PROFIT LOSS AND DISCOUNT, PARTNERSHIP & AVERAGE

PROFIT LOSS AND DISCOUNT

Suppose two shopkeepers buy an item at Rs.200 each. One marks the price on the item as Rs.250 but sells it at Rs.240, while the other labels the price as Rs.210 but sells it at Rs.190.

Rs.200 at which the shopkeepers buy the item is called the Cost Price.

Rs.250 and Rs.210 which are the price marked/labelled on the item are called Marked Price or List Price.

Rs.240 and Rs.190 at which the items are sold are called Selling Price.

Excess of selling price over cost price is known as Profit and excess of cost price over the selling price is known as Loss.

Since the first shopkeeper sells at a price higher than the price at which he buys, he makes a profit and profit amount is Rs.40, while the other sells at lower price than the cost price and so he makes a loss equal to Rs.10.

The difference between the marked/list price and the selling price is termed as Discount.

The difference between the marked/list price and the cost price is termed as Mark-up.

Profit, loss and mark-up are ALWAYS expressed as percentage of cost price and discount as percentage of marked price.

Let CP, SP and MP represent cost price, selling price and marked price respectively and P, L, M and D represent profit, loss, mark-up and discount amounts respectively.

Then,

SP = CP + P or CP – L
MP = CP + M
SP = MP – D

If the corresponding lower case letters represent percentage expressed in decimal form, then

p = P/CP
l = L/CP
d = D/MP
m = M/CP

Combining the two results,

SP = CP(1 + p) = CP (1 – l)
= MP(1– d)
MP = CP(1 + m)
CP = SP/(1 + p)
MP = SP/(1 – d)

Application 1

A trader marked up the product by 40% and gave a discount of 30%. What is the profit or loss percent?

Let c be the cost price.

Then, MP = 1.4c and

SP = 0.7 MP

= 0.7 × 1.4c

= 0.98c which is less than c implying a loss.

But SP = CP(1 – *l*) and hence

0.98c = (1 – *l*)c or

L = 0.02 or 2%

So, the final result of the transaction is a loss of 2%. [Answer]

Alternatively, let the cost price be 100.

Then, MP = 140 and

SP = 140 × 0.7

= 98

⇒ a loss of 2 over 100 or 2% loss.

Application 2

A shopkeeper buys two items at Rs.1800 each. One he sells at 20% profit and the other at 15% loss. What is the net profit or loss percent?

Profit = 1800 × 0.2

= 360

Loss = 1800 × 0.15

= 270

Net = 360 – 270

= Rs.90 profit

Loss percent = 90/(2 × 1800)

= 0.025 or 2.5% profit [Answer]

Alternatively, since the cost price is the same, 20% profit and 15% loss would mean a profit of 5% on one item which is equivalent to 2.5% profit on both the items put together.

Application 3

A fruit vendor buys some apples at 6 for Rs.100 and an equal number of a different variety at 4 for Rs.50. If she sells the entire stock at 15 for Rs.200, what is her profit or loss percent?

Let the number of apples of each variety be *n*.

Then, total cost price = {(100*n*/6) + (50*n*/4)}

= (200*n* + 150*n*)/12

= 350*n*/12

Noting that the entire stock is 2*n* apples, the total selling price = 400*n*/15

Since 350/12 < 400/15, the net is loss.

The loss = $(350n/12) - (400n/15)$

$= (1750n - 1600n)/60$

$= 150n/60$

$= 5n/2$

Loss percent = $\{(5n/2)/(350n/12)\} \times 100$

$= 60/7\%$

$= 8\frac{4}{7}\%$ [Answer]

Alternatively, by Speed Maths method

When equal quantities are bought and prices are expressed as certain number for certain amount, calculations can be done faster by taking the common quantity as the LCM of those numbers.

In the present case, the numbers are 6, 4 and 15, LCM of which is 60.

Cost price of 60 apples at 6 for Rs.100 = 1000

Cost price of 60 apples at 4 for Rs.50 = 750

Total cost price of 120 apples = 1750

Selling price of 120 apples at 15 for Rs.200 = 1600

Loss = 1750 – 1600

= 150

Loss percentage = (150/1750) × 100

$= 8\frac{4}{7}\%$

Successive discounts are given on discounted price only.

Application 4

A buys a shirt from a shop offering three successive discounts of 20%, 10% and 5%. B purchases from a shop giving 25%, 5% and 5% successively. Who gets a better deal – A or B?

Let m be the marked price.

For A:

First discounted price = 0.8 *m*

Second discounted price = 0.9 × 0.8*m* = 0.72 *m*

Final discounted price = 0.95 × 0.72*m* = 0.684 *m*

The total discount = 1 – 0.684 = 0.316 or 31.6%

For B:

First discounted price = 0.75 *m*

Second discounted price = 0.95 × 0.75 *m* = 0.7125 *m*

Final discounted price = 0.95 × 0.7125 *m* = 0.676875 *m*

The total discount = 1 – 0.676875 = 0.323125 or 32.31%

Clearly B gets a better deal. [Answer]

Application 5

If A = 40% of 872 and B = 900% of 42, then which of the following MUST be true?

