BOOLEAN ALGEBRA

*In 1854 George Boole introduced a systematic treatment of logic and developed for this purpose an algebraic system known as **symbolic logic**, or **Boolean algebra**.*

What is Boolean Algebra?

Boolean Algebra

- is algebra for the manipulation of objects that can take on only two values, typically **true** and **false**.
- It is common to interpret the digital value **0 as false** and the digital value **1 as true**.

Boolean Arithmetic

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = 1$$

Remember that in the world of Boolean algebra, there are only two possible values for any quantity and for any arithmetic operation: 1 or 0.

There is no such thing as “2” within the scope of Boolean values. Since the sum “1 + 1” certainly isn’t 0, it must be 1 by process of elimination.

What is OR GATE?

or gate

$$0 + 0 = 0$$

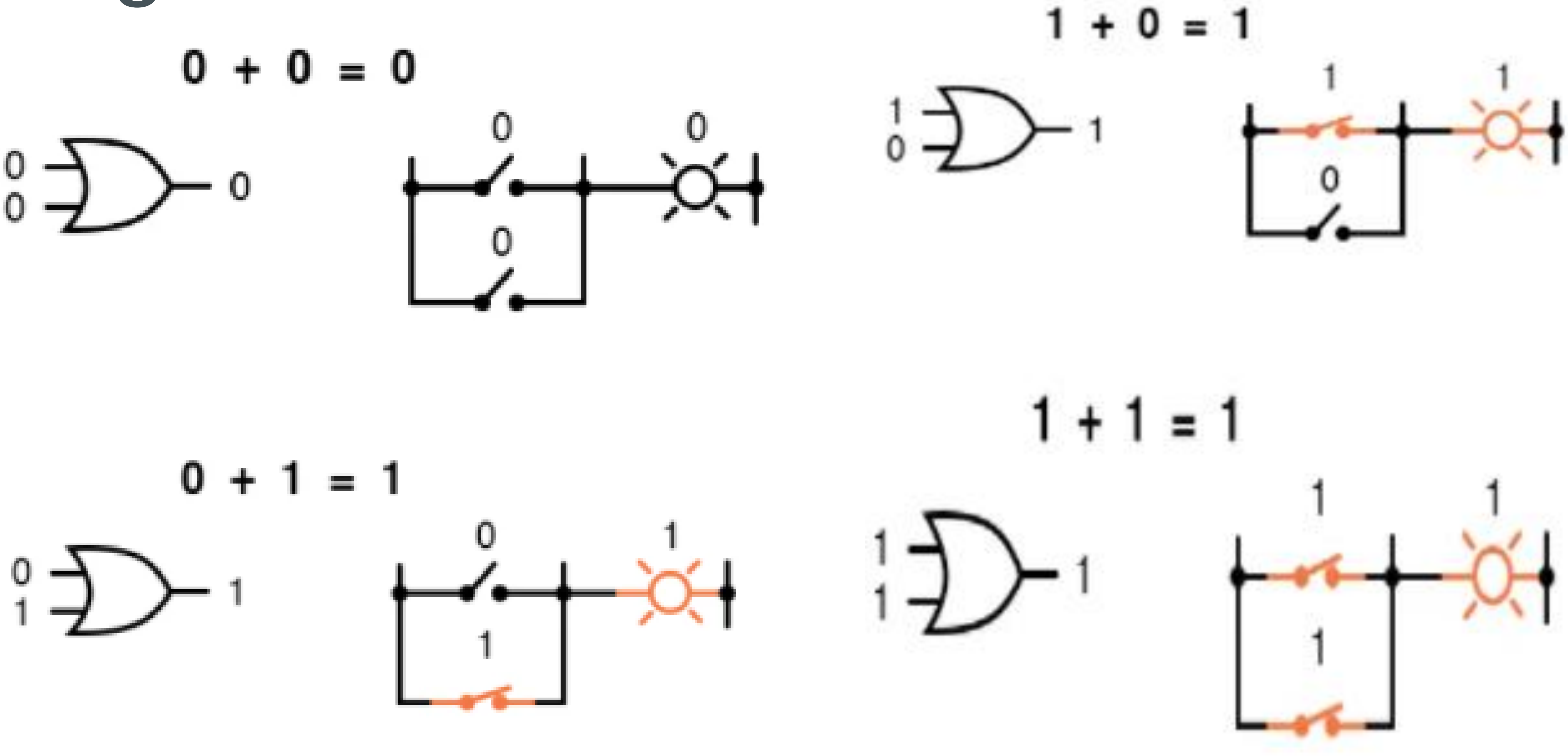
$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = 1$$

Boolean addition corresponds to the logical function of an “OR” gate, as well as to parallel switch contacts.

or gate



There is no such thing as subtraction in the realm of Boolean mathematics.

