#1

a)

$$(A)^{-3} = \begin{bmatrix} 125 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 64 \end{bmatrix}$$

$$(A^3)^{-1} = \begin{bmatrix} 125 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 64 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} \frac{1}{125} & 0 & 0 \\ 0 & \frac{1}{8} & 0 \\ 0 & 0 & \frac{1}{64} \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{1}{5} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$$

b) $(2A^T)^{-1} = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

$$\frac{(A^T)^{-1}}{2} = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$$

$$(A^T)^{-1} = \begin{bmatrix} 6 & 2 \\ 10 & 4 \end{bmatrix}$$

Inverse on both sides

$$A^T = \frac{1}{6 \times 4 - 10 \times 2} \begin{bmatrix} 4 & -2 \\ -10 & 6 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 4 & -2 \\ -10 & 6 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & -\frac{1}{2} \\ -\frac{5}{2} & \frac{3}{2} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -\frac{5}{2} \\ -1/2 & 3/2 \end{bmatrix}$$

$5 - 11 \times (-1)$
$5 + 11$

2)
a)
$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 11 & -7 & 5 \end{array}\right] \quad R_3 \to R_3 - 11R_2$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 4 & 16 \end{array}\right] \quad R_1 \to R_1 + 3R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & -4 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 4 & 16 \end{array}\right] \quad R_3 \to R_3/4$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & -4 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 4 \end{array}\right] \quad R_2 \to R_2 + R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & -4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{array}\right] \quad R_1 \to R_1 + R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

3)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ x & 0 & -4 \\ 0 & -1 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -2 & y \\ 2 & 4 & 1 \\ 5 & -1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 22 & 3 & 3 \\ -32 & 12 & -4 \\ 23 & -9 & -1 \end{bmatrix}$$

$$A \times B = C$$

$$A \times B = \begin{bmatrix} 1 & 2 & 3 \\ x & 0 & -4 \\ 0 & -1 & 5 \end{bmatrix} \times \begin{bmatrix} 3 & -2 & y \\ 2 & 4 & 1 \\ 5 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times3 + 2\times2 + 3\times5 & 1\times-2 + 2\times4 + 3\times-1 & 1\times y + 2\times1 + 3\times0 \\ x\times3 + 0\times2 + -4\times5 & x\times-2 + 0\times4 + -4\times-1 & x\times y + 0\times1 + -4\times0 \\ 0\times3 + -1\times2 + 5\times5 & 0\times-2 + -1\times4 + 5\times-1 & 0\times y + -1\times1 + 5\times0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 + 4 + 15 & (-2) + 8 + (-3) & y + 2 + 0 \\ x + 0 + (-20) & (-2x) + 0 + 4 & xy + 0 + 0 \\ 0 + (-2) + 25 & 0 + (-4) + (-5) & 0 + (-1) + 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 22 & 3 & 2+y \\ x-20 & 4-2x & xy \\ 23 & -9 & -1 \end{bmatrix} = \begin{bmatrix} 22 & 3 & 3 \\ -32 & 12 & -4 \\ 23 & -9 & -1 \end{bmatrix}$$

$2+y = 3$	$xy = -4$
$y = 3-2$	$x \times 1 = -4$
$y = 1$	$x = -4$

$-1\times0 + \ldots$

$0 + 63 + \ldots$

4)

$V = (-1, 7, -3)$

$W = (0, 9, 11x)$

$V \cdot W = 0$ (if orthogonal)

$-1\times0 + 7\times9 + -3\times11x = 0$

$0 + 63 - 33x = 0$

$33x = 63$

$x = \frac{\cancel{63}\,21}{\cancel{33}\,11}$

$x = \frac{21}{11}$

$x = 1.91$

5)

$$A = \begin{bmatrix} 5 & -2 & 3 \\ -1 & -2 & 1 \\ 1 & k & 3 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ -5 \\ 9 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 5 & -2 & 1 \\ -1 & -2 & -5 \\ 1 & k & 9 \end{bmatrix}$$

$\det(A_3)$

$= 1\left((-2)\times(-5) - (-2)\times 1\right) - k\left(5\times(-5) - (-1)\times 1\right) + 9\left(5\times(-2) - (-1)\times(-\right.$

$= (10+2) - k(-25+1) + 9(-10-2)$

$= 12 + 24k - 108$

$= 24k - 96$

$-384 = 24k - 96$

$24k = -384 + 96$

$k = \dfrac{-288}{24}$

$k = -12$

$\det(A) = 1\left((-2)\times 3 - (-2)\times 1\right) - k\left(5\times 1 - (-1)\times 3\right)$
$+ 3\left(5\times(-2) - (-1)\times(-2)\right)$

$= (-6+2) - k(5+3) + 3(-10-2)$

$= -4 - 8k - 36$

$= -40 - 8k$

$= -40 - 8\times(-12)$

$= -40 + 96$

$= 56$

$x = 3,\ y = -2,\ z = -6$

6)

