TIME AND WORK

Time taken to complete a work and the work force required to complete it are in inverse relation.

In another word, if x men can do a work in y days and p men can do the same work in q days, then we can write as $xy = pq$ or in general, if x men can do y works in z days and p men can do q works in r days, then we can write as $\frac{xz}{y} = \frac{pr}{q} \Rightarrow xqz = pyr$. This relation is known as men-day concept.

Note: Number of works and men or Number of works and days are directly proportional quantities.

UNITARY METHOD

The unitary method is the traditional approach to solve questions relating to Time and Work. Here, the work is defined as follows.

Work done (Total Unit) = LCM of the time taken by each person involved

Application 1

12 men can do a work in 15 days. If 10 men have come to work, how many days are required to complete the work?

Solution

Applying man-day concept

$$\frac{xz}{y} = \frac{pr}{q}$$

Initially 12 men worked for 15 days. Since the work is same, number of days required for 10 men can be found using the above formula.

$12 \times 15 = 10 \times x$

x = 18 days

Remember!

If m_1 and m_2 are number of men required to do a work in d_1 days and d_2 days then $\frac{m_1}{m_2} = \frac{d_2}{d_1}$.

Application 2

12 men can bind 80 note books in 5 days. If 9 men are working for 8 days, find the number of books they can bind.

Solution

12 men $\rightarrow$ 80 note books in 5 days.

Now, 9 men $\rightarrow x$ note books $\rightarrow$ 8 days.

Applying men-day concept

$$\frac{12 \times 5}{80} = \frac{9 \times 8}{x}$$

x = 96 note books

Application 3

24 men can make 12 toys in 2 days working 4 hours a day. Then how many men are required to make 40 toys in 4 days working 5 hours a day?

Solution

24 men $\rightarrow$ 12 toys in 2 days working 4 hrs/day

Now, x men $\rightarrow$ 40 toys in 4 days working 5 hrs/day

$$\frac{24 \times 2 \times 4}{12} = \frac{x \times 4 \times 5}{40}, \quad \text{or } x = 32$$ (converting days in hrs)

Remember!

If a man completes a work in m days, his efficiency can be given in percentage as $\frac{100}{m}$%.

Application 4

A can finish a work in 20 days and B can finish the same work in 10 days. If they work together, find the number of days required to complete the work.

Solution

Total units = 20 units (LCM of 20 and 10)

$$\text{A's 1 day work} = \frac{20 \text{ units}}{20 \text{ days}} = 1 \text{ unit}$$

$$\text{B's 1 day work} = \frac{20 \text{ units}}{10 \text{ days}} = 2 \text{ units}$$

$$(\text{A} + \text{B})\text{'s 1 day work} = (1 + 2) \text{ units}$$

$$\therefore \text{Time taken to complete the work} = \frac{20}{3} = 6\frac{2}{3} \text{ days}$$

Remember!

If a man's efficiency is x % then he will complete the work in $\frac{100}{x}$ days.

Application 5

A can do a piece of work in 10 days and B can do that in 15 days. They both started work together, but after 3 days A left for another job. If B alone stayed and completed the work, how long did the work last?

Solution

Total units = 30 units

$$\text{A's 1 day work} = \frac{30 \text{ units}}{10 \text{ days}} = 3 \text{ units}$$

$$\text{B's 1 day work} = \frac{30 \text{ units}}{15 \text{ days}} = 2 \text{ units}$$

A and B worked together for 3 days.

$$(\text{A} + \text{B})\text{'s 1 day work} = (3 + 2) \text{ units}$$

$$(\text{A} + \text{B})\text{'s 3 days work} = (5 \times 3) = 15 \text{ units}$$

They would have completed 15 units in 3 days.

Remaining units =15

Time taken by B to complete the remaining units $= \frac{15}{2} = 7.5$ days

Application 6

If A and B can do a work in 12 days, B and C in 20 days while C and A in 15 days, how long would B alone take to do the same work?

Solution

Work done = 60 units

$(A+B)'s\ 1\ day\ work = \frac{60\ units}{12\ days} = 5\ units$

$(B+C)'s\ 1\ day\ work = \frac{60\ units}{20\ days} = 3\ units$

$(C+A)'s\ 1\ day\ work = \frac{60\ units}{15\ days} = 4\ units$

$(A+B)+(B+C)+(C+A) \xrightarrow{u/d} 12\ units$

(u/d =units per day)

$2(A+B+C) \xrightarrow{u/d} 12\ units$

$A+B+C \xrightarrow{u/d} 6\ units$

$\therefore B's\ 1\ day\ work = 6-4 = 2\ units$

Time taken by $B = \frac{60}{2} = 30$ days

Application 7

If 5 men or 8 women or 10 boys can do a work in 40 days, how long will 20 men, 48 women and 20 boys take to complete the work?

