(12) Estimar el error cometido al aproximar la función $f(x)=\sqrt[3]{x}$ por su polinomio de Taylor de orden 2, centrado en $a=8$, para $7 \le x \le 9$.

$\cdot f(x) = x^{1/3}$, $f'(x) = \frac{1}{3} \cdot x^{-2/3}$, $f''(x) = \frac{1}{3} \cdot -\frac{2}{3} \cdot x^{-5/3}$, $f'''(x) = -\frac{2}{9} \cdot -\frac{5}{3} \cdot x^{-8/3}$

$* f(8) = \sqrt[3]{8} = 2$

$* f'(8) = 1/3 (8)^{2/3} = 1/12$

$* f''(8) = -2/9 (8)^{5/3} = -1/144$

$* f'''(8) = 10/27 (8)^{8/3} = 5/3456$

$\Longrightarrow T_{ax} = f(a) + \frac{f'(a)}{1!} \cdot (x-a) + \frac{f''(a)}{2!} \cdot (x-a)^2 \cdots + \frac{f^{(n)}}{n!} (x-a)^n$ ▲

▲ $T_{8,2} = 2 + \frac{1}{12}(x-8) - \frac{1/144}{2}(x-8)^2 \longrightarrow$ Polinomio de Taylor de grado 2

$*$ Sabiendo que $f(x) \approx T_{a,x} + R_{a,x}$, con lagrange: $R_{a,x} \doteq \frac{f^{(n+1)}_{(t)}}{(n+1)!} \cdot (x-a)^{(n+1)}$ ⊛

⊛ $\left| \frac{10/27 (t)^{8/3}}{3!} \cdot (x-a)^3 \right| \doteq \frac{10}{3! \, 27} \cdot \left| \frac{1}{t^{8/3}} \right|$, $x \in [7,9] \longrightarrow t \in (7,8) \vee t \in (8,9)$

$\cdot$ Veamos el valor máximo de t, entre 7 y 8, $7 < t < 8$, $1/t^8 < 1/7^8$, $\boxed{1/t^{8/3} < 1/7^{8/3} < 1}$ ⊠

⊠ $\frac{10}{27 \cdot 3!} \cdot 1 < 10/27 \cdot 3!$

⊛ $|(x-8)^3|$, $7 \le x \le 9$: $-1 \le (x-8)^3 \le 1 \doteq -1^3 \le (x-8)^3 \le 1^3$, $0 \le |(x-8)^3| \le 1$

$*$ Finalmente, el error : $\frac{10}{27! \, 3} \cdot 1$

(1) Calcular las siguientes integrales sobre regiones rectangulares.

(a) $\iint_R (x^2+y^2)\,dA$, donde R es el rectángulo $0 \le x \le 2$, $0 \le y \le 5$.

(b) $\iint_R (\operatorname{sen} x + \cos y)\,\mathrm{d}A$, donde R es el rectángulo $0 \le x \le \pi/2$, $0 \le y \le \pi/2$.

(c) $\iint_R x^2 y^2\,\mathrm{d}A$, donde R es el rectángulo $0 \le x \le a$, $0 \le y \le b$.

a) $\int_0^5\int_0^2 x^2+y^2\,dx\,dy \doteq \int_0^2 x^2+y^2\,dx \doteq \int_0^2 x^2\,dx + \int_0^2 y^2\,dx \doteq x^3/3\Big|_0^2 + y^2x\Big|_0^2$

$\doteq 8/3 + 2y^2 \;,\; \int_0^5 \frac{8}{3} + 2y^2\,dy \doteq \frac{8}{3}\cdot y\Big|_0^5 + 2\,y^3/3\Big|_0^5 \doteq 40/3 + 250/3$

$\doteq 290/3 \longrightarrow$ Área del Rectángulo

b) $\int_0^{\pi/2}\int_0^{\pi/2} \operatorname{sen}(x) + \cos(y)\,dx\,dy \doteq \int_0^{\pi/2} \operatorname{sen}(x) + \cos(y)\,dx \doteq \int_0^{\pi/2} \operatorname{sen}(x)\,dx +$

$\int_0^{\pi/2} \cos(y)\,dx \doteq -\cos(x)\Big|_0^{\pi/2} + x\cdot\cos(y)\Big|_0^{\pi/2} \doteq 1 + \pi/2\cos(y) \doteq \int_0^{\pi/2} 1 + \pi/2\cos(y)\,dy$

