Solution - Exercise [2]

Introduction to Computer Graphics - B-IT Master Course

[Melisa Cecilia] [Duy Khanh Gian] [Chenyu Zhao]

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Exercise 1

Given are two points p1,p2 on the unit sphere in ?3. Using a quaternion the point p1 is to be rotated onto the point p2.

a. Give a formula to determine the angle of rotation α

$$\alpha = \arccos\{\frac{p_1.p_2}{\|p_1\|.\|p_2\|}\}$$

b. Give a formula to detemine the rotation axis v

$$v = p_1 \times p_2$$

c. Write down the quaternion q which performs the rotation with angle α around v

$$a = \text{rotation angle}$$
$$v = \text{rotation axis}$$
$$q = \cos(\frac{a}{2}) + i(x \sin\frac{a}{2}) + j(y \sin\frac{a}{2}) + k(z \sin\frac{a}{2})$$

d. Write down the relationship between p1 and p2 using quaternion multiplication

$$(0, p_2) = q(0, p_1)q^{-1}$$

Exercise 2

Given a point p ϵ $\mathbb{R}$ 3 in homogenous coordinates (x y z 1)$^{\mathrm{T}}$:

Rotation Matrix

$$R_z = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) & 0 & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Translation Matrix

$$T_t = \begin{pmatrix} 1 & 0 & 0 & t_1 \\ 0 & 1 & 0 & t_2 \\ 0 & 0 & 1 & t_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

a. Derive a matrix M_1 which first rotates the point α degrees (α is given in radians) around the axis (0 0 1)T and then performs a translation with an offset of (t1 t2 t3)$^{\mathrm{T}}$

$$M_1 = \begin{pmatrix} 1 & 0 & 0 & t_1 \\ 0 & 1 & 0 & t_2 \\ 0 & 0 & 1 & t_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) & 0 & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) & 0 & t_1 \\ \sin(\alpha) & \cos(\alpha) & 0 & t_2 \\ 0 & 0 & 1 & t_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

b. Derive a matrix M_2 which first performs a translation with an offset of (t1 t2 t3)$^{\mathrm{T}}$ and then rotates the point α degrees (α is given in radians) around the axis (0 0 1)$^{\mathrm{T}}$

$$M_2 = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) & 0 & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & t_1 \\ 0 & 1 & 0 & t_2 \\ 0 & 0 & 1 & t_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) & 0 & t_1\cos(\alpha) - t_2\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) & 0 & t_1\sin(\alpha) + t_2\cos(\alpha) \\ 0 & 0 & 1 & t_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

c. In which way affects the order of operations the respective final transformation matrix in this case

The order matters when you need to apply rotation and translation to a Matrix. This transformation will involves rotation which is generally not commutable. Translation itself is commutable since its basically addition of vectors so the order doesn't matter. But rotation matrix is generally not commutable. This is why the result of doing translation first rotation second and rotation first translation second will be different.