Solution

5 men or 8 women or 10 boys $\Rightarrow$ 40 days

20 men, 48 women and 20 boys = ?

Work done = 40 units

5 men = 1 u/d 8 women = 1 u/d 10 boys = 1 u/d

then 20 men = 4 u/d 48 women = 6 u/d 20 boys = 2 u/d

together $\Rightarrow$ 12 units/day

So, time taken to complete the work $= \frac{\text{Work done}}{\text{Units / day}}$

$= \frac{40}{12} = 3\frac{4}{12} = 3\frac{1}{3}$ days

Application 8

If 12 men and 10 women can do a piece of work in 5 days and 13 men and 20 women can do the same work in 4 days, how long will 7 men and 5 women take to finish it?

Solution

12 men and 10 women = 5 days

13 men and 20 women = 4 days

Time for 7 men and 5 women = ?

Work done = 20 units

12 M + 10 W = 4 units/day ... (1)

13 M + 20 W = 5 units/day ... (2)

By solving equation (1) and (2)

1 Man = $\frac{3}{11}$ units/day

5 Women = $\frac{4}{11}$ units/day

Now, 7 Men and 5 Women $\Rightarrow \frac{21}{11} + \frac{4}{11} = \frac{25}{11}$ units / days

Time taken to complete the target $= \frac{20}{\frac{25}{11}} = \frac{220}{25} = 8\frac{4}{5}$ days

Remember!

If A's efficiency is a % and B's efficiency is b % then their combined efficiency is (a + b) %

PIPES AND CISTERNS

Pipes and Cisterns problems are similar to time and work problems. A pipe's filling/emptying capacity is inversely proportional to the time it takes to fill or empty the tank.

Inlet Pipe: A pipe used to fill the tank or cistern is known as Inlet Pipe.

Outlet Pipe: A pipe used to empty the tank or cistern is known as Outlet Pipe.

Remember!

If a pipe can fill/empty a tank in n hours, its efficiency can be given in percentage as $\frac{100}{n}$%.

Application 9

A tank is fitted with two inlet taps A and B. A can fill the tank in 5 hours and B can fill in 4 hours. If both the pipes are open, how long will it take to fill the tank completely?

Solution

Total units = 20 units

No. of units filled by A in an hour $= \frac{20\ units}{5\ hrs} = 4\ units$

No. of units filled by B in an hour $= \frac{20\ units}{4\ hrs} = 5\ units$

Together, they will fill 9 units/hr.

Time taken to fill 20 units $= \frac{20}{9} = 2\frac{2}{9}$ hrs

Application 10

The inlet pipe of a tank can fill the tank in 3 hours and its outlet pipe can empty it in 9 hours. If both the pipes are open, how long will it take to empty or fill the tank completely?

Solution

Total units = 9 units

No. of units filled by A in an hour $= \frac{9 \text{ units}}{3 \text{ hrs}} = 3 \text{ units}$

No. of units emptied by B in an hour $= \frac{9 \text{ units}}{9 \text{ hrs}} = 1 \text{ unit}$

No. of units filled when A and B are open together $= 3 - 1 = 2 \text{ units}$

Time taken to fill the tank $= \frac{9}{2} = 4.5 \text{ hrs}$

Application 11

A pipe can fill a tank in 21 hours. Due to a leak in the bottom, it is filled in 24 hours. If the tank is full, in how many hours the tank will be half emptied because of leak?

Solution

Total units = 168 units

No. of units filled by A in an hour $= \frac{168 \text{ units}}{21 \text{ hrs}} = 8 \text{ units}$

No. of units filled by A with the leak in an hour

$= \frac{168 \text{ units}}{24 \text{ hrs}} = 7 \text{ units}$

A	8 u
B	– 1
A – B	7 u

$\therefore$ Leak empties 1 unit in 1 hour.

$\therefore$ Time taken by leak to empty the whole tank $= \frac{168}{1} = 168$ hours

$\therefore$ To empty half the tank $\rightarrow \frac{168}{2} = 84$ hours

Application 12

Two pipes P and Q can fill a tank in 15 minutes and 20 minutes respectively. If both the pipes are opened simultaneously, after how much time should P be closed so that the tank is full in 10 minutes?

