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 Session ID: 2

## CMPE 240 2019 Experiment 2 Preliminary Work

### Truth Table

#	<u>r</u>	<u>c</u>	<u>g</u>	<u>p</u>	<u>b</u>
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	0
14	1	1	1	0	1
15	1	1	1	1	1

### Sum of Products (SOP)

$$b = (r'cgp') + (r'cgp) + (rc'gp') + (rcg'p') + (rcgp') + (rcgp)$$

### Minimized SOP

$$\begin{aligned}
 b &= (r'cgp') + (r'cgp) + (rc'gp') + (rcg'p') + (rcgp') + (rcgp) \\
 &= (r'cgp') + (r'cgp) + (rc'gp') + (rcg'p') + (rcgp') + (rcgp') + (rcgp) \\
 &\quad \text{(Idempotent)} \\
 &= (rcgp') + (rc'gp') + (r'cgp') + (r'cgp) + (rcg'p') + (rcgp') + (rcgp) \\
 &\quad \text{(Commutative)} \\
 &= (rgp').(c+c') + (r'cgp') + (r'cgp) + (rcg'p') + (rcgp') + (rcgp) \\
 &\quad \text{(Distributive)} \\
 &= (rgp').(1) + (r'cgp') + (r'cgp) + (rcg'p') + (rcgp') + (rcgp) \\
 &\quad \text{(Complement)} \\
 &= (rgp') + (r'cgp') + (r'cgp) + (rcg'p') + (rcgp') + (rcgp) \\
 &\quad \text{(Identity)} \\
 &= (rgp') + (r'cgp') + (r'cgp) + (rcg'p') + (rcgp') + (rcgp') + (rcgp) \\
 &\quad \text{(Idempotent)}
 \end{aligned}$$

$$\begin{aligned}
&= (rgp') + (rcg'p') + (rcgp') + (r'cgp') + (r'cgp) + (rcgp') + (rcgp) \\
&\text{(Commutative)} \\
&= (rgp') + (rcp').(g' + g) + (r'cgp') + (r'cgp) + (rcgp') + (rcgp) \\
&\text{(Distributive)} \\
&= (rgp') + (rcp').(1) + (r'cgp') + (r'cgp) + (rcgp') + (rcgp) \\
&\text{(Complement)} \\
&= (rgp') + (rcp') + (r'cgp') + (r'cgp) + (rcgp') + (rcgp) \\
&\text{(Identity)} \\
&= (rgp') + (rcp') + (cg).(r'p' + r'p) + (rcgp') + (rcgp) \\
&\text{(Distributive)} \\
&= (rgp') + (rcp') + (cg).(r'p' + r'p + rp') + (rcgp) \\
&\text{(Distributive)} \\
&= (rgp') + (rcp') + (cg).(r'p' + r'p + rp' + rp) \\
&\text{(Distributive)} \\
&= (rgp') + (rcp') + (cg).(r'(p' + p) + rp' + rp) \\
&\text{(Distributive)} \\
&= (rgp') + (rcp') + (cg).(r'(1) + rp' + rp) \\
&\text{(Complement)} \\
&= (rgp') + (rcp') + (cg).(r' + rp' + rp) \\
&\text{(Identity)} \\
&= (rgp') + (rcp') + (cg).(r' + r(p' + p)) \\
&\text{(Distributive)} \\
&= (rgp') + (rcp') + (cg).(r' + r(1)) \\
&\text{(Complement)} \\
&= (rgp') + (rcp') + (cg).(r' + r) \\
&\text{(Identity)} \\
&= (rgp') + (rcp') + (cg).(1) \\
&\text{(Complement)} \\
&= (rgp') + (rcp') + (cg) \\
&\text{(Identity)}
\end{aligned}$$

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### Product of Sums (POS)

$$b = (r+c+g+p).(r+c+g+p').(r+c+g'+p).(r+c+g'+p').(r+c'+g+p) \\ .(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c+g'+p').(r'+c'+g+p')$$

### Minimized POS

$$b = (r+c+g+p).(r+c+g+p').(r+c+g'+p).(r+c+g'+p').(r+c'+g+p) \\ .(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c+g'+p').(r'+c'+g+p')$$

$= (r+c+g+p).(r+c+g+p).(r+c+g+p').(r+c+g'+p).(r+c+g'+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c+g'+p').(r'+c'+g+p')$   
(Idempotent)

$= (r+c+g+p).(r+c+g+p).(r+c+g+p').(r+c+g+p').(r+c+g'+p).(r+c+g'+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c+g'+p').(r'+c'+g+p')$   
(Idempotent)

$= (r+c+g+p).(r+c+g+p).(r+c+g+p').(r+c+g+p').(r+c+g'+p).(r+c+g'+p').(r+c'+g+p).(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c+g'+p').(r'+c'+g+p')$   
(Idempotent)

$= (r+c+g+p).(r+c+g+p').(r+c+g'+p).(r+c+g'+p').(r+c+g+p).(r+c+g+p').(r+c+g'+p).(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c+g'+p').(r'+c'+g+p')$   
(Commutative)

