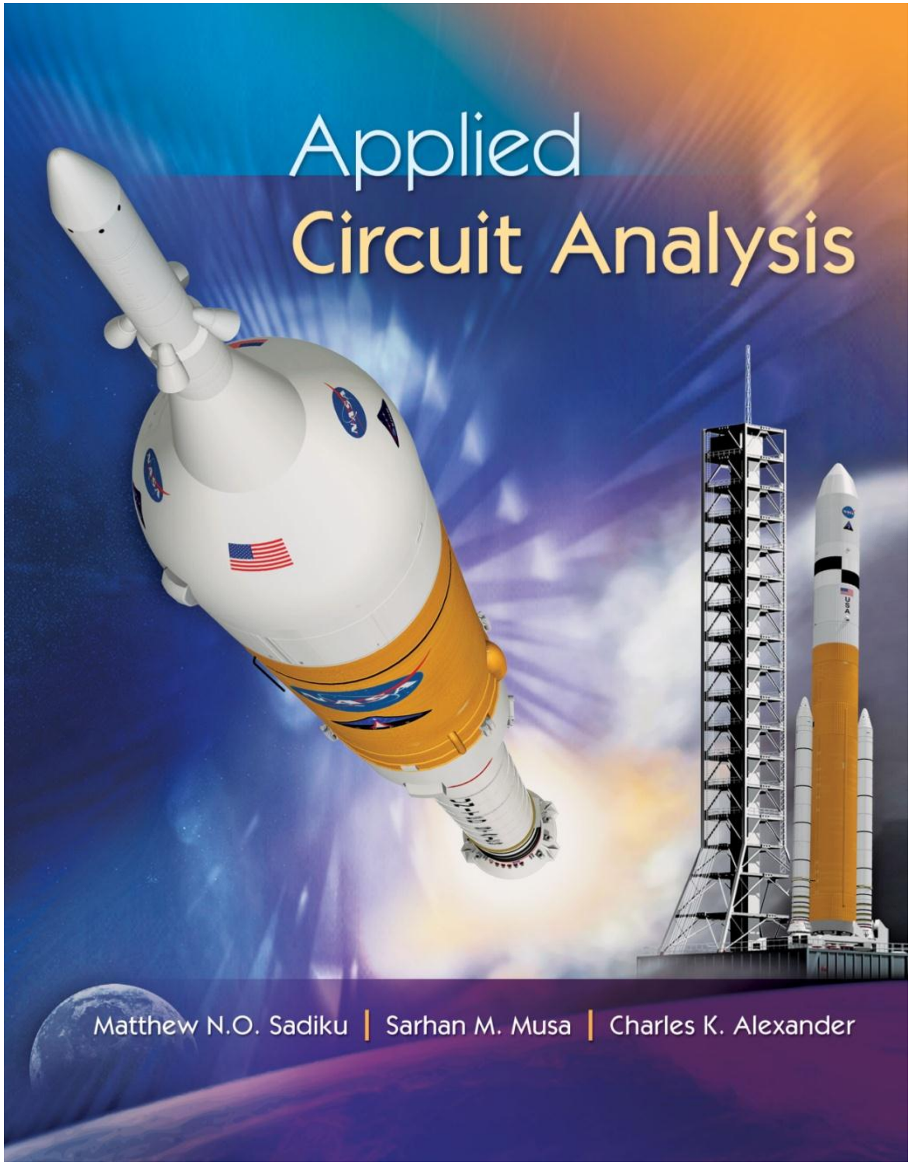
Applied Circuit Analysis
Chapter 9
Capacitance

