Governing eqⁿ ⇒

1) mass Conservatⁿ ⇒ $\nabla . u = 0$

2) momentum Conservation ⇒

$$\rho\left[\frac{Du}{Dt}\right] = -\nabla p + \nabla . \tau - \rho_E(\nabla \phi)$$

3) Poisson's eqⁿ ⇒

$$-\epsilon \nabla^2 \phi = \rho_E = FZC$$

4) PNP eqⁿ ⇒ $$\frac{\partial c}{\partial t} + \nabla . \left(cu - D\nabla c - \frac{Dc\nabla \phi}{V_T}\right) = 0$$

5) Conformation tensor ⇒

$$\frac{\partial A}{\partial t} + u . \nabla A - A . \nabla u - \nabla u^T . A = \frac{1 + \varepsilon(\mathrm{tr}A - 3)}{\lambda}(I - A).$$

Scalling parameters : $\bar{x} = \frac{x}{R}$ | $\bar{t} = \frac{tU}{R}$

$\bar{\nabla} = \nabla R$ | $\bar{\rho}_E = \frac{\rho_E}{FZC_0}$ | $\bar{c} = \frac{c}{C_0}$

$\bar{u} = \frac{u}{U}$ | $\bar{\phi} = \frac{\phi}{V_T}$ | $\bar{p} = \frac{p}{\rho U^2}$

where, $\bar{V} = \frac{ER}{V_T}$ | $E = \frac{2V}{L}$ | $U = \frac{\epsilon V_T^2}{\eta R}$ | $V_T = \frac{KT}{eZ}$

$\eta = \eta_s + \eta_p$ | $\beta = \frac{\eta_s}{\eta_s + \eta_p}$

* Dimensionless parameter used :

$$Re = \frac{\rho R U}{\eta} \quad \Big| \quad Wi = \frac{\lambda U}{R}$$

1) Continuity equation :

$$\nabla . u = 0$$

$$\Rightarrow \quad \underset{R}{\underline{\bar{\nabla}}} . \bar{u} U = 0 \quad \Big\} \quad \boxed{\bar{\nabla} . \bar{u} = 0}$$

2) Momentum Conservation :

$$\rho\left[\frac{\partial u}{\partial t} + u\cdot\nabla u\right] = -\nabla p + \nabla\cdot\tau - \rho_E\nabla\phi$$

a) $$\rho\left[\frac{\partial u}{\partial t} + u\cdot\nabla u\right] \longrightarrow \rho\left[\frac{\partial(\bar{u}U)}{\partial\left(\frac{\bar{t}R}{U}\right)} + \bar{u}U\cdot\frac{\bar{\nabla}}{R}(\bar{u}U)\right]$$

$$\Rightarrow \rho\left[\frac{U^2}{R}\left(\frac{\partial\bar{u}}{\partial\bar{t}}\right) + \frac{U^2}{R}\left(\bar{u}\cdot\bar{\nabla}\bar{u}\right)\right]$$

$$\Rightarrow \boxed{\frac{\rho U^2}{R}\left[\frac{\partial\bar{u}}{\partial\bar{t}} + \bar{u}\cdot\bar{\nabla}\bar{u}\right]}$$

b) $\nabla p \longrightarrow \dfrac{\bar{\nabla}}{R}, \ \bar{p}\,(\rho U^2) \Rightarrow \left(\dfrac{\rho U^2}{R}\right)\left[\bar{\nabla}\bar{p}\right]$

$\nabla \cdot \tau \rightarrow \nabla \cdot (\tau_s + \tau_p)$

PTT Model,

$$\boxed{\tau_s = 2\eta_s S}$$

Rra PTT

fluid

press

$f \nabla^2$

where;

$$S = \frac{1}{2}\left[\nabla u + \nabla u^T\right]$$

and

for $\tau_p \Rightarrow$

$$\tau_p\left[1 + \frac{\varepsilon \lambda \, tr(\tau_p)}{\eta_p}\right] + \lambda\left[\frac{\partial \tau_p}{\partial t} + u \cdot \nabla \tau_p - \tau_p \cdot \nabla u - \nabla u^T \cdot \tau_p\right] = 2\eta_p S$$

We are using Conformation tensor,

$$\Rightarrow \tau_p = \frac{\eta_p}{\lambda}(A - I)$$

$$\Rightarrow \nabla \cdot \tau = \nabla \tau_s + \nabla \tau_p$$

$$= \nabla(2\eta_s S) + \nabla\left[\frac{\eta_p}{\lambda}(A - I)\right]$$

c) $\nabla \cdot \tau = \nabla(2\eta_s S) + \nabla\left[\frac{\eta_p}{\lambda}(A - I)\right]$

$\nabla \cdot \tau = 2\eta_s \nabla S + \frac{\eta_p}{\lambda} \nabla A$

$= 2\eta_s \frac{\bar{\nabla}}{R}\left(\frac{U}{R}\right)\bar{S} + \frac{\eta_p}{\lambda}\frac{\bar{\nabla}}{R} A$