(a) A = B (b) A > B

(c) A < B (d) Cannot say

Since x% of y = y% of x, A = 40% of 872 = 872% of 40 which is clearly smaller than 900% of 42. So, A < B. [Answer]

Application 6

When the price on Tuesday is 60% more than the Monday price and the Wednesday price is 40% more than the Tuesday price, then the Wednesday price is

(a) 100% more than the Monday price

(b) more than double the Monday price

(c) less than double the Monday price

(d) None of the above

The most popular wrong answer would be (a). But the correct answer is (b). 60% over 40% = 1.6 × 1.4 = 2.24. So, the Wednesday price is 124% more than the Monday price ⇒ more than double. [Answer]

Application 7

A theatre owner, keen to enhance ticket sales, reduces the price per ticket by 25%. To ensure no fall in gate collection, the percentage increases required in the number of tickets sold must be

(a) 25% (b) > 25%

(c) < 25% (d) Cannot say

(4) of Guard Against Popular Errors clearly indicates the answer must be (b) [Answer].

[The actual increase required is $33\frac{1}{3}\%$.]

[In general, the percentage increase required in the balancing variable to offset a r% (in decimal form) decrease in the base variable is $\{1/(1 - r)\}$, again in decimal form.]

Application 8

When a shopkeeper gives a discount of 20% over a price which is marked up by 50% over the cost price, the profit percentage is

(a) more than 30% (b) equal to 30%

(c) less than 30% (d) Cannot say

(4) of *Guard Against Popular Errors* clearly indicates the answer must be (c). [Answer]

[In general, with all percentage in decimal form, *m*% mark-up and *d*% discount would give the actual profit percentage as $\{(m - d) - md\}$ = (0.5 – 0.2) – (0.5 × 0.2) = 0.2 or 20% in the above application.]

PARTNERSHIP

A partnership is an association of two or more persons who put their money together in order to carry on a certain business.

The persons are called **partners**.

Partnership is of 2 types

(1) Simple partnership

(2) Compound partnership

Simple Partnership

When investments of all the partners are for the same period, the partnership is called **simple partnership.**

The gain or loss is distributed among the partners in the ratio of their investments.

For example, three persons A, B, C invest Rs.1600, Rs.1800, Rs.2300 respectively in business. Then the profit is divided in the ratio 1600:1800:2300 = 16:18:23

If the profit is Rs.1938, then

$$\text{A's share} = \text{Rs.}1938 \times \frac{16}{57} = \text{Rs.}34 \times 16 = \text{Rs.}544$$

$$\text{B's share} = \text{Rs.}1938 \times \frac{18}{57} = \text{Rs.}34 \times 18 = \text{Rs.}612$$

$$\text{C's share} = \text{Rs.}1938 \times \frac{23}{57} = \text{Rs.}34 \times 23 = \text{Rs.}782$$

SMART Tip

If a partner retires during the one year period – In such case, only the period for which his capital was utilized is taken into consideration.

Application

P and Q started a business investing Rs.85000 and Rs.15000 respectively. In what ratio is the profit earned after 2 years be divided between P and Q?

Solution

P's investment = Rs.85000 for 2 years

Q's investment = Rs.15000 for 2 years

∴ P's share of profit : Q's share of profit = 85000 : 15000

= 17 : 3

Compound Partnership

When investments of the partners are for different time periods, the partnership is called **compound partnership.**

Here equivalent capitals are calculated for a certain unit of time by taking (capital × number of units of time). Now the gain or loss is divided in the ratio of these capitals.

For example, A, B and C enter into partnership. A advances Rs.1200 for 4 months, B advances Rs.1400 for 8 months and C, Rs.1000 for 10 months and they gain Rs.585 altogether.

Here the capitals are, Rs.1200 × 4, Rs.1400 × 8 and Rs.1000 × 10 or Rs.4800, Rs.11200 and Rs.10000

So, the profit should be divided in the ratio of 4800:11200:10000 = 48:112:100 = 12:28:25

$$\text{A's share} = \text{Rs.}585 \times \frac{12}{65} = \text{Rs.}9 \times 12 = \text{Rs.}108$$

$$\text{B's share} = \text{Rs.}585 \times \frac{28}{65} = \text{Rs.}9 \times 28 = \text{Rs.}252$$

$$\text{C's share} = \text{Rs.}585 \times \frac{25}{65} = \text{Rs.}9 \times 25 = \text{Rs.}225$$

Working or Sleeping Partners

A partner who manages the business is known as a **working partner** and the one who simply invests the money is a **sleeping partner.**

SMART Tip

Three partners invested their capitals in a business. If the timing of their investments is in the ratio $t_1 : t_2 : t_3$ and their profits are in the ratio $P_1 : P_2 : P_3$, then the ratio of their capitals invested is $\frac{P_1}{t_1} : \frac{P_2}{t_2} : \frac{P_3}{t_3}$.

