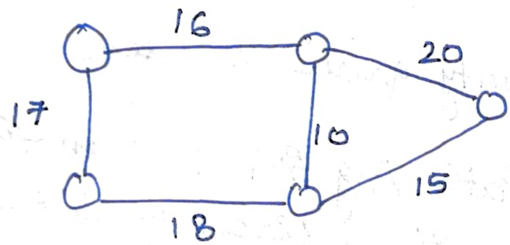
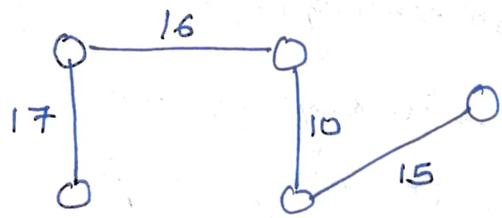
Ans 1 Minimum Spanning Tree:

A spanning tree of an undirected graph is a subgraph that is a tree & joined by all vertices. One of those tree which has minimum total cost would be its minimum spanning tree.

Eg:



Minimum cost spanning tree



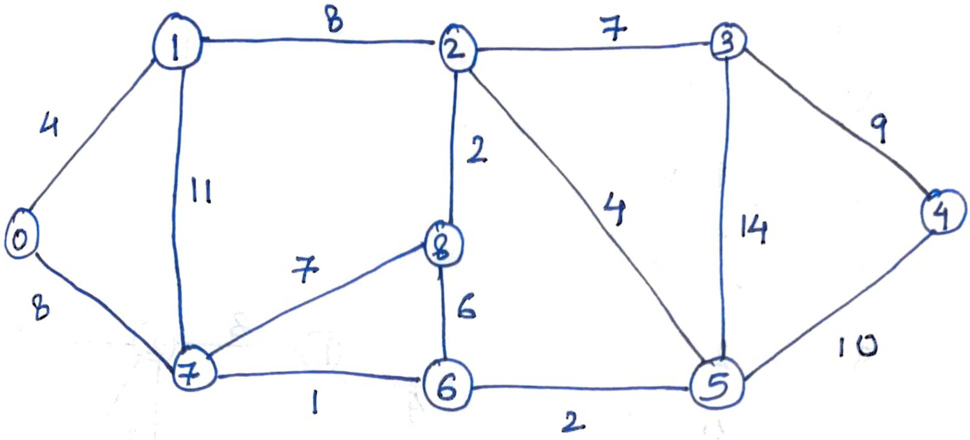
Applications of MST

It has direct applications in the design of networks including computer networks, telecommunication networks, transportation networks etc.

Ans 2

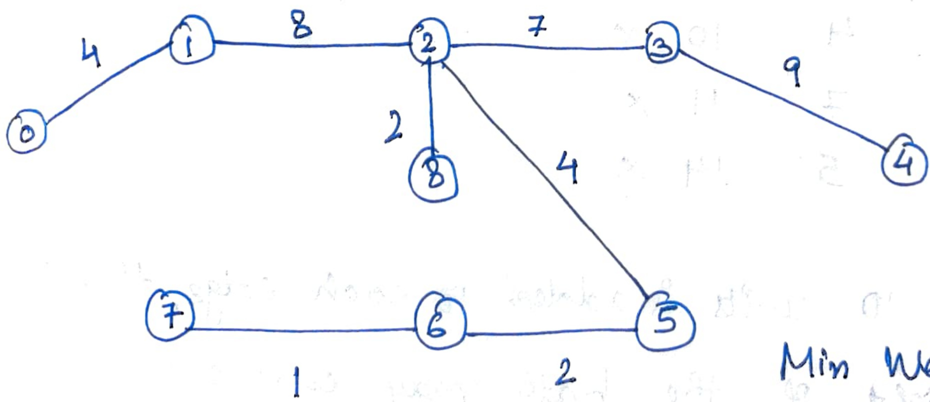
	Prim's Algorithm	Kruskal's Algorithm	Dijkstra's Algorithm	Bellman Ford's Algo
T.C.	$O(V^2)$	$O(E \log V)$	$O(V+E\log V)$	$O(VE)$
S.C.	$O(V+E)$	$O(\lvert E\rvert+\lvert V\rvert)$	$O(V^2)$	$O(V^2)$

