用数学方法优化时间复杂度

Actually, the problem is not so difficult. For a rope of length n, we are requested to cut the rope into several pieces (staying the original state is not ~~allowable~~ permitted). The target is to maximize the product of the lengths of the ropes that are cut into.

Obviously DP is a normal method to solve this problem. By the way, due to the constraint that we can't keep the original state, n=2 or n=3 need special handling. The code is also easy.

The reason why I ~~record~~ note down this problem is that it can be perfectly solved by math method.

The time complexity of DP method is $O(n^2)$, but after improved by math method, the time complexity can be $O(1)$.

For a rope of length n, if we cut it into a ropes of length x, the product of them will be $x^{\frac{n}{x}}$. Obviously we can ~~only~~ just explore $x^{\frac{1}{x}}$ and maximize it.

$$\begin{aligned} y &= x^{\frac{1}{x}} \\ \ln y &= \frac{1}{x} \ln x \\ \frac{y'}{y} &= \frac{1 - \ln x}{x^2} \\ y' &= \frac{1 - \ln x}{x^2} x^{\frac{1}{x}} \end{aligned}$$

So $x = e$ may be the answer, but $x \in Z$. After calculating, $x = 3$ is the best answer. So we can get a cutting strategy:

- if $x\%3 = 0$, then cut it into ropes of 3 length.
- if $x\%3 = 1$, then cut it into several ropes of 3 and 2 ropes of 2.
- if $x\%3 = 2$, then cut it into several ropes of 3 and 1 rope of 2.

Of course, $n = 2, n = 3$ is the exception.