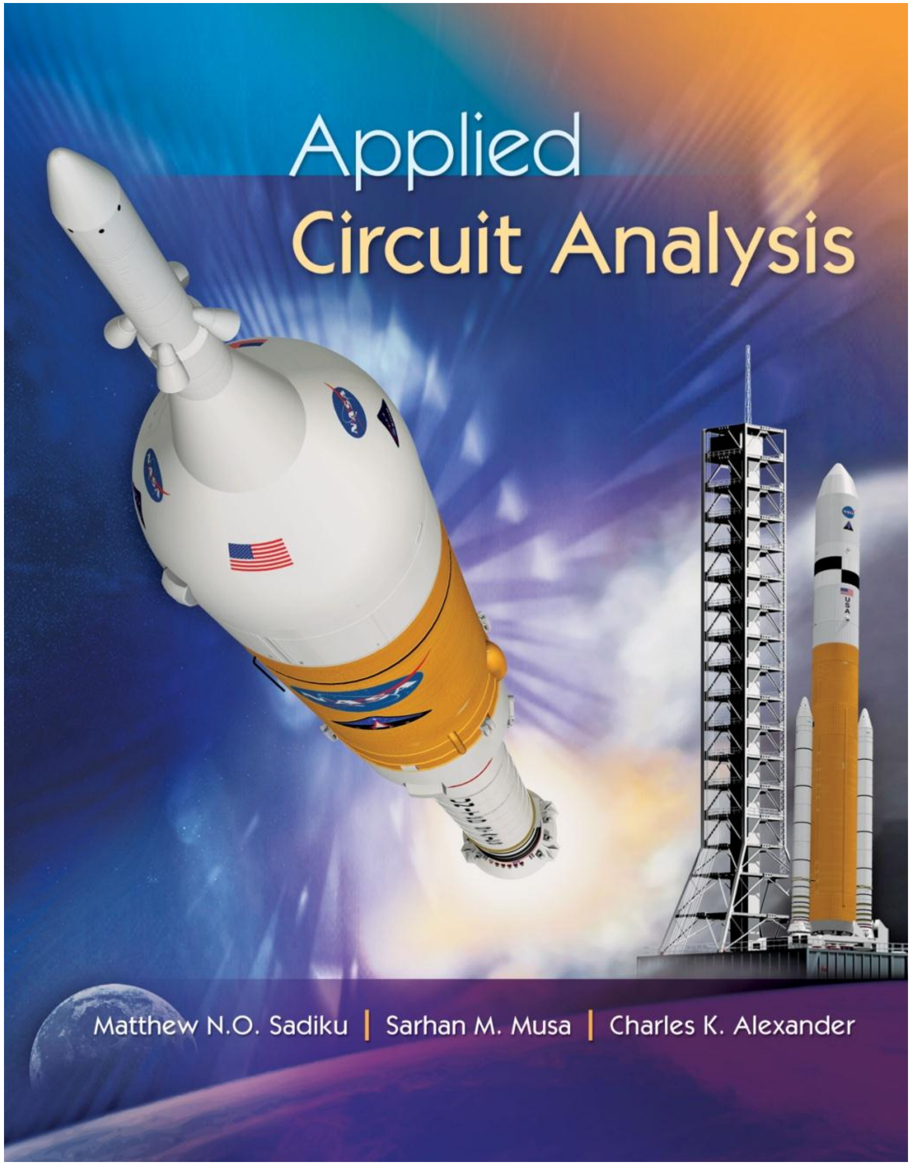


Applied Circuit Analysis

Chapter 10

Inductance

Overview

- This chapter will cover the inductor
- Like the capacitor, inductors do not dissipate energy
- The difference being that capacitors store their energy in electric fields and inductors use magnetic fields.

Electromagnetic Induction

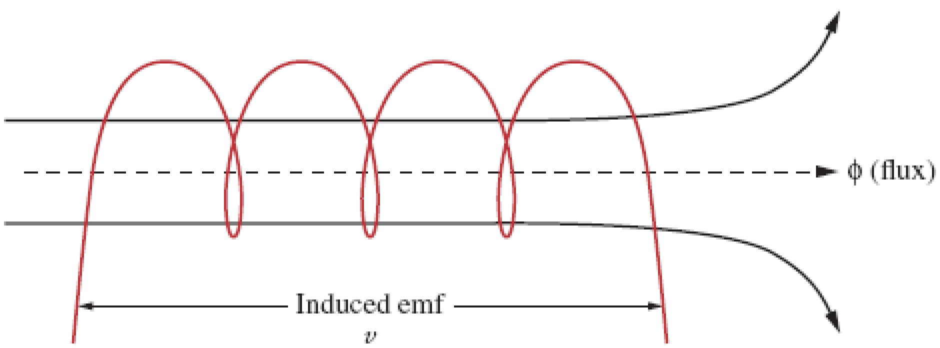
- When a magnetic flux passes through a coil or conductor, an electromotive force (emf) is generated.
- This is known as electromagnetic induction.
- The emf produces a voltage at the terminals of the conductor.
- This is known as Faraday's law.

