Solution - Exercise [2]

Introduction to Computer Graphics - B-IT Master Course

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Exercise 1

Given are two points p1,p2 on the unit sphere in ?3. Using a quaternion the point p1 is to be rotated onto the point p2.

a. Give a formula to determine the angle of rotation α

$$\alpha = \arccos\{\frac{q_1.q_2}{|q_1|.|q_2|}\}$$

b. Give a formula to detemine the rotation axis v

$$v = q_1.q_2$$

c. Write down the quaternion q which performs the rotation with angle α around v

$$a = rotationangle$$
$$x, y, z = rotationaxis$$
$$q = \cos(\frac{a}{2}) + i(x\sin\frac{a}{2}) + j(y\sin\frac{a}{2}) + k(z\sin\frac{a}{2})$$

d. Write down the relationship between p1 and p2 using quaternion multiplication

$$q_1 = q_{10} + \mathbf{i}q_{11} + \mathbf{j}q_{12} + \mathbf{k}q_{13}$$
$$q_2 = q_{20} + \mathbf{i}q_{21} + \mathbf{j}q_{22} + \mathbf{k}q_{23}$$

$$q_1 \times q_2 = (q_{10}q_{20} - q_{11}q_{21} - q_{13}q_{23} - q_{14}) + \mathbf{i}(q_{11}q_{20} + q_{10}q_{21} + q_{13}q_{23} - q_{14}q_{22})$$
$$+\mathbf{j}(q_{10}q_{22} - q_{11}q_{23} + q_{12}q_{20} + q_{13}q_{21}) + \mathbf{k}(q_{10}q_{23} + q_{11}q_{22} - q_{12}q_{21} + q_{13}q_{20})$$

Exercise 2

Given a point p ϵ $\mathbb{R}$ 3 in homogenous coordinates (x y z 1)$^{\mathrm{T}}$:

Rotation Matrix

$$R_z = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) & 0 & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Translation Matrix

$$T_t = \begin{pmatrix} 1 & 0 & 0 & t_1 \\ 0 & 1 & 0 & t_2 \\ 0 & 0 & 1 & t_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

a. Derive a matrix M_1 which first rotates the point α degrees (α is given in radians) around the axis (0 0 1)T and then performs a translation with an offset of (t1 t2 t3)$^{\mathrm{T}}$

$$M_1 = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) & 0 & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & t_1 \\ 0 & 1 & 0 & t_2 \\ 0 & 0 & 1 & t_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) & 0 & t_1\cos(\alpha) - t_2\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) & 0 & t_1\sin(\alpha) + t_2\cos(\alpha) \\ 0 & 0 & 1 & t_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

b. Derive a matrix M_2 which first performs a translation with an offset of (t1 t2 t3)$^{\mathrm{T}}$ and then rotates the point α degrees (α is given in radians) around the axis (0 0 1)$^{\mathrm{T}}$
s

$$M_2 = \begin{pmatrix} 1 & 0 & 0 & t_1 \\ 0 & 1 & 0 & t_2 \\ 0 & 0 & 1 & t_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) & 0 & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) & 0 & t_1 \\ \sin(\alpha) & \cos(\alpha) & 0 & t_2 \\ 0 & 0 & 1 & t_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$