Übung 2

2.4 Berechne die folgenden Integrale:

(a) $\int\limits_{-1}^{1} (3x^3 - 2x^2 + x - 1)dx$

$$f(x) = 3x^3 - 2x^2 + x - 1 \Rightarrow F(x) = \frac{3}{4}x^4 - \frac{2}{3}x^3 + \frac{1}{2}x^2 - x + c$$

$$\Rightarrow \int\limits_{-1}^{1} (3x^3 - 2x^2 + x - 1)dx = F(x)\Big|_{-1}^{1} = F(1) - F(-1) = -\frac{10}{3}$$

(b) $\int_{-1}^{1} \frac{1}{1+x^2}dx$

$$f(x) = \frac{1}{1+x^2} \Rightarrow F(x) = \arctan(x)$$

$$\Rightarrow \int_{-1}^{1} \frac{1}{1+x^2}dx = F(x)\Big|_{-1}^{1} = F(1) - F(-1) = \frac{\pi}{2}$$

(c) $\int_{-1}^{2} \frac{e^x-1}{e^x+1}dx$

$$\begin{aligned} &\int_{-1}^{2} \frac{e^x - 1}{e^x + 1}dx \\ =&\int_{-1}^{2} \frac{e^x}{e^x+1}dx - \int_{-1}^{2} \frac{1}{e^x+1}dx \end{aligned}$$

Mit $g(x) = e^x + 1 \Rightarrow g'(x) = e^x$ folgt aus $\int_a^b \frac{g'(x)}{g(x)}dx = \ln(|g(x)|)\Big|_a^b$:

$$= \ln(e^x + 1)\Big|_{-1}^{2} - \int_{-1}^{2} \frac{1}{e^x+1}dx$$

Mit $s := e^x$ folgt aus der Substitutionsregel:

$$\begin{aligned} &= \ln(e^x + 1)\Big|_{-1}^{2} - \int_{e^{-1}}^{e^2} \frac{1}{s+1}\frac{ds}{s} \\ &= \ln(e^x + 1)\Big|_{-1}^{2} - \int_{e^{-1}}^{e^2} \frac{1}{s(s+1)}ds \end{aligned}$$

Aus der Partialbruchzerlegung von $\frac{1}{s(s+1)}$ zu $\frac{1}{2} - \frac{1}{s+1}$ folgt:

$$= \ln(e^x+1)\Big|_{-1}^{2} - \int_{e^{-1}}^{e^2} \frac{1}{s} ds + \int_{e^{-1}}^{e^2} \frac{1}{s+1} ds$$

Mit $u \coloneqq s+1$ folgt aus der Substitutionsregel:

$$\begin{aligned} &= \ln(e^x+1)\Big|_{-1}^{2} - \int_{e^{-1}}^{e^2} \frac{1}{s} ds + \int_{e^{-1}+1}^{e^2+1} \frac{1}{u} du \\ &= \ln(e^x+1)\Big|_{-1}^{2} - \ln(s)\Big|_{e^{-1}}^{e^2} + \ln(u)\Big|_{e^{-1}+1}^{e^2+1} \\ &= -2\ln(e^2+1) + 2\ln(e+1) - 1 \end{aligned}$$

$\Rightarrow \int_{-1}^{2} \frac{e^x-1}{e^x+1} dx = -2\ln(e^2+1) + 2\ln(e+1) - 1$