Solution

Total units = 60 units

No. of units filled by P in a minute $= \frac{60 \text{ units}}{15 \text{ min}} = 4 \text{ units}$

No. of units filled by Q in a minute $= \frac{60 \text{ units}}{20 \text{ min}} = 3 \text{ units}$

Q is working for 10 minutes.

$\therefore 10 \times 3 = 30 \text{ units}$

Remaining 30 units has to be completed by P in $\frac{30}{4} \text{ min} = 7\frac{2}{4} \text{ min}$

$\Rightarrow$ 7 min 30 sec

$\therefore$ P should be closed after $7\frac{1}{2}$ minutes.

Application 13

A, B and C are three pipes connected to a tank. A and B together fill the tank in 4 hours. B and C together fill the tank in 5 hours. A and C together fill the tank in 3 hours. In how much time will A, B and C fill the tank together?

Solution

Work done = 60 units

$(A+B)\text{'s 1 day work} = \frac{60 \text{ units}}{4 \text{ days}} = 15 \text{ units}$

$(B+C)\text{'s 1 day work} = \frac{60 \text{ units}}{5 \text{ days}} = 12 \text{ units}$

$(C+A)\text{'s 1 day work} = \frac{60 \text{ units}}{3 \text{ days}} = 20 \text{ units}$

$A + B \rightarrow 15 \text{ units / hr}$ ... (1)

$B + C \rightarrow 12 \text{ units / hr}$ ... (2)

$A + C \rightarrow 20 \text{ units / hr}$ ... (3)

Adding (1), (2) and (3),

2(A + B + C) = 47 units/hr

$A + B + C = \frac{47}{2} \text{ units / hr}$

So, no. of hours required by A, B and C to fill the tank

$= \frac{60}{\frac{47}{2}} = \frac{120}{47} = 2\frac{26}{47} \text{ hrs}$

CLASS WORK

1. A man works for 2 days and then rests for a day, then works for 2 days and rests 1 day and so on. For every day he works he is paid Rs.10. He can complete a piece of work in 20 working days. Find the fraction of the work done if he has earned Rs.140.

   (a) $\frac{1}{6}$ (b) 1

   (c) $\frac{7}{10}$ (d) Cannot say

2. A contractor engaged 100 labourers for completing a certain project within a fixed time. When $\frac{3}{5}^{th}$ of the time limit had expired, he found that $\frac{3}{4}^{th}$ of the originally assigned work had been already completed. The number of labourers who could have been removed by the contractor for this project so that the project could be completed by the remaining labourers within the given time limit would be

   (a) 20 (b) 30

   (c) 40 (d) 50

3. 12 monkeys can eat 12 bananas in 12 minutes. In how many minutes can 4 monkeys eat 4 bananas?

   (a) 4 mins (b) 8 mins

   (c) 10 mins (d) 12 mins

4. Gopal has engaged a man to work for 20 days on the condition that for every day he works, he will be paid Rs.20 per day and for every day of his absence, he will be fined Rs.5. If he receives only Rs.275, for how many days was he absent?
(a) 5 (b) 6
(c) 9 (d) 11

5. A group of men decided to do a job in 4 days but 20 men dropped out everyday, the job was completed at the end of the 7^{th} day. Find the men who are in the work initially?
(a) 110 (b) 120
(c) 130 (d) 140

6. 30 men are supposed to do a work in 38 days. After 25 days, 5 more men were employed on work for which the work is completed in 1 day before. If 5 more men were not worked then how many days took in delay?
(a) 1 day (b) 2 days
(c) 3 days (d) 4 days

7. 8 workers can build a wall 18 m long, 2 m broad and 12 m high in 10 days, working 9 hours a day. Find how many workers will be able to build a wall 32 m long, 3 m broad and 9 m high in 8 days working 6 hours a day.
(a) 10 (b) 20
(c) 30 (d) 40

8. A contractor was engaged to construct a road in 16 days. After working for 12 days with 20 labours it was found that only $\frac{5^{th}}{8}$ of the road had been constructed. To complete the work in stipulated time the number of extra labours required is
(a) 4 (b) 8
(c) 16 (d) 24

9. A certain number of men can do a piece of work in 40 days. If there were 45 men more the work could have been finished in 25 days. Find the original number of men employed in the work.
(a) 50 (b) 65
(c) 75 (d) 80