$\doteq \int_0^{\pi/2} dy + \pi/2\int_0^{\pi/2}\cos(y)\,dy \doteq \pi/2 + \pi/2\left(\operatorname{sen}(\pi/2) - \operatorname{sen}(0)\right) \doteq \pi/2 + \pi/2 = \frac{2\pi}{2} = \pi$

c) $\int_0^b\int_0^a x^2y^2\,dx\,dy \doteq \int_0^a x^2y^2\,dx \doteq y^2\cdot x^3/3\Big|_0^a \doteq y^2\cdot\frac{a^3}{3} \;,\; \int_0^b \frac{a^3}{3}y^2\,dy \doteq \frac{a^3}{3}\int_0^b y^2\,dy \doteq$

$\frac{a^3}{3}\cdot y^3/3\Big|_0^b \doteq a^3/3\cdot b^3/3 \doteq (ab)^3/9$

(2) Calcule las siguientes integrales iteradas.

(a) $\int_0^2 \int_0^4 y^3 e^{2x}\, dy\, dx.$

(b) $\int_1^3 \int_1^5 \frac{\ln(y)}{xy}\, dy\, dx.$

(c) $\int_0^1 \int_0^1 \sqrt{s+t}\, ds\, dt.$

a) $\int_0^2 \int_0^4 y^3 e^{2x}\, dy dx \doteq \int_0^4 y^3 e^{2x} dy \doteq e^{2x} \int_0^4 y^3 dy \doteq e^{2x} \cdot y^4/4 \Big|_0^4 \doteq e^{2x} (256)$ (*)

(*) $\int_0^2 e^{2x} 256\, dx \doteq 256 \int_0^2 e^{2x} dx \doteq 256 \int_0^2 e^{u} \frac{du}{2} \doteq \begin{matrix} u = 2x \\ du = 2dx \end{matrix} \doteq \frac{256 \cdot e^{2x}}{2} \doteq \frac{128 e^{2x}}{2}$

b) $\int_1^5 \frac{\ln(y)}{x y} dy \doteq \frac{1}{x} \cdot \int_1^5 \frac{\ln(y)}{y} dy \doteq \begin{matrix} u = \ln(y) \\ du = \frac{1}{y} dx \end{matrix} \doteq \frac{1}{x} \int_1^5 \frac{u}{y} du\, y \doteq \frac{1}{x} \frac{u^2}{2} \Big|_1^5 \doteq \frac{1}{x} \frac{(\ln(y))^2}{2} \Big|_1^5 \doteq \frac{1}{x} \left(\frac{\ln(5)^2 - \ln(1)^2}{2} \right) \doteq \frac{1}{x} \left(\frac{\ln(5)^2 - 0}{2} \right)$

$\frac{\ln(5)^2}{2x}$, $\int_1^3 \frac{\ln(5)^2}{2x} dx \doteq \frac{\ln(5)^2}{2} \cdot \int_1^3 \frac{1}{x} dx \doteq \frac{\ln(5)^2}{2} \cdot \left(\ln(3) - \ln(1)\right) \doteq \frac{\ln(5)^2 - \ln(3)}{2}$

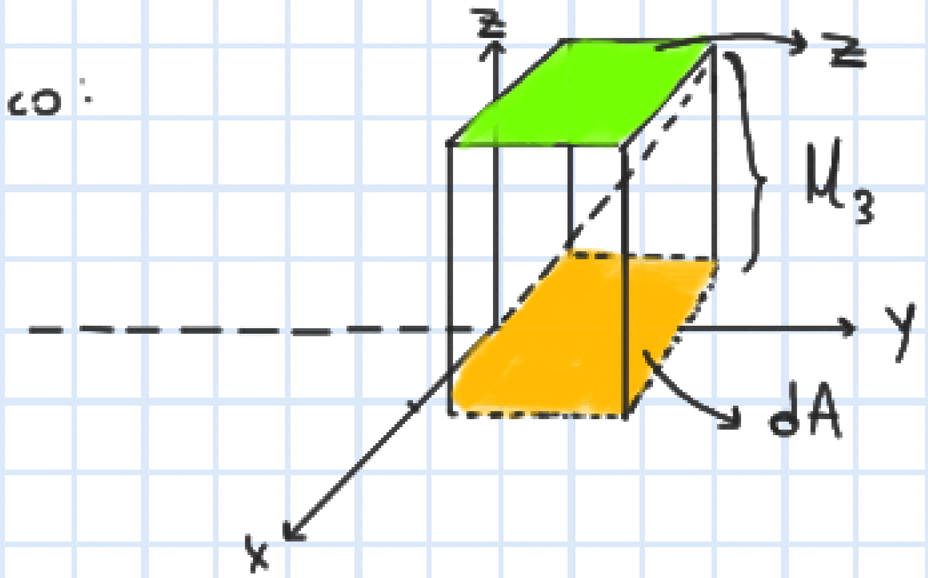
c) $\int_0^1 \int_0^1 \sqrt{s+t}\, ds\, dt \doteq \int_0^1 \sqrt{s+t}\, ds = \int_0^1 (s+t)^{1/2} ds \doteq \begin{matrix} u = s+t \\ du = 1\, dx \end{matrix} \doteq \int_0^1 u^{1/2} du \doteq \frac{u^{3/2}}{3/2} \Big|_0^1 \doteq \frac{2(s+t)^{3/2}}{3} \Big|_0^1 \doteq$

