PERMUTATION AND COMBINATION & PROBABILITY

PERMUTATION AND COMBINATION

1. Factorial:

The factorial can be defined as the product of the number (for which we have to find factorial) by its successor till it reaches to one.

We can write it as, **n! (Factorial of n) = n (n-1) (n-2)....1**

For example: The factorial of 3:

3! = 3*2*1 = 6

Note: The factorial of zero (0) is always 1 because an empty set can arrange in one way only.

2. Permutation: It refers to the number of ways a particular set can be arranged, where order of the arrangement matters. A combination lock can be called a permutation lock.

For example:

(i) Let we have three letters a, b, and c and we have to arrange two letters at a time.

So, in this case, the permutations of the two letters = ab, ba, bc, cb, ac, and ca.

(ii) If we have to arrange all letters (a,b,(c) simultaneously, the permutation would be: abc, acb, bac, bca, cab, and cba.

Formula for calculating number of possible permutations of things, from a set of n at a time is as follows:

$$ {}^nP_r = n(n-1)(n-2)(n-3)\dots(n-r+1) = \frac{n!}{(n-r)!} $$

For example:

(i) $${}^8P_3 = \frac{8!}{(8-3)!} = \frac{8*7*6*5}{5!} = (8*7*6) = 336$$

(ii) $${}^7P_5 = \frac{7!}{(7-5)!} = \frac{7!}{2!} = \frac{7*6*5*4*3*2!}{2!} = 2520$$

(iii) The number of permutations or arrangements of all n things at a time = n! (Factorial of n).

i.e., n = 3, so 3! = 3*2*1 = 6 (the number of permutations = 6)

3. Combinations:

It refers to the number of ways a particular set can be arranged, where order of the arrangement does not matter which means for a combination of the n number of things there may be different orders.

Formula for calculating the possible combination for r things, from a set of n objects at a time is as follows:

$$ {}^nC_r = \frac{n!}{r!(n-r)!} = \frac{n(n-1)(n-2)\dots\dots\dots(n-r+1)}{r!} $$

Note:

(i) ${}^nC_n = 1$

(ii) ${}^nC_0 = 1$

(iii) ${}^nC_r = {}^nC_{(n-r)}$

For examples:

(i) $${}^8C_3 = \frac{8!}{3!(8-3)!} = \frac{8*7*6*5!}{3!5!} = \frac{8*7*6}{3*2*1}$$

$$= (8*7) = 56$$

Or, $${}^8C_3 = {}^8C_{(8-3)} = {}^8C_5 = \frac{8*7*6*5*4}{5*4*3*2*1} = 8*7 = 56$$

(ii) $${}^7C_5 = {}^7C_{(7-5)} = {}^7C_2 = \frac{7*6}{2*1} = 21$$

Note:

- When the number of things is x, y, and z then the number of combinations taking two at a time will be xy, yz, and zx.

PROBABILITY

1. Probability:

It is the quantitative measure of the chance of occurrence of a particular event. It tells about the chance or likelihood of the occurrence of a particular event. It can be expressed as proportions that range from 0 to 1, or can also be expressed as percentages that range from 0% to 100%. For example, a probability of 40 % (0.40) indicates that there are 40 chances out of 100 of the occurrence of an event.

2. Experiment:

It is an operation that can be infinitely repeated and has well-defined set of possible outcomes, which is known as a sample space. Each outcome is known as an event. For example, tossing a coin is an experiment that produces two well-defined outcomes: Head and Tail.

3. Random Experiment:

It refers to an experiment whose possible outcomes are known but the exact outcome cannot be predicted in advance. For example:

(i) Tossing of a coin:

When we toss a coin, the outcome will be either (H) or Tail (T)

(ii) Throwing a dice:

A dice has six faces, each with a different number of dots from one to six. When a dice is thrown, any number from one to six can appear on its upper face. So the outcome can be 1 or 2 or 3 or 4 or 5 or 6.

(iii) Drawing a card from a pack of 52 playing cards:

A pack of playing cards has 52 cards which are divided into four categories which are as follows:

- Spades
- Clubs
- Hearts
- Diamonds

Each category has 13 cards out of which nine cards are numbered from 2 to 10; remaining cards include an Ace, a King, a Queen, and a Jack.

The hearts and diamonds are red in colour, whereas spades and clubs are black in colour. Furthermore, kings, queens, and jacks are called face cards.

(iv) Taking a ball randomly from a bag that contains balls of different colours.