$= ((r+c)+(g+p).(g+p')).(r+c+g'+p).(r+c+g'+p').(r+c+g+p).(r+c+g+p').(r+c+g'+p).(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c+g'+p').(r'+c'+g+p')$   
(Distributive)

$= ((r+c)+(g+p).(g+p')).(g'+p).(g'+p')).(r+c+g'+p).(r+c+g+p).(r+c+g+p').(r+c+g'+p).(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c+g'+p').(r'+c'+g+p')$   
(Distributive)

$= ((r+c)+(g+p).(g+p')).(g'+p).(g'+p')).(r+c+g+p).(r+c+g+p').(r+c+g'+p).(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c+g'+p').(r'+c'+g+p')$   
(Distributive)

$= ((r+c)+(g+0).(g'+p).(g'+p')).(r+c+g+p).(r+c+g+p').(r+c+g'+p).(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c+g'+p').(r'+c'+g+p')$   
(Complement)

$= ((r+c)+g.(g'+p).(g'+p')).(r+c+g+p).(r+c+g+p').(r+c+g'+p).(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c+g'+p').(r'+c'+g+p')$

(Identity)

$$=((r+c)+g.(g'+pp')).(r+c+g+p).(r+c+g+p').(r+c+g'+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c+g'+p').(r'+c'+g+p')$$

(Distributive)

$$=((r+c)+g.(g'+0)).(r+c+g+p).(r+c+g+p').(r+c+g'+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c+g'+p').(r'+c'+g+p')$$

(Complement)

$$=((r+c)+gg').(r+c+g+p).(r+c+g+p').(r+c+g'+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c+g'+p').(r'+c'+g+p')$$

(Identity)

$$=((r+c)+0).(r+c+g+p).(r+c+g+p').(r+c+g'+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c+g'+p').(r'+c'+g+p')$$

(Complement)

$$=(r+c).(r+c+g+p).(r+c+g+p').(r+c+g'+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c+g'+p').(r'+c'+g+p')$$

(Identity)

$$=(r+c).(r+c+g+p).(r+c+g+p').(r+c+g'+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c+g'+p').(r'+c'+g+p')$$

(Idempotent)

$$=(r+c).(r+c+g+p).(r+c+g+p').(r+c+g+p').(r+c+g'+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c+g'+p').(r'+c'+g+p')$$

(Idempotent)

$$=(r+c).(r+c+g+p').(r+c+g'+p').(r'+c+g+p').(r'+c+g'+p').(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c'+g+p')$$

(Commutative)

$$=(r+c).((c+p')+(r+g).(r+g')).(r'+c+g+p').(r'+c+g'+p').(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c'+g+p')$$

(Distributive)

$$=(r+c).((c+p')+(r+g).(r+g')).(r'+c+g+p').(r'+c+g'+p').(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c'+g+p')$$

(Distributive)

$$=(r+c).((c+p')+(r+g).(r+g')).(r'+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c'+g+p')$$

(Distributive)

$$=(r+c).((c+p')+(r+gg')).(r'+g).(r'+g')).(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c'+g+p')$$

(Distributive)

$$=(r+c).((c+p')+(r+0).(r'+g).(r'+g')).(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c'+g+p')$$

(Complement)

$$=(r+c).((c+p')+r.(r'+g).(r'+g')).(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c'+g+p')$$

(Identity)

$$=(r+c).((c+p')+r.(r'+gg')).(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c'+g+p')$$

(Distributive)

$$=(r+c).((c+p')+r.(r'+0)).(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c'+g+p')$$

(Complement)

$$\begin{aligned}
&= (r+c).((c+p')+rr').(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c'+g+p') \\
&\text{(Identity)} \\
&= (r+c).((c+p')+0).(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c'+g+p') \\
&\text{(Complement)} \\
&= (r+c).(c+p').(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c'+g+p') \\
&\text{(Identity)} \\
&= (r+c).(c+p').(r+c+g+p).(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c'+g+p') \\
&\text{(Idempotent)} \\
&= (r+c).(c+p').(r+c+g+p).(r+c+g+p).(r+c+g+p').(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c'+g+p') \\
&\text{(Idempotent)} \\
&= (r+c).(c+p').(r+c+g+p).(r+c+g+p).(r+c+g+p').(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p).(r'+c+g+p').(r'+c'+g+p') \\
&\text{(Idempotent)} \\
&= (r+c).(c+p').(r+c+g+p).(r'+c+g+p).(r+c+g+p').(r'+c+g+p').(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p').(r'+c'+g+p') \\
&\text{(Commutative)} \\
&= (r+c).(c+p').(r+c+g+p).(r'+c+g+p).(r+c+g+p').(r'+c+g+p').(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p').(r'+c'+g+p') \\
&\text{(Distributive)} \\
&= (r+c).(c+p').((c+g)+(r+p).(r'+p)).(r+c+g+p').(r'+c+g+p').(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p').(r'+c'+g+p') \\
&\text{(Distributive)} \\
&= (r+c).(c+p').((c+g)+(r+p).(r'+p).(r+p')).(r'+c+g+p').(r+c+g+p).(r+c+g+p').(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p').(r'+c'+g+p') \\
&\text{(Distributive)} \\
&= (r+c).(c+p').((c+g)+(r+p).(r'+p).(r+p')).(r'+c+g+p').(r+c+g+p).(r+c+g+p').(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p').(r'+c'+g+p') \\
&\text{(Distributive)} \\
&= (r+c).(c+p').((c+g)+(rr'+p).(r+p').(r'+p')).(r+c+g+p).(r+c+g+p').(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p').(r'+c'+g+p') \\
&\text{(Distributive)} \\
&= (r+c).(c+p').((c+g)+(0+p).(r+p').(r'+p')).(r+c+g+p).(r+c+g+p').(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p').(r'+c'+g+p') \\
&\text{(Complement)} \\
&= (r+c).(c+p').((c+g)+p.(r+p').(r'+p')).(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p').(r'+c'+g+p') \\
&\text{(Identity)} \\
&= (r+c).(c+p').((c+g)+p.(rr'+p')).(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p').(r'+c'+g+p') \\
&\text{(Distributive)} \\
&= (r+c).(c+p').((c+g)+p.(0+p')).(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p').(r'+c'+g+p') \\
&\text{(Complement)}
\end{aligned}$$

