Assignment 1

Group 11
The Date

0. Load Libraries and Dependencies

```
library(tidyverse)
library(nFactors)
```

1. Problem Statement

The presented data from a conducted study shows the recovery status for 100 participants after taking two different drugs with differing doses. Recovery is measured as the percentage drop in body pathogens before and after taking the respective drug. In order to judge on the effectiveness of the drugs the principal component analysis is applied, to shrink the information to only a subset of the variables' dimensions. The associated identification of useful factor brandings and the reflection of the data on the identified subdimensions should help to evaluate the initial question.

The second section starts with the data preparation and a brief description of the data. Afterwards, in section three, the assumptions for using principal component analysis are described including the way we deal with them. In section four the principal component analysis is applied and the most relevant outputs are described. The last section covers the answering of first, how the factors in the subspace can be labled appropriately and second, which drugs were effective.

2. Descriptive Statistics

Before dealing with the principal component analysis (PCA), it is important to prepare the data. After importing the data and performing the glimpse-function, one can see that the data is stored in a data.frame object with the dimension 100x9.

\$ R500 <int> 15, 15, 15, 5, 15, 15, 20, 5, 15, 5, 20, 10, 5, 20, 20, ...
\$ R1000 <int> 20, 20, 15, 10, 5, 20, 20, 10, 15, 10, 15, 10, 5, 20, 10...
\$ R2000 <int> 20, 20, 25, 5, 5, 15, 20, 15, 5, 5, 5, 20, 5, 15, 15, 20...
\$ R4000 <int> 30, 30, 40, 25, 65, 35, 25, 20, 25, 30, 20, 30, 25, 65, ...

First of all the first column has to be eliminated, as it only stores the ID values. It would be a major mistake taking this column into account when performing PCA, as the results would be heavily biased.

```
drugdata <- drugdata[-1]
glimpse(drugdata)</pre>
```

```
## Observations: 100
## Variables: 8
## $ L500 <int> 15, 10, 10, 10, 10, 20, 15, 5, 15, 10, 5, 20, 15, 20, 20...
## $ L1000 <int> 20, 15, 15, 15, 10, 20, 15, 5, 15, 10, 10, 20, 15, 30, 1...
## $ L2000 <int> 25, 5, 30, 5, 5, 20, 15, 5, 15, 5, 10, 25, 5, 20, 20, 20...
## $ L4000 <int> 30, 15, 30, 5, 25, 5, 35, 10, 55, 35, 20, 40, 30, 75, 30...
## $ R500 <int> 15, 15, 15, 5, 15, 15, 20, 5, 15, 5, 20, 10, 5, 20, 20, ...
## $ R1000 <int> 20, 20, 15, 10, 5, 20, 20, 10, 15, 10, 15, 10, 5, 20, 10...
## $ R2000 <int> 20, 20, 25, 5, 5, 15, 20, 15, 5, 5, 5, 20, 5, 15, 15, 20...
## $ R4000 <int> 30, 30, 40, 25, 65, 35, 25, 20, 25, 30, 20, 30, 25, 65, ...
```

One can also see that all the remaining values are of class integer. This is important to know, as correlation matrices can only be calculated using either integers or numerical values. After selecting the relevant variables, it is also useful to check for missing values. This is done by using the apply-function, which outputs the amount of missing values for each column.

```
drugdata %>% apply(2, function(x){
   sum(is.na(x))
})

## L500 L1000 L2000 L4000 R500 R1000 R2000 R4000
## 0 0 0 0 0 0 0 0
```

Hence, there arent any missing values presented in the data. Even though the PCA has no distributional assumptions regarding the data it might still be useful to get an idea of the distribution of the variables

