February 6, 2020

1

1.1

(plasma).

1.2

$$\frac{m_\alpha d\boldsymbol{v}_\alpha}{dt} = q_\alpha(E_\alpha \ + \boldsymbol{v} + \alpha \times \boldsymbol{B}_\alpha)$$

$$\frac{\partial f_\alpha}{\partial t} + \boldsymbol{v} \cdot \nabla f_\alpha + \frac{q_\alpha}{m_\alpha}(\boldsymbol{E}_\alpha + \boldsymbol{v}_\alpha \times \boldsymbol{B}_\alpha) \cdot \nabla_v f_\alpha = \sum_\beta C_{\alpha\beta}(f_\alpha, f_\beta)$$

1.3

n

$$n_e = \sum Z_\alpha n_\alpha$$

$n_e n_\alpha$ αZ_α

T $E_k \sim kT$.

d$d = n^{-1/3}$.

λ_L, $\lambda_L = 1.67 \times 10^{-5} Z_\alpha Z_\beta T^{-1}$.

d:

$$\lambda \sim \frac{h}{\sqrt{mkT}} << d \Rightarrow n^{1/3} T^{-1/2} << 1$$

$$\bar{E}_p << E_k \quad \Rightarrow \quad n\lambda_L^3 << 1$$

1.4

λ_D

$$\lambda_D = 7430\Big(\frac{T_e[eV]}{n_e[m^{-3}}\Big)^{1/2}$$

$$\lambda_D = 69\Big(\frac{T_i[K]}{n_i[m^{-3}}\Big)^{1/2}$$

$$\varphi(r) = \frac{q}{4\pi\epsilon_0 r} exp\big(-\frac{r}{\lambda_D}\big)$$

λ_D $L >> \lambda_D$.

1.5

$$\omega_{pe} = \big(\frac{n_e e^2}{\epsilon_0 m_e}\big)^{1/2}, \quad \omega_{pi} = \big(\frac{n_i Z_i^2 e^2}{\epsilon_0 m_i}\big)^{1/2}$$

$m_i >> m_e$ $\omega_{pe} >> \omega_{pi}$. $\omega_p = \omega_{pe}$.

$$\lambda_D = \frac{v_{th}}{\omega_{pe}}$$

$$t_D = \frac{\lambda_D}{v_{th}} = \frac{1}{\omega_{pe}}$$

ω_{pe}^{-1}

2

3

3.1

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0$$

$$\rho \frac{d\boldsymbol{u}}{dt} = \nabla \cdot \boldsymbol{P} + \rho \boldsymbol{g}$$

$$\rho \frac{d}{dt}\big(\epsilon + \frac{\boldsymbol{u}^2}{2}\big) = \nabla \cdot (\boldsymbol{P} \cdot \boldsymbol{u}) + \rho \boldsymbol{g} \cdot \boldsymbol{u} - \nabla \cdot \boldsymbol{q}$$

3.2

$$\begin{cases} \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0 \\ \rho \frac{d\boldsymbol{u}}{dt} = \nabla \cdot \boldsymbol{P} + \boldsymbol{J} \times \boldsymbol{B} \\ \rho \frac{d}{dt}\big(\epsilon + \frac{\boldsymbol{u}^2}{2}\big) = \nabla \cdot (\boldsymbol{P} \cdot \boldsymbol{u}) + \boldsymbol{E} \cdot \boldsymbol{J} - \nabla \cdot \boldsymbol{q} \\ p = p(\rho, T) \\ \nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \\ \nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} \\ \nabla \cdot \boldsymbol{B} = 0 \\ \boldsymbol{J} = \sigma(\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}) \end{cases}$$

$$\begin{cases} \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0 \\ \rho \frac{d\boldsymbol{u}}{dt} = -\nabla p + \boldsymbol{J} \times \boldsymbol{B} \\ p\rho^{-\gamma} = constant \\ \nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \\ \nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} \\ \boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B} = 0 \end{cases}$$

:

$$\boldsymbol{J} = \sigma\big[(\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}) - \frac{1}{en}\boldsymbol{J} \times \boldsymbol{B} + \frac{1}{en}\nabla p_e\big]$$

3.3

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B}) + \eta_m \nabla^2 \boldsymbol{B}$$

$$\eta_m = \frac{1}{\mu_0 \sigma}$$

).

$$Rm = \frac{UL}{\eta_m}$$

$Rm << 1$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \eta_m \nabla^2 \boldsymbol{B}$$

η_m

$Rm >> 1 \sigma \to \infty$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B})$$