$\frac{2}{3} \cdot (s+t) \Big|_0^1 \doteq \frac{2}{3} (s+1) \doteq \int_0^1 \frac{2}{3} (s+1) ds \doteq \frac{2}{3} \frac{(s+1)^2}{2} \Big|_0^1 \doteq \frac{2}{6} \cdot (1+1) \doteq \frac{1}{3} \cdot 2 = 2/3$

(3) Encuentre el volumen del sólido que está debajo del plano $4x+6y-2z+15=0$ y arriba del rectángulo $R=\{(x,y)\,|\,-1\le x\le 2,\quad -1\le y\le 1\}$.

(4) Determine el volumen del sólido que está debajo del paraboloide hiperbólico $z=3y^2-x^2+2$ y arriba del rectángulo $R=\{(x,y)\,|\,-1\le x\le 1,\quad 1\le y\le 2\}$.

• Analisis gráfico:



a) $\iint_R 4x+6y-2z+15\,dA \doteq \int_{-1}^{2}\int_{-1}^{1}\frac{4x}{2}+\frac{6y}{2}+\frac{15}{2}\,dy\,dx \doteq$

$\int_{-1}^{1}\frac{4x}{2}\,dy+\int_{-1}^{1}\frac{6y}{2}\,dy+\int_{-1}^{1}\frac{15}{2}\,dy \doteq \frac{4x}{2}(y)\Big|_{-1}^{1}+3\cdot y^2/2\Big|_{-1}^{1}+\frac{15}{2}\cdot y\Big|_{-1}^{1}$

$\doteq 4x+3+15$, $\int_{-1}^{2}4x+3+15\,dx=\int_{-1}^{2}4x\,dx+\int_{-1}^{2}3\,dx+\int_{-1}^{2}15\,dx$

$\doteq 4\left(x^2/2\right)\Big|_{-1}^{2}+3(x)\Big|_{-1}^{2}+15\cdot(x)\Big|_{-1}^{2}\doteq 4\left(\frac{4}{2}+\frac{1}{2}\right)+3(2+1)+15(2+$

$\doteq 10+9+45=\boxed{64}\ u_3$

b) $\int_{-1}^{1}\int_{1}^{2}3y^2-x^2+2\,dy\,dx \doteq \int_{1}^{2}3y^2-x^2+2\,dy \doteq \int_{1}^{2}3y^2\,dy-\int_{1}^{2}x^2\,dy+\int_{1}^{2}2\,dy$ ⊛

⊛ $3\,y^3/3\Big|_{1}^{2}-x^2\cdot y\Big|_{1}^{2}+2\,y\Big|_{1}^{2}\doteq 3\left(\frac{8}{3}-\frac{1}{3}\right)-x^2(2-1)+2(2-1)\doteq$

$7-x^2+2$, $\int_{-1}^{1}7-x^2+2\,dx \doteq \int_{-1}^{1}7\,dx-\int_{-1}^{1}x^2\,dx+2\int_{-1}^{1}dx$ ▲

▲ $7(1-(-1))-\left(\frac{(1)^3}{3}-\frac{(-1)^3}{3}\right)+2(1-(-1))\doteq 14-\frac{2}{3}+4=\frac{42-2+12}{3}$

