Deformed Wing Virus (DWV), which is a deadly pathogen that causes various symptoms, that kills brood bees and decreases the lifespan of adult bees.

To find the magnitude of impact by the Varroa mites, we modeled the percentage of mites in a colony with respect to time using a differential logistic equation.

$$\frac{dM}{dt} = 0.024M\left(1 - \frac{M}{0.05 \cdot 40,000}\right) \tag{3.3}$$

In equation 3.3, the rate of proportionality is 0.024 (2.4%) because the mite population increase by 2.4% each day[12], the mite population is M, the maximum population is 0.05 (5%) of the beehive population because that is when it is most potent[13], and the 40,000 represents the average beehive population.

$$M(t) = \frac{0.05 \cdot 40,000}{1 + \left(\frac{0.05 \cdot 40,000}{100}\right) e^{-0.024t}} \tag{3.4}$$

Integrating the logistic differential equation (equation 3.3 we get the Varroa Mite population equation (equation 3.4)

$$M_p \Rightarrow \begin{cases} 25\% \ \text{ Death} & \Rightarrow 0.25\left(M_p(t)\right) = 0.25\left(\frac{M(t)}{B(t)}\right) \\ 75\% \ \text{ Decrease lifespan} & \Rightarrow 0.75\left(M_p(t)\right) = 0.75\left(\frac{M(t)}{B(t)}\right) \end{cases} \tag{3.5}$$

Using research and observations we defined that 25% of the infected bees would die 75% would have a decreased life span. On the right of the Death and Decrease lifespan is the percentage of bees affected by Death or Decrease lifespan.

$$B_{mitedie} = 0.25\left(\frac{M(t)}{B(t)}\right)(B(t)) \tag{3.6}$$

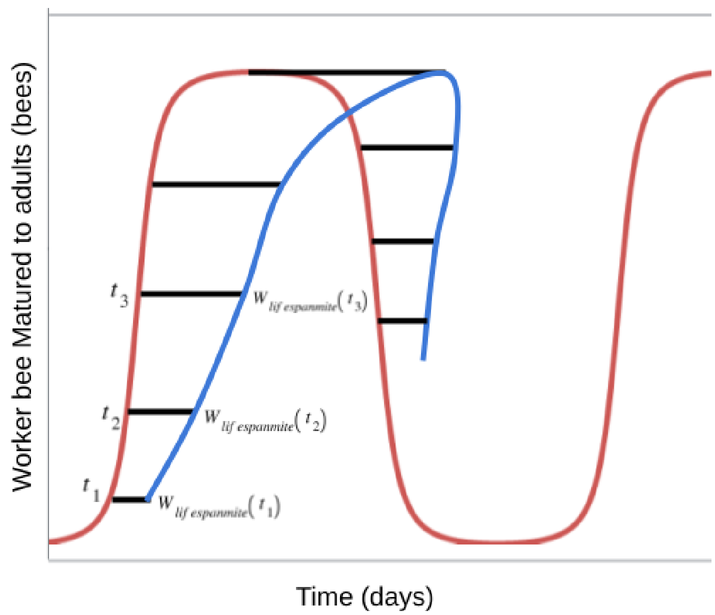
To find the amount of bees that die from mites $B_{mitedie}$ we take the percentage of bees prone to die $(0.25\left(\frac{M(t)}{B(t)}\right))$ and multiple by the number of bees $(B(t))$.

$$W_{lifespanmite}(t) = \left(1 - \frac{M(t)}{B(t)}\right)(W_{lifespan}(t)) + 0.75\left(\frac{M(t)}{B(t)}\right)(W_{lifespan} - 5) \tag{3.7}$$

In equation 3.7 we used expected value to determine the expected lifespan as a function of t. $\left(1 - \frac{M(t)}{B(t)}\right)$ is the percentage of non-infected bees multiplied by the seasonal life span equation 3.1. Similarly, $0.75\left(\frac{M(t)}{B(t)}\right)$ is the probability of an infected bee decreasing in lifespan multiplied by the $(W_{lifespan} - 5)$, the regular seasonal lifespan minus an average of 5 day decrease in lifespan caused by the mite and DWV.

3.3.3 Combining lifespan

Using our lifespan equation and Adult Mature rate equation mentioned in we can determine the death by "natural" lifespan.



In the figure above, there are several arbitrary example t values ($t = t_1, t_2, t_3$) and its respective $W_{lifespanmite}$ value, which is shown as the length of the horizontal lines. To find the deaths we need to graph the function made by the Adult matured bees equation and the changing lifespan values.

$$W_{mature} = \int_{t-1}^{t} W_{rate}(t-21) \tag{3.8}$$

First we transformed equation (1.1) from Section Reproduction Rate to find out how many rate of Worker bees maturing on that day t. Taking the integral of that value, we can quantify how many worker bees matured on that day t.

$$W_{lifespandie} = \Big\langle t + W_{lifespanmite}(t), W_{mature} \Big\rangle \tag{3.9}$$

We are able to graph the death equation $W_{lifespandie}$ using parametric equations based on the variable t. Parametric equations can have multiple outputs for an x input. Therefore, to find the number of deaths on a given day we need to sum up all of the W_{mature} present on the same day.