$\Rightarrow$

$\nabla \cdot \tau = \left(\frac{U}{R^2}\right) 2\eta_s \left(\bar{\nabla}\bar{S}\right) + \left(\frac{1}{R}\right)\frac{\eta_p}{\lambda}\left(\bar{\nabla} A\right)$

$S = \frac{1}{2}(\nabla u + \nabla u^T)$

$\bar{S} = \frac{U}{R} S$

$= \frac{U}{R}\left[\frac{1}{2}(\bar{\nabla}\bar{u} + \bar{\nabla}\bar{u}^T)\right]$

d) $\rho_E \nabla\phi \longrightarrow \bar{\rho}_E \left(\frac{FZC_0}{R}\right) \cdot \bar{\nabla} \bar{\phi} V_T$

$$\Rightarrow \rho_E \nabla\phi = \left(\frac{FZC_0V_T}{R}\right) \bar{\rho}_E \bar{\nabla}\bar{\phi}$$

put everything in the eqn :→

$$\Rightarrow \frac{\rho U^2}{R}\left[\frac{D\bar{u}}{D\bar{t}}\right] = -\frac{\rho U^2}{R}\left[\bar{\nabla}\bar{P}\right] + \frac{U}{R^2}\, 2n_s\left(\bar{\nabla}\bar{S}\right) + \frac{1}{R}\frac{n_p}{\lambda}\left(\bar{\nabla}A\right)$$

$$- \frac{F z C_0 V_T}{R}\left(\bar{\rho}_E \bar{\nabla}\bar{\phi}\right)$$

$$\frac{D\bar{u}}{D\bar{t}} = -\bar{\nabla}\bar{P} + \frac{\left(2n_s \cancel{U}/R^{\cancel{2}}\right)}{\left(\rho U^{\cancel{2}}/\cancel{R}\right)}\,\bar{\nabla}\bar{S} + \frac{\left(n_p/\lambda \cancel{R}\right)}{\left(\rho U^2/\cancel{R}\right)}\left(\bar{\nabla}A\right)$$

$$- \left[\frac{F z C_0 kT}{\cancel{R} e z} \times \frac{1}{\left(\rho U^2/\cancel{R}\right)}\right] \bar{\rho}_E \bar{\nabla}\bar{\phi}$$

$$\frac{D\bar{u}}{D\bar{t}} = -\bar{\nabla}\bar{P} + \left[\frac{2n_s}{\rho U R} \times \frac{\eta}{\eta}\right] \bar{\nabla}\bar{S} + \left[\frac{n_p}{\rho U^2 \lambda} \times \frac{\eta}{\eta} \times \frac{R}{R}\right] \bar{\nabla}A$$

$$- \left[\frac{F z C_0 kT}{e z \rho \left(\frac{\epsilon V_T^2}{\eta R}\right) U}\right] \bar{\rho}_E \bar{\nabla}\bar{\phi}$$

$$\frac{D\bar{u}}{D\bar{t}} = -\bar{\nabla}\bar{P} + \left[2\left(\frac{n_s}{\eta}\right)\left(\frac{\eta}{\rho U R}\right)\right] \bar{\nabla}\bar{S} + \left[\left(\frac{n_p}{\eta}\right)\left(\frac{\eta}{\rho U R}\right)\left(\frac{R}{\lambda U}\right)\right] \bar{\nabla}A$$

$$- \left[\frac{F z C_0}{\rho\left(kT/ez\right) \times \epsilon \left(\frac{\eta R}{U}\right)} \times \frac{R}{R}\right] \bar{\rho}_E \bar{\nabla}\bar{\phi} \times \frac{2}{2}$$

$$\Rightarrow \frac{D\bar{u}}{D\bar{t}} = -\bar{\nabla}\bar{p} + \frac{2\beta}{Re}\left(\bar{\nabla}\bar{S}\right) + \frac{(1-\beta)}{(Re)(W_i)} \times \bar{\nabla}A$$

$$- \underbrace{\left(\frac{2FeZ^2 C_0 R^2}{\epsilon KT}\right)}_{K^2 \text{ (given)}} \times \left(\frac{\eta}{\rho U R}\right) \times \left[\frac{\bar{\rho}_E \bar{\nabla}\bar{\phi}}{2}\right]$$

$$\Rightarrow \boxed{\frac{D\bar{u}}{D\bar{t}} = -\bar{\nabla}\bar{p} + \frac{2\beta}{Re}\left(\bar{\nabla}\bar{S}\right) + \frac{(1-\beta)}{(Re)(W_i)} \times \bar{\nabla}A - \frac{K^2}{2Re}\left(\bar{\rho}_E \bar{\nabla}\bar{\phi}\right)}$$

