CmpE 343
Introduction to Probability and Statistics for Computer Engineers
Fall 2019
Homework 3

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Solution 1

$$
\begin{aligned}
f(t) &= a_0 + \sum_n^{\infty} a_n cos(2\pi f_0 nt) + \sum_n^{\infty} b_n sin(2\pi f_0 nt) \\
a_0 &= \frac{1}{T_0}\int_0^{T_0} f(t)dt \\
&= \frac{1}{0.04}\int_0^{0.04} f(t)dt \\
&= \frac{1}{0.04}(\text{area of the first triangle}) \\
&= \frac{1}{0.04}\frac{0.04*1}{2} \\
&= \frac{1}{2} \\
a_n &= \frac{1}{T_0}\int_0^{T_0} x(t)e^{-j\omega_0 nt}dt \\
&= \frac{1}{T_0}\left(\int_0^{T_0/2} te^{-j\omega_0 nt}dt + \int_{T_0/2}^{T_0} -te^{-j\omega_0 nt}dt\right) \\
&= \frac{1}{T_0}\left(\left[\frac{te^{-j\omega_0 nt}}{-j\omega_0 n} - \frac{e^{-j\omega_0 nt}}{(-j\omega_0 n)^2}\right]_0^{T_0/2} - \left[\frac{te^{-j\omega_0 nt}}{-j\omega_0 n} - \frac{e^{-j\omega_0 nt}}{(-j\omega_0 n)^2}\right]_{T_0/2}^{T_0}\right) \\
&= \frac{1}{T_0}\left(\left[\left(\frac{T_0 e^{-j\pi n}}{-2j\omega_0 n} - \frac{e^{-j\pi n}}{(-j\omega_0 n)^2}\right) - \left(\frac{1}{(-j\omega_0 n)^2}\right)\right] - \left[\left(\frac{T_0 e^{-j2\pi n}}{-j\omega_0 n} - \frac{e^{-j2\pi n}}{(-j\omega_0 n)^2}\right) - \left(\frac{T_0 e^{-j\pi n}}{-2j\omega_0 n} - \frac{e^{-j\pi n}}{(-j\omega_0 n)^2}\right)\right]\right) \\
&= \frac{1}{T_0}\left(\left[\left(\frac{T_0(-1)^n}{-2j\omega_0 n} - \frac{(-1)^n}{(-j\omega_0 n)^2}\right) - \left(\frac{1}{(-j\omega_0 n)^2}\right)\right] - \left[\left(\frac{T_0}{-j\omega_0 n} - \frac{1}{(-j\omega_0 n)^2}\right) - \left(\frac{T_0(-1)^n}{-2j\omega_0 n} - \frac{(-1)^n}{(-j\omega_0 n)^2}\right)\right]\right) \\
&= \frac{1}{T_0}\left(\frac{-j\omega_0 nT_0[1-(-1)^n] - 2(-1)^n}{(-j\omega_0 n)^2}\right) \\
&= \frac{T_0^2}{T_0}\left(\frac{-j2\pi n[1-(-1)^n] - 2(-1)^n}{(-j2\pi n)^2}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= T_0\left(\frac{-j2\pi n[1-(-1)^n]-2(-1)^n}{(-j2\pi n)^2}\right)\\
a_{n=2k} &= T_0\left(\frac{-j2\pi n[1-1]-2}{(-j2\pi n)^2}\right)\\
&= T_0\left(\frac{-2}{(-j2\pi n)^2}\right)\\
&= \frac{-2*0.04}{(-j2\pi n)^2}\\
a_{n=2k} &= \frac{-0.08}{(-j2\pi n)^2}\\
a_{n=2k+1} &= T_0\left(\frac{-j2\pi n[1-(-1)]-2(-1)}{(-j2\pi n)^2}\right)\\
&= T_0\left(\frac{-j2\pi n[1+1]+2}{(-j2\pi n)^2}\right)\\
&= T_0\left(\frac{-j4\pi n+2}{(-j2\pi n)^2}\right)\\
a_0 &= 1/2\\
a_{n=2k} &= \frac{-0.08}{(-j2\pi n)^2}\\
a_{n=2k+1} &= \frac{-j0.16\pi n+0.08}{(-j2\pi n)^2}
\end{aligned}
$$

Solution 2

$$F = \frac{S_1^2.\sigma_2^2}{S_2^2.\sigma_1^2}$$
$$F = \frac{S_1^2.4}{S_2^2.5}$$

If $S_1 < S_2$, then $F < \frac{4}{5}$;

$$P(F < \frac{4}{5}) = 1 - f_\alpha(9,4)$$

When we check the table, $\alpha = 0,64$

$$P(F < \frac{4}{5}) = 1 - 0,64$$
$$P(F < \frac{4}{5}) = 0,36$$

Since 0,36 is not that high these sapmles don't support the claim.

Solution 3

$$e = Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$
$$0,01 = Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

When we check the table;

$$P(Z < Z_{\alpha/2}) = 1 - 0,005$$
$$Z_{\alpha/2} = 2,575$$

$\sigma^2 = 1$, $\sigma = 1$;

$$0,01 = 2,575.\frac{1}{\sqrt{n}}$$
$$\sqrt{n} = \frac{2,575}{0,01}$$
$$n = 257,5^2$$
$$n = \lceil 66306,25 \rceil$$
$$n = 66307$$

Solution 4

$$
\begin{aligned}
\bar{X} &= \frac{1}{5}(274.1641 + 237.4870 + 243.3493 + 227.5179 + 237.8315) \\
\bar{X} &= \frac{1}{5}\sum_{i=1}^{5} X_i \\
\bar{X} &= 244.07 \\
S &= \sqrt{\frac{1}{5-1}\sum_{i=1}^{5}(X_i - \bar{X})^2} \\
S &= 17,77 \\
\alpha &= 0,05 \\
t_{\alpha/2} &= 2,776 \\
T &= \frac{\bar{X} - \mu}{S/\sqrt{n}}
\end{aligned}
$$

$$
\begin{gathered}
-t_{\alpha/2} < T < t_{\alpha/2} \\
-t_{\alpha/2} < \frac{\bar{X} - \mu}{S/\sqrt{n}} < t_{\alpha/2} \\
-t_{\alpha/2}\frac{S}{\sqrt{n}} < \bar{X} - \mu < t_{\alpha/2}\frac{S}{\sqrt{n}} \\
-\bar{X} - t_{\alpha/2}\frac{S}{\sqrt{n}} < -\mu < -\bar{X} + t_{\alpha/2}\frac{S}{\sqrt{n}} \\
\bar{X} + t_{\alpha/2}\frac{S}{\sqrt{n}} > \mu > \bar{X} - t_{\alpha/2}\frac{S}{\sqrt{n}} \\
244,07 + 2,776\frac{17,77}{\sqrt{5}} > \mu > 244,07 - 2,776\frac{17,77}{\sqrt{5}} \\
266,13 > \mu > 222,00
\end{gathered}
$$

Since 259 in the range of 266,1 to 222 this random sample support the claim.