At a given $t = q$, there may be multiple previous t values that have the same death time, we represent these as $t_1, t_2, t_3 \cdots$ as needed (not all will have more than one death value).

$$q \to t_1, t_2, t_3 \cdots \tag{3.10}$$

$$W_{lifespandie}(q) = \sum_{n=1}^{k} W_{mature}(t_n) \tag{3.11}$$

Where $W_{lifespandie}(q)$ is the number of deaths on $t = q$ and k is the preceding t values that have the same death time. The equation 3.11 outlines how to find the number of deaths on a particular day. We can take that equation and apply it over time to get the number of deaths over time.

3.4 Predation

3.4.1 Striped skunk

We use the Leslie Matrix defined by Scottish Mathematician Patrick Holt Leslie to model the population of the striped skunk. The standard implementation of the Leslie Matrix demonstrates the recursive relationship from one-time cycle to the next for a specie's population, it's defined as below (Wikipedia).

$$\begin{bmatrix} n_0 \\ n_1 \\ \vdots \\ n_{\omega-1} \end{bmatrix}_{t+1} = \begin{bmatrix} f_0 & f_1 & f_2 & \dots & f_{\omega-2} & f_{\omega-1} \\ s_0 & 0 & 0 & \dots & 0 & 0 \\ 0 & s_1 & 0 & \dots & 0 & 0 \\ 0 & 0 & s_2 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & s_{\omega-2} & 0 \end{bmatrix} \begin{bmatrix} n_0 \\ n_1 \\ \vdots \\ n_{\omega-1} \end{bmatrix}_t \tag{4.1}$$

n_{skunk_x}, the count of individuals (n) of each age class x

s_{skunk_x}, the fraction of individuals that survives from age class x to age class $x+1$

f_{skunk_x}, fecundity, the per capita average number of female offspring reaching n_0 born from mother of the age class x, essentially this is the reproductive rates for different age classes

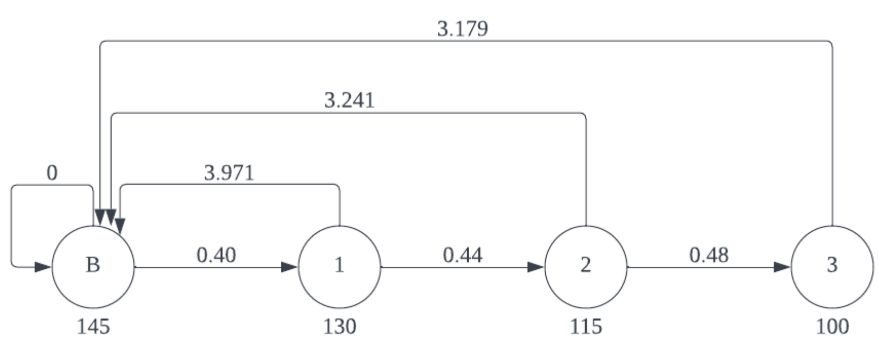
s_{skunk_x}, the survival rate for population from an age class to the next age class. Example: the survival rate from 1 year old to 2 years old.

More precisely, it can be viewed as the number of offspring produced at the next age class b_{x+1} weighted by the probability of reaching the next age class. Therefore, $f_x = s_x b_{x+1}$.

In short it can be written as:

$$\mathbf{n}_{t+1} = \mathbf{L}\mathbf{n}_t \tag{4.2}$$

The most important step is to determine the eigenvalue of L, which can be translated from the figure below with specific values being calculated in the appendix.



We considered the typical life expectancy of a wild skunk is 3 years, as such we can separate the Skunk population into 4 age classes: B for the born/babies class, 1 for the 1 year old age class, 2 for 2 years old age class, and 3 for 3 years old age class [14].

The values on 4 arrows pointing towards B represent the reproductive rates per individual for each age class, for example, 3.971 means for every skunk in that age group gave birth to on average 3.971 baby skunks. (Note that this number takes into consideration the gender distribution of the skunk population, calculations in the appendix). These four values correspond to the f_{skunk_0}, f_{skunk_1}, f_{skunk_2}, and f_{skunk_3} eigenvalues of L_{skunk}.

The values on the arrow between each circle represent the survival rate from 1 age class to the next age class, for example, 0.40 means 40 percent of baby skunks managed to survive and became sexually mature 1 year after their birth. Similarly, these three values correspond to the s_{skunk_0}, s_{skunk_1}, s_{skunk_2}, and s_{skunk_3} eigenvalues of L_{skunk}.

Plug in the resulting values into the set location of the matrix we can get the following Leslie Matrix L_{skunk} for skunk population dynamics.

$$L_{skunk} = \begin{bmatrix} 0 & 3.971 & 3.24 & 3.179 \\ 0.40 & 0 & 0 & 0 \\ 0 & 0.44 & 0 & 0 \\ 0 & 0 & 0.48 & 0 \end{bmatrix} \tag{4.3}$$

Since the Leslie Matrix reflects a recursive relationship of a population, it's necessary to have an initial vector value denoted by n_0 to show the initial population for each age class.

