CmpE 343
Introduction to Probability and Statistics for Computer Engineers
Fall 2019
Homework 2

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Solution 1

$$\mu = E[X] = \int_0^1 x\frac{6x(x+1)}{5}dx$$
$$\mu = \int_0^1 x\frac{6x(x+1)}{5}dx$$
$$\mu = \frac{6}{5}\left[\frac{x^4}{4} + \frac{x^3}{3}\right]_0^1$$
$$\mu = 0,7$$

$$\sigma^2 = E[X^2] - \mu^2$$
$$\sigma^2 = \int_0^1 x^2\frac{6x(x+1)}{5}dx - \mu^2$$
$$\sigma^2 = \frac{6}{5}\left[\frac{x^5}{5} + \frac{x^4}{4}\right]_0^1 - (0,7)^2$$
$$\sigma^2 = 0,54 - 0,49$$
$$\sigma^2 = 0,05$$
$$\sigma = \frac{1}{2\sqrt{5}}$$

$$P(\mu - \sqrt{5}\sigma < X < \mu - \sqrt{5}\sigma) = P(0,7 - \frac{\sqrt{5}}{2\sqrt{5}} < X < 0,7 + \frac{\sqrt{5}}{2\sqrt{5}})$$
$$P(\mu - \sqrt{5}\sigma < X < \mu - \sqrt{5}\sigma) = P(0,7 - \frac{1}{2} < X < 0,7 + \frac{1}{2})$$
$$P(\mu - \sqrt{5}\sigma < X < \mu - \sqrt{5}\sigma) = P(0,2 < X < 1,2)$$
$$P(\mu - \sqrt{5}\sigma < X < \mu - \sqrt{5}\sigma) = \int_{0,2}^{1,2} \frac{6x(x+1)}{5}dx$$
$$P(\mu - \sqrt{5}\sigma < X < \mu - \sqrt{5}\sigma) = \int_{0,2}^{1} \frac{6x(x+1)}{5}dx + \int_{1}^{1,2} \frac{6x(x+1)}{5}dx$$
$$P(\mu - \sqrt{5}\sigma < X < \mu - \sqrt{5}\sigma) = \frac{6}{5}\left[\frac{x^3}{3} + \frac{x^2}{2}\right]_{0,2}^1 + 0$$
$$P(\mu - \sqrt{5}\sigma < X < \mu - \sqrt{5}\sigma) = 0,9728$$

Solution 2

There are two dice are rolling so sample space has 6.6=36 elements with equal probability of $\frac{1}{36}$.

$$\left.\begin{array}{llll} (1,1) & (2,1) & (3,1) & (4,1) \\ (1,2) & (2,2) & (3,2) & \\ (1,3) & (2,3) & & \\ (1,4) & & & \end{array}\right\} \rightarrow \text{there are 10 cases that X=1; at others X=0}$$

$$\mu_x = \sum_x x.g(x)$$
$$\mu_x = 0.\frac{1}{36}.26 + 1.\frac{1}{36}.10$$
$$\mu_x = \frac{10}{36}$$
$$\mu_x = \frac{5}{18}$$

$$\left.\begin{array}{lll} (1,1) & (3,1) & (5,1) \\ (1,3) & (3,3) & (5,3) \\ (1,5) & (3,5) & (5,5) \end{array}\right\} \rightarrow \text{there are 9 cases that Y=1; at others Y=0}$$

$$\mu_x = \sum_y y.h(y)$$
$$\mu_x = 0.\frac{1}{36}.27 + 1.\frac{1}{36}.9$$
$$\mu_x = \frac{9}{36}$$
$$\mu_x = \frac{1}{4}$$

$$\text{XY} = \begin{cases} 1, & \text{if X=1 and Y=1} \\ 0, & \text{otherwise} \end{cases}$$

$$\left.\begin{array}{ll} (1,1) & (3,1) \\ (1,3) & \end{array}\right\} \rightarrow \text{there are 3 cases that XY=1; at others XY=0}$$

$$E[XY] = \sum_x \sum_y x.y.f(x,y)$$
$$E[XY] = 1.1.\frac{1}{36}.3$$
$$E[XY] = \frac{3}{36}$$
$$E[XY] = \frac{1}{12}$$

$$Cov(X,Y) = E[XY] - \mu_x.\mu_y$$
$$Cov(X,Y) = \frac{1}{12} - \frac{5}{18}.\frac{1}{4}$$
$$Cov(X,Y) = \frac{1}{72}$$

Solution 3

3.a

Geometric Distribution
Since, it seeks for probability of first success of Bernoulli trials occuring at k'th one.

$$g(k;p) = (1-p)^{k-1}.p$$
$$g(20;\frac{7}{25}) = \left(1-\frac{7}{25}\right)^{20-1}.\frac{7}{25}$$
$$g(20;\frac{7}{25}) = \left(\frac{18}{25}\right)^{19}.\frac{7}{25}$$

3.b

Negative Binomial Distribution
Since, it seeks for probability of k'th success of Bernoulli trials occuring at x'th one.

$$b*(x;k,p) = \binom{x-1}{k-1}.p^k.(1-p)^{x-k}$$
$$b*(10;4,\frac{10}{25}) = \binom{10-1}{4-1}.\left(\frac{2}{5}\right)^4.\left(1-\frac{2}{5}\right)^{10-4}$$
$$b*(10;4,\frac{10}{25}) = \binom{9}{3}.\left(\frac{2}{5}\right)^4.\left(\frac{3}{5}\right)^6$$

3.c

Multivariate Hypergeometric Distribution
Since, it seeks for probability of successes in selections, without replacement.
Also selections have 3 outcomes which makes it multivariate.

$$f(x_1,x_2,x_3;a_1,a_2,a3,N,n) = \frac{\binom{x_1}{a_1}.\binom{x_2}{a_2}.\binom{x_3}{a_3}}{\binom{N}{n}}$$
$$f(8,10,7;2,4,4,25,10) = \frac{\binom{8}{2}.\binom{10}{4}.\binom{7}{4}}{\binom{25}{10}}$$

Solution 4

$$
\begin{aligned}
P(A < X < B) &= P(A-\mu < X-\mu < B-\mu) \\
P(A < X < B) &= P(\frac{A-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{B-\mu}{\sigma}) \\
P(A < X < B) &= P(\frac{A-\mu}{\sigma} < Z < \frac{B-\mu}{\sigma}) \\
P(A < X < B) &= 0,95 \\
P(\frac{A-\mu}{\sigma} < Z < \frac{B-\mu}{\sigma}) &= 0,95 \\
P(Z < 1,65) &= 0,9505 \\
P(Z < -3,32) &= 0,0005 \\
P(-3,32 < Z < 1,65) &= 0,9505 - 0,0005 \\
P(-3,32 < Z < 1,65) &= 0,95 \\
P(\frac{A-\mu}{\sigma} < Z < \frac{B-\mu}{\sigma}) &= P(-3,32 < Z < 1,65)
\end{aligned}
$$

$$
\begin{aligned}
\frac{A-\mu}{\sigma} &= -3,32 \\
A &= -3,32.\sigma + \mu \\
A &= -3,32.5 + 63 \\
A &= 46,4
\end{aligned}
$$

$$
\begin{aligned}
\frac{B-\mu}{\sigma} &= 1,65 \\
B &= 1,65.\sigma + \mu \\
B &= 1,65.5 + 63 \\
B &= 71,25
\end{aligned}
$$