$\doteq 52/3\ u_3$

$$\int \frac{x^2-3x+1}{x+1}\,dx \;, \quad \begin{array}{l|l} x^2-3x+1 & x+1 \\ \hline -x^2-x & x-4 \\ 0-4x+1 & \\ \quad +4x+4 & \\ \qquad 5 & \end{array} \Bigg\} \; \frac{x^2-3x+1}{x+1} = \frac{\cancel{(x+1)}(x-4)}{\cancel{x+1}} + \frac{5}{x+1} \;\circledast$$

$$\circledast \int (x-4)\,dx + \int \frac{5}{x+1} \doteq \int x\,dx - \int 4\,dx + 5\int \frac{1}{x+1}\,dx \doteq \frac{x^2}{2} - 4x + 5\cdot \ln(|x+1|) + C$$

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* Factores lineales distintos

$$\int \frac{7x+3}{(x+4)(x-1)}\,dx \doteq \frac{A}{(x+4)} + \frac{B}{(x-1)} \doteq \frac{A(x-1)+B(x+4)}{(x+4)(x-1)} \doteq \frac{Ax-A+Bx+4B}{(x+4)\cdot(x-1)} \doteq \frac{x\overbrace{(A+B)}^{7} + \overbrace{(-A+4B)}^{3}}{(x+4)\cdot(x-1)} \;\therefore$$

$$\therefore \begin{cases} A+B=7 \\ -A+4B=3 \end{cases} \doteq \begin{cases} A=7-B \\ -A+4B=3 \end{cases} \doteq \begin{array}{l} -7+B+4B=3,\; B+4B=10,\; 5B=10 = \boxed{B=2} \\ A=7-2,\; \boxed{A=5} \end{array}$$

$$* \int \frac{7x+3}{(x+4)(x-1)}\,dx = \int \frac{5}{x+4}\,dx + \int \frac{2}{(x-1)}\,dx = 5\ln(|x+4|) + 2\ln(|x-1|) + C$$

$$\cdot \int \frac{x^2+2x-1}{2x^3+3x^2-2x}\,dx \doteq \frac{x^2+2x-1}{x(2x^2+3x-2)} \;, \quad x(2x^2-x+4x-2) = x(x(2x-1)+2(2x-1)) = x((2x-1)(x+2))$$

$$= \frac{A}{x} + \frac{B}{2x-1} + \frac{C}{x+2} \doteq \frac{A(2x-1)(x+2)+B(x)(x+2)+C(x)(2x-1)}{x(2x-1)(x+2)} = \frac{A2x^2-4Ax-Ax-2A+Bx^2+2Bx+2Cx^2-Cx}{x(2x-1)(x+2)} = \frac{(A2x^2+Bx^2+2Cx^2)+(-5Ax+2Bx-Cx)+(-2A)}{x(2x-1)(x+2)}$$

$$\frac{x^2\overbrace{(2A+B+2C)}^{1} + x\overbrace{(-5A+2B-C)}^{2} + \overbrace{(-2A)}^{-1}}{x(2x-1)(x+2)} \doteq$$

$$\begin{cases} 2A+B+2C=1 \\ -5A+2b-C=2 \\ -2A=-1 \end{cases}$$

$A = 1/2$, $\cancel{2}\, 1/\cancel{2} + B + 2C = 1$, $1+2c = 1-B$, $2c=-B$, $c = -B/2$

$\rightarrow -\frac{5}{2} + 2B - \frac{B}{2} \doteq \frac{-5+4B-B}{2} \doteq -\frac{5}{2} + \frac{3B}{2} \doteq \quad B = \frac{5/2}{3/2} \doteq \frac{10}{-6}$

$c = \frac{10/-6}{2} \doteq 10/-12$

$$* \frac{1/2}{x} + \frac{10/6}{2x-1} + \frac{10/-12}{x+2} \doteq \int \frac{1/2}{x}\,dx + \int \frac{10/6}{2x-1}\,dx + \int \frac{10/-12}{x+2}\,dx \doteq \frac{1/2}{1} \cdot \int \frac{1}{x}\,dx + \frac{10/6}{1}\int \frac{1}{2x-1}\,dx + \frac{10/-12}{1}\int \frac{1}{x+2}\,dx$$

$$= \frac{1/2}{1} \cdot \ln(|x|) + \frac{10/6}{1} \cdot \ln(|2x-1|) + \frac{10/-12}{1} \cdot \ln(|x+2|)$$

. Vector posición, tangente, recta tangente.

$f(t) = (2 \cdot \cos(t), \mathrm{sen}(t))$, en $\pi/4$.

1) Vector posición = $(2\cos(\pi/4), \mathrm{sen}(\pi/4)) \rightarrow$ Pto. que pertenece a la curva.