Subtraction implies the existence of negative numbers: 5 - 3 is the same thing as 5 + (-3), and in Boolean algebra negative quantities are forbidden.

There is no such thing as division in Boolean mathematics, either, since division is really nothing more than compounded subtraction, in the same way that multiplication is compounded addition.

What is AND GATE?

and gate

$$0 \times 0 = 0$$

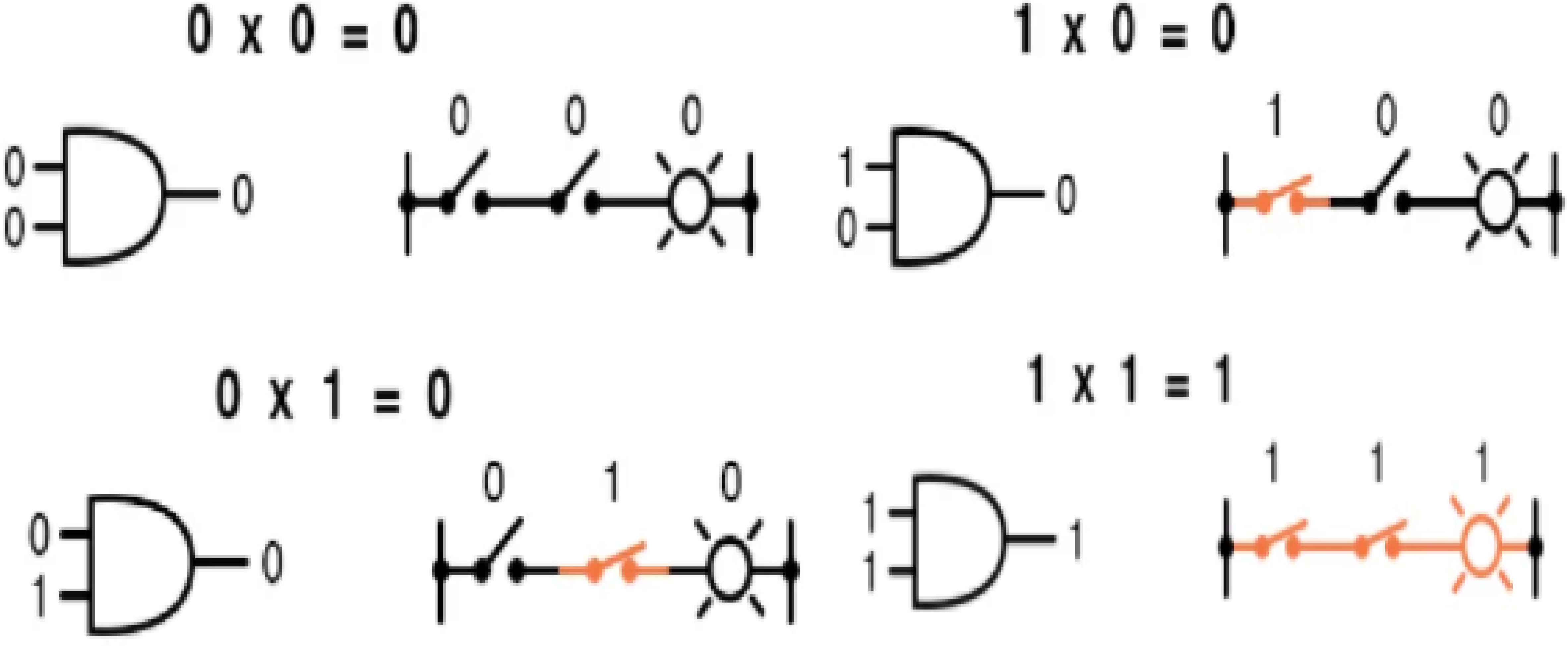
$$0 \times 1 = 0$$

$$1 \times 0 = 0$$

$$1 \times 1 = 1$$

Boolean multiplication corresponds to the logical function of an "AND" gate, as well as to series switch contacts.

and gate



Like “normal” algebra, Boolean algebra uses alphabetical letters to denote variables.

Unlike “normal” algebra, though, Boolean variables are always CAPITAL letters, never lower-case.