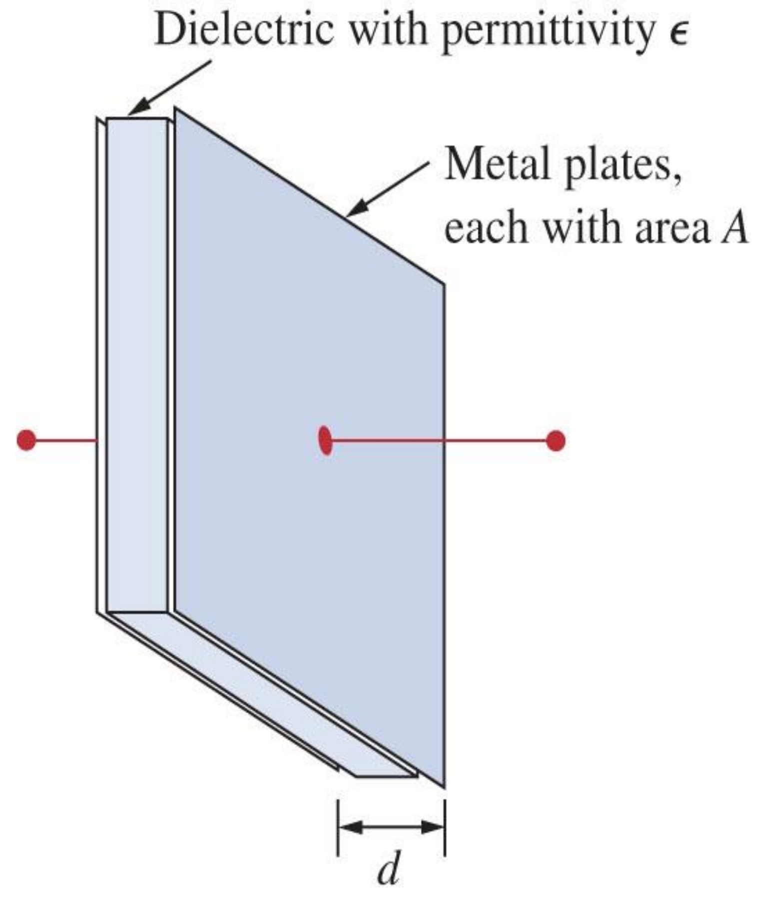


Overview

- The next two chapters will introduce two new linear circuit elements:
- The capacitor
- The inductor
- Unlike resistors, these elements do not dissipate energy
- They instead store energy
- We will also look at how to analyze them in a circuit

Capacitors

- **A capacitor is a passive element that stores energy in its electric field**
- **It consists of two conducting plates separated by an insulator (or dielectric)**
- **The plates are typically aluminum foil**
- **The dielectric is often air, ceramic, paper, plastic, or mica**



Capacitors II

- **When a voltage source *v* is connected to the capacitor, the source deposits a positive charge *q* on one plate and a negative charge –*q* on the other.**
- **The charges will be equal in magnitude**
- **The amount of charge is proportional to the voltage:**

$$q = Cv$$

- **Where *C* is the capacitance**

Capacitors III

- The unit of capacitance is the Farad (F)
- One Farad is 1 Coulomb/Volt
- Most capacitors are rated in picofarad (pF) and microfarad (μF)
- Capacitance is determined by the geometry of the capacitor:

Stored Energy

- **The current passing into a capacitor during charging for a time, *t*, can be defined as:**

$$I = \frac{CV}{t}$$

- **The energy stored in the capacitor will be:**

$$W = \frac{1}{2}CV^2$$

Electric Fields

- **Capacitors store their energy in electric fields**
- **The fields originate from the charges stored on the plates.**
- **The electric flux, *Ψ*, between parallel plates of area *A* with charge *Q* is:**

$$\Psi = Q$$

- **The electric flux density is:**

$$D = \frac{\Psi}{A} = \frac{Q}{A}$$

Fields II

- **The electric field strength *E* is inversely proportional to the plate spacing.**

$$E = \frac{V}{d}$$

- **The field and the flux density are related:**

$$D = \varepsilon E$$

- **ε is the permittivity of the dielectric**

Fields III

- **These relationships may be combined in order to develop a formula for the capacitance of a parallel plate capacitor**

$$C = \frac{Q}{V} = \frac{\Psi}{V} = \frac{DA}{Ed} = \frac{\varepsilon A}{d}$$

- **Or**

$$C = \frac{\varepsilon A}{d}$$

Capacitance Value

- As can be seen, geometry and materials determine the value of the capacitor:
  - Larger surface area means bigger capacitance.
  - The smaller spacing between the plates also increases capacitance
  - Higher permittivity increases capacitance

Dielectric Properties

- **Here is a list of common materials and their dielectric constants**

Dielectric constants of common materials.