Defined the initial population vector for skunk as followed,

$$n_{skunk} = \begin{bmatrix} 0 \\ 15 \\ 0 \\ 0 \end{bmatrix} \tag{4.4}$$

This number is justified as, "The family group breaks up in the fall and the young move to new territory"[15].

Therefore, the impact on the bee colony as a result of the skunk's population will be calculated by taking the number of skunks for each age group and multiplying each one of the population by its corresponding daily consumption defined as the number below each circle. (145,130,115,100)

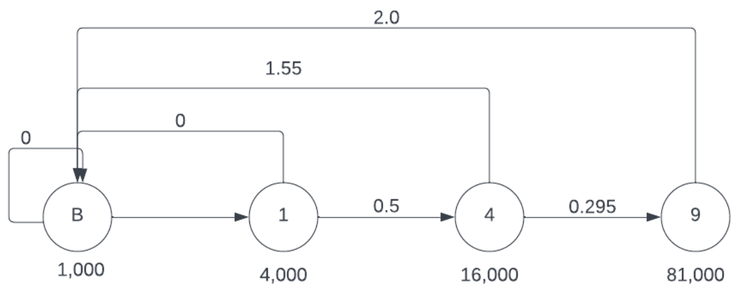
3.4.2 Bear

$$L_{bear} = \begin{bmatrix} 0 & 1.55 & 2.0 & 2.0 \\ 0.625 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.295 & 0 \end{bmatrix} \tag{4.5}$$

This Leslie Matrix, similar to the skunk population model, takes the survivability of bears at three ages, 1 year, 4 years, and 6 years and stores the average number of bears being reproduced by each bear at that age. The first row has the average number of cubs produced per bear at their age. Starting on the left with bears surviving to the age of 1 years old, and one column to the right which shows the average reproduction rate of 4 year old bears. Then, the third column from the left shows the average reproduction rate of 6 year old bears. The reproduction rate of 1 year old bears is non-existent or 0 because female bears at that age are not sexually mature and can't produce cubs.

$$n_{bear} = \begin{bmatrix} 0 \\ 1.0 \\ 0 \\ 0 \end{bmatrix} \tag{4.6}$$

The value above is the initial population for bears in the area around a bee hive. Bears are typically solitary animals, only interacting with other bears for breeding and raising cubs. This is why the initial population of the bears is 1.0; eventually, the population will increase as more bears are introduced into a family unit. Then, population will increase naturally through reproduction.



The graphic above is a visual representation of the matrix, L_{bear}, but also includes the potential quantities of bees killed by bears of different ages; noted under each circle. Generally, bear cubs do not seek out as many bee hives to take honey and bee larvae from. This is why the number of bees eaten per attack on a hive is lower for bears under one year old than 9 year old bears.

3.5 Hurricane and Bees

We developed a stochastic model to estimate the damage dealt to a bee colony by a hurricane.

3.5.1 Probability

H_p and H_{wp} are decision variables to determine whether a hurricane occurs for a particular location and in what category. We represent these two variables in ordered pairs as below.

$$(H_p, H_{wp})$$

H_p decides on whether there is a hurricane in the inputted state. This decision is dependent on the probability of a hurricane occurring in a particular state. We calculate the probability of a hurricane occurring in a particular state daily using the total number of hurricanes occurring in that state between 1851 and 2020 and divide it by 169 x 365 = 61685 (number of days between), this probability is specified using the table below.

Table 5: Chance of Hurricane in States

State	C_1	C_2	C_3	C_4	C_5	**All**	**Frequency**
Florida	47	36	24	11	2	120	0.001934
Texas	29	16	12	7	0	64	0.001031
Louisiana	24	20	13	4	1	62	0.000999

Continued on next page

Table 5 – continued from previous page

State	C_1	C_2	C_3	C_4	C_5	All	Frequency
North Carolina	32	19	6	1	0	58	0.000935
Mississippi	3	5	5	0	1	14	0.000226
South Carolina	17	9	2	3	0	31	0.000500
Alabama	12	6	5	0	0	23	0.000371
Georgia	14	4	2	1	0	21	0.000338
New York	9	3	3	0	0	15	0.000242
Rhode Island	5	2	3	0	0	10	0.000161
Connecticut	7	2	2	0	0	11	0.000177
Massachusetts	7	4	1	0	0	12	0.000193
Virginia	11	2	0	0	0	13	0.000210
New Jersey	4	0	0	0	0	4	0.000064
Maine	2	1	0	0	0	3	0.000048
Maryland	2	0	0	0	0	2	0.000032
Delaware	2	0	0	0	0	2	0.000032
Pennsylvania	1	0	0	0	0	1	0.000016
New Hampshire	0	1	0	0	0	1	0.000016
United States	228	130	78	27	4	467	0.007526

Source: [16]

H_{wp}, on the other hand, takes into consideration the probability for different categories of hurricanes in a particular state by taking the number of hurricanes for a category and dividing that by the total number of hurricanes that happened at that state. For example, the chance of a hurricane in Florida being a category 5 hurricane is $2/120 = 0.01666$. The value of this variable H_{wp} will ultimately be the result of a random generation whose probability of outcomes is defined as above.