Induction

- **This can be expressed mathematically as:**

$$v = N\frac{d\phi}{dt} = N \times \text{Rate of change in flux}$$

- **Where *v* is the induced voltage and *N* is the number of turns in the coil.**



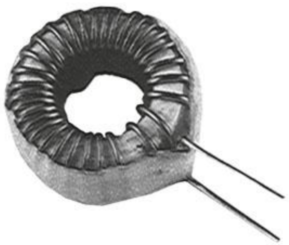
Lenz's Law

- The polarity of the induced emf and the direction of the current flow can be determined by applying Lenz's law:
- The induced current always develops a flux that opposes the change producing the current.
- The combination of Faraday's law and Lenz's law allows us to calculate induced voltage and polarity.

Inductors

- An inductor is a passive element that stores energy in its magnetic field
- They have applications in power supplies, transformers, radios, TVs, radars, and electric motors.
- Any conductor has inductance, but the effect is typically enhanced by coiling the wire up.

(a)



(b)

(c)

Courtesy of Tech America

Inductors II

- If a current is passed through an inductor, the voltage across it is directly proportional to the time rate of change in current

$$v = L\frac{di}{dt}$$

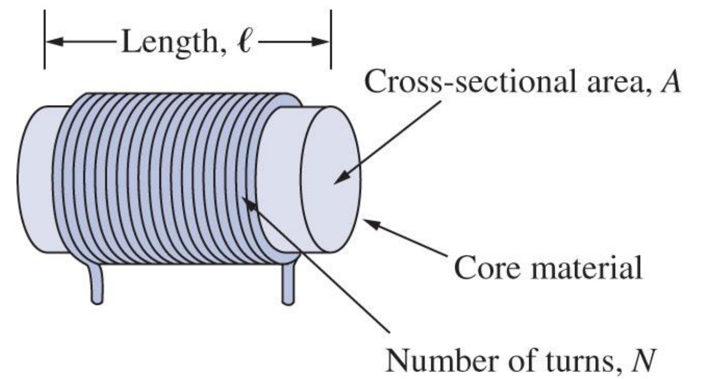
- Where, *L*, is the unit of inductance, measured in Henries, H.
- On Henry is 1 volt-second per ampere.
- The voltage developed tends to oppose a changing flow of current.

Inductors III

- **Calculating the inductance depends on the geometry:**
- **For example, for a solenoid the inductance is:**

$$L = \frac{N^2 \mu A}{l}$$

- **Where *N* is the number of turns of the wire around the core of cross sectional area *A* and length *l*.**



Permeability

- The material used for the core has a magnetic property called the permeability, μ.
- Permeability is the ability of a material to support a magnetic flux.
- Free space has a fixed permeability, μ_0
- Other materials are compared to this by the relative permeability μ_r

Permeability II

- The permeability of a material is the product of μ_0 and μ_r.
- Here are the relative permeability of some common materials:

Relative permeability of some common materials*.

Material	Relative permeability (μ_r)
Air	1
Cobalt	250
Nickel	600
Soft iron	5,000
Silicon-iron	7,000
Nonmagnetic metals	1

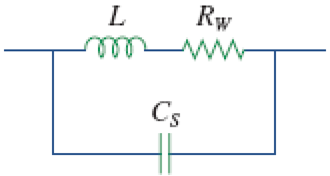
*The values given are typical; they vary from one published source to another owing to different varieties of most materials.

Increasing Inductance

- There are a few ways to increase inductance:
  - Increase the number of turns
  - Increase the cross sectional area
  - Decrease the coil length
  - Change the core material

Realistic Inductor

- Inductors in the real world have non-ideal behavior.
- Most significantly, being made of wire, they do not have zero resistance.
- What may be surprising is that they also have some capacitance due to the turns being insulated but adjacent to each other.



Energy Storage

- **The energy stored in an inductor's magnetic field is the average value of the current**

$$W = v \times 0.5 I_m \times t$$

- **The power delivered to the inductor is:**

$$p = vi = \left(L \frac{di}{dt} \right) i$$

- **The energy stored is:**

$$w = \frac{1}{2} L I_m^2$$

Properties of Inductors

- If the current through an inductor is constant, the voltage across it is zero
- Thus an inductor acts like a short for DC
- The current through an inductor cannot change instantaneously
- If this did happen, the voltage across the inductor would be infinity!
- This is an important consideration if an inductor is to be turned off abruptly; it will produce a high voltage

Properties of Inductors II

- Like the ideal capacitor, the ideal inductor does not dissipate energy stored in it.
- Energy stored will be returned to the circuit later
- In reality, inductors do have internal resistance due to the wiring used to make them.
- A real inductor thus has a winding resistance in series with it.
- There is also a small winding capacitance due to the closeness of the windings
- These two characteristics are typically small, though at high frequencies, the capacitance may matter.