AVERAGE

- The average of n quantities of the same kind is equal to the sum of all the quantities divided by the number of quantities;
- $\text{Average} = \frac{\text{Sum of quantities}}{\text{Number of quantites}}$
- Sum of quantities = Average ∗ Number of quantities
- $\text{Number of quantities} = \frac{\text{Sum of quantities}}{\text{Average}}$
- Average of two or more than two groups;
- If the number of quantities in two groups are n_1 and n_2 respectively and their individual average is X and Y respectively, the combined average of the two groups is given by;

- $= \frac{n1X - n2Y}{n1 = n2}$
- If the average of n_1 quantities is X and the average of n_2 quantities out of n_1 quantities is Y, the average of the remaining quantities is given by;
- $= \frac{n1X - n2Y}{n1 - n2}$
- The average of n quantities is equal to X. One quantity of value P is replaced with a new quantity having value Q the average of quantities becomes Y. Then Q is given by;
- Q = P + n (Y – X)
- The average of n quantities is equal to X. If we remove one quantity the average becomes Y, the value of the removed quantity is given by;
- = n(X – Y) + Y
- And, if the average of n quantities is equal to X and on adding a new quantity the average becomes Y, the value of new quantity is given by;
- = n(Y – X) + Y

Average of numbers:

Natural numbers:

- The average of first n consecutive natural numbers is given by; $= \frac{n + 1}{2}$
- The average of square of first n consecutive natural numbers is given by; $= \frac{(n + 1)(2n + 1)}{6}$
- The average of cubes of first n consecutive natural numbers is given by; $= \frac{n(n + 1)^2}{4}$

Even numbers:

- The average of first n consecutive even numbers is given by; = n + 1
- Also, the average of first n consecutive even numbers starting from 2 to X, where the last even number is X, is given by; $= \frac{X + 2}{2}$
- The average of square of first n consecutive even numbers is given by; $= \frac{2(n + 1)(2n + 1)}{3}$
- Also, the average of square of first n consecutive even numbers starting from 2 to X, where the last even number is X, is given by; $= \frac{(X + 1)(X + 2)}{3}$

Odd numbers:

- The average of first n consecutive odd numbers is equal to n.
- Also, the average of first n consecutive odd numbers starting from 1 to X, where the last odd number is X is given by; $= \frac{X + 1}{2}$
- The average of square of first n consecutive odd numbers starting from 1 to X, where the last odd number is X, is given by; $= \frac{X(X + 2)}{3}$

CLASS WORK

1. The cost price of item B is Rs.200/- more than the cost price of item A. Item A was sold at a profit of 20% and item B was sold at a loss of 30%. If the respective ratio of selling prices of items A and B is 6:7, what is the cost price of item B?
   (a) Rs.520 (b) Rs.430
   (c) Rs.400 (d) Rs.360

2. A dealer offers a cash discount of 20% and still makes a profit of 20%, when he further allows 16 articles to a dozen to a particularly sticky bargainer. How much per cent above the cost price were his wares listed?
   (a) 100% (b) 80% (c) 75% (d) 85%

3. The percentage profit earned by selling an article for Rs.2120 is equal to the percentage loss incurred by selling the same article for Rs.1520. At what price should the article be sold to make 25% profit?
   (a) Rs.2275 (b) Rs.2100
   (c) Rs.2650 (d) Rs.2400

4. In a certain store, the profit is 270% of the cost. If the cost increases by 30% but the selling price remains constant, approximately what percentage of the selling price is the profit?
   (a) 68% (b) 72% (c) 50% (d) 65%

5. A person X sold an Item to Y at 40% loss, then Y sold it to third person Z at 40% profit and finally Z sold it back to X at 40% profit. In this whole process what is the percentage loss or profit of X?
   (a) 70% (b) 62.5% (c) 57.6% (d) 55%

6. Navya buys a certain number of toys at 12 per Rs.9 and the same number at 18 per Rs.9. If she sells them at 18 per Rs.15 does she gain or lose and by what percentage?
   (a) 33 (1/3)% loss (b) 12% gain
   (c) 33 (1/3)% gain (d) 12% loss

7. The marked price of an article is 30% higher than the cost price. If a trader sells the articles allowing 10% discount to customer, then the gain percent will be
   (a) 17 (b) 20 (c) 19 (d) 15

8. A merchant marked the price of an article by increasing its production cost by 40%. Now he allows 20% discount and gets a profit of Rs.48 after selling it. The production cost is
(a) Rs.320 (b) Rs.360
(c) Rs.400 (d) Rs.440