A possible outcome for these decision variables:

$$(Yes, 3)$$

3.5.2 Severity

$$d_{h_n} = 2r \arcsin\left(\sqrt{\sin^2\left(\frac{B_\varphi - H\varphi_n}{2}\right) + \cos B_\varphi \cdot \cos H\varphi_n \cdot \sin^2\left(\frac{B_\lambda - H_{\lambda_n}}{2}\right)}\right) \tag{5.1}$$

d_{h_n} is the haversine distance from the bee colony which defined by the earth's radius (r), the colony's longitude (B_φ) and latitude (B_λ) to the eye of the hurricane (Longitude $H\varphi_n$ and latitude H_{λ_n}) at the time.

$$H_{demage_n} = H_p \cdot H_{wp} \cdot \left\lceil \frac{d_{h_n} - r_n}{{r_h}^3} \right\rceil \left(1 - \frac{hav_{h_n}}{r_h}\right) B_{pop_n} \cdot W_s \tag{5.2}$$

This formula determines the severity of a storm, hence it's damage to the bee colony at time n if the bee colony is influenced.

$$\left\lceil \frac{d_{h_n} - r_n}{{r_h}^3} \right\rceil \tag{5.2.1}$$

The simple ceiling function 5.2.1, will returns a value of 0 if $d_{h_n} - r_h$ is negative means the location of the beehive is not in the range of the hurricane defined by the radius of the hurricane (r_h. if $d_{h_n} - r_h$ is positive, then the term would returns "1" indicating that the bee colony are affected by the hurricane.

$$\left(1 - \frac{hav_{h_n}}{r_h}\right) B_{pop_n} \cdot W_s \tag{5.2.2}$$

For equation 5.2.2, we divide the haversine distance by the radius of the hurricane (haversine) to find out the distance from the bee colony to the eye relative to the hurricane's size. Then we subtract this fraction from 1 to establish the "influence" factor of this relationship which is closer to the eye would have meant more destruction to the bee colony due to the hurricane. This "destruction" variable is measured by W_s defined using the Saffir-Simpson Scale.

Table 6: Hurricane Damage to Worker Bees

Category	Wind Speed	Bee Response
1	74 - 95 mph	5% of drones+workers die
2	96 - 110 mph	20% of drones+workers die
3	111 - 129 mph	45% of drones+workers die
4	130 - 156 mph	65% of drones+workers die
5	> 157 mph	85% of drones+workers die

3.6 Pollination

We simulate using a 20 acres apple trees orchard to model the pollination process of honey bees. The following variables are significant to model the process of pollination and determine the number of beehives needed.

Table 7: Acres Pollination

Variables	Definition	Values
$tree_x$	Number of trees in a acre using normal distribution	$tree_x = \frac{1}{\sqrt{2\pi 159.375^2}} e^{\frac{-(x-462.5)^2}{2159.375^2}}$
bee_{num}	Randomised Number of bees on an apple tree	sample(20,21,22,23,24) size = 1
bee_{req}	Number of bees required in a acre of land	$bee_{num} \cdot tree_x z$
$Euclid_d$	Euclidean Distance between beehive and a location	$\sqrt{(l_i - bh_{li})^2 + w_i - bh_{wi})^2}$
$Euclid_{index}$	An distance index that match with Euclidean Distance	Reference the table on the right
$w(t)$	The population of worker bees at a given time period	defined by section "Reproductive Rates"

x =

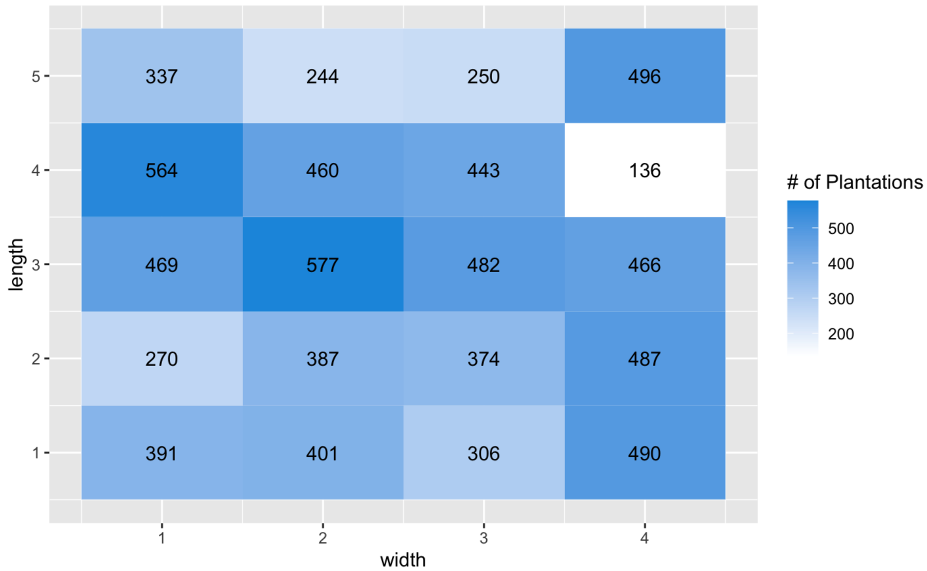
$Euclid_d{}^2$	$Euclid_{index}$
0	1
1	2
2	3
4	4
5	5
8	6
9	7
10	8
13	9
16	10
17	11
18	12
20	13
25	14