Because they are allowed to possess only one of two possible values, either 1 or 0, each and every variable has a complement: the opposite of its value.

What is NOT GATE?

not gate

If: $A = 0$

Then: $\overline{A} = 1$

If: $A = 1$

Then: $\overline{A} = 0$

Boolean complementation finds equivalency in the form of the NOT gate, or a normally-closed switch or relay contact.

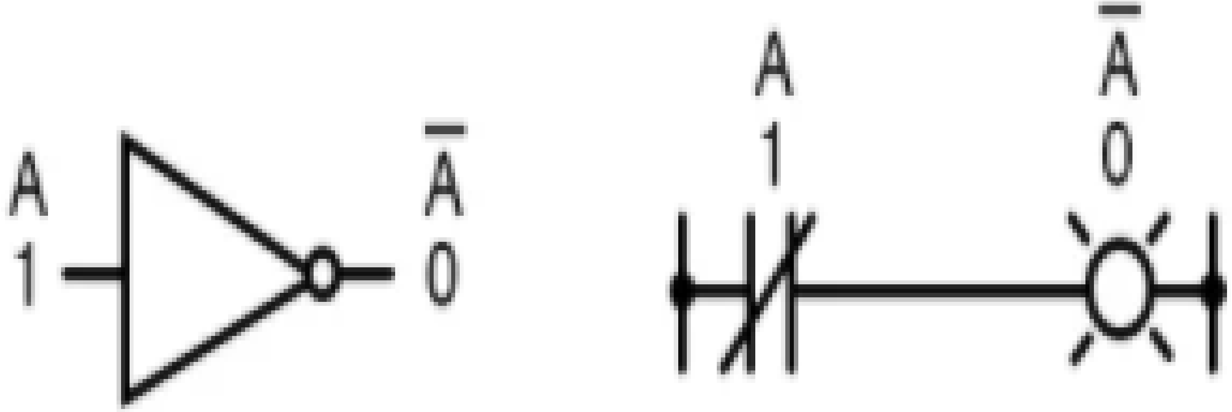
not gate

If: $A = 0$

Then: $\overline{A} = 1$

If: $A = 1$

Then: $\overline{A} = 0$



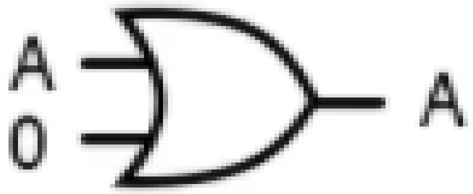
What is identity?

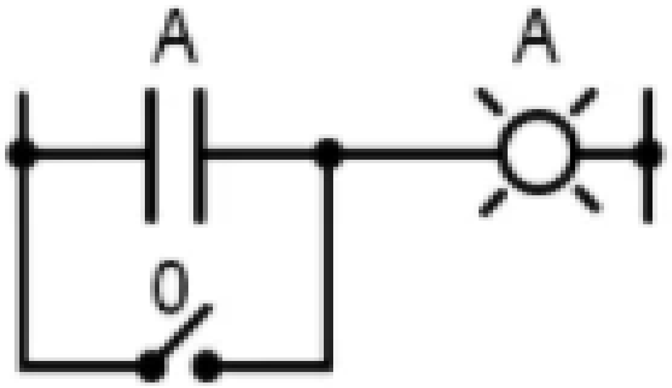
Identity *is a statement true for all possible values of its variable or variables.*

The algebraic identity of $x + 0 = x$ tells us that anything (x) added to zero equals the original "anything," no matter what value that "anything" (x) may be.