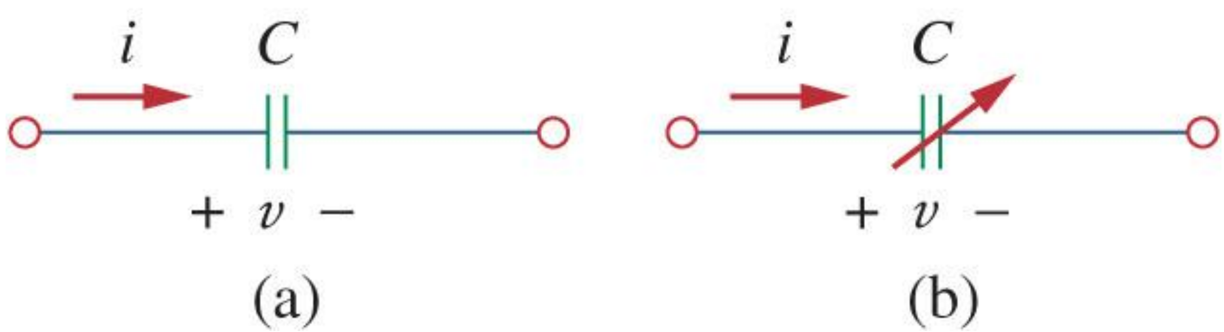
Material	Dielectric constant (ε_r)
Vacuum	1.0
Air	1.0006
Teflon	2.0
Paper (dry)	2.5
Polystyrene	2.5
Rubber	3.0
Oil (transformer)	4.0
Mica	5.0
Porcelain	6.0
Glass	7.5
Tantalum oxide	30
Water (distilled)	80
Ceramic	7,500

Types of Capacitors

- The most common types of capacitors are film capacitors with polyester, polystyrene, or mica.
- To save space, these are often rolled up before being housed in metal or plastic films
- Electrolytic caps (shown here) have a very high capacitance

Types of Caps II

- **Trimmer caps have a range of values that they can be set to**
- **Variable air caps can be adjusted by turning a shaft attached to a set of moveable plates**



Applications for Capacitors

- Capacitors have a wide range of applications, some of which are:
  - Blocking DC
  - Passing AC
  - Shift phase
  - Store energy
  - Suppress noise
  - Start motors

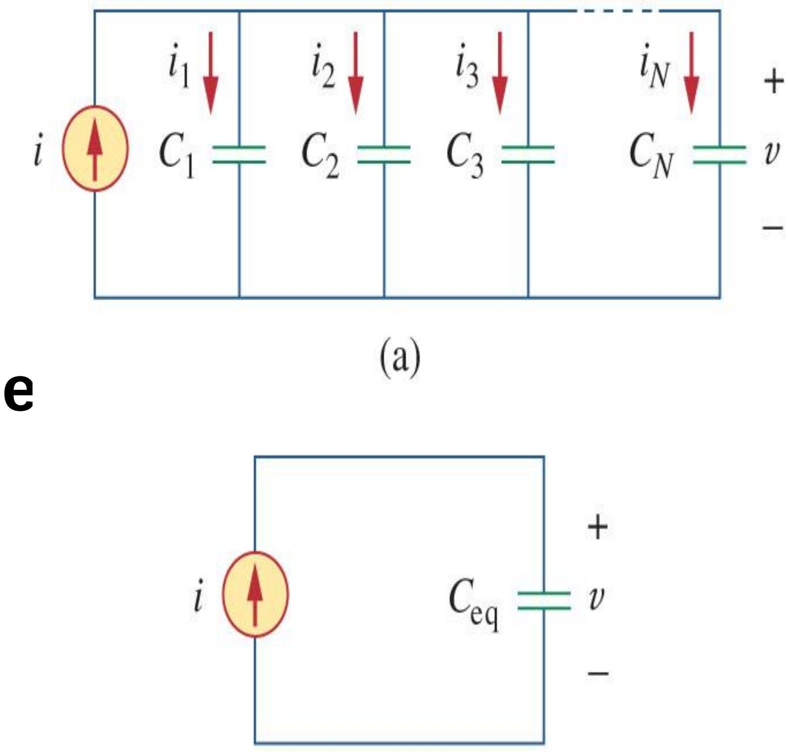
(a) (b) (c)