Ans 3



Prim's Algo

0	1	2	3	4	5	6	7	8
$\boxed{0}$	$\cancel{\infty}$	∞	∞	∞	∞	∞	$\cancel{\infty}$	∞
	$\boxed{4}$	$\boxed{8}$	$\boxed{7}$	$\cancel{10}$	$\boxed{4}$	$\cancel{8}$	$\cancel{8}$	$\boxed{2}$
				$\boxed{9}$		$\boxed{2}$	$\cancel{7}$	
							$\boxed{1}$	



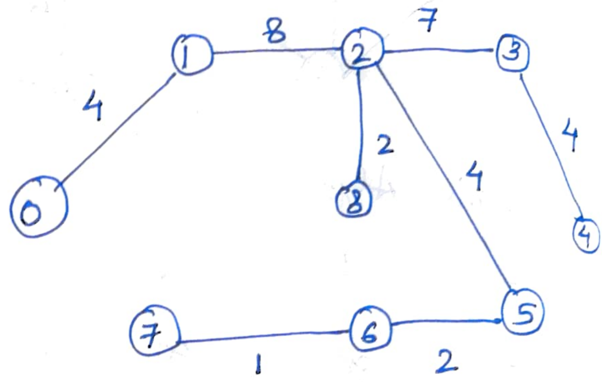
Min Weight = 37

Parent	0	1	2	3	4	5	6	7	8
	-1	$\cancel{-1}$	$\cancel{-1}$	$\cancel{-1}$	-1	$\cancel{-1}$	-1	$\cancel{-1}$	$\cancel{-1}$
		0	1	2		2		$\cancel{0}$	2
							$\cancel{8}$	$\cancel{8}$	
					$\cancel{5}$		5	6	
					3				

Parent : -1 0 1 2 3 2 5 6 2

Kruskal's Algo

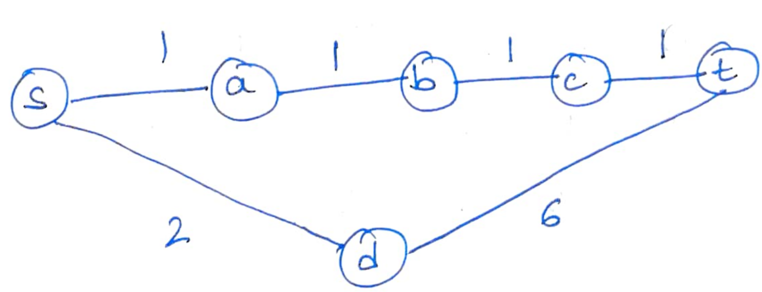
u	v	w	
7	6	1	✓
6	5	2	✓
2	8	2	✓
2	5	4	✓
0	1	4	✓
8	6	6	✗
7	8	7	✗
2	3	7	✓
1	2	8	✓
0	7	8	✗
3	4	9	✓
5	4	10	✗
1	7	11	✗
3	5	14	✗



Weight = 37

Ans 4 i) If 10 units is added to each edge, the overall weight of the path may change.

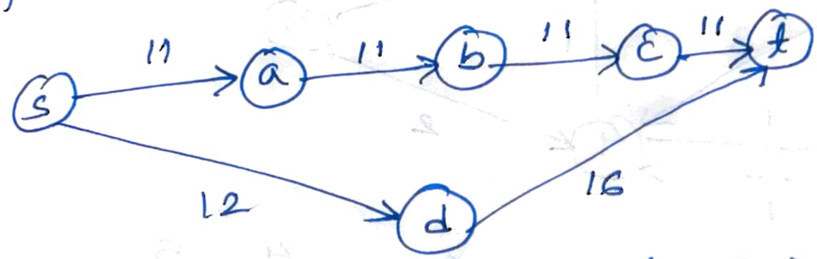
Eg:



Shortest path is $s \to a \to b \to c \to t$

weight $1+1+1+1 = 4$

now if 10 unit weight is added to each edge.