Additive Identities: Adding Zero

$$A + 0 = A$$

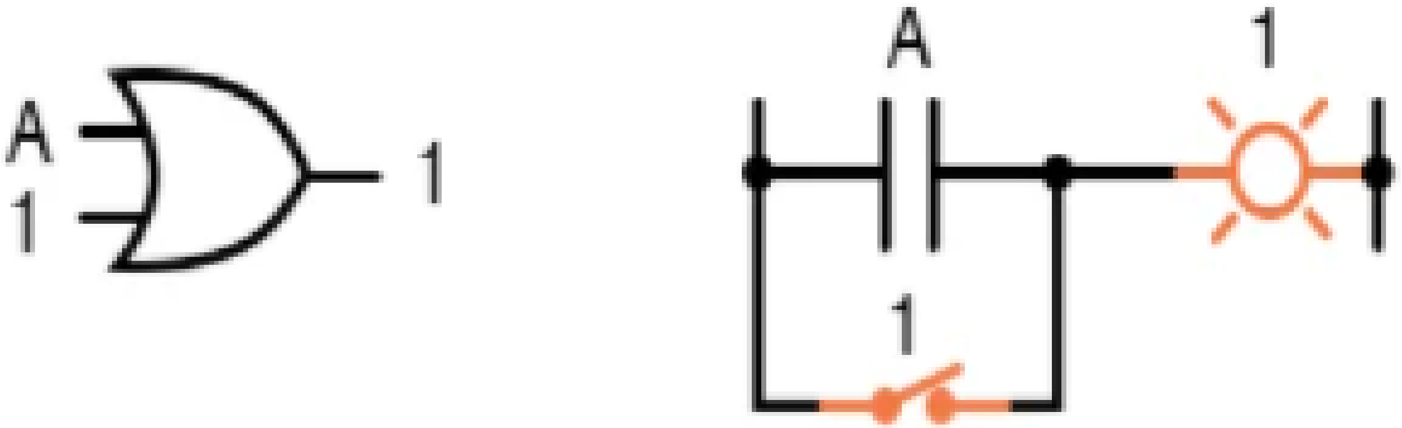




No matter what the value of A, the output will always be the same: when A=1, the output will also be 1; when A=0, the output will also be 0.

Additive Identities: Adding One

$$A + 1 = 1$$

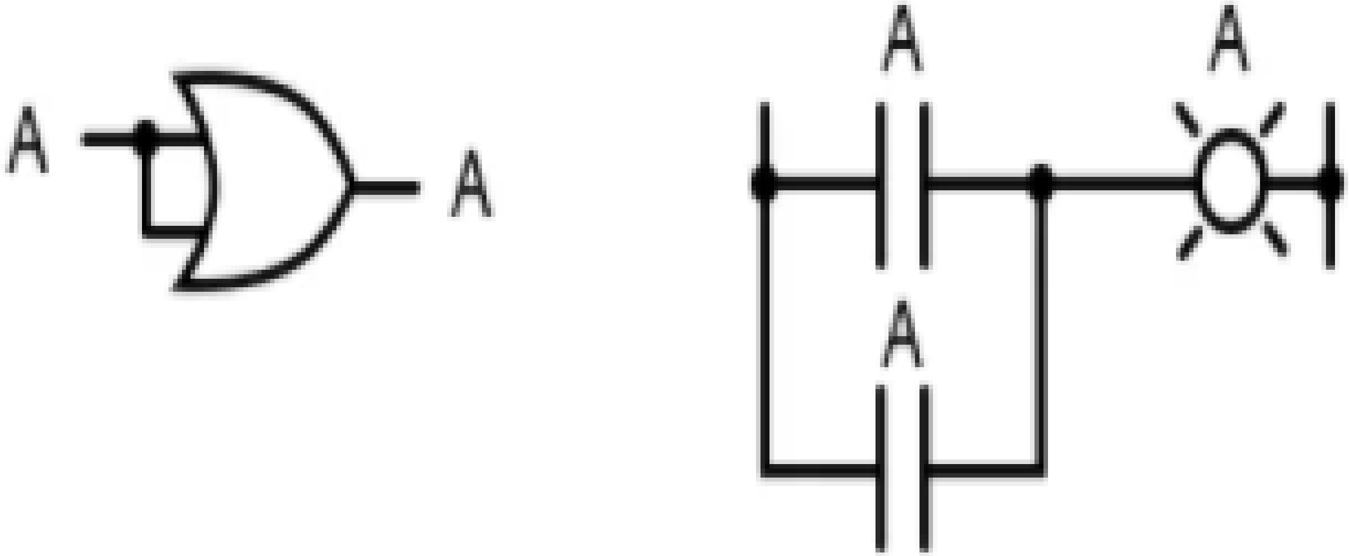


No matter what the value of A, the sum of A and 1 will always be 1.

In a sense, the “1” signal overrides the effect of A on the logic circuit, leaving the output fixed at a logic level of 1.