In addition to the variable $Euclid_d$ we derive the $Euclid_{index}$ which is defined as followed:

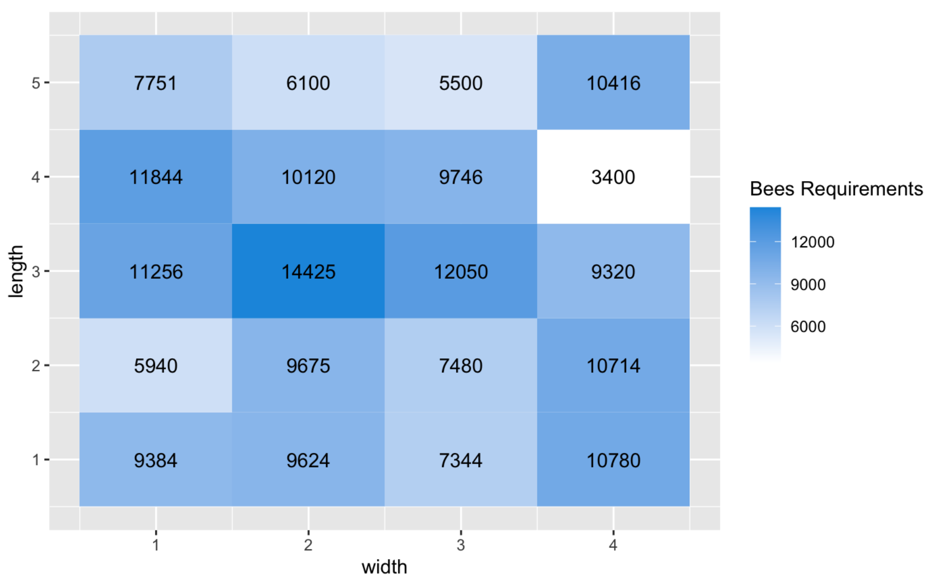
First we figure out how many trees with their corresponding frequency using the function 'rnorm' in R. The data and the distribution is as follow:

391 270 469 564 337 401 387 577 460 244 306 374 482 443 250 490 487 466 136 496

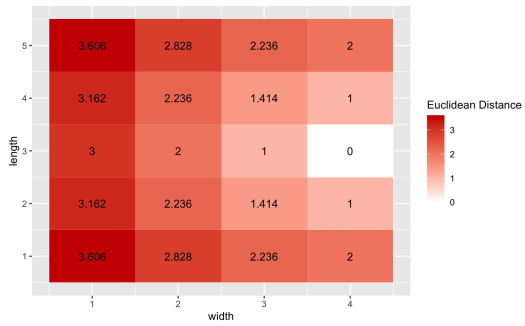
Then we can plot this data into a heat map that correlates with the 20 acres apple orchard.



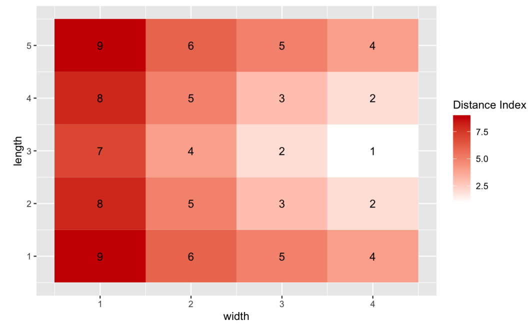
We randomize the number of bees per apple tree which is the random sample of 6 numbers: 20, 21, 22, 23, 24, 25, and applied this randomization to the above heat map to get the total number of bees needed for the pollination of all the trees in an acre of land for every single acre/tile in the apple orchard. [17]



The next step is to place our first beehive using the (length, width) coordinate system. For this example, we define our first beehive as the coordinate (3,4) hence the tile that required 9320 bees to pollinate. We then construct a separate heat map that determines the ranked Euclidean distance index ($Euclid_{index}$) from the first beehive, so that a tile closer to the beehive will have smaller $Euclid_{index}$, with itself being 0.