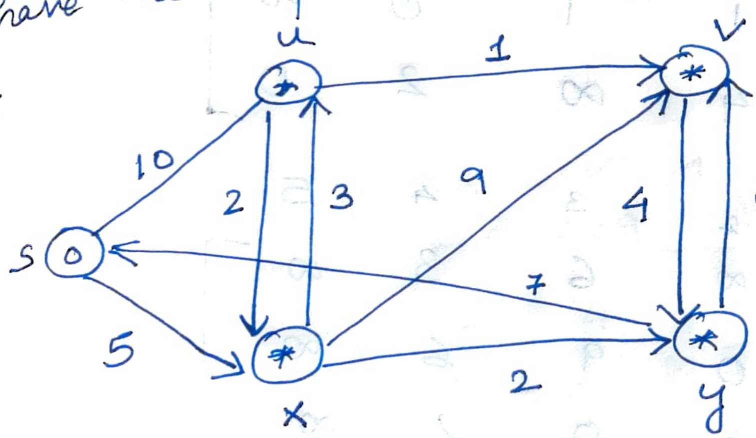


Shortest path changed to $s \to d \to t$

Weight $= 28$

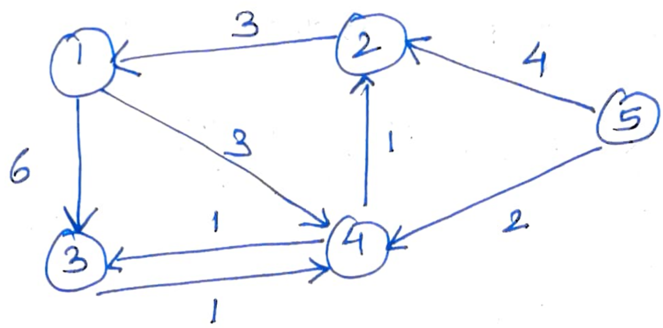
ii) Multiplying the weight of each edge by 10 will have no impact on the shortest path.

Ans 5



s	u	v	x	y
0	∞	∞	∞	∞
0	10	∞	5	∞
0	10	11	5	∞
0	10	11	5	$\cancel{\infty}$
0	10	11	5	7

Ans 6 All pair shortest path algorithm - Floyd Warshall



$$A^0 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & 0 \end{bmatrix} \end{array}$$

$$A^1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & 0 \end{bmatrix} \end{array}$$

$A^0[2,3] = \infty$

$A^0[2,1] + A^0[1,3] = 3+6 = 9$

$9 < \infty$

Similarly $A^0[2,4] = \infty$

$$A^0[2,1] + A^0[1,4] = 3+3 = 6$$

$$\Rightarrow 6 < \infty$$

$$A^0[2,5] = \infty$$

$$A^0[2,1] + A^0[1,5] = 3+\infty$$

$$A^2 = \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ 7 & 4 & 13 & 2 & 0 \end{bmatrix} \end{array}$$

$$A'[1,3] = 6$$

$$A'[1,2] + A'[2+3] = \infty + 9$$

$$6 < \infty + 9$$

$$A^3 = \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ 7 & 4 & 13 & 2 & 0 \end{bmatrix} \end{array}$$

$$A^4 = \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 4 & 4 & 3 & \infty \\ 3 & 0 & 7 & 6 & \infty \\ \infty & 3 & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ 7 & 3 & 3 & 2 & 0 \end{bmatrix} \end{array}$$

$$A_5 = \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 4 & 4 & 3 & \infty \\ 3 & 0 & 7 & 6 & \infty \\ \infty & 3 & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ 7 & 3 & 3 & 2 & 0 \end{bmatrix} \end{array}$$