Additive Identities: Adding a Quantity to Itself

$$A + A = A$$



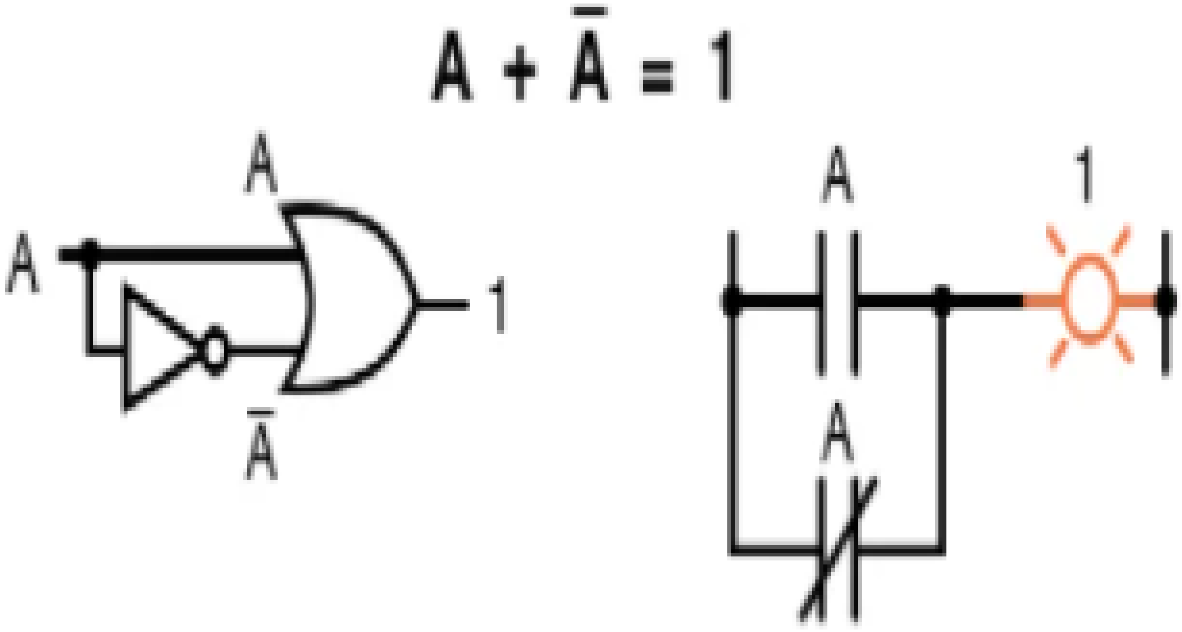
Adding A and A together, which is the same as connecting both inputs of an OR gate to each other and activating them with the same signal.

Additive Identities: Adding a Quantity to Itself

In real-number algebra, the sum of two identical variables is twice the original variable’s value ($x + x = 2x$), but remember that there is no concept of “2” in the world of Boolean math, only 1 and 0, so we cannot say that $A + A = 2A$.

Thus, when we add a Boolean quantity to itself, the sum is equal to the original quantity: $0 + 0 = 0$, and $1 + 1 = 1$.

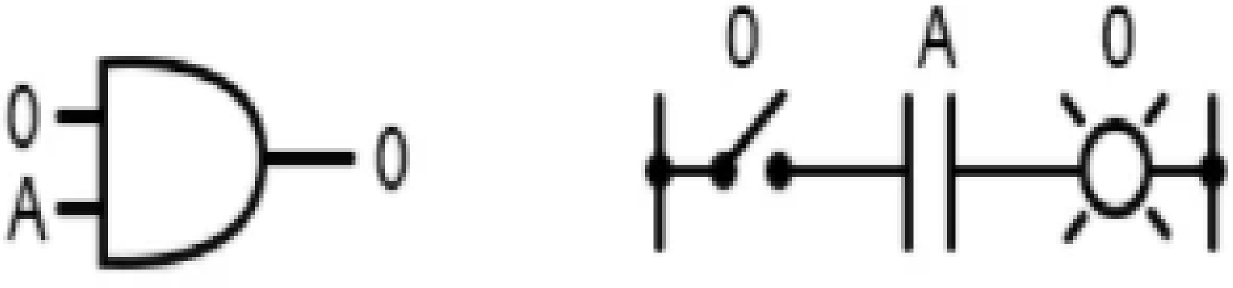
Additive Identities: Adding a Quantity to Its Complement