(a) Euclidean Distance

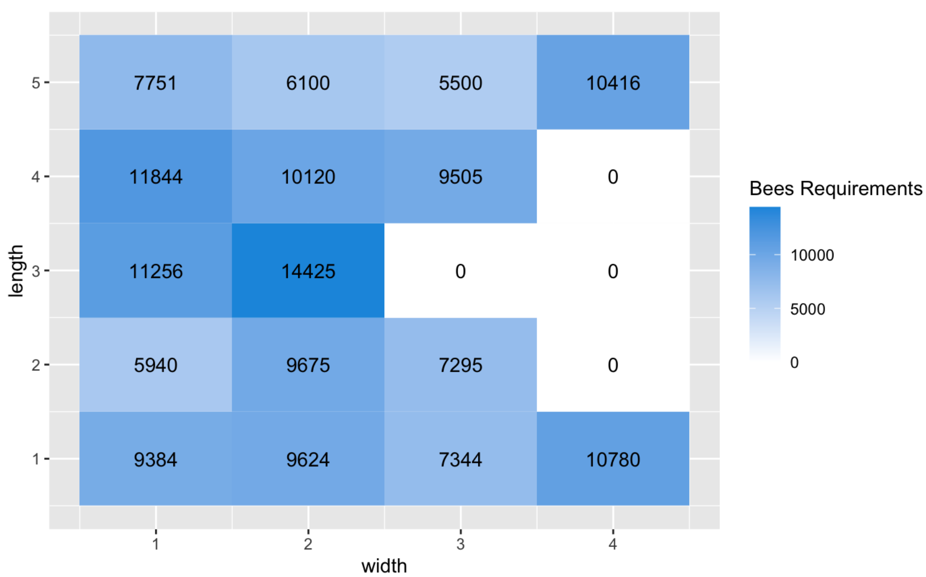


(b) Ranked Euclidean Distance

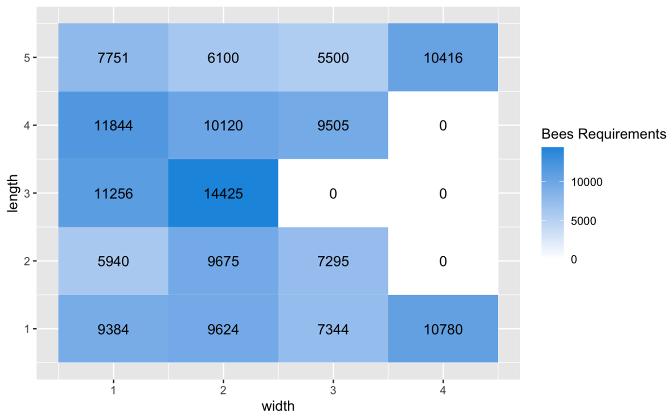
The Bee pollination algorithm is defined as below:

1. Determines the population of worker bees at a given time period $w(t)$ (Set at 40000 workers bee, the average number of worker bees during summer).
2. The bees would first pollinate the acre of land that of its beehive.
3. If the current pollination tile is fulled, then the rest of the bees will equally distributed into the most adjacent non pollinated tile defined by Euclidean Distance.
4. Repeat step number two until all bees locate itself into a tile of acre.

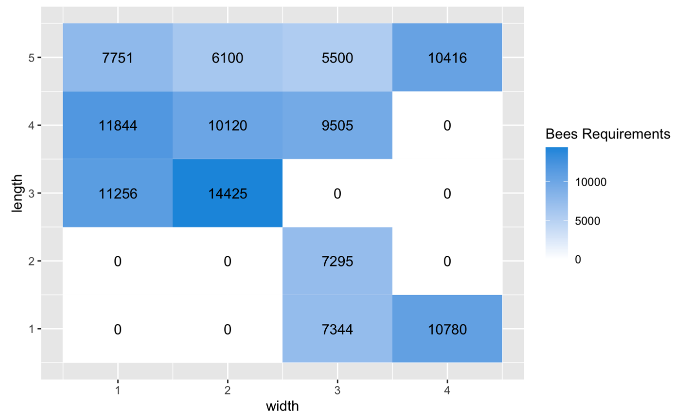
Program this algorithm into R and apply it with both the "Rank Euclidean Distance" heat map and the "Bees Requirements" heat map, we get another "Bees Requirements" heat map but this time the number within each tile changes as bees from the first beehives already fulfill some of the pollination requirements for the apple trees.



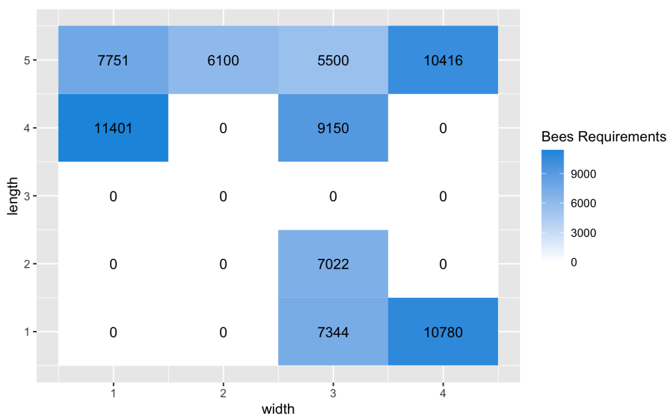
We then apply the same technique using the updated Bees Requirement for each tile multiple times until all values within each tile became 0. This means that all requirements for pollination are fulfilled, and the number of repetitions or number of beehives located in a different location is the number of beehives we need to fully pollinate 20 acres of land.