$$=(r+c).(c+p').((c+g)+pp')).(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p').(r'+c'+g+p')$$

(Identity)

$$=(r+c).(c+p').((c+g)+0)).(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p').(r'+c'+g+p')$$

(Complement)

$$=(r+c).(c+p').(c+g).(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p').(r'+c'+g+p')$$

(Identity)

$$=(r+c).(c+p').(c+g).(r+c+g+p).(r+c+g+p').(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p').(r'+c'+g+p')$$

(Idempotent)

$$=(r+c).(c+p').(c+g).(r+c+g+p).(r+c+g+p').(r+c+g+p').(r+c'+g+p).(r+c'+g+p').(r'+c+g+p').(r'+c'+g+p')$$

(Idempotent)

$$=(r+c).(c+p').(c+g).(r+c+g+p').(r'+c+g+p').(r+c'+g+p').(r'+c'+g+p').(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p')$$

(Commutative)

$$=(r+c).(c+p').(c+g).((g+p')+(r+c).(r'+c)).(r+c'+g+p').(r'+c'+g+p').(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p')$$

(Distributive)

$$=(r+c).(c+p').(c+g).((g+p')+(r+c).(r'+c).(r+c')).(r'+c'+g+p').(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p')$$

(Distributive)

$$=(r+c).(c+p').(c+g).((g+p')+(r+c).(r'+c).(r+c')).(r'+c'+g+p').(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p')$$

(Distributive)

$$=(r+c).(c+p').(c+g).((g+p')+(r+c').(r'+c')).(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p')$$

(Distributive)

$$=(r+c).(c+p').(c+g).((g+p')+(0+c).(r+c').(r'+c')).(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p')$$

(Complement)

$$=(r+c).(c+p').(c+g).((g+p')+c.(r+c').(r'+c')).(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p')$$

(Identity)

$$=(r+c).(c+p').(c+g).((g+p')+c.(r+c')).(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p')$$

(Distributive)

$$=(r+c).(c+p').(c+g).((g+p')+c.(0+c')).(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p')$$

(Complement)

$$=(r+c).(c+p').(c+g).((g+p')+cc').(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p')$$

(Identity)

$$=(r+c).(c+p').(c+g).((g+p')+0).(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p')$$

(Complement)

$$\begin{aligned}
&= (r+c).(c+p').(c+g).(g+p').(r+c+g+p).(r+c+g+p').(r+c'+g+p).(r+c'+g+p') \\
&\quad ) \\
&\text{(Identity)} \\
&= (r+c).(c+p').(c+g).(g+p').((r+g)+(c+p).(c+p')).(r+c'+g+p).(r+c'+g+p') \\
&\text{(Distributive)} \\
&= (r+c).(c+p').(c+g).(g+p').((r+g)+(c+p).(c+p')).(c'+p).(r+c'+g+p') \\
&\text{(Distributive)} \\
&= (r+c).(c+p').(c+g).(g+p').((r+g)+(c+p).(c+p')).(c'+p).(c'+p')) \\
&\text{(Distributive)} \\
&= (r+c).(c+p').(c+g).(g+p').((r+g)+(c+0).(c'+p).(c'+p')) \\
&\text{(Complement)} \\
&= (r+c).(c+p').(c+g).(g+p').((r+g)+c.(c'+p).(c'+p')) \\
&\text{(Identity)} \\
&= (r+c).(c+p').(c+g).(g+p').((r+g)+c.(c'+pp')) \\
&\text{(Distributive)} \\
&= (r+c).(c+p').(c+g).(g+p').((r+g)+c.(c'+0)) \\
&\text{(Complement)} \\
&= (r+c).(c+p').(c+g).(g+p').((r+g)+cc') \\
&\text{(Identity)} \\
&= (r+c).(c+p').(c+g).(g+p').((r+g)+0) \\
&\text{(Complement)} \\
&= (r+c).(c+p').(c+g).(g+p').(r+g) \\
&\text{(Identity)}
\end{aligned}$$

### Circuit