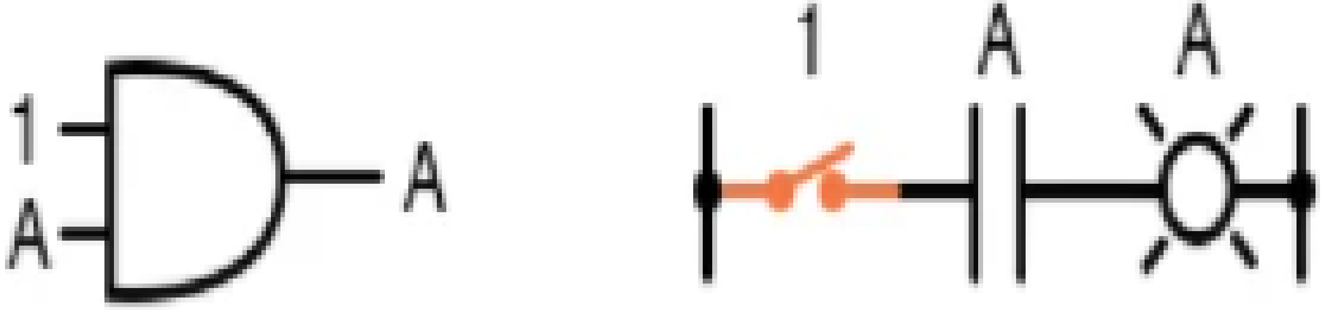
Since there must be one "1" value between any variable and its complement, and since the sum of any Boolean quantity and 1 is 1, the sum of a variable and its complement must be 1.

Multiplicative Identities: Multiplying by 0 or 1

$0A = 0$



$1A = A$



Multiplicative Identities: Multiplying a Quantity by Itself

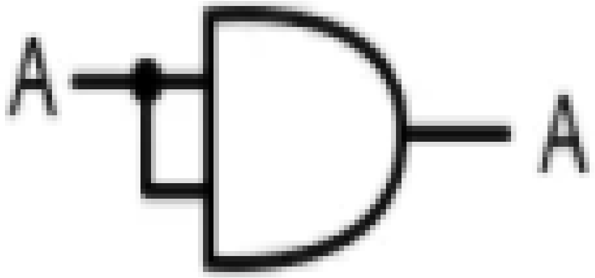
In normal algebra, the product of a variable and itself is the square of that variable (3 x 3 = 9).

However, the concept of square implies a quantity of 2, which has no meaning in Boolean algebra, so we cannot say that A x A = A2.

Multiplicative Identities: Multiplying a Quantity by Itself

The product of a Boolean quantity and itself is the original quantity, since 0 x 0 = 0 and 1 x 1 = 1.

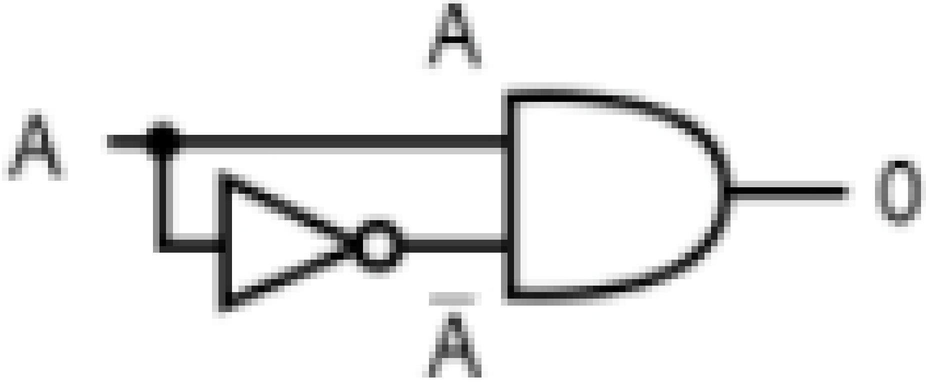
$$AA = A$$



Multiplicative Identities: Multiplying a Quantity by Its Complement

Since there must be one “0” value between any variable and its complement, and since the product of any Boolean quantity and 0 is 0, the product of a variable and its complement must be 0.

$$A\bar{A} = 0$$



BASIC BOOLEAN ALGEBRAIC IDENTITIES

Additive	Multiplicative
$A + 0 = A$	$0A = 0$
$A + 1 = 1$	$1A = A$
$A + A = A$	$AA = A$
$A + \overline{A} = 1$	$A\overline{A} = 0$

LOGIC GATES

LOGIC GATES

- We see that Boolean functions are implemented in digital computer circuits called gates.
- A gate is an electronic device that produces a result based on two or more input values.
- In reality, gates consist of one to six transistors, but digital designers think of them as a single unit.
- Integrated circuits contain collections of gates suited to a particular purpose.

