TFL115 - Exercise 6

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1 Introduction

In this exercise there are assignments which should help you understand what we covered in the lecture. This exercise is not mandatory and should not be delivered, however it will help you manage the mandatory exercise which is introduced later on. Please let us know if there are any problems with the exercise or you feel unable to solve the tasks.

Feel free to use Github or Bitbucket when solving the tasks, but please do use a private repository for the solutions.

2 System of linear equations

First, what is a linear equation? It is an equation of a line, for example

$$y = 5x + 2.$$

A system of linear equations are more of these lines together, where they depend on each-other. The solution to these systems are the point of intersection between them, in 2 dimensions it is where the two lines intersect. The system can be in 2,3,4 or more dimensions, and are not always guaranteed to have a solution. It can be 1 solution, 0 solutions or infinitely many solutions. In 2 dimensions, there are a solution when the two lines are intersecting, hence 1 solution. If the lines are parallel, there are 0 solutions. If the two lines (again in 2D), are the same line (e.g. y = 5x+2), there is an infinite number of solutions. It is worth mentioning this is only for simple equations, so X squared and other complex equations are not valid.

A practical example of a system of linear equations can be:

Let's say we want to find out how fast I can run before you are able to drive past me with your car. Let's say I run at 1 kilometre pr minute, your car can drive 5 kilometre pr minute. You give me a 10 minute head-start. This can be divided into two equations:

$$d = 1t$$

- 1 kilometre pr minute

$$d = 5(t - 10)$$

- 5 kilometre pr minute, but we remove 10 minutes (The head-start)

The solution will be where these two lines intersect.

After 12.5 minutes you drive past me. This can be shown in an online graphing tool, such as Geogebra.

There is only 1 solution to this problem, because this will be the only place these two lines will ever meet. This example is easy to solve as a graph, but can be tricky to solve as equations, especially if we have many unknowns (many dimensions). If we were to solve a system in 3 dimensions, it will be where the three planes are intersecting. Let's say we have the following three equations:

$$3x + 2y - z = 1$$

$$2x - 2y + 4z = -2$$

$$-x + 0.5y - z = 0$$

What is x, y and z? It is not necessarily so easy to see by looking at the equations, it is not necessarily so easy by looking at the 3D figure as well. But it can be solved in multiple ways. The one we will look at here uses matrices in order to solve it, as we did in class. This is easy for us when we know how numpy works. It can of course be solved in other ways, but this will be the easiest for us, using Python.

So, when we are solving a system like this, we need to put it into different matrices. The three equations:

$$3x + 2y - z = 1$$

$$2x - 2y + 4z = -2$$

$$-x + 0.5y - z = 0$$

Is actually the same as writing:

$$\begin{bmatrix} 3 & 2 & -1 \\ 2 & -2 & 4 \\ -1 & 0.5 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$$

Why is it the same? Because, we have just split it up in matrices, and if we take matrix multiplication of the two matrices to the left (which is here denoted as the dot between the matrices), we get the left side of "=" back. See this link for a howto on matrix multiplication: `https://www.mathsisfun.com/algebra/matrix-multiplying.html`

So - from having it on this form, which is actually, $A * x = b$ (where A is the 3x3 matrix, x is the 1x3 matrix with x,y,z and b is the 3x1 matrix with the solutins) we can solve it by manipulating the equation. First we then have to remove A from the left side of the equation, in order to have x alone. This can be done by taking the inverse of matrix A, since the inverse of a matrix times the matrix itself is 1 (if you don't believe us - look it up :-)).

By doing that (multiplying with the inverse of A on both sides of the equation), we get the following:

$$x = A^{-1} * b$$

This can be solved by us, and if we use matrix multiplication on the inverse of A and b, we get the answers for x, y and z.

Info above is based on, so take a look here for more information: `https://www.mathsisfun.com/algebra/systems-linear-equations.html`

- Use the information above and the code from lecture 6 to solve the system of linear equations stated above
- Draw it in Geogebra 3D or some other online graphing tool to see what you have solved.

3 Number array

Download the file *exercise_5_numbers.txt* from Canvas.

- Use the file to generate a numpy array. *Hint*: `np.genfromtxt()`
- Find the row with the highest sum (adding all the numbers). What is the value of this sum?
- Find the column with the highest sum. What is the value of this sum?
- What is the single highest value in the array?
- What is the sum of the two last rows?

4 Emperors

Download the file *roman_emperors.csv* from Canvas.

- Use the file to generate a numpy array. *Hint*: delimiter, dtype
- How many emperors were born in Rome?
- What was the most common way of rising to power? How many rose to power this way?
- Which emperors committed suicide?

5 Solutions

5.1 System of linear equations: X=1, Y=-2, Z=-2

5.2 Number array: 91692, 93310, 10004, 163183

5.3 Emperors: 9, Birthright: 35, [Maximian, Otho, Valentinian II, Nero, Gordian I]