9. A person sells two fans for Rs.6800. The cost price of the first fan is equal to the selling price of the second fan. If the first is sold at 30% loss and the second at 100% gain, what is total profit or loss (in rupees)?
(a) 750 (b) 800 (c) 670 (d) 580

10. A dealer marked the price of an item 20% above cost price. He allowed two successive discounts of 20% and 25% to a customer. As a result he incurred a loss of Rs.1400. At what price (in rupees) did he sell the item to the customer?
(a) 3600 (b) 4200 (c) 3850 (d) 4125

11. A and B enter into a partnership. A contributes Rs.8000 and B contributes Rs.10000. At the end of six months they introduce C who contributes Rs.6000. After the lapse of three years they find that the firm has made a profit of Rs.9660. Find the shares of each (in Rupees).
(a) 3360, 4200, 2100 (b) 3000, 4000, 2000
(c) 2050, 1000, 2500 (d) 2800, 3500, 2100

12. Kavitha and Sunitha are partners in a business. Kavita invests Rs.35000 for 8 months and Sunitha invests Rs.42000 for 10 months. Out of a profit of Rs.31570, Kavita's share is
(a) Rs.15300 (b) Rs.14200
(c) Rs.13486 (d) Rs.12628

13. A, B and C enter into partnership. A contributes 1/3 of the capital while B contributes as much as A and C together contribute. If the profit at the end of the year amounts to Rs.840. What would each receive? (in Rupees)
(a) 160,180, 240 (b) 100, 250, 300
(c) 150, 200, 350 (d) 280, 420, 140

14. A, B and C start a business each investing Rs.20000. After 5 months A withdrew Rs.5000, B withdrew Rs.4000 and C invests Rs.6000 more. At the end of the year, a total profit of Rs.69900 was recorded. Find the share of each.
(a) Rs.20500, Rs.21200, Rs.28200
(b) Rs.20000, Rs.21000, Rs.28000
(c) Rs.21200, Rs.20000, Rs.28000
(d) Rs.20000, Rs.21000, Rs.28000

15. Vinoth started a business by investing Rs.36000. After 3 months Sofia joined him by investing Rs.36000. Out of an annual profit of Rs.37100, find the share of each.
(a) Rs.20000, Rs.15000 (b) Rs.21200, Rs.15900
(c) Rs.16500, Rs.19450 (d) Rs.22500, Rs.15600

16. Kamal and Sam entered into partnership with capitals in the ratio 4:5. After 3 months, Kamal withdrew $\frac{1}{4}$ of his capital and Sam withdrew $\frac{1}{5}$ of his capital. The gain at the end of 10 months was Rs.760. What is Kamal's share of profit?
(a) Rs.300 (b) Rs.500
(c) Rs.450 (d) Rs.330

17. The average of 5 consecutive odd numbers is 61. Find the difference between the highest and the lowest numbers.
(a) 2 (b) 5 (c) 8 (d) 10

18. The average of 6 numbers is 3.95. The average of 2 of them is 3.4, the average of the other two is 3.85. Find the average of the remaining 2 numbers.
(a) 4.5 (b) 4.6 (c) 4.7 (d) 4.8

19. Of the three numbers, the average of the first and the second is greater than the average of the second and the third by 15, find the difference between the first and the third of the three numbers.
(a) 15 (b) 45 (c) 60 (d) 30

20. The mean of 50 observations was 36. It was found later that an observation 48 was wrongly taken as 23. Find the corrected new mean.
(a) 35.2 (b) 36.1 (c) 36.5 (d) 39.1

21. The average of 15 numbers is 54. The average of the first 8 numbers is 64 and the average of the last 8 numbers is 60. Find the 8^{th} number.
(a) 182 (b) 152 (c) 214 (d) 180

22. A motorist travels to a place 150 km away at an average speed of 50 km/hr and returns at 30 km/hr. Find his average speed for the whole journey.
(a) 35 km/hr (b) 37 km/hr
(c) 37.5 km/hr (d) 40 km/hr

23. The average salary of all the workers in a workshop is Rs.8000. The average salary of 7 technicians is Rs.12000 and the average salary of the rest is Rs.6000. Find the total number of workers in the workshop.
(a) 20 (b) 21 (c) 22 (d) 23

24. The average age of the students of a class is 15.8 years. The average age of boys in the class is 16.4 years and that of girls is 15.4 years. Find the ratio of the number of boys to the number of girls.
(a) 1:2 (b) 2:3 (c) 3:4 (d) 3:5

25. The average age of husband, wife and their child 3 years ago were 27 years and that of wife and the child 5 years ago was 20 years. Find the present age of the husband.
(a) 35 years (b) 40 years
(c) 50 years (d) 45 years