10. Some staff promised to do a job in 18 days, but 6 of them went on leave. So the remaining men took 20 days to complete the job. How many men were there originally?
(a) 40 (b) 60
(c) 80 (d) 90

11. A and B can do a piece of work in 10 days in 15 days. They work for 5 days. The rest of the work had finished by C in 2 days. If they get Rs.1500 for the whole work, the daily wages of B and C are?
(a) Rs.150 (b) Rs.200
(c) Rs.225 (d) Rs.250

12. A sum of money is sufficient to pay A's wages for 21 days and B's wages for 28 days. The same money is sufficient to pay the wages of both for?
(a) 12 days (b) 14 days
(c) $12\frac{1}{4}$ days (d) $24\frac{1}{2}$ days

13. If 6 men and 8 boys can do a piece of work in 10 days while 26 men and 48 boys can do the same in 2 days, the time taken by 15 men and 20 boys in doing the same type of work will be?
(a) 4 days (b) 5 days
(c) 6 days (d) 7 days

14. If 12 men and 2 boys working together can do four times as much work as a man and a boy. Working capacities of a men and a boy are in the ratio?
(a) 2:1 (b) 1:4
(c) 3:2 (d) 5:4

15. A machine P can print one lakh books in 8 hours, machine Q can print the same number of books in 10 hours while machine R can print them in 12 hours. All the machines are started at 9 a.m. While machine P is closed at 11 am and the remaining two machines complete the work. Approximately at what time will the work be finished?
(a) 11:30 am (b) 12:30 pm
(c) 12 noon (d) 1 pm

16. A and B can do a piece of work in 30 days, while B and C can do the same work in 24 days and C and A in 20 days. They all work together for 10 days when B and C leave. How many days more will A take to finish the work?
(a) 18 days (b) 36 days
(c) 24 days (d) 30 days

17. 38 Boys working 6 hours a day can do a piece of work in 12 days. Find the number of days in which 57 Girls working in 8 hours a day can do twice the work. Assume that 2 Boy do as much work in 1 hour as 3 Girl do in 1.5 hours.
(a) 24 days (b) 28 days
(c) 32 days (d) 27 days

18. Vinay is thrice as good a work man as Wilex. If together they can complete a task in 12 days, in how many days can Vinay alone complete it?
(a) 48 (b) 16
(c) 24 (d) 12

19. Ajith working alone can make a cabinet in 12 days. Bala will take 6 days more than Aajith to do the same work. Ajith and Bala along with the help of Charlie completes it in 5 days. If they are paid Rs.9000 for the job, find Charlie's share

(a) Rs.2750 (b) Rs.2500

(c) Rs.2250 (d) Rs.2000

20. Anand and Deepan together can complete a work in 12 hours. Deepan and Elwin together can complete a same work in 15 hours. Elwin and Anand together can complete a same work in 20 hours. In how many hours Anand & Deepan & Elwin together complete the work?

(a) 5 (b) 6

(c) 10 (d) 12

21. Two partners Mahdhav and Nivas can do a work alone in 29 days. Madhav takes the rest of one day after every 4 days and Nivas takes the rest of one day after every 5 days. If Madhav and Nivas starts working together, then the work will be completed on

(a) 15^{th} day (b) 16^{th} day

(c) 17^{th} day (d) 18^{th} day

22. Lokesh would take three times as long as Kailash and Manish together and Manish twice as long as Kailash and Lokesh together to do a certain piece of work. Kailash, Lokesh, and Manish working together can complete the work in 10 days. How long would Kailash take by himself to complete the same piece of work?

(a) 24 days (b) 30 days

(c) 40 days (d) 36 days

23. 5 Workers from Group 1 and 8 workers from Group 2 can do a job in 8 days, while 4 workers from Group 1 and 6 workers from Group 2 can do it in 10 days. How many days will take 10 workers from Group 2 take to finish the job?

(a) 10 days (b) 40 days

(c) 20 days (d) Cannot be determined

24. A pump can fill a tank with water in 4 hours. Because of a leak, it took $5\frac{1}{3}$ hours to fill the tank. The leak can drain all the water of the tank in?

(a) 16 hrs (b) 7 hrs

(c) 8 hrs (d) 14 hrs

25. Three taps A, B and C can fill a tank in 12, 24 and 36 hours respectively. If A is open all the time and B and C are open for one hour each alternately, the tank will be full in:

(a) 6 hours (b) $8\frac{2}{3}$ hours

(c) 7 hours (d) $7\frac{1}{2}$ hours