Compact operator notation for finite differences

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This is a overview of operator notation used in the course.

Forward difference approximation operator:

$$[D_t^+ u]^n = \frac{u^{n+1} - u^n}{\Delta t} \tag{1.1}$$

Centered difference approximation operator:

$$[D_t u]^n = \frac{u^{n+\frac{1}{2}} - u^{n-\frac{1}{2}}}{\Delta t} \tag{1.2}$$

Backward difference approximation operator:

$$[D_t^- u]^n = \frac{u^n - u^{n-1}}{\Delta t} \tag{1.3}$$

Averaging operator:

$$[\overline{u}^t]^n = \frac{1}{2}(u^{n-\frac{1}{2}} + u^{n+\frac{1}{2}}) \tag{1.4}$$

Note that

$$[D_t D_t u]^n = [D_t(D_t u)]^n = \frac{[D_t(u^{\frac{1}{2}} - u^{-\frac{1}{2}}]^n}{\Delta t}$$

$$= \frac{1}{\Delta t}(\frac{u^{n+\frac{1}{2}+\frac{1}{2}} - u^{n+\frac{1}{2}-\frac{1}{2}}}{\Delta t} - \frac{u^{n-\frac{1}{2}+\frac{1}{2}} - u^{n-\frac{1}{2}-\frac{1}{2}}}{\Delta t})$$

$$= \frac{u^{n+1} - 2u^n + u^{n-1}}{\Delta t^2}$$