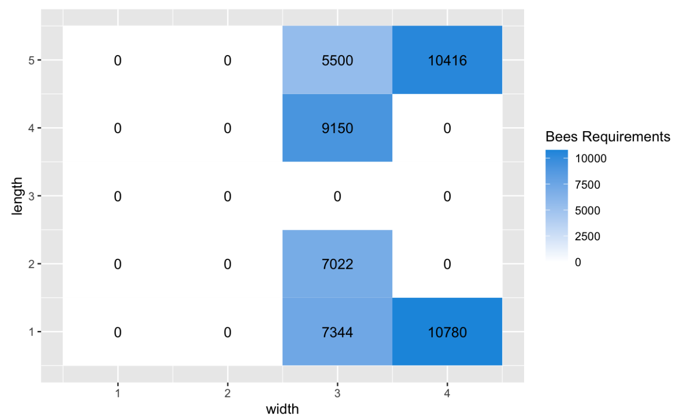
(a) 1 Beehive Placement



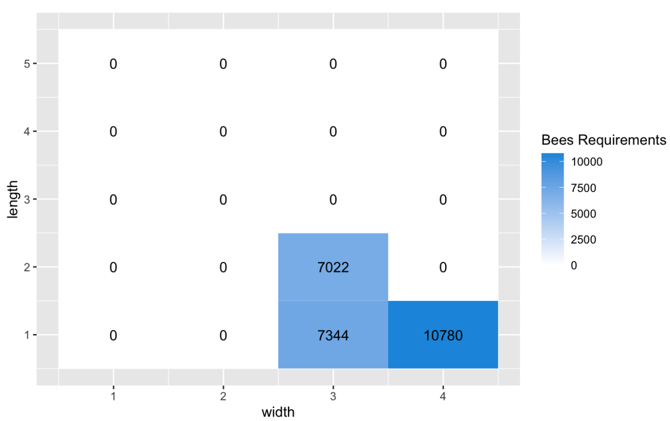
(b) 2 Beehives Placement



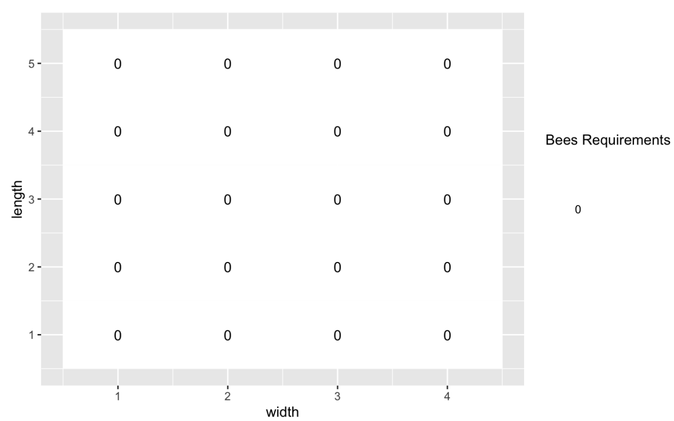
(c) 3 Beehives Placement



(d) 4 Beehives Placement



(e) 5 Beehives Placement



(f) 6 Beehives Placement

Program this recursion relationship into R we get in a total of 6 beehives (Including the above) needed to turn every single tile to 0. The length-width coordinate for these 6 beehives are: (3,4),(1,1),(3,2),(5,1),(5,3),(1,3)

R Program Citation: [18]

4 Conclusion

4.1 Results

To put together the final result we combined all of the different models from the different sections into one.

$$B(t) = B_0 + \text{Birth} - \text{Death}$$

$$\text{Birth} = \int_0^t W_{rate}(x)dx + \int_0^t D_{rate}(x)dx$$

The birth is the sum of the birth of worker and drone bees. Where B_0 is the initial bee population, W_{rate} is the worker birth rate (equation 1.2) and D_{rate} is the drone birth rate (equation 1.3).

$$\text{Death} = \begin{pmatrix}\text{Worker bee death}\\\text{by lifespan \& mite}\end{pmatrix} + \begin{pmatrix}\text{Death by}\\\text{Starvation}\end{pmatrix} + \begin{pmatrix}\text{Death by}\\\text{mites (+DMV)}\end{pmatrix} + \begin{pmatrix}\text{Death by}\\\text{lifespan}\end{pmatrix} + \begin{pmatrix}\text{Death by}\\\text{Predation}\end{pmatrix} + \begin{pmatrix}\text{Death by}\\\text{Hurricane}\end{pmatrix}$$

Each factor correlates to a specific model throughout the paper.

4.2 Sensitivity Analysis

Overall this model is a birth - death model where we interconnect different ecological/biological factors with each other and ultimately with the size of the bee colony. The following are the evaluations of each factor on the size of the bee population.

