Assignment 1: EFA

Group 11

The Date

1. Problem Statement Task 2

exploratory factor analysis: • explain the correlation structure among observed variables • try to find underlying dimensions that can explain the observed correlations • example: the correlation between scores on mathematics, statis- tics and physics exams can be explained because they all measure somehow quantitative intelligence

1. State the problem
2. Descriptive Statistics (to check data, to find outliers)
3. Test (or at least state) the assumptions of the method, if any
4. Conduct the method (describe in more detail the "best" approach you have found)
5. Interpret the solution
6. Compare the results briefly with alternative solutions, if any
7. Conclusion

2. Descriptive Statistics

```
library(psych)

corr <-read.delim("data/screening.txt", header = TRUE, sep="",dec = ".", skipNul = FALSE)
corr <- subset(corr, select = -c(X_name_))

m <- matrix(NA,20,20)
m[lower.tri(m,diag=TRUE)] <- 1:10

makeSymm <- function(m) {
   m[upper.tri(m)] <- t(m)[upper.tri(m)]
   return(m)
}

corr <- makeSymm(corr)
```

3. Assumptions of Methods

assuming standardized data and factors + uncorrelated factors

```
# Perform Kaiser's MSA to evaluate appropriateness of data
KMO(corr)
```

```
## Kaiser-Meyer-Olkin factor adequacy
## Call: KMO(r = corr)
## Overall MSA =  0.95
## MSA for each item =
##   x1   x2   x3   x4   x5   x6   x7   x8   x9  x10  x11  x12  x13  x14  x15
## 0.95 0.94 0.94 0.95 0.89 0.95 0.95 0.96 0.97 0.97 0.96 0.96 0.95 0.96 0.96
##  x16  x17  x18  x19  x20
## 0.96 0.93 0.96 0.94 0.95
```

```
# Kaiser MSA = 0.95 > 0.8 --> appropriate data
```

4. Method

There are different methods of obtaining a factor model, such as principal component factoring, iterative principal components factoring or the maximum likelihood method. First, the number of relevant factors to be extracted has to be determined. This can be inferred from the eigenvalues λ_i and eigenvectors ϵ_i of the observed correlation matrix R^{obs}. There are a number of rules of thumb which can be applied to the computed values:

- Retain only those factors with an eigenvalue larger than 1 (Guttman-Kaiser rule)
- Horn's parallel procedure
- Make a scree-plot and extract the amount of factors before the knee point of the slope

```
# Define the amount of factors
eval <- eigen(corr)$values

plot(eval, xlab = "Principal Component", ylab = "Eigenvalue",
     type = "b", main = "Scree Plot")
```

Considering the Scree Plot we should only retain the first factor. However, we also took the Kaiser-Guttman rule into account and decided to keep two factors. Factor analysis can be executed with different factoring methods to extract the latent variables. We used principal factoring and maximum likelihood. They both eomploy iterative approaches of estimating the correlation matrix from the oberserved Matrix. Principal factoring assumes that the intitial communalities are 1, meaning that there is no error at the starting point. In each iteration, these values then replace the diagonal in the correlation matrix which is used to compute a new set of factors. Maximum likelihood assumes a normal distribution of the dataset and iteratively adjusts distribution parameters to better fit the model to the observed data.
As there is an infinite number of different factoring solutions, rotations are applied to find the best possible interpretation of the model. For orthogonal models, which is one of our assumptions for this task, the most common procedures are varimax and quartimax. The latter focuses on identifing factor structure such that all variables have fairly high loadings on a few factors and have near zero loadings on the other factors. Varimax on the other hand tries to maximize the variance of loadings for each factor, such that every factor has high loadings on a few variables and low loadings for the other variables.

```
# Perform Factor Analysis
fa.out.pa <- fa(r = corr, nfactors = 2, fm="pa", rotate = "varimax", residuals = TRUE, SMC=FALSE)
fa.out.pa
```

```
## Factor Analysis using method =  pa
## Call: fa(r = corr, nfactors = 2, rotate = "varimax", residuals = TRUE,
##     SMC = FALSE, fm = "pa")
## Standardized loadings (pattern matrix) based upon correlation matrix
##      PA1  PA2   h2   u2 com
## x1  0.54 0.31 0.39 0.61 1.6
## x2  0.60 0.24 0.41 0.59 1.3
## x3  0.29 0.51 0.35 0.65 1.6
## x4  0.47 0.47 0.44 0.56 2.0
## x5  0.12 0.70 0.51 0.49 1.1
## x6  0.28 0.45 0.28 0.72 1.7
## x7  0.38 0.64 0.55 0.45 1.6
## x8  0.67 0.26 0.51 0.49 1.3
## x9  0.66 0.22 0.48 0.52 1.2
## x10 0.45 0.22 0.25 0.75 1.5
## x11 0.70 0.28 0.58 0.42 1.3
## x12 0.58 0.31 0.43 0.57 1.5
## x13 0.52 0.30 0.36 0.64 1.6
## x14 0.40 0.53 0.45 0.55 1.9
## x15 0.68 0.42 0.64 0.36 1.7
## x16 0.73 0.28 0.61 0.39 1.3
## x17 0.31 0.65 0.52 0.48 1.4
## x18 0.62 0.28 0.47 0.53 1.4
## x19 0.57 0.33 0.43 0.57 1.6
## x20 0.51 0.42 0.44 0.56 1.9
##
##                        PA1  PA2
## SS loadings           5.60 3.48
## Proportion Var        0.28 0.17
## Cumulative Var        0.28 0.45
## Proportion Explained  0.62 0.38
## Cumulative Proportion 0.62 1.00
##
## Mean item complexity =  1.5
## Test of the hypothesis that 2 factors are sufficient.
```

