

Solution - Exercise [2]

Introduction to Computer Graphics - B-IT Master Course

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Exercise 1

Given are two points p_1, p_2 on the unit sphere in \mathbb{R}^3 . Using a quaternion the point p_1 is to be rotated onto the point p_2 .

- a. Give a formula to determine the angle of rotation α

$$\alpha = \arccos \left\{ \frac{q_1 \cdot q_2}{|q_1| \cdot |q_2|} \right\}$$

- b. Give a formula to determine the rotation axis v

$$v = q_1 \cdot q_2$$

- c. Write down the quaternion q which performs the rotation with angle α around v

$$\begin{aligned} a &= \text{rotationangle} \\ x, y, z &= \text{rotationaxis} \\ q &= \cos\left(\frac{a}{2}\right) + i\left(x \sin \frac{a}{2}\right) + j\left(y \sin \frac{a}{2}\right) + k\left(z \sin \frac{a}{2}\right) \end{aligned}$$

- d. Write down the relationship between p_1 and p_2 using quaternion multiplication

$$\begin{aligned} q_1 &= q_{10} + \mathbf{i}q_{11} + \mathbf{j}q_{12} + \mathbf{k}q_{13} \\ q_2 &= q_{20} + \mathbf{i}q_{21} + \mathbf{j}q_{22} + \mathbf{k}q_{23} \end{aligned}$$

$$\begin{aligned} q_1 \times q_2 &= (q_{10}q_{20} - q_{11}q_{21} - q_{13}q_{23} - q_{14}q_{24}) + \mathbf{i}(q_{11}q_{20} + q_{10}q_{21} + q_{13}q_{23} - q_{14}q_{22}) + \\ &\quad \mathbf{j}(q_{10}q_{22} - q_{11}q_{23} + q_{12}q_{20} + q_{13}q_{21}) + \mathbf{k}(q_{10}q_{23} + q_{11}q_{22} - q_{12}q_{21} + q_{13}q_{20}) \end{aligned}$$

Exercise 2

And some more text ...