1. Nectar Production Starvation: This is a necessity for all bees. If the nectar production dropped by 10% there would not be enough nectar supply for the winter, dropping the population by many folds.

2. Bees Reproduction: Bee reproduction is integral to sustaining the bee hive. We considered the maximum reproduction rate to be 2000, however, if this number were to drop by 500 to have a maximum rate of 1500 the reproduction rate could drop by 25%. That 25% makes bees more susceptible to starvation and predators, quicker spread of mites and problems overall.

3. Lifespan Mites: Lifespan and mites are one of the leading causes of colony collapse disorder. The infection of mites will have a death and an impact of shortening bee's lifespan.

4. Natural Disasters: This abiotic density-independent factor has an extremely high magnitude of potential impact (wipes out 85% of the bee population, however the probability for this massive impact to materialize is minimal (less than 1% nation wide per day). Therefore, the expected value for this factor is significant to consider but not the greatest actor to determines the size of the population.

5. Predation: This is biotic density-dependent factor have an considerable impact on a daily basis, and it's gradually increasing in the short term for both predators of bees: skunks and bears. In the case of skunks, at year 4, the result of the Leslie matrix for age distribution is at [68,42,4,5] multiply each value of it's corresponding consumption rate of bees we get a total consumption of 16280 daily if left untreated. We consider this as one of the greater factor due to it's large and consistent impact.

4.3 Strength and Weaknesses

4.3.1 Strength

1. The model cover the majority aspects of the ecological factors that could influenced the size of the bee colonies. This would increase the precision of our model in estimating the size of the bees population.

2. The model was specific to each individual days with the consideration of the patterns of seasonality. This would result in a very detailed prediction down to individuals days instead of a general time-series forecasting that only models trends.

4.3.2 Weaknesses

1. Even though we assume the beehives were located in the wild and were untreated by the farmers, we didn't include pesticides as one of our factors which could have an impact in bee's reproduction and bee's survivability. This could led to an underestimation of the result of our model.

2. We didn't considers the possibility for intraspecific interaction between beehives and the possibility for robbery between bees population to met their own nutrient requirements. This could have both positive and negative implications for a individual bee colony, and led to an increase in variability for the empirical distribution of population compare to our mathematical distribution.

5 Appendix

5.1 Flat peak cosine

$$\sqrt{\frac{1+b^2}{1+b^2\cos^2 x}}\cos(x) \tag{5.1}$$

The flat peak cosine function (equation 5.1) is a sinusoidal function that has variable peak flatness or sharpness. Changing the b value allows us to flatten (b increase) or sharpen (b decrease) the peaks and troughs.

5.2 Zero to Original Equation

$$F_1(x) = \frac{1}{2}\left(\frac{|-x|}{-x} - 1\right)(-x) \tag{5.2}$$

The Zero-Original equation (equation 5.2) is a function that transitions from a constant value to a linear equation at a single break point. In this equation the constant is 0 for all negative numbers and 0 and the linear slope is 1 for any positive number.

5.3 Zero or One Equation

$$F_2(x) = \frac{1}{2}\left(\frac{|x|}{x} + 1\right) \tag{5.3}$$

The Zero or One Equation (above) is a Boolean equation that gives either a 0 or 1. If $x < 0$ then $F_2(x < 0) = 0$ and if $x > 0$ then $F_2(x > 0) = 0$. If the function is $x = 0$, it is mathematically undefined, however, we defined that if $x = 0$ then $F_2(0) = 0$.

5.4 Skunks Leslie Matrix Values

Table 8: Skunk Population Dynamics

Constant	Calculation/Explanation
0	Baby Skunk doesn't have reproductive abilities
3.971	$0.95 \cdot 0.55 \cdot (7.2 + 0.4)$
3.241	$0.90 \cdot 0.55 \cdot (7.2 + 0)$
3.179	$0.85 \cdot 0.55 \cdot (7.2 - 0.4)$
0.40	$0.44 - 0.4$
0.44	$0.44 + 0$
0.48	$0.44 + 0.4$

Sources: [19], [20], [15], [21], [22], [23]

5.5 Sunlight hours calculation (Using R)

```
df <-
  getSunlightTimes(
    date = seq.Date(as.Date("2021-12-01"), as.Date("2023-12-31"), by = 100),
    keep = c("sunrise", "sunriseEnd", "sunset", "sunsetStart"),
    lat = 41.140259,
    lon = -89.384445,
    tz = "EST"
  )
```

Table 9: Sunlight Hours for States

State	State capitals	Latitude	Longitude	Equation
Alabama	Montgomery	32.377716	-86.300568	$S_{AL}(t) = 2.087 \sin(0.017t - 1.889) + 12.169$
Alaska	Juneau	58.301598	-134.420212	$S_{AK}(t) = 5.725 \sin(0.017t - 1.872) + 12.334$
Arizona	Phoenix	33.448143	-112.096962	$S_{AZ}(t) = 2.176 \sin(0.017t - 1.888) + 12.172$

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