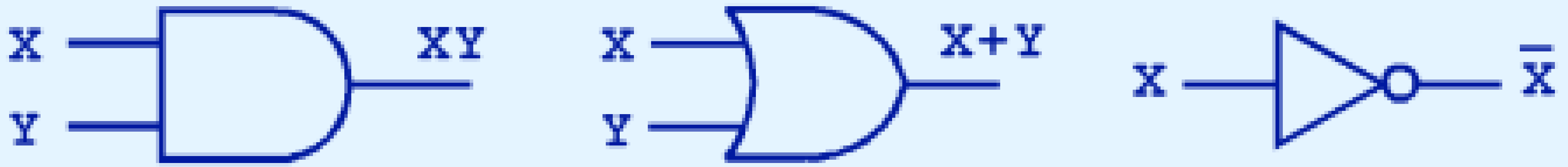
What is TRUTH TABLE?

Truth table

- shows the relationship, in tabular form, between the input values and the result of a specific Boolean operator or function on the input variables.

What are the 3 basic LOGIC GATES?

3 basic LOGIC GATES



X AND Y

X	Y	XY
0	0	0
0	1	0
1	0	0
1	1	1

X OR Y

X	Y	X+Y
0	0	0
0	1	1
1	0	1
1	1	1

NOT X

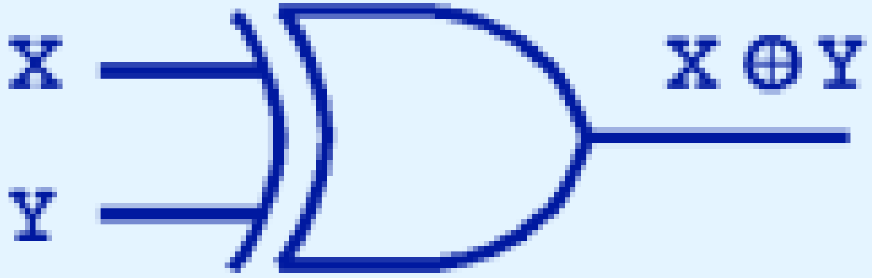
X	$\overline{X}$
0	1
1	0

XOR (EXCLUSIVE OR) GATE

- The output of the XOR operation is true only when the values of the inputs differ.