4. Sample Space: It refers to the all possible outcomes of an experiment. It is denoted by S.

For example:

When a coin is tossed, the possible outcomes include Head and Tail. So, S in this case, = (H, T)

When two coins are tossed, there are four possible outcomes, i.e. S= (HH, HT, TH, TT)

When a dice is thrown, there are six possible outcomes, i.e. S = (1, 2, 3, 4, 5, and 6)

5. Event:

It refers to a subset of a Sample Space. It is generally denoted by a capital letter "E". For example:

(a) When a coin is tossed, the outcome head or tail is called an event. Total number of events in this case, n (E) =2 (head and tail).

(b) When a dice is rolled, the outcome 1 or 2 or 3 or 4 or 5 or 6 is an event. Total number of events in this case, n (E) =6 (1 to 6).

6. Probability of an Event

Let E be an event and S is the sample space. Then the probability of the event E is given by:

$$P(E) = \frac{n(E)}{n(S)}$$

P (E) = Probability of an Event

n (E) = number of ways in which an event can occur

n (S)= Total number of possible outcomes

Example: Let us find out the probability of getting Head when a coin is tossed once.

Total number of possible outcomes = n (S) = 2 (head or tail)

Total number of ways in which the event can occur = n (H) = 1

So, P (E) = n (E)/n(S) = 1/2 or 50%

7. Equally Likely Events:

The events in which there is no preference for a particular event over the other are known as equally likely events.

Examples:

(I) When a coin is tossed, the head and tail are equally likely events.

(II) When a dice is thrown, all the six outcomes (1, 2, 3, 4, 5, and 6) are equally likely to occur, so they are equally likely events.

8. Mutually Exclusive Events:

The two or more events in which the occurrence of one of the events excludes the occurrence of the other event are known as the mutually exclusive events.

For example:

(i) When a coin is tossed, the outcome is head or tail. Head and tail cannot appear simultaneously. So, in this case, the occurrence of Head and Tail are mutually exclusive events.

(ii) When a dice is rolled, all the numbers cannot appear simultaneously, so they are mutually exclusive events.

(iii) Let a dice is thrown and A be the event of getting 2 or 4 or 6 and B be the event of getting 4 or 5 or 6. Then

A = (2, 4, 6) and B = (4, 5, 6)

So, A and B are not mutually exclusive events, as 4 and 6 are present in both the events.

9. Independent Events:

The events in which the occurrence or non-occurrence of one event does not influence the occurrence or non-occurrence of the other event.

For example: When a coin is tossed twice, the event of getting Head (H) in the first toss and the event of getting Head (H) in the second toss are independent events. This is due to the fact that the occurrence of getting Head (H) in the first toss does not influence the occurrence of getting Head (H) in the second toss.

10. Simple Events:

It refers to the events where one experiment happens at a time and it has a single outcome. The probability of simple events is denoted by P (E) where E is the event. In the case of simple events, we consider the probability of occurrence of single events.

For example:

(i) Probability of getting a tail (T) when a coin is tossed.

(ii) Probability of getting 6 when a dice is thrown.

11. Compound Event:

It refers to an event in which there is more than one possible outcome. In other words, the event in which we take the probability of the joint occurrence of two or more events is known as a compound event.

For example:

(i) When two coins are tossed, the probability of joint occurrence of Head (H) in the one coin and Tail (T) in another coin is a compound event.

12. Exhaustive Events:

The mutually exclusive events that form the sample space collectively are called the exhaustive events. For example, when a coin is tossed, either Head or Tail appears and they collectively form the sample space. So, there are two exhaustive events.

13. Algebra of Events

Let A and B are two events and S is the sample space when a dice is thrown. Then

Let A = (2, 4, 6) and B = (4, 5, 6), then:

(i) $A \cup B$ is the event in which either A or B or both A and B occur. For example, $A \cup B = (2, 4, 5, 6)$

(ii) $A \cap B$ is the event in which both A and B occurs. For example, $A \cap B = (4, 6)$

(iii) $\bar{A}$ is the event in which A does not occur. For example, $\bar{A} = (1, 3, 5)$

(iv) $\bar{B}$ is the event in which B does not occur. For example, $\bar{B} = (1, 2, 3)$

(v) $\bar{A} \cap \bar{B}$ is the event in which none of A and B occurs. $\bar{A} \cap \bar{B} = (1, 3)$

14. Additional Theorem

Let A and B are two events associated with a random experiment. Then

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$, if $P(A \cap B) \neq 0$

If A, and B are mutually exclusive events, $P(A \cup B) = P(A) + P(B)$ as $P(A \cap B) = 0$ for mutually exclusive events.