2) Vector tangente = $f'(t) = (-2\,\mathrm{sen}(t), \cos(t))$, $(-2\,\mathrm{sen}(\pi/4), \cos(\pi/4))$

3) Recta tangente $\doteq \underbrace{(2\cos(\pi/4), \mathrm{sen}(\pi/4))}_{P_0} + t \cdot \underbrace{(-2\,\mathrm{sen}(\pi/4), \cos(\pi/4))}_{\text{Vector director}}$

$$\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}} \doteq \sum \frac{1}{\sqrt{n}} \cdot x^n$$, criterio del cociente, $\left|\frac{a_{n+1}}{a_n}\right| \doteq \frac{1/\sqrt{n+1}}{1/\sqrt{n}} \doteq \frac{\sqrt{n}}{\sqrt{n+1}}$, $\lim_{n\to\infty} \sqrt{n}/\sqrt{n+1} = \sqrt{\frac{n}{n+1}} = \sqrt{\frac{n/n}{n/n + 1/n}} =$

$\lim_{n\to\infty} \sqrt{\frac{1}{1+1/n}} = \frac{1}{1+1/\infty} = 1$, $0 < 1 < \infty \Rightarrow R = 1/L$, $1/1 = \boxed{1} \rightarrow$ Converge, $|x| < 1$, $-1 < x < 1$, $I = \{-1, 1\}$

$$\int_{-\infty}^{\infty} e^{-c|x|}\,dx = \underbrace{\int_{-\infty}^{0} e^{-c|x|}\,dx}_{A} + \underbrace{\int_{0}^{\infty} e^{-c|x|}\,dx}_{B}$$

Compruebo las integrales para c = 0, c > 0 y c < 0

A $\rightarrow$ • $c = 0$ ✓, $c < 0$ ✓, $c > 0$ ✗ ; B $\rightarrow$ $c = 0$ ✓, $c < 0$ ✗, $c > 0$ ✓ } Converge en $c = 0$

• A $\int_{-\infty}^{0} e^{-c|x|}\,dx$, $c = 0 \rightarrow \int_{-\infty}^{0} e^{0}\,dx \doteq \lim_{t\to-\infty} \int_{t}^{0} e^{0}\,dx = \lim_{t\to-\infty}\left(e^{0}\int_{t}^{0} dx\right) \doteq \lim_{t\to-\infty} x\Big|_{t}^{0} \doteq (x^{0} - x^{t}) \doteq 1 - x^{t} \doteq 1 - t^{-\infty} = 1 - \frac{1}{t^{\infty}}$

$\doteq 1 - 0 \doteq \boxed{1} \rightarrow$ Converge para $c = 0$.

• $\int_{-\infty}^{0} e^{-c|x|}\,dx$, $c < 0 \rightarrow b = -c$, $\int_{-\infty}^{0} e^{b|x|}\,dx \doteq \begin{matrix} u = bx \\ du = b \end{matrix} \doteq \int_{-\infty}^{0} e^{u}\,du/b \doteq \frac{1}{b} \cdot \int_{-\infty}^{0} e^{u}\,du$

$\doteq \frac{1}{b} \cdot \lim_{t\to-\infty} \int_{t}^{0} e^{u}\,du = \frac{1}{b} \cdot \left(e^{bx}\Big|_{t}^{0}\right) \doteq \frac{1}{b} \cdot \left(1 - e^{bt}\right) = \frac{1}{b} \cdot \left(1 - e^{-\infty}\right) = \frac{1}{b} \cdot \left(1 - \frac{1}{e^{\infty}}\right) \doteq \frac{1}{b}$, converge

• $\int_{-\infty}^{0} e^{-c|x|}\,dx \doteq -\frac{1}{c} \cdot e^{u}\Big|_{t}^{0}$, $-\frac{1}{2} \cdot \left(e^{-c(0)} - e^{-ct}\right) = -\frac{1}{2} \cdot \left(1 - e^{\infty}\right)$, No converge

$f(x) = 2x^4 + y^2 - x^2 - 2y$

$f_x(x,y) = 8x^3 + 0 - 2x = 8x^3 - 2x,\ f_y(0,2) = 0$
$f_y(x,y) = 2y - 2,\ f_y(0,2) = 2$ $\Big\}\ \nabla f(0,2) = (0,2)$

$\cdot < \nabla f(x,y), (u_1, u_2) > = (0,2)\cdot(u_1,u_2) = 1,\ 0u_1 + 2u_2 = 1,\ u_2 = 1/2$