Courtesy of Tech America

Parallel Capacitors

- **We learned with resistors that applying the equivalent series and parallel combinations can simply many circuits.**
- **Starting with *N* parallel capacitors, one can note that the voltages on all the caps are the same**
- **Applying KCL:**

$$i = i_1 + i_2 + i_3 + \cdots + i_N$$



(a)
(b)

Parallel Capacitors II

- **Taking into consideration the current voltage relationship of each capacitor:**

$$i = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt} + \cdots + C_N \frac{dv}{dt}$$

$$= \left(\sum_{k=1}^{N} C_k \right) \frac{dv}{dt} = C_{eq} \frac{dv}{dt}$$

- **Where**

$$C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N$$

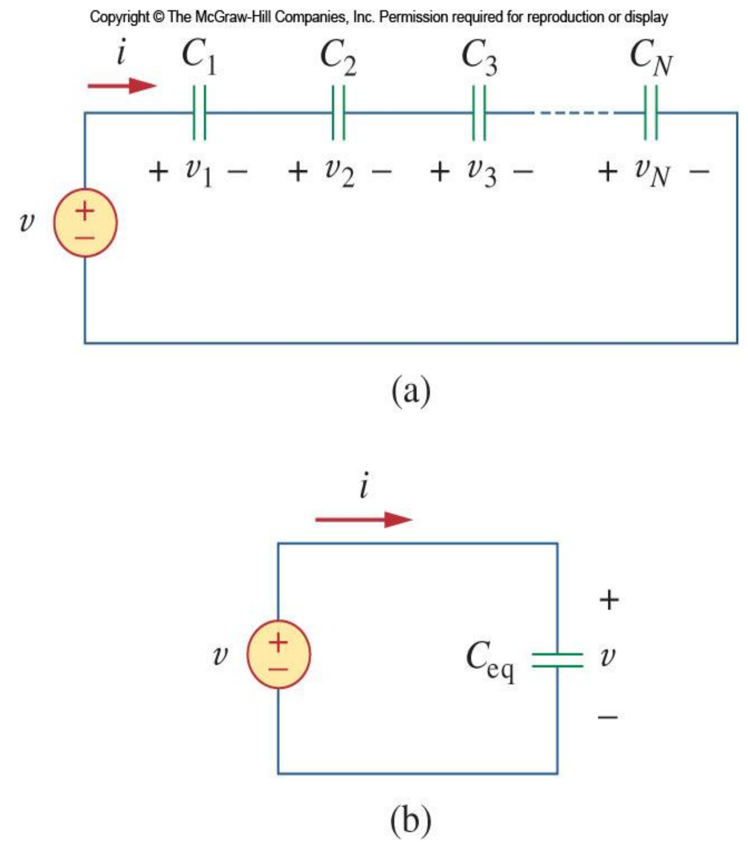
- **From this we find that parallel capacitors combine as the sum of all capacitance**

Series Capacitors

- **Turning our attention to a series arrangement of capacitors:**
- **Here each capacitor shares the same current**
- **Applying KVL to the loop:**

$$v = v_1 + v_2 + v_3 + \cdots + v_N$$

- **Now apply the voltage current relationship**



Series Capacitors II

$$v = \frac{1}{C_1}\int_{t_0}^{t} i(\tau)\,d\tau + v_1(t_0) + \frac{1}{C_2}\int_{t_0}^{t} i(\tau)\,d\tau + v_2(t_0) + \frac{1}{C_3}\int_{t_0}^{t} i(\tau)\,d\tau + v_3(t_0) + \cdots + \frac{1}{C_N}\int_{t_0}^{t} i(\tau)\,d\tau + v_N(t_0)$$

$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}\right)\int_{t_0}^{t} i(\tau)\,d\tau + v_1(t_0) + v_2(t_0) + v_3(t_0) + \cdots + v_N(t_0)$$

$$= \frac{1}{C_{eq}}\int_{t_0}^{t} i(\tau)\,d\tau + v(t_0)$$

- **Where** $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}$

- **From this we see that the series combination of capacitors resembles the parallel combination of resistors.**