$u = (3, 2, 0)$

$v = (-4, 0, -5)$

$w = (1, 0, 4)$

Volume $= |u \cdot (v \times w)|$

$$v \times w = \begin{bmatrix} i & j & k \\ -4 & 0 & -5 \\ 1 & 0 & 4 \end{bmatrix}$$

$$= i(0 \times 4 - 0 \times (-5)) - j(-4 \times 4 - 1 \times (-5)) + k(-4 \times 0 - 1 \times 0)$$

$$= -j(-16 + 5)$$

$$= 11j$$

Volume $= u \cdot 11j$

$$= 11 \times (u_j)$$

$$= 11 (3, 2, 0) \cdot (0, 1, 0)$$

$$= 11 (3 \times 0 + 2 \times 1 + 0 \times 0)$$

$$= 11 \times 2$$

$$= 22$$

Since, triple scale not 0, they are not Coplanar

7)

$u = (5, -1, 0)$
$v = (2, 0, 3)$

a) $u \cdot v = (5 \times 2 + -1 \times 0 + 0 \times 3)$
$= 10 + 0 + 0$
$= 10$

b) $u \cdot v = |u| \cdot |v| \cdot \cos\theta$

$\cos\theta = \frac{u \cdot v}{|u| \cdot |v|}$

$|u| = \sqrt{(5)^2 + (-1)^2 + 0^2}$
$= \sqrt{25 + 1}$
$= \sqrt{26}$

$\cos\theta = \frac{10}{\sqrt{26 \times 13}}$

$\cos\theta = \frac{10}{\sqrt{338}}$

$|v| = \sqrt{(2)^2 + 0^2 + (3)^2}$
$= \sqrt{4 + 9}$
$= \sqrt{13}$

$\theta = \cos^{-1}(10/\sqrt{338})$

$= 57.05$

8)

$A(7,1,0)$
$B(6,-2,3)$
$C(0,1,-1)$

$\overrightarrow{AB} = B - A = (6-7, (-2)-1, 3-0) = (-1,-3,3)$

$\overrightarrow{AC} = C - A = (0-7, 1-1, -1-0) = (-7,0,-1)$

Area $= \overrightarrow{AB} \times \overrightarrow{AC}$

$$= \begin{bmatrix} i & j & k \\ -1 & -3 & 3 \\ -7 & 0 & -1 \end{bmatrix}$$

$= i(-3\times-1 - 0\times3) - j(-1\times-1 - (-7)\times3) + k(-1\times0 - (-7)\times-3)$
$= i(3-0) - j(1+21) + k(0-21)$
$= 3i - 22j - 21k$
$= (3, -22, -21)$

Area $= \sqrt{3^2 + (-22)^2 + (-21)^2}$

$= \sqrt{9 + 484 + 441}$

$= \sqrt{934} = 30.56$

9)

$u = (-2, 0)$

$v = (4, 5)$

a)

$5u = 5(-2, 0)$

$= (-10, 0)$

$\|5u\| = \sqrt{(-10)^2 + 0^2}$

$= 10$

$\|v\| = \sqrt{4^2 + 5^2}$

$= \sqrt{16 + 25}$

$= \sqrt{41}$

$= \|5u\| - 2\|v\|$

$= 10 - 2\sqrt{41}$

$= 10 - 12.80$

$= -2.8$ //

b)

$$\text{proj}_v(u) = \frac{u \cdot v}{\|v\|} \cdot \frac{v}{\|v\|}$$

$$u \cdot v = -2 \times 4 + 0 \times 5$$
$$= -8 + 0$$
$$= -8$$

$$\|v\| = \sqrt{16+25}$$
$$= \sqrt{41}$$

$$\text{proj}_v(u) = \frac{-8}{\sqrt{41}} \cdot \left(\frac{4}{\sqrt{41}}, \frac{5}{\sqrt{41}} \right)$$

$$= \left(\frac{-32}{41}, \frac{-40}{41} \right)$$

Q(i)

10) $c_{ij}(A) = \begin{bmatrix} -2 & -1 & 1 \\ 0 & b & 2 \\ a & 0 & 2 \end{bmatrix}$

$C_{ij}(A) = \begin{bmatrix} 10 & -8 & 20 \\ 2 & 0 & 4 \\ c & 4 & -10 \end{bmatrix}$

(a) $C_{11}(A) = (-1)^{1+1} \cdot M_{11}$

$M_{11} = 10$, $a_{11} = -2$

(b) $a_{21} = 0$

$C_{21}(A) = (-1)^{2+1} \cdot M_{21}$

$M_{21} = -2$

(c) $a_{31} = a$

$C_{31}(A) = (-1)^{3+1} \cdot M_{31} = c$

$a = M_{31} = C$

$b = M_{21} = -2$

$c = M_{11} = 10$

$a = 10$
$b = -2$
$c = 10$

b) $Adj(A) = \left[C_{ij}(A) \right]^T$

$$= \begin{bmatrix} 10 & 2 & c \\ -8 & 0 & 4 \\ 20 & 4 & -10 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 2 & 10 \\ -8 & 0 & 4 \\ 20 & 4 & -10 \end{bmatrix}$$

last Column $= [10, 4, -10]$

~~det(A)=~~

$$A = \begin{bmatrix} -2 & -1 & 1 \\ 0 & b & 2 \\ a & 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -1 & 1 \\ 0 & -2 & 2 \\ 10 & 0 & 2 \end{bmatrix}$$

c) $\det(A) = -2(-2\times2 - 0\times2) - (-1)(0\times2 - 10\times2) + 1(0\times0 - (-2\times10))$

$= -2(-4) + 1(-20) + 1(20)$

$= 8 - 20 + 20$

$= 8$