$\|u_1,u_2\| = 1,\ \sqrt{u_1^2 + (1/2)^2} = 1 = \sqrt{u_1^2 + 1/4} = \sqrt{u_1^2 + 1/4}^2 = 1^2 = u_1 = \pm\sqrt{3}/4$

$\rightarrow u = \cdot\ (\sqrt{3}/4,\ 1/2)$
$\quad\cdot\ (-\sqrt{3}/4,\ 1/2)$

$-\frac{1}{2} - \frac{2}{2}$

$\cdot |x-1| \le 1,\ -1^3 \le (x-4)^3 \le 1^3 = (x-4)^3 \le 1$

$f(x) = \sqrt{x},\ x^{1/2}$ $\begin{cases} f'(x) = x^{-1/2} \rightarrow 1/x^{1/2} \\ f''(x) = -\frac{1}{2}\cdot x^{-3/2} = -\frac{1}{2}\cdot\frac{1}{x^{3/2}} = -1/2x^{3/2} \end{cases}$

$\cdot T_{2,4}(x) = \sqrt{4} + 1/(4)^{1/2}(x-4) + \frac{1/2(4)^{3/2}}{2}(x-4)^2 = 2 + 1/2(x-4) + -1/32(x-4)^2$

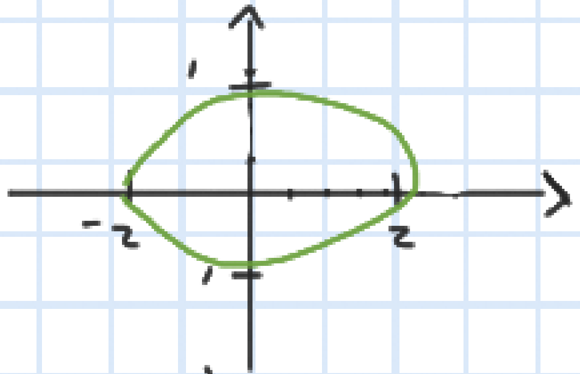
$\cdot \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1},\ \frac{\frac{3}{4x^{5/2}}(c)}{3!}\cdot(x-a)^3 = \frac{3}{24x^{5/2}}(c)\cdot(x-a)^3$ con c entre a y x

$\frac{3}{24x^{5/2}}(c)\cdot(x-4)^3,\ c \in \{3,4\},\ \left|\frac{3}{24c^{5/2}}\right|\cdot|(x-4)^3| = \left|\frac{3}{24}\right|\cdot\left|\frac{1}{c^{5/2}}\right|\cdot|(x-4)^3| = \frac{3}{24}\cdot\left|\frac{1}{c^{5/2}}\right|\cdot|(x-4)^3|$

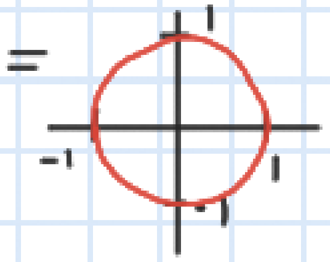
$= 3/24 \cdot 1/c^{5/2} \cdot |(x-4)^3| = 3^{5/2} \le c^{5/2} \le 4^{5/2} = 1/c^{5/2} < 1/3^{5/2} < 1$

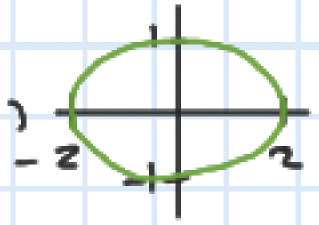
$3/24 \cdot 1 \cdot |(x-4)^3| *$ Error: $3/24$

$r(t) = (2\cos(t), \operatorname{sen}(t))$



$(\cos(t), \operatorname{sen}(t)) =$





- $\left.\begin{array}{l} \cos'(t) = -\operatorname{sen}(t) \\ \operatorname{sen}'(t) = \cos(t) \end{array}\right\} \; r'(t) = (-2\operatorname{sen}(t), \cos(t))$

  $\longrightarrow t_0 = (-2\operatorname{sen}(\pi/4), \cos(\pi/4))$

- $(2\cdot\cos(\pi/4), \operatorname{sen}(\pi/4)) + t\cdot(-2\operatorname{sen}(\pi/4), \cos(\pi/4)) =$

  $(\sqrt{2}, \sqrt{2}/2) + t\cdot(-\sqrt{2}, \sqrt{2}/2)$