CLASS WORK

1. Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?

(a) 25200 (b) 52000
(c) 120 (d) 24400

2. In how many different ways can the letters of the word 'LEADING' be arranged in such a way that the vowels always come together?

(a) 520 (b) 720
(c) 700 (d) 750

3. A committee of 5 persons is to be formed from 6 men and 4 women. In how many ways can this be done when at least 2 women are included?

(a) 196 (b) 186
(c) 190 (d) 200

4. How many arrangements can be made out of the letters of the word COMMITTEE, taken all at a time, such that the four vowels do not come together?

(a) 216 (b) 45360
(c) 1260 (d) 43200

5. A college has 10 basketball players. A 5-member team and a captain will be selected out of these 10 players. How many different selections can be made?

(a) 1260 (b) 1400
(c) 1250 (d) 1600

6. In how many ways can a group of 5 men and 2 women be made out of a total of 7 men and 3 women?

(a) 63 (b) 64
(c) 125 (d) 135

7. In how many different ways can the letters of the word 'OPTICAL' be arranged so that the vowels always come together?

(a) 360 (b) 120
(c) 700 (d) 720

8. 12 people at a party shake hands once with everyone else in the room. How many handshakes took place?

(a) 64 (b) 66
(c) 72 (d) 76

9. If the letters of the word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word 'SACHIN' appears at serial number:

(a) 600 (b) 601
(c) 602 (d) 603

10. How many 4-letter words with or without meaning, can be formed out of the letters of the word, 'LOGARITHMS', if repetition of letters is not allowed?

(a) 4050 (b) 3600
(c) 1200 (d) 5040

11. How many 7 digit numbers can be formed using the digits 1, 2, 0, 2, 4, 2, 4?

(a) 120 (b) 240
(c) 360 (d) 424

12. In how many different ways can the letters of the word 'DETAIL' be arranged in such a way that the vowels occupy only the odd positions?
(a) 25 (b) 36
(c) 42 (d) 120

13. If the letters of the word CHASM are rearranged to form 5 letter words such that none of the word repeat and the results arranged in ascending order as in a dictionary what is the rank of the word CHASM?
(a) 24 (b) 32
(c) 36 (d) 72

14. In how many ways can 5 different toys be packed in 3 identical boxes such that no box is empty, if any of the boxes may hold all of the toys?
(a) 24 (b) 25
(c) 36 (d) 72

15. How many ways can 10 letters be posted in 5 post boxes, if each of the post boxes can take more than 10 letters?
(a) 5^{10} (b) 10^5
(c) ${}^5P_5$ (d) ${}^5C_5$

16. A bag contains 6 white and 4 black balls. 2 balls are drawn at random. Find the probability that they are of same colour.
(a) 1/2 (b) 7/15
(c) 8/15 (d) 1/9

17. Two cards are drawn at random from a pack of 52 cards. What is the probability that either both are black or both are queen?
(a) 52/221 (b) 55/190
(c) 55/221 (d) 19/221

18. A bag contains 4 white, 5 red and 6 blue balls. Three balls are drawn at random from the bag. The probability that all of them are red, is:
(a) 2/91 (b) 1/22
(c) 3/22 (d) 2/77

19. In a lottery, there are 10 prizes and 25 blanks. A lottery is drawn at random. What is the probability of getting a prize?
(a) 2/7 (b) 5/7
(c) 1/5 (d) 1/2

20. Two dice are tossed. The probability that the total score is a prime number is:
(a) 5/12 (b) 1/6
(c) 1/2 (d) 7/9

21. From a pack of 52 cards, two cards are drawn together at random. What is the probability of both the cards being kings?
(a) 1/15 (b) 1/221
(c) 25/57 (d) 35/256

22. In a box, there are 8 red, 7 blue and 6 green balls. One ball is picked up randomly. What is the probability that it is neither red nor green?
(a) 1/3 (b) 3/5
(c) 8/21 (d) 7/21

23. One card is drawn at random from a pack of 52 cards. What is the probability that the card drawn is a face card (Jack, Queen and King only)?
(a) 3/13 (b) 1/13
(c) 3/52 (d) 9/52

24. A bag contains 6 black and 8 white balls. One ball is drawn at random. What is the probability that the ball drawn is white?
(a) 3/7 (b) 4/7
(c) 1/8 (d) ¾

25. Two cards are drawn together from a pack of 52 cards. The probability that one is a spade and one is a heart, is:
(a) 3/20 (b) 29/34
(c) 47/100 (d) 13/102