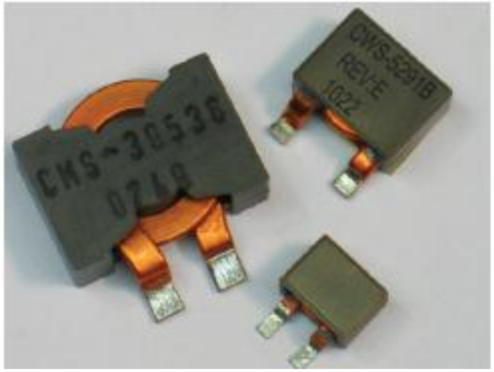
Types of Inductors

- Inductors can range in value from μH to tens of Henries.
- The low values are used in communications systems, while the large values are used in power systems
- They may be fixed or variable
- The circuit symbol for fixed and variable inductors is shown here:

(a) (b)

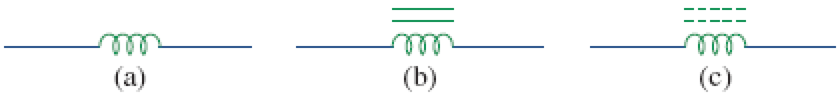
Types of Inductors II

- Inductors are also referred to as chokes sometimes.
- They can be linear or toroidal, where the core is circular.



Cores

- There are three common types of cores used in inductors: air (a), iron (b), and ferrite (c)

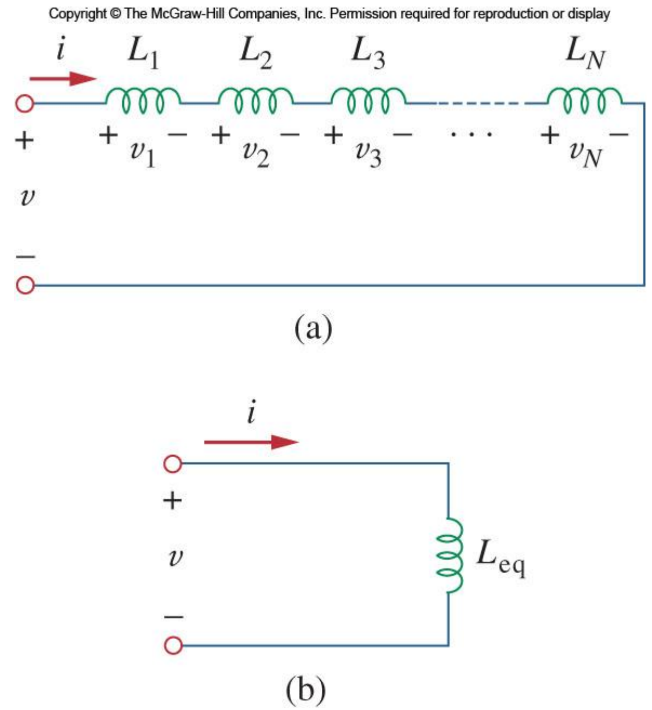


- The type of core used is determined by the application.
- Iron core inductors have large values associated with power applications.
- Air and ferrite cores are often used at high frequency.

Series Inductors

- **We now need to extend the series parallel combinations to inductors**
- **First, let's consider a series combination of inductors**
- **Applying KVL to the loop:**

$$v = v_1 + v_2 + v_3 + \cdots + v_N$$



Series Inductors II

- **Factoring in the voltage current relationship**

$$v_k = L_k \frac{di}{dt}$$

- **We get**

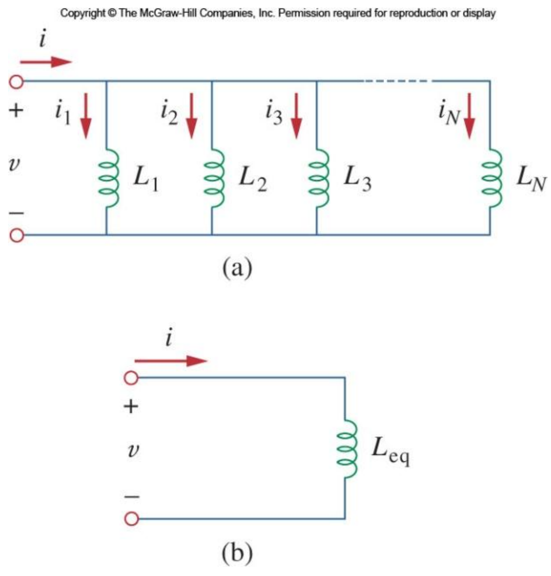
$$L_{eq} = L_1 + L_2 + L_3 + \cdots + L_N$$

