## February 6, 2020

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## 1.1

(plasma).

1.2

$$\frac{m_{\alpha}d\boldsymbol{v}_{\alpha}}{dt} = q_{\alpha}(E_{\alpha} + \boldsymbol{v} + \alpha \times \boldsymbol{B}_{\alpha})$$

$$rac{\partial f_{lpha}}{\partial t} + oldsymbol{v} \cdot 
abla f_{lpha} + rac{q_{lpha}}{m_{lpha}} (oldsymbol{E}_{lpha} + oldsymbol{v}_{lpha} imes oldsymbol{B}_{lpha}) \cdot 
abla_v f_{lpha} = \sum_{eta} C_{lphaeta} (f_{lpha}, f_{eta})$$

1.3

 $\mathbf{n}$ 

$$n_e = \sum Z_{\alpha} n_{\alpha}$$

$$n_e n_\alpha \ \alpha Z_\alpha$$

$$\mathbf{T} \quad E_k \sim kT.$$

$$\mathbf{d}d = n^{-1/3}.$$

$$\lambda_L, \ \lambda_L = 1.67 \times 10^{-5} Z_\alpha Z_\beta T^{-1}.$$

$$d:$$

$$\lambda \sim \frac{h}{\sqrt{mkT}} << d \Rightarrow n^{1/3} T^{-1/2} << 1$$

$$\bar{E}_p << E_k \ \Rightarrow \ n\lambda_L^3 << 1$$

1.4

$$\lambda_D$$

$$\lambda_D = 7430 \left(\frac{T_e[eV]}{n_e[m^{-3}]}\right)^{1/2}$$
 
$$\lambda_D = 69 \left(\frac{T_i[K]}{n_i[m^{-3}]}\right)^{1/2}$$
 
$$\varphi(r) = \frac{q}{4\pi\epsilon_0 r} exp\left(-\frac{r}{\lambda_D}\right)$$

 $\lambda_D L >> \lambda_D$ .

1.5

$$\omega_{pe} = \left(\frac{n_e e^2}{\epsilon_0 m_e}\right)^{1/2}, \quad \omega_{pi} = \left(\frac{n_i Z_i^2 e^2}{\epsilon_0 m_i}\right)^{1/2}$$

 $m_i >> m_e \ \omega_{pe} >> \omega_{pi}. \ \omega_p = \omega_{pe}.$ 

$$\lambda_D = \frac{v_{th}}{\omega_{pe}}$$

$$t_D = \frac{\lambda_D}{v_{th}} = \frac{1}{\omega_{pe}}$$

 $\omega_{pe}^{-1}$ 

 $\mathbf{2}$ 

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3.1

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0$$

$$\rho \frac{d\boldsymbol{u}}{dt} = \nabla \cdot \boldsymbol{P} + \rho \boldsymbol{g}$$

$$\rho \frac{d}{dt} \left( \epsilon + \frac{\boldsymbol{u}^2}{2} \right) = \nabla \cdot (\boldsymbol{P} \cdot \boldsymbol{u}) + \rho \boldsymbol{g} \cdot \boldsymbol{u} - \nabla \cdot \boldsymbol{q}$$

$$\begin{cases} \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0 \\ \rho \frac{d\boldsymbol{u}}{dt} = \nabla \cdot \boldsymbol{P} + \boldsymbol{J} \times \boldsymbol{B} \\ \rho \frac{d}{dt} (\epsilon + \frac{\boldsymbol{u}^2}{2}) = \nabla \cdot (\boldsymbol{P} \cdot \boldsymbol{u}) + \boldsymbol{E} \cdot \boldsymbol{J} - \nabla \cdot \boldsymbol{q} \\ p = p(\rho, T) \\ \nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \\ \nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} \\ \nabla \cdot \boldsymbol{B} = 0 \\ \boldsymbol{J} = \sigma(\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}) \end{cases}$$

$$\begin{cases} \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0 \\ \rho \frac{d\boldsymbol{u}}{dt} = -\nabla p + \boldsymbol{J} \times \boldsymbol{B} \\ p \rho^{-\gamma} = constant \\ \nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \\ \nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} \\ \boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B} = 0 \end{cases}$$

:

$$\boldsymbol{J} = \sigma \left[ (\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}) - \frac{1}{en} \boldsymbol{J} \times \boldsymbol{B} + \frac{1}{en} \nabla p_e \right]$$

## 3.3

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B}) + \eta_m \nabla^2 \boldsymbol{B}$$

$$\eta_m = \frac{1}{\mu_0 \sigma}$$
).
$$Rm = \frac{UL}{\eta_m}$$

$$Rm << 1$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \eta_m \nabla^2 \boldsymbol{B}$$

$$\eta_m$$

$$Rm >> 1\sigma \to \infty$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B})$$

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