Series and Parallel Caps

- Another way to think about the combinations of capacitors is this:
- Combining capacitors in parallel is equivalent to increasing the surface area of the capacitors:
- This would lead to an increased overall capacitance (as is observed)
- A series combination can be seen as increasing the total plate separation
- This would result in a decrease in capacitance (as is observed)

Current Voltage Relationship

- **Using the formula for the charge stored in a capacitor, we can find the current voltage relationship**
- **Take the first derivative with respect to time gives:**

$$i = C\frac{dv}{dt}$$

- **This assumes the passive sign convention**

Properties of Capacitors

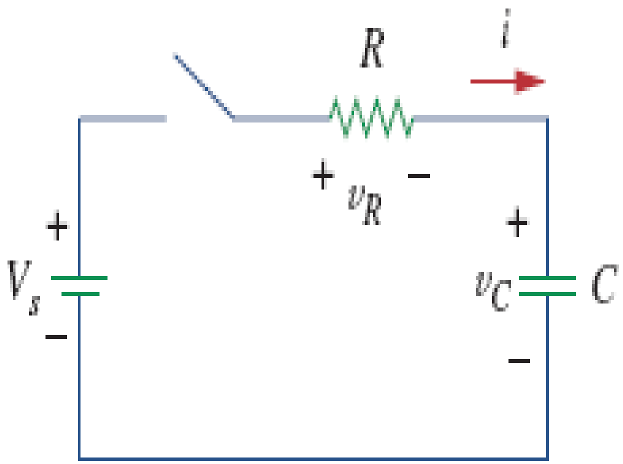
- Ideal capacitors all have these characteristics:
- When the voltage is not changing, the current through the cap is zero.
- This means that with DC applied to the terminals no current will flow.
- <u>Except</u>, the voltage on the capacitor's plates can't change instantaneously.
- An abrupt change in voltage would require an infinite current!
- This means if the voltage on the cap does not equal the applied voltage, charge will flow and the voltage will finally reach the applied voltage.

Properties of capacitors II

- An ideal capacitor does not dissipate energy, meaning stored energy may be retrieved later
- A real capacitor has a parallel-model leakage resistance, leading to a slow loss of the stored energy internally
- This resistance is typically very high, on the order of 100 MΩ and thus can be ignored for many circuit applications.

Charging Cycle

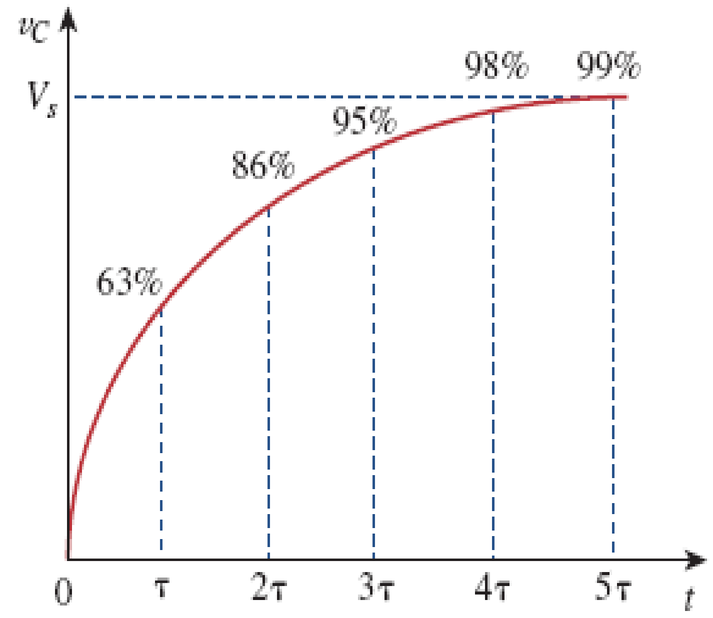
- **Consider a battery connected in series to a capacitor and a resistor.**
- **Assume that the capacitor is not charged at first.**
- **At *t=0* the switch is closed.**