- **Here we can see that the inductors have the same behavior as resistors.**
- **Therefore the equivalent inductance is the sum of the individual inductors.**

Parallel Inductors

- **Now consider a parallel combination of inductors:**
- **Using the same approach we did for resistors, the equivalent inductance will be:**

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}$$



(a)

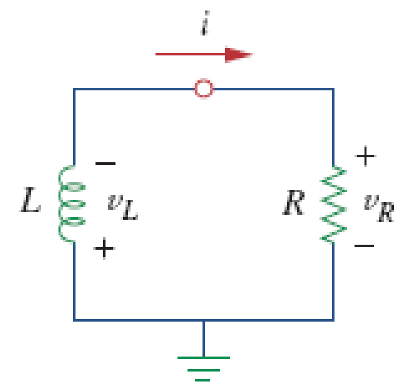
(b)

- **For two inductors:**

$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

Transient RL Circuits

- A RL circuit only contains resistors and inductors.
- Let us consider the circuit shown here:
- At *t=0* we will assume the inductor has an initial current.
- Applying KVL:



$$v_L + v_R = 0$$

Transient RL Circuits II

- **Applying differential calculus shows that:**

$$i(t) = I_0 e^{-Rt/L}$$

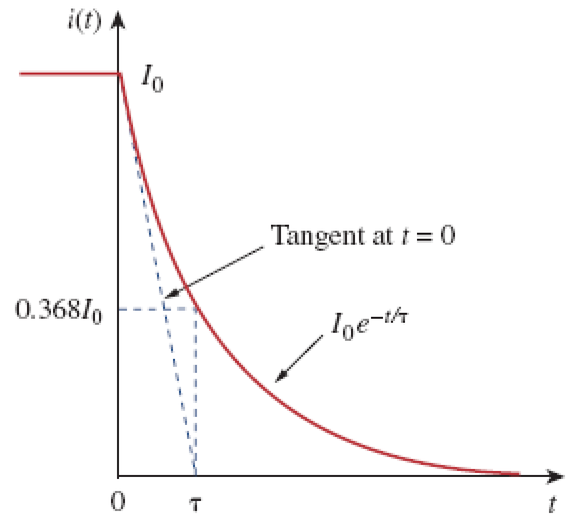
- **The natural response of the RL circuit is an exponential decay.**
- **Much like the capacitor, there is a time constant:**

$$\tau = \frac{L}{R}$$

Transient RL Circuits III

- **The time constant again has units of seconds.**
- **Knowing the current, we can say the voltage across the resistor is:**

$$v_R(t) = I_0 \, \mathrm{R}e^{-t/\tau}$$



Applications

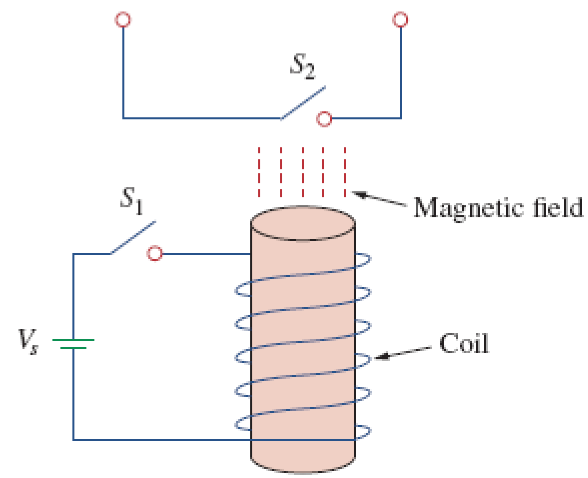
- Due to their bulky size, inductors are less frequently used as compared to capacitors, however they have some applications where they are best suited.
- They can be used to create a large amount of current or voltage for a short period of time.
- Their resistance to sudden changes in current can be used for spark suppression.
- Along with capacitors, they can be used for frequency discrimination.

Relays

- **Many applications of inductors are exploiting the magnetic field generated and the inductance is a side effect of using a coil of wires to create the field.**
- **A relay is a good example of this:**
- **A coil of wire is used to make an electromagnet.**
- **When energized the magnet moves a contact within a switch.**

Relays II

- Thus a relay is capable of opening or closing an isolated switch by way of current passing through the coil.
- This can enable a host of applications



Relays III

- The fact that the magnet has an inductance affects the behavior of the relay:
- The inductor's resistance to the introduction of current means that there will be a delay from when the current is applied to the time the contact moves.

$$t_d = \tau \ln \frac{i(0) - i(\infty)}{i(t_d) - i(\infty)}$$

Caution about Relays

- **Another side effect of using a relay is the inductance will generate a high voltage when the input current is turned on or off.**
- **This can damage some transistors or chips that are connected to the same circuit.**