Q1) $E[X] = E[\sum_{i=1}^{m} X_i]$

$$X_i = \begin{cases} 1, & \text{cup } i \text{ is empty} \\ 0, & \text{otherwise} \end{cases}$$

$$E[\sum_{i=1}^{m} X_i] = \sum_{i=1}^{M} E[X_i]$$

$E[X_i] = P(\text{cup } i \text{ is empty})$

$= \left(\frac{m-1}{m}\right)^n$; n is balls

Since each of n ball goes in $m-1$ cups

Thus $E[X] = m\left(1-\frac{1}{m}\right)^n$

$= \left(\frac{50-1}{50}\right)^{100} \times 50$

$= 0.1326 \times 50$

≈ 6.63

≈ 7 approximately.

Q2) a)

P(Bit ERROR) = 10^{-10}

This is error in transmission of bit

$1 \to 0 \quad P = 10^{-10}$

$0 \to 1 \quad P = 10^{-10}$

so,

for block of 1000 bits

Probability of bit error = 1000×10^{-10}

$P = 10^{-7}$

b) using Markov or Chebyshev's inequality

$P\{N \geq 10\} \leq \frac{P}{10} = 10^{-8}$

Upperbound = 10^{-8}

Q3) Spade cards = 13

Queens = 4 (including Queen of spades)

so, winning cards = 13 + 3 = 16

losing cards = 36

so,

Probability (winning) = $\frac{16}{52}$

so

wins after 30 days :

$$30\left(\frac{16}{52} \times 4 - \frac{36}{52} \times 1\right)$$

$$30\left(\frac{16}{13} - \frac{9}{13}\right) = 30 \times \frac{7}{13}$$

wins = $16.153