Charging II

- **At any time the capacitor voltage is:**

$$v_C = V_S - \left(V_S - V_0\right)e^{-t/RC}$$



- **Where V_0 is the initial voltage.**
- **Since $V_0=0$ this can be written as:**

$$v_C = V_S\left(1 - e^{-t/RC}\right)$$

Charging III

- **From this the charging current is:**

$$i = \frac{v_R}{R} = \frac{V_S}{R} e^{-t/RC}$$

- **The time it takes to charge to *1/e* of the full charge, or 36.8% is called the time constant τ for the circuit.**

$$\tau = RC$$

Charging Current

- **Once again we can determine the charging current from the resistor's voltage drop**

$$v_R = V_S - v_C = V_S - V_s(1 - e^{-t/RC}) = V_S e^{-t/RC}$$

- **Or**

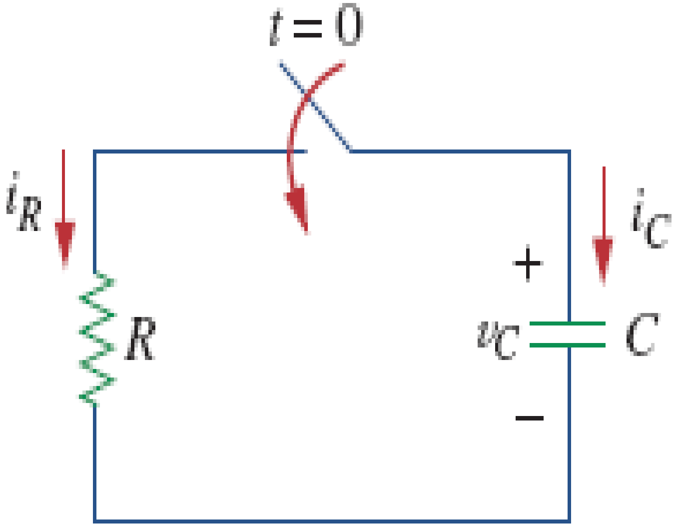
$$i_C = \frac{V_S}{R} e^{-t/RC}$$

- **As can be seen, the current decreases towards zero as time goes on.**

Discharging

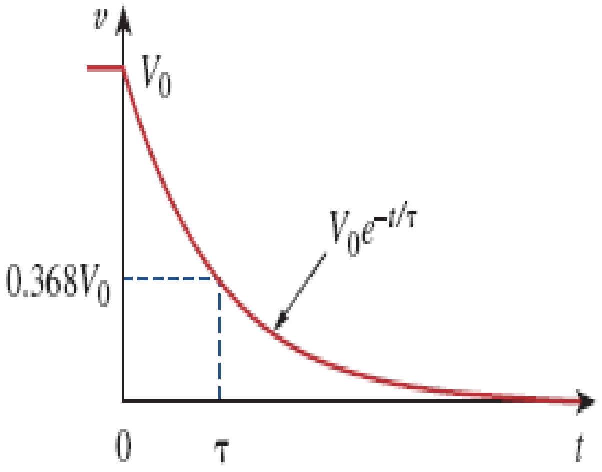
- For discharging we consider the source free RC circuit shown here.
- The initial voltage on the capacitor is V_0.
- The resulting voltage as a function of time is:

$$v_C(t) = V_0 e^{-t/\tau}$$



Discharging II

- **After five time constants, the capacitor has discharged to less than 1% of its starting value.**
- **This is regarded as the transient time.**
- **The capacitor is typically considered to be fully charged or discharged after this amount of time.**



Troubleshooting

- Capacitors are the most common component in a circuit to fail.
- Certain types, such as paper, molded paper, and electrolytic are failure-prone.
- Others, such as mica and ceramic almost never need replacement.

Troubleshooting II

- **Capacitors may fail as short or open circuits.**
- **If a lead become disconnected internally, the cap fails as an open.**
- **If the dielectric becomes compromised, the failure will be as a short.**
- **A DMM may be used to identify capacitors failed as shorts.**

Troubleshooting III

- One should expect the resistance of the capacitor to appear as an open on a meter.
- It is important to note that for polarized capacitors, the meter polarity must match the capacitor’s otherwise the cap may become damaged.