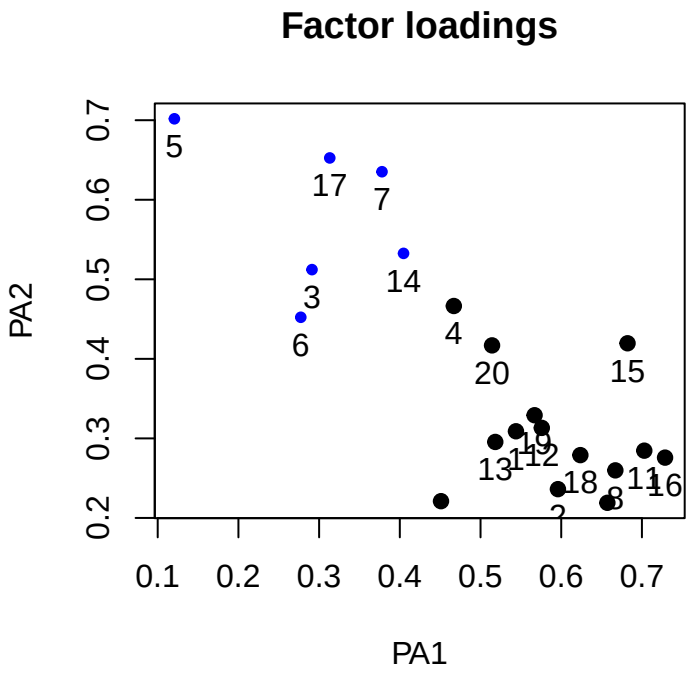
```
##
## The degrees of freedom for the null model are  190  and the objective function was  9.5
## The degrees of freedom for the model are 151  and the objective function was  0.89
##
## The root mean square of the residuals (RMSR) is  0.04
## The df corrected root mean square of the residuals is  0.04
##
## Fit based upon off diagonal values = 0.99
## Measures of factor score adequacy
##                                                  PA1  PA2
## Correlation of (regression) scores with factors   0.91 0.86
## Multiple R square of scores with factors          0.83 0.73
## Minimum correlation of possible factor scores     0.66 0.47
```

```
fa.out.ml <- fa(r = corr, nfactors = 2, fm="ml", rotate = "varimax", residuals = TRUE, SMC=FALSE,
                max.iter = 10)
```

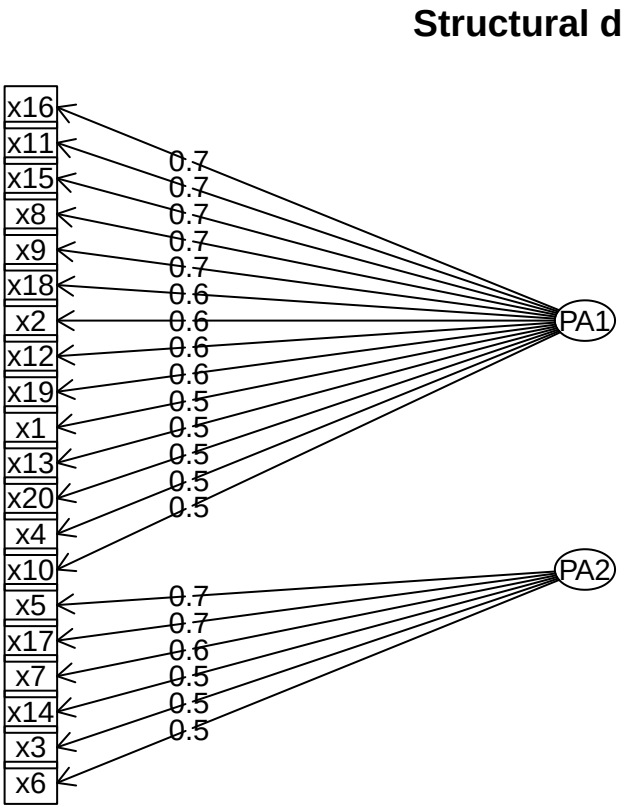
5. Interpretation of Solution

As it is the objective of EFA to explore the variable structure, the labeling of meaningful latent factors is open for interpretation and ambigious. Maximum Likelihood and Principal Factoring achieved very similar results, both generating a root mean square of the residuals (RMSR) of 0.04 and the same variable groups. Varimax provided the better possible factor explanation as 14 variables were loading higher on the first Factor and the rest on the second. We labeled the first one including statements such as "Lack of confidence during tests" (x1), "Heart beating fast during tests" (x18), "Screening bothers me" (x12) as "high intense anxiety" and the latent factor of variables such as "Thinking about test results" (x3), "The harder I try to contain myself, the less assured I get" (x6), "Defeat myself during tests" (x14) as "self-manipulative thoughts" or "low intense anxiety". A full structural diagram and the respective factor laodings plot can be seen below.

```
# Plot of factor loadings
plot(fa.out.pa, title = "Factor loadings")
```



```r
# Structural diagram
fa.diagram(fa.out.pa, main = "Structural diagram")
```



6. Comparison with alternative solutions

7. Conclusion