X XOR Y

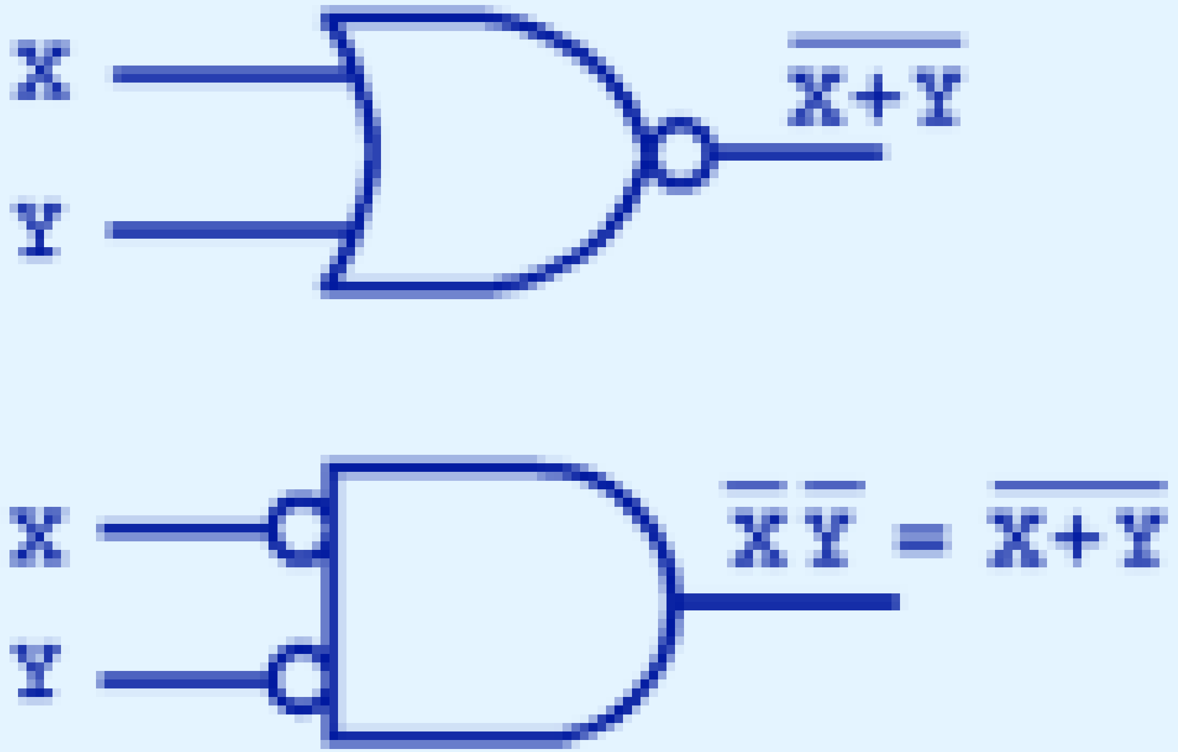
X	Y	X ⊕ Y
0	0	0
0	1	1
1	0	1
1	1	0



NOR (COMPLEMENTARY OR) GATE

X NOR Y

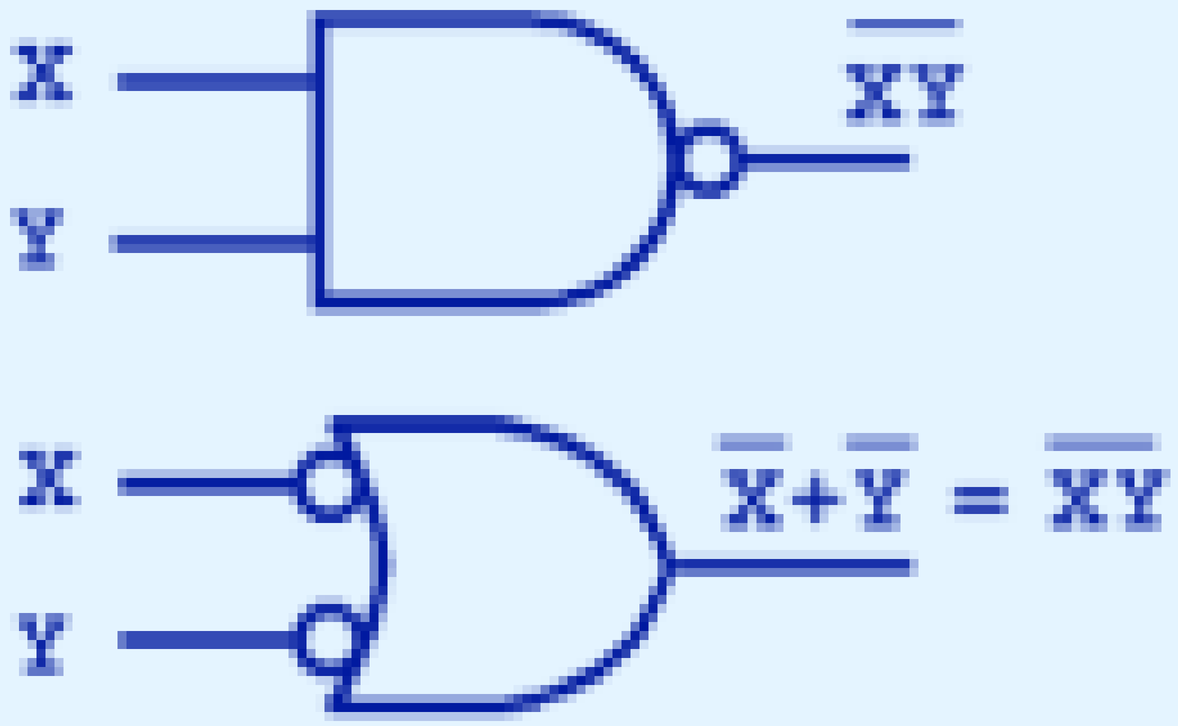
X	Y	X NOR Y
0	0	1
0	1	0
1	0	0
1	1	0



NAND (COMPLEMENTARY AND) GATE

X NAND Y

X	Y	X NAND Y
0	0	1
0	1	1
1	0	1
1	1	0



Applications???

Any questions???

REFERENCES:

- Boolean Algebra and Digital Logic. (n.d.). https://www2.southeastern.edu/Academics/Faculty/kyang/2018/Spring/CMPS375/ClassNotes/CMPS375ClassNotesChap03.pdf
- Boolean Arithmetic.(n.d.). https://www.allaboutcircuits.com/textbook/digital/chpt-7/boolean-arithmetic/
- Boolean Algebraic Identities. (n.d.). https://www.allaboutcircuits.com/textbook/digital/chpt-7/boolean-algebraic-identities/