summary(drugdata)

```
##
         L500
                         L1000
                                         L2000
                                                           L4000
##
            : 5.0
                                             : 5.00
                                                               : 5.00
    Min.
                            : 5.0
                                     Min.
                                                      Min.
                    Min.
##
    1st Qu.: 5.0
                     1st Qu.:10.0
                                     1st Qu.:10.00
                                                       1st Qu.:23.75
                                                      Median :35.00
    Median:10.0
                     Median:15.0
                                     Median :15.00
##
##
    Mean
            :12.2
                            :14.5
                                             :17.00
                                                               :36.35
                     Mean
                                     Mean
                                                      Mean
##
    3rd Qu.:20.0
                     3rd Qu.:20.0
                                     3rd Qu.:21.25
                                                      3rd Qu.:50.00
            :30.0
                            :35.0
                                             :60.00
                                                               :85.00
##
    Max.
                     Max.
                                     Max.
                                                      Max.
##
         R500
                         R1000
                                         R2000
                                                          R4000
##
    Min.
            : 5.0
                    Min.
                            : 5.0
                                     Min.
                                             : 5.0
                                                     Min.
                                                             : 5.00
    1st Qu.: 5.0
                                     1st Qu.:10.0
##
                     1st Qu.:10.0
                                                      1st Qu.:20.00
##
    Median:10.0
                     Median:15.0
                                     Median:15.0
                                                     Median :30.00
##
    Mean
            :12.4
                            :14.3
                                             :16.6
                                                             :36.35
                     Mean
                                     Mean
                                                     Mean
##
    3rd Qu.:15.0
                     3rd Qu.:20.0
                                     3rd Qu.:20.0
                                                      3rd Qu.:45.00
            :40.0
                            :35.0
                                             :50.0
                                                             :90.00
##
    Max.
                     Max.
                                     Max.
                                                     Max.
```

As one would expect, the range of the variables increase as the dose increases. Taking into consideration that the minimum of each variable is equal to five, this means that due to the higher dose there might be at least some participants whose percentage drop in body pathogens was quite high compared to the participants who got a lower dose. This can also be derived when looking at the mean values or the outlier resistant median values. This might give rise to the assumption that higher dose is more effective. But let's see what is the outcome of the PCA.

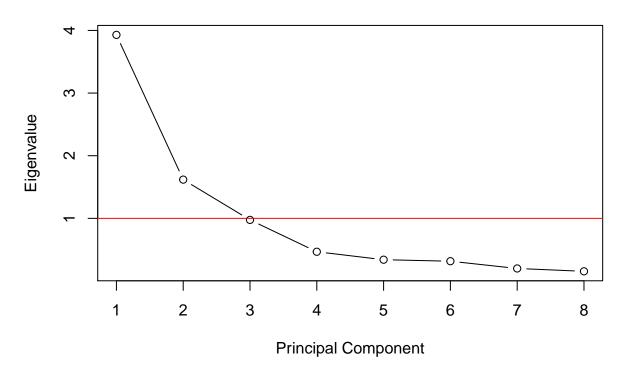
3. Assumptions

- 1. scaling
- 2. Finding the right amount of PCA

Determine the appropriate number of components that can be used to effectively summarize the information in the data. Explain how you settled on the reported number of components.

• Scree Plot

Scree Plot

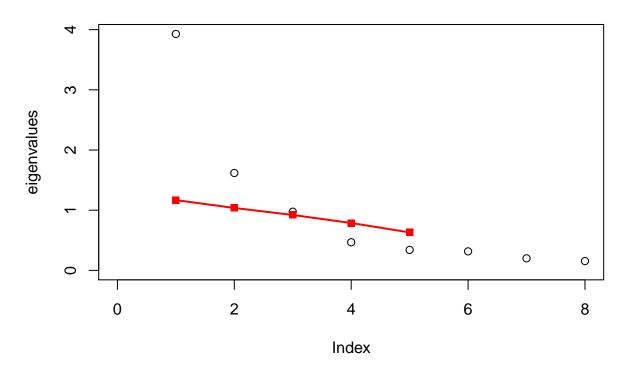


• Horn's Parallel procedure

Compute Eigenvalues associated with many simulated uncorrelated normal variables - retain the ith PC if the corresponding eigenvalue is larger than the 95th percentile of the distribution of the ith largest eigenvalue of the random data (same idea as the previous rule but taking random variation into account)

```
ap <- parallel(subject = nrow(drugdata), var = 5, rep= 1000,cent = 0.05)
plot(eval, xlim=c(0,ncol(drugdata)), ylim = c(0,max(eval)), ylab = "eigenvalues")
lines(ap$eigen$qevpea, type = "o",pch = 15, lwd = 2,col = "red")</pre>
```