3) Poisson's eqn:

$$-\epsilon\nabla^2\phi = \rho_e = FZC$$

$$-\nabla^2\phi = \frac{FZC}{\epsilon}$$

$$\Rightarrow \quad -\frac{\bar{\nabla}^2(\bar{\phi}V_T)}{R^2} = \frac{FZ\bar{C}C_0}{\epsilon}$$

$$\Rightarrow \quad \bar{\nabla}^2\bar{\phi} = \frac{FZ\bar{C}C_0}{\epsilon}\left(\frac{R^2}{V_T}\right)$$

$$= \frac{FZ\bar{C}C_0}{\epsilon} \times \frac{R^2}{(KT/eZ)}$$

$$\Rightarrow \quad \bar{\nabla}^2\bar{\phi} = \frac{FeZ^2\bar{C}C_0 R^2}{\epsilon KT} = -\underbrace{\left(\frac{2FZ^2R^2eC_0}{\epsilon KT}\right)}_{=K}\frac{\bar{C}}{2}$$

(given in paper)

$$\Rightarrow \quad \bar{\nabla}^2\bar{\phi} = -\left(\frac{K^2}{2}\right)\bar{C}$$

4)

4) PNP eqn

$$\frac{\partial c}{\partial t} + \nabla \cdot \left(c u - D \nabla c - \frac{D}{V_T} c \nabla \phi \right) = 0$$

$$\Rightarrow \frac{\partial (\bar{c} c_0)}{\partial (\bar{t} R / U)} + \frac{\bar{\nabla}}{\cancel{R}} \cdot \left(\bar{c} c_0 \bar{u} U - D \frac{\bar{\nabla}}{R} \bar{c} c_0 - \frac{D}{\cancel{V_T}} \bar{c} c_0 \frac{\bar{\nabla}}{R} \bar{\phi} \cancel{V_T} \right) = 0$$

$$\Rightarrow \cancel{c_0} U \left(\frac{\partial \bar{c}}{\partial \bar{t}} \right) + \bar{\nabla} \cdot \cancel{(c_0)} \left(\bar{c} \bar{u} U - D \frac{\bar{\nabla}}{R} \bar{c} - D \bar{c} \frac{\bar{\nabla}}{R} \bar{\phi} \right) = 0$$

$$\Rightarrow \frac{\partial \bar{c}}{\partial \bar{t}} + \bar{\nabla} \cdot \left[\bar{c} \bar{u} - \left(\frac{D}{RU} \right) \bar{\nabla} \bar{c} - \left(\frac{D}{RU} \right) \bar{c} \bar{\nabla} \bar{\phi} \right] = 0$$

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DAY 172-193

$$\Rightarrow \frac{D}{RU} = \frac{1}{Pe}$$ (peclet no.)

$$\Rightarrow \boxed{\frac{\partial \bar{c}}{\partial \bar{t}} + \bar{\nabla} \cdot \left[\bar{c} \bar{u} - \frac{1}{Pe} \bar{\nabla} \bar{c} - \frac{1}{Pe} \bar{c} \bar{\nabla} \bar{\phi} \right] = 0}$$

Page No.:
Date:

5) <u>Conformation tensor</u> ($A \rightarrow$ dimensionless)

$$\frac{\partial A}{\partial t} + u \cdot \nabla A - A \cdot \nabla u - \nabla u^T \cdot A = \frac{1+\varepsilon(\mathrm{tr}A - 3)}{\lambda}(I - A)$$

$$\Rightarrow \frac{\partial A}{\partial (\bar{t} R/U)} + \bar{u} U \cdot \frac{\bar{\nabla}}{R} A - A \cdot \frac{\bar{\nabla}}{R} \bar{u} U - \frac{\bar{\nabla}}{R} \bar{u}^T U \cdot A$$

$$= \frac{1+\varepsilon(\mathrm{tr}A - 3)}{\lambda}(I - A)$$

$$\Rightarrow \frac{U}{R}\left[\frac{\partial A}{\partial \bar{t}} + \bar{u} \cdot \bar{\nabla} A - A \cdot \bar{\nabla} \bar{u} - \bar{\nabla} \bar{u}^T \cdot A\right] = \frac{1+\varepsilon(\mathrm{tr}A - 3)}{\lambda}(I - A)$$

$$\Rightarrow \frac{\partial A}{\partial \bar{t}} + \bar{u} \cdot \bar{\nabla} A - A \cdot \bar{\nabla} \bar{u} - \bar{\nabla} \bar{u}^T \cdot A = 1+\varepsilon(\mathrm{tr}A - 3)(I - A) \times \underbrace{\frac{R}{\lambda U}}_{Wi}$$

$$\Rightarrow \boxed{\frac{\partial A}{\partial \bar{t}} + \bar{u} \cdot \bar{\nabla} A - A \cdot \bar{\nabla} \bar{u} - \bar{\nabla} \bar{u}^T \cdot A = \frac{1+\varepsilon(\mathrm{tr}A - 3)}{Wi}(I - A)}$$