Horn's parallel procedure - 5% lower percentile



4. Method

• Perform PCA

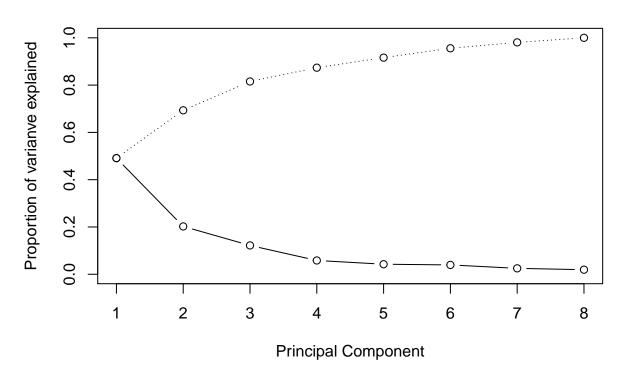
```
pr.out <- prcomp(drugdata, scale = TRUE)</pre>
```

• Variance explained

```
summary(pr.out)
```

```
## Importance of components:
                             PC1
                                     PC2
                                            PC3
                                                    PC4
                                                            PC5
## Standard deviation
                           1.9822 1.2721 0.9876 0.68321 0.58317 0.56204
## Proportion of Variance 0.4911 0.2023 0.1219 0.05835 0.04251 0.03949
## Cumulative Proportion 0.4911 0.6934 0.8153 0.87368 0.91619 0.95568
                                       PC8
##
                              PC7
## Standard deviation
                          0.44734 0.39303
## Proportion of Variance 0.02501 0.01931
## Cumulative Proportion 0.98069 1.00000
# First two principal components explain 69.34% of the variance
# Variance Explained Plot
prop_varex <- eval / sum(eval)</pre>
```

Variance Explained



5. Interpretation

```
# Eigenvectors
evec <- pr.out$rotation

# Correlation coefficients between PC's and initial variables
varnames <- names(drugdata)
corrcoef <- matrix(nrow = 2, ncol = ncol(drugdata), byrow = T)
colnames(corrcoef) <- varnames
rownames(corrcoef) <- c("PC1", "PC2")

for( n in 1:length(rownames(corrcoef)) ){
   for( m in 1:nrow(evec) ){
      corrcoef[n,m] <- evec[m,n] * sqrt(eval)[n]
   }
}
corrcoef</pre>
```

```
## PC2 0.4032200 0.2868202 -0.3035224 -0.6031874 0.4910683 0.2948454

## R2000 R4000

## PC1 -0.6175423 -0.5039101

## PC2 -0.4033411 -0.6532555

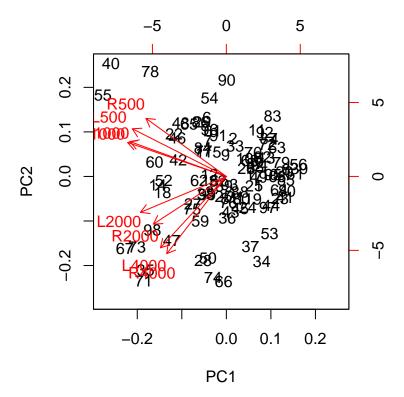
# First principal component is significantly negatively correlated

# with all variables

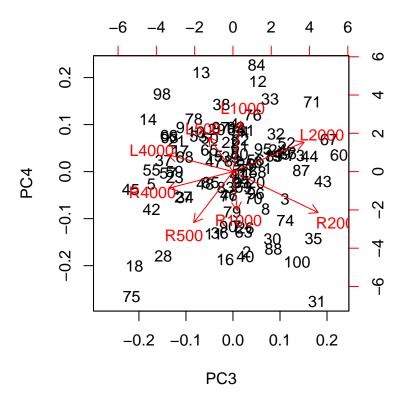
# Second component discriminates between L2000, L4000, R2000, R4000 on one hand

# (high dose) and L500, L1000, R500 and R1000 on the other hand (low dose)

biplot(pr.out)
```



biplot(pr.out, choices = c(3,4))



first plot shows two groupings of dosage amount independant of the drug

second plot barely shows any relevant information as there is barely any group seperation -> was to be expected since they explain small portion of the variance only

For the majority of the patients, both drugs had barely any effect on the percentage drop. Lower dose seems to be more effective compared to higher dose, as with a lower dose more patients had a percentage drop.